Modeling Optimal Intervention Timing and Allocation for Contagious Disease Control in Colorado Under Budgetary Constraints

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1 Introduction

The COVID-19 pandemic demonstrated the importance of timely and strategic intervention deployment in managing a global health crisis. Following the rapid development of highly effective vaccines, a new challenge emerged: how to best allocate a finite, initially scarce resource across a large population. This challenge was explored in the literature whit the article by Jack Buckner, et al. serving as a starting point form where our exploration.[2] Thus, the core question of intervention time and prioritization became the topic we wish to explore in this paper. To focus our exploration, we decided to limit the interventions to a simple model of vaccination, social distancing, and anti-viral. The scope was further limited to model the population based of the demographics of Colorado, in order to use real number for a population so that the modeling of our hypothetical disease could be linked to the real world. [1]

2 Methodology

2.1 Epidemiological Model

We base our analysis on a modified Susceptible-Exposed-Infected-Recovered-Dead (SEIRD) compartmental model, which is appropriate for diseases like COVID-19 characterized by a latent period. The population N of Colorado is partitioned into four classes whose populations evolve over time t:

- $S(t)$: Susceptible individuals.
- $E(t)$: Exposed (latently infected) individuals.
- $I(t)$: Infected (and infectious) individuals.
- $R(t)$: Recovered (immune or deceased) individuals.
- $D(t)$: Dead individuals.

The total population is assumed to decrease as people die, and N is the total living population thus, $N = S + E + I + R$. The model only accounts for the people who die as a result of the disease, rather than natural birth and death. The dynamics are governed by the following system of ordinary differential equations (ODEs):

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta \frac{SI}{N} - u(t)S \\
\frac{dE}{dt} &= \beta \frac{SI}{N} - \sigma E \\
\frac{dI}{dt} &= \sigma E - \gamma I - \mu I \\
\frac{dR}{dt} &= \gamma I + u(t)S \\
\frac{dD}{dt} &= \mu I
\end{aligned}
$$

where:

- Λ: Birth rate (or new susceptible population inflow).
- μ: Mortality rate of the disease.
- β: Effective contact rate.
- σ: Rate of progression from E to I.
- γ: Rate of recovery from infection.
- $u(t)$: The vaccination control rate (proportion of susceptibles vaccinated per unit time).

The vaccination control $u(t)$ is the core intervention we seek to optimize.

To capture age-specific risk and intervention targeting, we also consider an age-structured extension of this model. We partition the population into A age groups indexed by $i = 1, \dots, A$ (in our simulations, children, adults, and elderly). For each age group i we track

$$S_i(t),\ E_i(t),\ I_i(t),\ R_i(t),\ D_i(t),$$

with the living population in group i given by

$$N_i(t) = S_i(t) + E_i(t) + I_i(t) + R_i(t)$$

$$N(t) = \sum_{i=1}^{A} N_i(t).$$

2.1.1 Force of infection and contact matrix with social distancing.

Let $C_{ij}(t)$ denote the average number of potentially infectious contacts per unit time that a person in age group i has with individuals in age group j at time t. The force of infection acting on age group i is

$$\lambda_i(t) = \beta \sum_{j=1}^{A} C_{ij}(t)\, \frac{I_j(t)}{N_j(t)}. \tag{1}$$

We model social distancing as a multiplicative reduction in a fixed baseline contact matrix $C_{ij}^{(0)}$. Let $d_i(t) \in [0, 1]$ denote the fraction of contacts reduced for age group i at time t. The effective contact matrix is

$$C_{ij}(t) = \big(1 - d_i(t)\big)\big(1 - d_j(t)\big)\, C_{ij}^{(0)}. \tag{2}$$

In our implementation we use piecewise-constant distancing policies of the form

$$d_i(t) = \begin{cases} 0, & t < T_i^{\text{dist}}, \\ d_i^*, & t \geq T_i^{\text{dist}}, \end{cases} \tag{3}$$

where T_i^{dist} is the time at which distancing begins for age group i, and d_i^* is the target reduction level (e.g., $d_i^* = 0.2$ for a 20% reduction in contacts).

