COMP6212 Assignment 1

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1 Efficient Mean-Variance frontier

1.a

Generate 100 random portfolios and plot their expected return and variance.
The whole procedure could be described as:

- Construct 100 vectors X whose length is 3, and the elements x_1, x_2, x_3 satisfy $\sum\limits_{i=1}^{3} x_i = 1$ and $x_i \geq 0$.
- For each vector:
  - Calculate the expected return $E = \sum_{i=1}^{3} X_i \mu_i$,
  - Calculate the expected variance $V = \sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j \sigma_{ij}$.

Plot a scatter of diagram, we can get the picture like figure 1.

1.b

Matlab no longer support the function `frontcon` in version R2016b, I used the `Portfolio` object as replacement according to the offical documnetation.

From the figure 2 we can find that the `frontier` gives a left bound of portfolios.

The figure 3 shows the three two-asset models, the all exist on a single line, and the frontier is also part of the line.

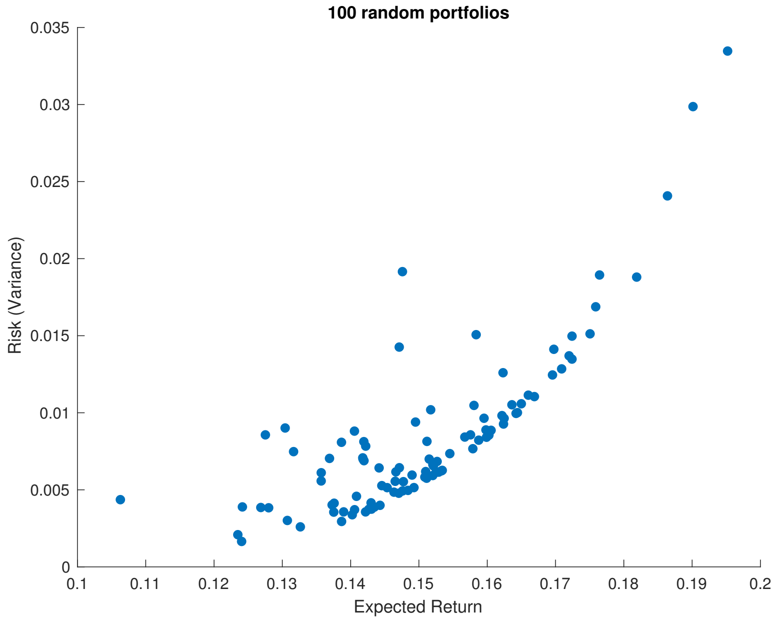

Figure 1: E-V plot of 100 randomly selected portfolios

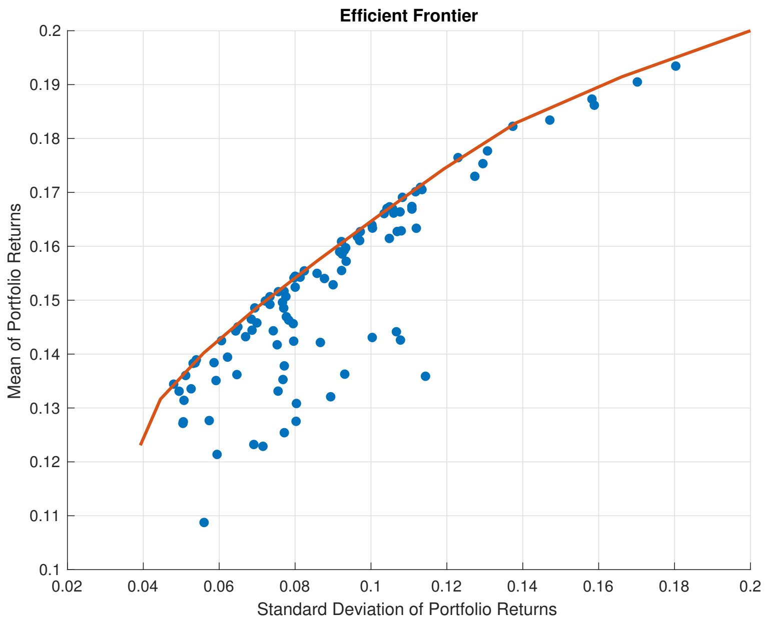

Figure 2: Efficient frontier for three-asset model

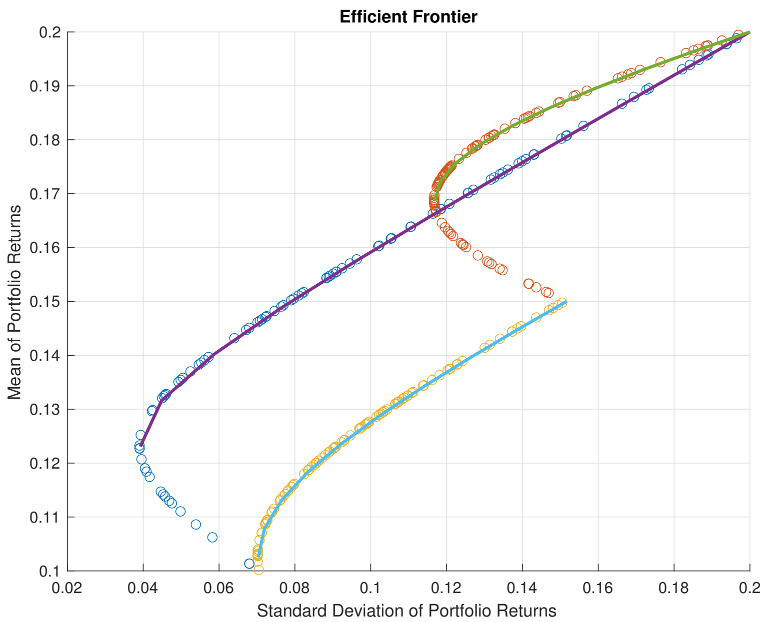

Figure 3: Efficient frontiers for two-asset models

1.c

The `NativeMV` function claculate the status of Maximum return and Minimum risk, divide the range of return accroding to the given number, and calculate the minimum risk of each state. The `linprog` function is used by `NativeMV` to find out the maximum return at first.

