Factor Timing

Jonas Neller, 21-730-676

Lorena Tassone, 18-700-237

Lucas Letulé, 21-703-327

November 13, 2022

Department of Banking & Finance

Quantitative Asset Management

Prof. Markus Leippold & Philippe Rohner

Abstract

This paper aims to replicate and critically evaluate the contribution made by Haddad et al. [2020]. Therefore, the methodology described by the authors is employed at first and thereafter possible extensions are considered. The findings presented by Haddad et al. [2020] cannot be replicated in full due to data gaps and a partially ambiguous description of the methodology. Moreover, possible stumbling blocks when implementing the strategy in practice are identified. The original approach is then extended in various ways, most notably by replacing the book to market ratio with the annualized volatility as a predictor. The results are interpreted in the light of the problems encountered in the replication.

Keywords: Factor Timing, Practical Implication, Investment Strategy

Contents

List of Figures

List of Tables

1 Motivation

The paper by Haddad et al. [2020] deals with the intersection of factor investing and the timing of investments. They use a cross-section of 50 equity factor portfolios and combine a prediction thereof with a time-series prediction of returns. To get a hold of the multi-dimensionality, they use the first five principal components (PC) of the 50 anomalies. These PCs capture most of the variability of anomaly returns. They find the PCs to be strongly predictable. The predicted PCs in turn are used to predict the anomaly returns. This is possible, since the PCs are a linear combination of the anomalies and capture the "common variation in risk premiums". Using these predictions, they form optimal factor timing portfolios. Haddad et al. [2020] suggest that these findings lay support to practical implementations. However, to be applied in a practical environment, an academic investment strategy needs to fulfill some requirements. We have identified three necessary basic properties. In our paper, we will explore these characteristics and critically place Haddad et al. [2020] into the following framework.

First, an investment should be accessible, at least for an institutional investor audience. These large investor types typically have a broader access to investments than plain retail or intermediary investors. If they are clients of large banks or size permits it they can undoubtedly access or build most investment universes. However, it is questionable if this justifies very individualized or complex investment universes. Commonly used data sets in academia are usually highly individualized and can contain a larger number of small- and micro-caps than universally accessible investment vehicles. High costs to construct these universes are a factor that can prohibit such constructions in practice [Cazalet and Roncalli, 2014]. Secondly, the approach suggested should at least partially be adaptable to common investment restrictions faced by some investors face and nonetheless generate positive and robust results. Long-only portfolios as well as a no-leverage constraint should be therefore be included in the analysis. These considerations make an interpretation of scientific results much more applicable to practitioners. Otherwise, high costs can harden a practical implementation [Cazalet and Roncalli, 2014]. Lastly and most importantly, the strategy should be replicable, so the analysis conducted and theoretical approach followed is comprehensible. A good position to start would surely be a good data availability or an easily reconstructable data set. Since real money could be at stake, this is particular important, so asset managers know and understand the signals they would be using. In their paper, Haddad et al. [2020] did not account for any of the above mentioned characteristics. Nevertheless, they suggest that their approach can be feasible in practice. The portfolios constructed and provided by the authors are heavily individualized. This would add implementation complexity in a live environment and tremendous costs if these portfolios should be re-constructed. They also don't apply any restrictions which professional investors face in their day-to-day business or conduct robustness tests accounting for these. Furthermore, we found multiple irregularities in the paper, that make it near impossible to recreate the paper´s results exactly[1].

These characteristics motivate certain extensions to the findings and approach of Haddad et al. [2020]. The investment universe should be changed, so it is accessible to a broader audience. The presented long-short version should also be accompanied by implementable long versions similar to additional practical restrictions. To provide a better replicability, the authors would additionally need to revise the paper and clarify certain steps in their methodology. One of our contribution to the research conducted by Haddad et al. [2020] is the back-testing with long-only portfolios. The results are however delivered with a grain of salt, since we run into a few issues when replicating the strategy. Throughout the paper there are multiple occasions where the exact replication step is ambiguous or unclear. In addition, one of the key data inputs is not provided and needs to be derived. These issues seem small themselves, but combined and after running dimensionality reduction and multiple predictive regressions, they shift the final results by a significant margin.