2.1.2 Antivirals and time-varying mortality.

We allow the disease-induced mortality rate to be reduced by antivirals. Let μ_i denote the baseline mortality rate for age group i, and let $a_i(t) \in [0, 1]$ denote the antiviral efficacy (fractional reduction in mortality) in that group at time t. The effective mortality rate is

$$\mu_i^{\text{eff}}(t) = \mu_i\big(1 - a_i(t)\big). \tag{4}$$

We again use a piecewise-constant representation:

$$a_i(t) = \begin{cases} 0, & t < T_i^{\text{anti}}, \\ a_i^*, & t \geq T_i^{\text{anti}}, \end{cases} \tag{5}$$

where T_i^{anti} is the time at which antivirals become available in age group i, and a_i^* is the maximal antiviral effect (e.g., $a_i^* = 0.5$ corresponds to a 50% reduction in mortality).

2.1.3 Age-specific vaccination.

In the age-structured setting we write $u_i(t)$ for the vaccination control applied to susceptibles in age group i. Analogously to the aggregate model, vaccination moves individuals directly from S_i to R_i:

$$u_i(t) = \begin{cases} 0, & t < T_i^{\text{vax}}, \\ u_i^*, & t \geq T_i^{\text{vax}}, \end{cases} \tag{6}$$

where T_i^{vax} is the vaccination start time for group i and u_i^* encodes the per-capita vaccination rate (including vaccine efficacy).

2.1.4 Age-structured SEIRD dynamics with interventions.

Combining the definitions above, the age-structured SEIRD dynamics for each age group $i = 1, \dots, A$ are

$$\frac{dS_i}{dt} = -\lambda_i(t)\, S_i(t) - u_i(t)\, S_i(t), \tag{7}$$

$$\frac{dE_i}{dt} = \lambda_i(t)\, S_i(t) - \sigma\, E_i(t), \tag{8}$$

$$\frac{dI_i}{dt} = \sigma\, E_i(t) - \gamma\, I_i(t) - \mu_i^{\text{eff}}(t)\, I_i(t), \tag{9}$$

$$\frac{dR_i}{dt} = \gamma\, I_i(t) + u_i(t)\, S_i(t), \tag{10}$$

$$\frac{dD_i}{dt} = \mu_i^{\text{eff}}(t)\, I_i(t), \tag{11}$$

with the aggregate compartments obtained by summing over age groups, e.g. $S(t) = \sum_{i=1}^{A} S_i(t)$ and similarly for $E(t), I(t), R(t)$, and $D(t)$. In the numerical experiments, we vary the start times T_i^{vax}, T_i^{anti}, and T_i^{dist} to study how the timing of vaccination, antivirals, and social distancing jointly influences epidemic outcomes.

2.2 The Optimal Control Problem

The objective of our study is to find the time-dependent optimal control $u(t)$ that minimizes the cost $J(u)$ over a fixed time $[0, T]$, which is subject to the SEIRD dynamics described above and a budgetary constraint. To account for economic costs, we introduce an additional state variable $C(t)$ that represents the cumulative monetary cost of the interventions up to time t. The instantaneous cost rate has three components:

- **Vaccination cost:** let c_i^{vax} denote the cost (e.g., dollars) per vaccinated individual in age group i. Since the flow of individuals vaccinated per unit time in group i is $u_i(t)S_i(t)$, the vaccination cost rate is

$$\text{cost}^{\text{vax}}(t) = \sum_{i=1}^{A} c_i^{\text{vax}} \, u_i(t) \, S_i(t).$$

- **Antiviral cost:** let c_i^{anti} denote the cost per infectious individual effectively treated with antivirals in group i. We approximate the treated flow as $a_i(t)I_i(t)$, so the antiviral cost rate is

$$\text{cost}^{\text{anti}}(t) = \sum_{i=1}^{A} c_i^{\text{anti}} \, a_i(t) \, I_i(t).$$