The maximum problem:

$$\max \mathbf{w}^T\bar{\mathbf{r}} \text{ subject to } \sum_{i=1}^{N} w_i = 1, w_i \geq 0$$

be rewritten as:

$$\min \text{-}\mathbf{w}^T\bar{\mathbf{r}} \text{ subject to } \sum_{i=1}^{N} w_i = 1, w_i \geq 0$$

to use the `linprog` function to get the ansewer.

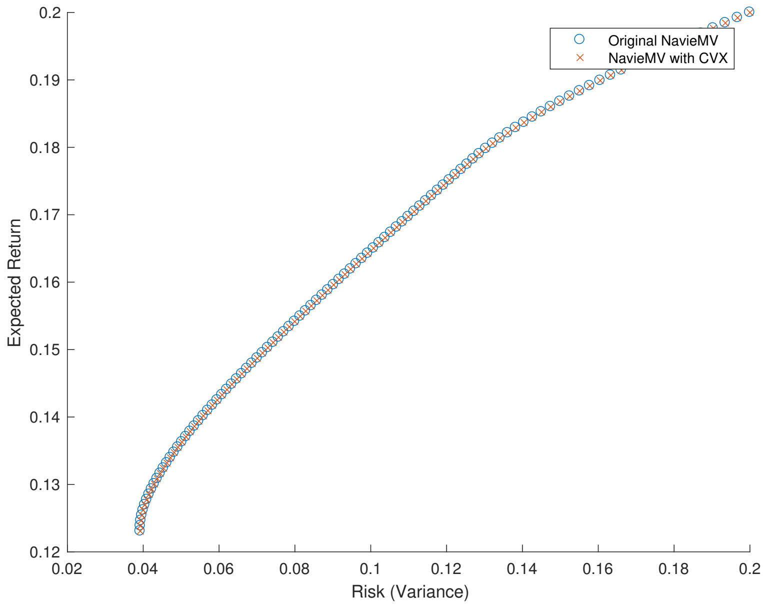

Figure 4: Performance of CVX

1.d

The figure 4 shows the result of using original `NativeMV` with `linprog`,`quadprog` and the result of using `CVX` are identical. This is because that the problem they are solving are identical, just using different methods. CVX is suitable for solving more generic problems. However, CVX cost much more time, which could be easily observed while running code.

2 Evaluation of performance

Downloaded the three-year data from `Yahoo! Finance`, using the close price to stand for the price of every single day, the days with no data are. For some situations like no transaction, I used the same price as the previous day standing for that day will have no retuen if you invest on it. Used MATLAB `tick2ret` function to get the return ratio.

Calculated the mean retuen of the stocks got, and choosed three positive return stocks to do the next steps.

Based on the data generated and the works done in `Question 1`, generate the frontier of the portfolios. Using the function `estimateFrontier` get a

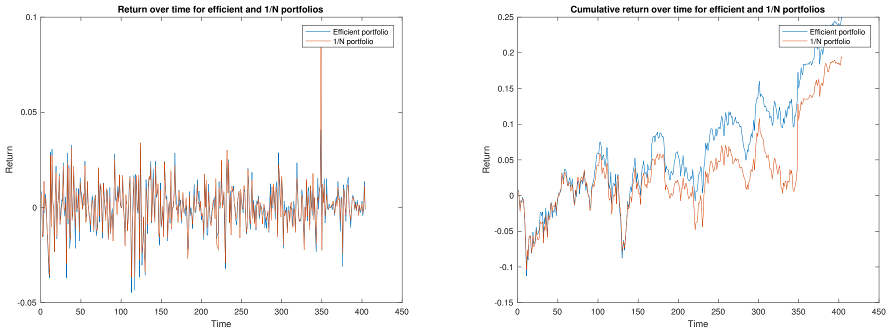

Figure 5: Compare of Mean-Variance and Navie $\frac{1}{N}$ strategy

set of efficient portfolios and used `sharpe` function to find out the one with the hightest `Sharpe Ratio`.