[1] Examples include: market book-to-market ratios are not provided by the authors, on page 1992 it is unclear if the adjustments of the variance are only conducted in- or also out-of-sample; on page 1993 it is unclear which "difference between this quantity" is referred to; different R^2 are introduced, on multiple occasions it is unclear which R^2 is used.

The paper continues as follows. We review relevant literature and present the theoretical framework of key characteristics of the authors´ methodology. We then dive into the factor timing strategy of the authors and contrast the extensions we included to the original paper. We extend the original methodology in the following ways: We use 4 or 6 PCAs instead of 5, we run the analysis using a longer out-of-sample (OOS) time-period until December 2019, unadjusted book-market-ratios as well as factor returns of long-only portfolios. Lastly, we use the annualized volatility of past returns as a predictor of future returns instead of the book-market-ratio. Lastly we critically discuss our results and those of Haddad et al. [2020] and conclude.

2 Literature Review

The discovery of factors and subsequently using factors to get exposed to certain risk premia is a major development in financial academia and asset management. A whole branch of research is based on factor investing. Explaining stock returns with an exposure to certain risk factor has proven to be very elegant. Starting out from the three-factor-model from Fama and French [1993], over time a zoo of over 400 different factors has accumulated [Harvey and Liu, 2019]. All these factors claim to explain stock returns with a different reasoning [Harvey and Liu, 2019]. However, there are voices claiming that most of them are a product of factor mining and rely on pure luck [Harvey and Liu, 2019]. The later discovered factors have in common that none of them where able to compete with the earlier found factors, let alone redeem one of Fama and French's three original factors. In factor investing one generally tries to find independent patterns in the cross-section of factors, to explain dispersion of asset returns [Cazalet and Roncalli, 2014]. This materializes in over-/under-weighting the superior factors from a cross-section for a given time. This is regularly conducted in the asset management industry (one think of "Value" or "Growth" investments) and can be extended to a multitude of significant factors. Haddad et al. [2020] use 50 factors for their paper, which in combination have explanatory power over future expected returns.

Another key finding of financial academia and goal of most asset managers is the successful timing of asset markets. Various strategies and methodologies have been applied and discussed in academia to forecast markets. They are mostly based on the groundbreaking work of Shiller [1980], Fama and French [1988], who found a predictability in stock markets. Their contribution is one of the important pillars to successfully generate excess returns in markets and has therefore received a lot of attention from practitioners as well. Building on their work, large fields like technical or fundamental analysis have been developed and deployed to infer future price moves and directionality of different asset classes based on their concept.

Timing investments accurately has later been expanded to different market anomalies. Famously, Cohen et al. [2003] expanded timing to the value anomaly and Cooper et al. [2004] to momentum. They helped to lay the basis for what is today called factor timing. It includes both of the approaches outlined above. Namely basic factor investing and timing these investments in addition. In practice and academia most strategies use a multitude of factors that ideally complement each other, and identify anomalies that influence the performance of each factor individually. Using the anomalies, the expected return of the factors is then forecasted. Investments decisions are then carried out based on the results of these forecasts. The goal of this strategy is to beat naively weighted factors and the general market.

Later work touches on an application of factor timing to a wider cross-section of factors. Akbas et al. [2015] evaluates the use of a single variable to forecast anomaly returns. With their approach they want to identify common components of a cross-section. Other approaches where changing the factor structure and approach to receiving factors. Lochstoer and Tetlock [2020] deconstruct common factors into cash flow and discount rate news, to receive additional information about the behaviour and drivers of factors. Taking opposing positions, Arnott et al. [2016] use valuation ratios to forecast anomaly returns, while Asness et al. [2017] argues that not valuation ratios, but rather the value factor makes value timing work.