- **Social distancing cost:** let c_i^{dist} denote the daily economic loss per person at full distancing in age group i (e.g., "earnings missed" from not attending work in person). If $d_i(t)$ is the distancing intensity (fraction of contacts reduced), then the distancing cost rate is

$$\text{cost}^{\text{dist}}(t) = \sum_{i=1}^{A} c_i^{\text{dist}} \, d_i(t) \, N_i(t).$$

The cumulative cost state $C(t)$ therefore evolves according to

$$\frac{dC}{dt} = \text{cost}^{\text{vax}}(t) + \text{cost}^{\text{anti}}(t) + \text{cost}^{\text{dist}}(t), \qquad C(0) = 0. \tag{12}$$

In addition to these flow costs, we include fixed up-front research and development (R&D) costs for vaccines and antivirals. Let K_{vax} denote the vaccine R&D cost and K_{anti} the antiviral R&D cost. These are incurred once if the corresponding intervention is deployed at any point during the horizon $[0, T]$. Formally, we may define indicator variables

$$\mathbb{I}_{\text{vax}}(u) = \begin{cases} 1, & \text{if } u_i(t) > 0 \text{ for some } i, t \in [0, T], \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathbb{I}_{\text{anti}}(a) = \begin{cases} 1, & \text{if } a_i(t) > 0 \text{ for some } i, t \in [0, T], \\ 0, & \text{otherwise.} \end{cases}$$

The total monetary cost associated with a control strategy (u, a, d) over the horizon $[0, T]$ is then

$$\mathcal{C}(u, a, d) = C(T) + K_{\text{vax}} \, \mathbb{I}_{\text{vax}}(u) + K_{\text{anti}} \, \mathbb{I}_{\text{anti}}(a), \tag{13}$$

where $C(T)$ is given by the solution of (12).

Let $D(T) = \sum_{i=1}^{A} D_i(T)$ denote the cumulative number of deaths at the end of the time horizon. One natural formulation is to minimize deaths subject to an explicit budget constraint,

$$\min_{u,a,d} D(T) \quad \text{subject to} \quad \mathcal{C}(u, a, d) \le B, \tag{14}$$

for a given budget $B > 0$. In our numerical implementation, we instead consider a cost-effectiveness formulation that maximizes the number of lives saved per dollar spent. Let $D_{\text{baseline}}(T)$ denote the deaths under a baseline scenario with no interventions. The lives saved by a given control strategy (u, a, d) are

$$L(u, a, d) = D_{\text{baseline}}(T) - D(T),$$

and we evaluate the ratio

$$\frac{L(u, a, d)}{\mathcal{C}(u, a, d)}.$$

The grid search over intervention start times and intensities in the simulation code selects the combination that maximizes this lives-per-dollar metric, providing a discrete approximation to the optimal cost-effective allocation of vaccination, antivirals, and social distancing.

3 Results

Our numerical simulations, constrained by a hard annual flow budget of \$20 billion and utilizing the age-structured SEIRD model with targeted allocation, reveal three key findings related to cost-effectiveness ($\frac{L(u,a,d)}{\mathcal{C}(u,a,d)}$). When controlling for when the vaccinations and anti-vials would be introduced to the public, the duration of the social distance could be analyzed when compared to the budget, deaths, and total cost.

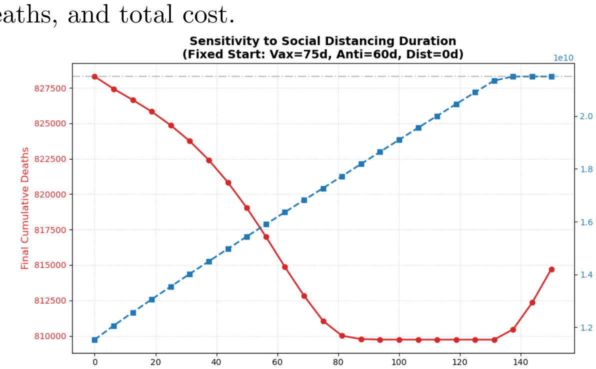

Figure 1: Time and Duration of Social Distancing

It can be seen in Figure 1, that when a vaccine is introduced to our system at 75 days and anti-viral at day 60, the optimal value for social distancing was 80 days, which is before the budget reaches its maximum, showing that social distancing is effective at reducing deaths to a point, and that the money would be best spent on other interventions such as vaccination after that point.