With the portfolio, we can compare it with the simple $\frac{1}{N}$ portfolio. As shown in the the figure 5, the figure on the left represting daily return can not show which is better, for the `cumulative` one we can find that the efficient portfolio performs better than the navie one for a long window.

3 Greedy and sparse index tracking

3.a

For the greedy strategy, get the stock with the smallest error. After that, continue finding the one which will give the smallest error when added by the exist set. Repeatly doing so until I have `seven` stocks in my basket(I have 35 stocks to view and one-fifth is 7). According the stocks selected are `4,7,11,13,16,18,27`. As the greedy search their weight are the same. The result of tracking is shown as figure 6.

3.b

The second strategy to select stocks is to use index tracking algorithm to sole the problem:

$$\hat{\mathbf{w}} = \min_{w}[\|\mathbf{y} - \mathbf{R}\mathbf{w}\|_2^2$$

To achieve a sparse result, using the technique of L^1-regularization. Use CVX toolkit to solve the problem:

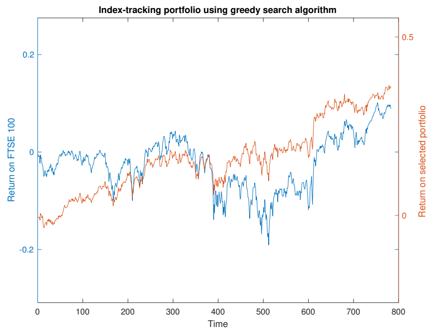

Figure 6: Index tracking portfolio selected via greedy search

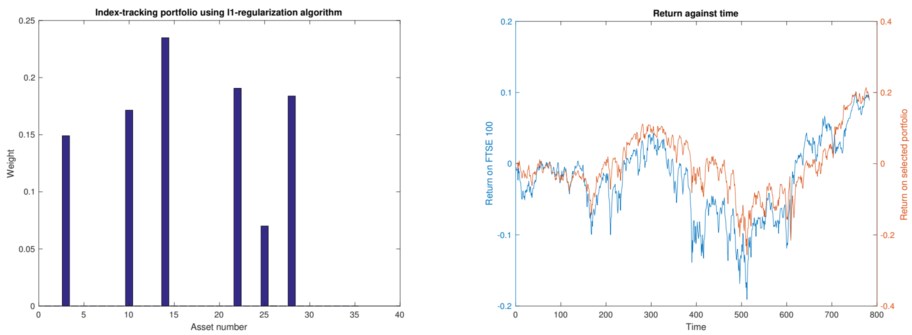

Figure 7: Index tracking portfolio selected with L^1-regularization

$$\hat{\mathbf{w}} = \min_{w}[\|\mathbf{y} - \mathbf{R}\mathbf{w}\|_2^2 + \tau\|\mathbf{w}\|_1]$$

A value of $\tau = 0.1$ was selected to produce a 6 asset portfolio, whe result is shown in figure 7. The stocks welected are `3,10,14,22,28`, which is totally different from greddy method.

To find a small number of stocks to invest and act as passive invester, we can find that the optimization method is more precise than the greedy method, it looks closer to the line of FTSE 100.

4 Discussion of Lobo et al.

To analysis the transaction cost and how it will influence the portfolio, we use a new variable x, for a holding portfolio w, the new portfolio will be

$w + x$. For an observed return of the assets $a = (a1...a2)$, we can get the mean $\bar{a}$ and the variance Σ. So we have an objective function:

$$maximize \qquad \bar{a}^T(w + x)$$

And we have serval constraints on the function.

- The cost of transaction and buying new aseests should be covered by selling shares. ($1^T x + \phi(x) \leq 0$)
- Some rules based hobbies or experience. Like what's the maximum amount to invest.
- Money will be lost during transaction, a low bound is needed to limit the frequent shortselling. ($w_i + x_i \geq -s_i, i = 1, ..., n.$)
- The risk should not be too high. Setting an up bound for variance. ($(w + x)^T \Sigma (w + x) \leq \sigma_{max}$)
- If the return $a \sim \mathcal{N}(\bar{a}, \Sigma)$., the return will probably larger than a certain amount. ($Prob(W \geq W^{low}) \geq \eta$)

According to this rules, we can write CVX's constraints to use CVX solve the optimization problem in an acceptable time.