

A Visual Proof Of The Pythagorean Theorem

Pythagoras theorem

Pythagoras

September 9, 2013

Outline

- 1 Introduction to Pythagoras Theorem
- 2 Theorem, Definition And Relation
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 Proof of the Pythagorean Theorem using Algebra
 - Figure
 - Area of Whole Square
 - Area of Other Pieces
 - Both Areas Must Be Equal
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Outline

- 1 Introduction to Pythagoras Theorem
- 2 Theorem, Definition And Relation
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 Proof of the Pythagorean Theorem using Algebra
 - Figure
 - Area of Whole Square
 - Area of Other Pieces
 - Both Areas Must Be Equal
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Introduction

The theorem is of fundamental importance in Euclidean Geometry where it serves as a basis for the definition of distance between two points. *The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides.*

Outline

- 1 Introduction to Pythagoras Theorem
- 2 **Theorem, Definition And Relation**
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 Proof of the Pythagorean Theorem using Algebra
 - Figure
 - Area of Whole Square
 - Area of Other Pieces
 - Both Areas Must Be Equal
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Theorem

Pythagoras Theorem

The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides

Definition

The theorem that the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse.

Relation

- In mathematics, the Pythagorean theorem or Pythagoras theorem is a relation in Euclidean geometry among the three sides of a right triangle.

Relation

- In mathematics, the Pythagorean theorem or Pythagoras theorem is a relation in Euclidean geometry among the three sides of a right triangle.
- The Pythagorean Theorem states that for a right triangle, the sum of the squares of the sides is equal to the square of the hypotenuse.

Relation

- In mathematics, the Pythagorean theorem or Pythagoras theorem is a relation in Euclidean geometry among the three sides of a right triangle.
- The Pythagorean Theorem states that for a right triangle, the sum of the squares of the sides is equal to the square of the hypotenuse.
- Pythagoras Theorem can also be stated as the square formed on the hypotenuse of a right-angled triangle has the same area as the sum of the areas of the squares formed on the other two sides.

The theorem can be written as an equation relating the lengths of the sides a , b and c , often called the Pythagorean equation:

$$a^2 + b^2 = c^2 \quad (1)$$

where c represents the length of the hypotenuse, and a and b represent the lengths of the other two sides.

Outline

- 1 Introduction to Pythagoras Theorem
- 2 Theorem, Definition And Relation
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 Proof of the Pythagorean Theorem using Algebra
 - Figure
 - Area of Whole Square
 - Area of Other Pieces
 - Both Areas Must Be Equal
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Importance in Triangles

There are many uses for the Pythagorean Theorem, that is the main reason why it is important. You can use the Pythagorean Theorem to find out if a triangle is an acute triangle, obtuse triangle or a right triangle. If the Theorem works and the side lengths squared is equal to the hypotenuse squared, then it is a right triangle, if the hypotenuse squared is longer than the two side lengths squared and added together then the triangle is obtuse. If the hypotenuse squared is shorter than the two side lengths squared and added together then the triangle is acute.

Importance in Finding Missing Side Lengths

Another reason the Pythagorean Theorem is imported is it can help you find missing side lengths. Not only can it help you find the third side length to a right triangle, but it can also help you find the missing side lengths to squares and rectangles when the triangles are pushed together. The pythagorean Theorem can help build rectangles and squares.

Outline

- 1 Introduction to Pythagoras Theorem
- 2 Theorem, Definition And Relation
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 **Proof of the Pythagorean Theorem using Algebra**
 - **Figure**
 - **Area of Whole Square**
 - **Area of Other Pieces**
 - **Both Areas Must Be Equal**
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Figure

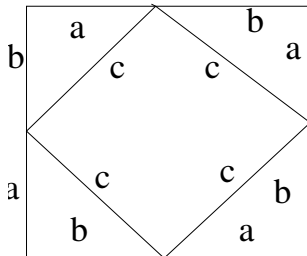


Figure: Figure for pythagoras theorem

We can show that using Algebra. Take a look at the diagram above. It has that "abc" triangle in it (four of them actually). Figure 4 shows a photo of a square contains various figures. We will calculate the areas of the respective figures in the below subsections.

Area of The Square

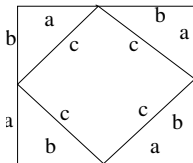


Figure: Figure for pythagoras theorem

It is a big square, with each side having a length of $a+b$, so the total area is:

$$A = (a + b)(a + b)$$

Area of Other Pieces....(continue)

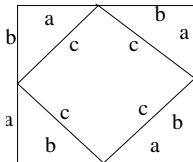


Figure: Figure for pythagoras theorem

Now let's add up the areas of all the smaller pieces:
First, the smaller (tilted) square has an area of:

$$A = c^2 \quad (2)$$

And there are four triangles, each one has an area of

$$A = \frac{1}{2}ab \quad (3)$$

Area of Other Pieces....(continue)

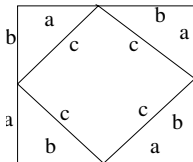


Figure: Figure for pythagoras theorem

So all four of them combined is

$$A = 4\left(\frac{1}{2}ab\right) = 2ab \quad (4)$$

So, adding up the tilted square and the 4 triangles gives:

$$A = c^2 + 2ab \quad (5)$$

Both Areas Must Be Equal

The area of the large square is equal to the area of the tilted square and the 4 triangles. This can be written as (from equation 5):

$$(a + b)(a + b) = c^2 + 2ab \quad (6)$$

Now, let us rearrange equation 6 to see if we can get the pythagoras theorem:

$$(a + b)(a + b) = c^2 + 2ab \quad (7)$$

Expand $(a+b)(a+b)$ in equation 7:

$$a + 2ab + b = c^2 + 2ab \quad (8)$$

Subtract "2ab" from both sides in equation 8:

$$a^2 + b^2 = c^2$$

The following above statements gives us the proof of pythagoras theorem. Also, we can see the table below to see the correctness of theorem.

Outline

- 1 Introduction to Pythagoras Theorem
- 2 Theorem, Definition And Relation
 - Theorem
 - Definition
 - Relation
- 3 Importance of Pythagoras theorem
 - Importance in Triangles
 - Importance in Finding Missing Side Lengths
- 4 Proof of the Pythagorean Theorem using Algebra
 - Figure
 - Area of Whole Square
 - Area of Other Pieces
 - Both Areas Must Be Equal
- 5 Examples and References of Pythagoras Theorem
 - Examples of Pythagoras theorem
 - References(Books)
 - References(Links)

Examples of Pythagoras theorem

Few examples on Pythagoras theorem

Lenght a	Height b	Hypotenuse c	a^2	b^2	c^2	a^2+b^2
4	3	5	16	9	25	25
8	15	17	64	225	289	289
9	40	41	81	1600	1681	1681
5	12	13	25	144	169	169
7	24	25	49	576	625	625

Table: Examples

References(Books)



Helmut Kopka and Patrick W. Daly.
Guide to LaTeX.
4 edition, 2003.



Leslie Lamport.
LaTeX: A Document Preparation System.
2 edition, 1994.



Irene Hyna Tobias Oetiker, Hubert Partl and Elisabeth Schlegl.
Introduction to Latex.
2 edition, 2011.

References(Links)

Here are some links used to make this pdf:

[http://www.mathsisfun.com/geometry/
pythagorean-theorem-proof.html](http://www.mathsisfun.com/geometry/pythagorean-theorem-proof.html)

<http://en.wikibooks.org/wiki/LaTeX/Basics>

<http://www.maths.tcd.ie/~dwilkins/LaTeXPrimer/>