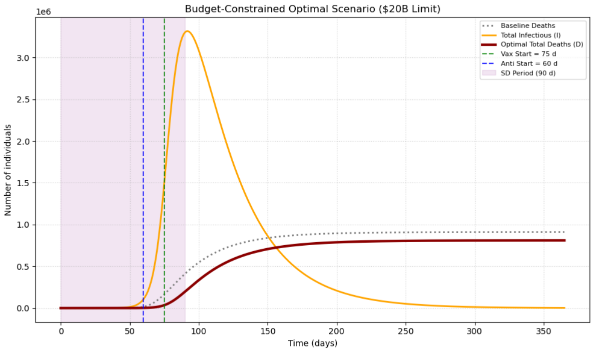

Figure 2: Budget-Constrained Optimal Scenario with $20 B Cost Limit

Using this constraint, it can be seen that the most cost-effective solution, Figure 2, involved strategically staggered deployment:

- **Antivirals (Day 60):** Started relatively early to immediately reduce the high age-specific mortality μ_3 ($\mu = 0.02$) in the high-risk Elderly group ($a_3^* = 0.3$).
- **Vaccination (Day 75):** Held until $T = 75$ days to maximize its impact on reducing the susceptible population (S_i) just before the peak acceleration phase.
- **Social Distancing (Day 0, 90-day duration):** Initiated immediately but strictly limited to 90 days, targeting the high-contact Adults ($d_2^* = 0.2$).

It is important to note that these interventions end up resulting in $532,839$ being saved as compared to the non-interventions. The population that saw the most significant effects of the interventions was the elderly population, with the number of each population shown below.

- Children (0-17): 107,556 lives saved
- Adults (18-64): 84,348 lives saved
- Elderly (65+): 340,935 lives saved

This result, saving the majority of lives in the elderly population, is directly attributed to the model's design: the highest baseline mortality rate ($\mu_3 = 0.02$) was assigned to this group, and the antiviral intervention was specifically targeted to the elderly ($\text{anti\_eff}_{\text{opt}} = [0.0, 0.0, 0.3]$). This was done to mirror the way anti-virals are utilized in real life, as they are the most effective for this subset of the population given their risk [3]. By reducing the effective mortality rate ($\mu_i^{\text{eff}}(t)$) where the disease was most lethal, the intervention achieved its maximum possible effect on total deaths, validating the cost-effectiveness principle of targeting high-risk populations. It is important to note that this is also a lagging intervention where it is implemented at 60 days, but still has the most substantial effect.

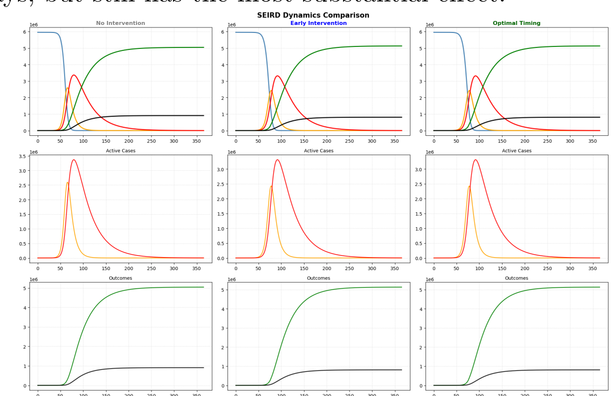

Figure 3: Optimal Timing vs Early Intervention

As seen in **Figure 3**, the total number of deaths between the early intervention column and the optimal timing of countermeasures column is nearly the same. This means that our early introduction of interventions is very close to the optimal deployment when considering costs. A likely explanation of this result is the cost of social distancing and the similar timing between models. Social distancing is by far the most expensive per day per person impact on total costs. Therefore, having as small of a social distancing window is critical to reducing the total expenditure of the intervention plan. Similarly, in **Figure 2** we see that the total duration of social distancing is very early on in the epidemic. This helps prevent rebounds of the disease and helps to reduce the total number of infections. In reality, it is difficult to know when the start of an infection really occurs, so social distancing windows will be larger and more expensive.