Further, more statistically based approaches have also been introduced to deal with a wider cross-section of factors. Freyberger et al. [2020] use adaptive group LASSO to dissect the characteristics non-parametrically. In doing so, they were able to extract incremental information from different characteristics for the cross-sectional and to show how the examined characteristics impact expected returns. Other explorations by Kozak [2020] and Kozak et al. [2020] utilize the statistical discount factor (SDF) to capture the joint explanatory power of of large cross-sectional stock return predictors. These two papers lay the foundation for Haddad et al. [2020]. Similarly, the application of Principal Component Analysis (PCA) to a large cross-section of stock returns laid out by Kozak et al. [2018] contributed with to the development of Haddad et al. [2020]. Kozak et al. [2018] assume no near-arbitrage to implement the PCA and use this assumption to argue that the SDF can be interpreted as the dominant source of variation.

3 Theoretical Framework

The theoretical framework is mostly based on the framework used by Haddad et al. [2020]. Hence, the focus of this empirical analysis lies in the assessment of the benefits of timing strategies for a cross-section of excess returns $\{R_{i,t}\}_{i \in I}$.

Haddad et al. [2020] illustrate the connection between factor timing benefits, the stochastic discount factor (SDF) and predictability in the following decomposition by showing in their Internet Appendix Section I, that the average maximum conditional Sharpe ratio can be expressed as follows, if asset returns are uncorrelated:

$$E\left(\mathrm{SR}_t^2\right) = E\left[\mathrm{var}_t\left(m_{t+1}\right)\right] = \sum_i \frac{E\left[R_{i,t+1}\right]^2}{\sigma_i^2} + \sum_i \left(\frac{R_i^2}{1 - R_i^2}\right) \tag{1}$$

As described by Haddad et al. [2020], the first equation illustrates that the expected variance of m_{t+1} equals the average maximum squared Sharpe ratio, where m_{t+1} represents the minimum variance SDF that prices the set of returns. The combination of these two elements is shown in the second equality. The first term is the sum of ratios of the squared average return to the conditional variance of the asset return (σ_i^2) representing an unconditional part reminiscent of static Sharpe ratios[2]. The second term, which represents predictability and is of particular interest to us, increases in R_i^2, which is the maximum predictive R-squared when forecasting asset i, $R_i^2 = 1 - \sigma_i^2 / \mathrm{var}\left(r_{i,t+1}\right)$.

We want to be able to characterize expected returns[3] by measuring the predictability of the largest principal components of the set of factors. To be able to create robust forecasts for returns, a set of restrictions helps us to overcome the problem of spurious results. Hence, we use the stock characteristics to reduce the dimensionality of the return space. Therefore, in accordance with the reference paper of Haddad et al. [2020], we make the two following assumptions: First, we follow previous contributions to the literature and assume that pricing-relevant information is captured only by a relatively small number of stocks characteristics. Further,

[2] In line with Haddad et al. [2020], we focus on a homoscedastic setting.

[3] The expected return of any asset is the product of the asset's conditional loading on the components and the expected return of these few components.

we address the primary motivation behind factor timing portfolio strategies by supposing that assets are conditionally priced by a factor model. Second, we assume that prices exhibit no near-arbitrage opportunities. These assumptions are further discussed in the following subsections.

3.1 Factor model, factor investing, and factor timing

First, a structure on the pricing kernel is imposed. As done by Haddad et al. [2020], in the span of N individual asset excess returns R_{t+1} we start with the minimum variance SDF [Hansen and Jagannathan, 1991]:

$$m_{t+1} = 1 - b_t' \left(R_{t+1} - E_t \left[R_{t+1}\right]\right) \tag{2}$$

Since we use stock characteristics to reduce the dimensionality of the return space identical to Haddad et al. [2020], we assume that cross-sectional heterogeneity in risk prices b_t can be largely captured by K observable stock characteristics, C_t, with K≪N and N being the number of individual stock excess returns R_{t+1}. The vector δ_t of size $K \times 1$ summarizes the time-series variation in each characteristic. Hence, Stock-level SDF loadings can be represented as

$$\underbrace{b_t}_{N\times 1} = \underbrace{C_t}_{N\times K}\underbrace{\delta_t}_{K\times 1}, \tag{3}$$

where C_t is a $N \times K$ matrix of stock characteristics and δ_t is a $K \times 1$ vector of (possibly) time-varying coefficients, and $K \ll N$.

