A Deep Dive into Exponentially Distributed Wait Times in SIR Models

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1 Background

We all know, thanks to one semester of Computational and Mathematical Modeling of Infectious Diseases, the important role that compartmental ordinary differential equations (ODE) models play in infectious disease modeling and decision making. The SEIR model is arguably the backbone of infectious disease modeling, given its relative simplicity to parameterize and provision of closed form analytical solutions. However, the SEIR model is only a model, and is thus characterized by some flaws in how it represents the real world it attempts to emulate. One of these flaws is the exponentially distributed latent and infectious period, which means that most individuals spend very little time in the exposed (E) or infectious (I) state, but a few individuals spend a very long time there. This does not necessarily match our real world observations, where most people spend close to the average time in either compartment.

2 Scholarly Questions

In this project, I propose to take a deep dive into the literature that studies the ramifications of exponentially distributed wait times of compartmental ODE's, with a specific focus on latent and infectious periods. I will begin with the recommended reading of Wearing, *et al.* and see where the wind takes me from there [1]. I am specifically interested in what measures this assumption may impact, and how the intervention of adding multiple instances of E and I states to approximate gamma distributed wait times impacts these measures. For example, does this assumption impact only timing of infections or magnitude of infection counts? How does it impact our approximations of R_0?

3 Leaning Goals and Anticipated Findings

From this literature deep dive, I hope to learn which common measures derived from SEIR models may be impacted by the exponential distribution of wait times and which are not. This can give me a sense of when it is safe to use the classic SEIR framework vs when I must be sure to convert to gamma distributed wait times, depending on the question I am answering with my model. I am further interested in exploring if there is any disadvantage to implementation as gamma distributed wait times. I anticipate that measures of timing will be more heavily impacted by this wait time distribution than measures of total number of infections, etc. However, I am excited to see what the literature holds!

4 So what does this have to do with class anyways?

Remember how we have spent that past 2-3 months discussing the SEIR model and its various flavors? This model is foundational to infectious disease research, and my research, but is often implemented with the exponentially distributed wait times. I think it will make me a better researcher and academic paper reviewer to understand what questions you can ask that are relatively safe under these assumptions vs

questions that require a different implementation. This will also contribute to my understanding of how ethical our infectious disease policy suggestions are based on the models that inform them.

References

[1] Helen J Wearing, Pejman Rohani, and Matt J Keeling. Appropriate models for the management of infectious diseases. *PLoS medicine*, 2(7):e174, 2005.