4 Discussion

4.1 Limitations and Future Work

This study utilizes a simple aggregated SEIRD model for the entire state of Colorado. Future work should incorporate:

1. **Spatial Heterogeneity:** Incorporating metapopulation models to account for the

movement between urban centers (e.g., Denver, Colorado Springs) and rural areas, which would influence the effective contact rate β and the optimal distribution logistics [1].

2. **Stochasticity and Uncertainty:** Utilizing stochastic optimal control to account for uncertain factors like vaccine efficacy, parameter estimation errors, and compliance rates.

Our model also has a very primitive cost evaluation that struggles to be an accurate reflection of real world considerations to cost of development and prevention measure use. In future iterations of this exploration, it would be valuable to incorporate metrics that more accurately represent the large number of factors that go into determining the cost of specific countermeasures. These factors could include:

1. **Dynamic Social Distancing Cost:** Our model assumes that people who are socially separated lose all possible income. In reality, some people work from home and are financially unaffected by social distancing protocols. Similarly, front line workers are also not affected by social distancing and continue to work when this countermeasure is in place.

2. **Vaccine & Antiviral Costs:** The current implementation only has a "cost per dose" and does not take into account for fluctuating costs of distribution from production facilities to final destinations. We also have not included varying supplies of vaccines in this study.

Seasonality is another core feature of epidemic modeling that should be improved upon in future works. As seen in our results section, the early intervention model was approximately equal to the optimal cost model. This could change when social distancing has a larger impact for more seasonal epidemics as social distancing times grow and react to infection rates. In addition, it is difficult to know when a seasonal spike in infections will occur, making it hard to predict when to enact social distancing policies. Early vaccinations has been proven to be a good start to reducing seasonal infections as a preventative measure due to their low costs. The same cannot be said about social distancing, it is not feasible to have many individuals not attend work for the sake of stopping the spread of possible infections when the phase of an epidemic is not known, or suspected to be in the off season.

5 Conclusion

Through the use of this model and the skills developed in class, we conclude that it is critical to have a wide range of intervention measures to prevent deaths during an epidemic. Although each countermeasure has some impact on their own, each in different ways, their combined influence allows an optimal reduction in deaths per dollar spent. We also observed that it is crucial that the costs of each intervention strategy be weighted with their effectiveness. Particularly with social distancing and its immense per-day costs. Although both vaccines and antivirals have large upfront costs, the cost per day/dose is much less than the cost of a single day of social distancing. This result implies that vaccine mandates and medication should be pursued as the primary means of flattening the curve before social distancing is employed. Vaccination and medication policies are much simpler to enforce, cost less to implement, and do not disrupt the social environment during epidemics or pandemics.

References

[1] U.S. Census Bureau. (n.d.). *Colorado profile* [Data profile]. https://data.census.gov/profile/Colorado?g=040XX00US08

[2] Buckner, J. H., Chowell, G., & Springborn, M. R. (2021). Dynamic prioritization of COVID-19 vaccines when social distancing is limited for essential workers. *Proceedings of the National Academy of Sciences of the United States of America, 118*(16), e2025786118. https://doi.org/10.1073/pnas.2025786118

[3] Centers for Disease Control and Prevention. (2024). *Differences in COVID-19 Outpatient Antiviral Treatment Among Adults Aged ≥65 Years by Age Group — National Patient-Centered Clinical Research Network, United States, April 2022–September 2023.* MMWR Morbidity and Mortality Weekly Report, 73(39), 834–840. https://www.cdc.gov/mmwr/volumes/73/wr/mm7339a3.htm