We substitute Equation (3) into Equation (2) and obtain the following alternative SDF representation [Hansen and Jagannathan, 1991]:

$$m_{t+1} = 1 - \delta_t' \left(F_{t+1} - E_t \left[F_{t+1}\right]\right) \tag{4}$$

In this case, $F_t = C_{t-1}' R_t$ are factor portfolios based on the characteristics. δ_t can now be interpreted as time-varying prices of risk on these factor portfolios.

Not only does the first assumption allow us to describe the cross-section, but it also permits the time variation in a relatively small number of factor risk prices. This leads to a dimensionality reduction of variables determining the SDF and considers meaningful variation in factor expected returns.

As a consequence, as stated by Haddad et al. [2020], a conditional factor model holds:

$$E_t\left[R_{j,t+1}\right]=\beta_{jt}^{\prime}\Sigma_{F,t}\delta_t=\beta_{jt}^{\prime}E_t\left[F_{t+1}\right] \tag{5}$$

The factor model and Equation (4) equivalence is given by $\delta_t^{\prime}=\Sigma_{F,t}^{-1}E_t\left(F_{t+1}\right)$, where $\Sigma_{F,t}$ represents the conditional covariance matrix of the factors.

Haddad et al. [2020] argue, that this relation highlights that the model can generate interesting risk premium variation for returns, even in a homoscedastic setting, where $\Sigma_{F,t}$ and $\beta_{it}^{\prime}$ are constant over time. This result appears because δ_t controls how the factors are loaded by the SDF, and hence the price of risk is changed.

In this framework, optimal portfolios can be constructed using only these few factors, since the sources of risks of concern to investors are completely captured by them. The changing properties of the factors lead to according adjustments in the portfolio weights over time. The maximum conditional Sharpe ratio return as an example is hence obtained by

$$R_{t+1}^{\mathrm{opt}}=E_t\left[F_{t+1}\right]^{\prime}\Sigma_{F,t}^{-1}F_{t+1}. \tag{6}$$

3.2 Absence of near-arbitrage

The idea of "no near-arbitrage" opportunities is applied in various previous contributions, adding "economic discipline to statistical exercise" [Haddad et al., 2020], by imposing an upper bound on the variance of the SDF Hansen and Jagannathan [1991]. Haddad et al. [2020] set this equal to a bound on the maximum squared Sharpe ratio. In our time-varying risk premium setting, there is no maximum Sharpe ratio but rather at each point in time there is a conditional maximum Sharpe ratio. Haddad et al. [2020] argue that average maximum conditional squared Sharpe ratio $E\left[\mathrm{SR}_t^2\right]$ is an appropriate metric for this setting. Hence, as imposed by Haddad et al. [2020], our second assumption states that there are no near-arbitrage opportunities on average, meaning that the average conditional squared Sharpe ratios are bounded above by a constant. Assumption 2 also leads to additional dimensionality reduction. Since the maximum conditional Sharpe ratio is invariant to rotations of the asset space, Equation (1) can be applied with the PC decomposition of returns.

$$E\left(\mathrm{SR}_t^2\right) = \sum_{i=1}^{K} \frac{E\left[PC_{i,t+1}\right]^2}{\lambda_i} + \sum_{i=1}^{K} \left(\frac{R_i^2}{1-R_i^2}\right) \tag{7}$$

$PC_{i,t+1}$ being the ith principal component portfolios of factors F, and λ_i the corresponding eigenvalue, we sum across all K PC portfolios. The first term illustrates the squared Sharpe ratio of an optimal static factor portfolio and represents the benefits of static factor investing. Again our focus lies in the second term, which identifies the amount of predictability for each PC and represents the incremental benefit of optimally timing the factors. Hence, the portfolio performance one can obtain increases in PC predictability. To calculate the contribution of each PC portfolio to the total predictability of returns we use the R^2 defined by Haddad et al. [2020]:

$$\begin{aligned} R_{\text{total}}^2 &\equiv \frac{\operatorname{tr}\left[\operatorname{cov}\left(E_t\left[F_{t+1}\right]\right)\right]}{\operatorname{tr}\left[\operatorname{cov}\left(F_{t+1}\right)\right]} = \frac{\operatorname{tr}\left[\operatorname{cov}\left(E_t\left[PC_{t+1}\right]\right)\right]}{\operatorname{tr}\left[\operatorname{cov}\left(PC_{t+1}\right)\right]} \\ &= \sum_{i=1}^{K} \left(\frac{R_i^2}{1-R_i^2}\right) \frac{\lambda_i}{\lambda} \end{aligned} \tag{8}$$

where $\lambda = \sum \frac{\lambda_i}{1-R_i^2} \approx \sum \lambda_i$ is the total unconditional variance of returns. The second line indicates that the R^2 comes from the predictability of each of the PCs, weighted by their importance in explaining the factors. The combination of total R^2 and maximum squared Sharpe ratio relations indicates that small PCs cannot meaningfully contribute to predictability. The intuition herefore is that while it is possible that each of the proposed factors by Haddad et al. [2020] are predictable, it is unlikely that they all capture independent risk sources. Otherwise, investors could obtain extremely large Sharpe ratios by diversifying across the sources of risk. Hence, the few large PC portfolios must capture cross-sectional and time-series variation in expected factor returns. As done by Haddad et al. [2020], we define Z_{t+1} as the vector containing the largest principal components portfolios of F_{t+1}. We use Proposition 1 of Haddad et al. [2020] to summarize the implications of this result for the SDF and the optimal factor timing portfolio.

Proposition 1. Under Assumption 1 and Assumption 2, the SDF can be approximated by a combination of the dominant factors:

$$m_{t+1} \approx 1 - E_t\left[Z_{t+1}\right]' \Sigma_{Z,t}^{-1}\left(Z_{t+1} - E_t\left[Z_{t+1}\right]\right). \tag{9}$$

Equivalently, the maximum Sharpe ratio factor timing portfolio can be approximated by

$$R_{t+1}^{\text{opt}} \approx E_t\left[Z_{t+1}\right]' \Sigma_{Z,t}^{-1} Z_{t+1}. \tag{10}$$

According to Haddad et al. [2020] the two assumptions are complementary, since Assumption 1 implies the sufficiency of factor timing (timing individual stocks is not needed) and Assumption 2 provides a way to time a set of factors, even without the first assumption.

Eventually, we can estimate the conditional means and variance of the largest principal components. As done by Haddad et al. [2020], we estimate the mean forecasts $E_t\left[Z_{t+1}\right]$ using standard forecasting methods for individual returns, since we only consider a few components.

To summarize, the above explained theoretical framework leads to the following five steps, whose empirical application will be discussed further in Section 4:

1. Start from a set of pricing factors F_{t+1}.
2. Reduce this set of factors to a few dominant components, Z_{t+1}, using principal component analysis.
3. Produce separate individual forecasts of each of the Z_{t+1}, that is measures of $E_t\left[Z_{t+1}\right]$.
4. To measure the conditional expected factor returns, apply these forecasts to factors using their loadings on the dominant components.
5. To engage in factor timing, use these forecasts to construct the portfolio given in Equation (10).

4 Replication and Extensions

In the next section we start with the replication of the original methodology. We discuss the data used, show and discuss the dimensionality reduction, forecast the dominant factors, use these forecasts to conduct a more granular forecast of factor returns and lastly create optimal factor portfolios. We proceed by presenting our extensions to Haddad et al. [2020]. The extensions tackle various issues of the original paper. Using long-only portfolios makes the results much more viable to practitioners, taking different amounts of principle components into account is a robustness check to some of the authors assumptions, adjusting the time-horizon helps to determine a continued meaningfulness, not adjusting our data sets increases the delivers improved results and changing the predictor expands the research of Haddad et al. [2020].

All computations for the replication, the extensions and the data used is made available online[4].

4.1 Replication - Factor Return Predictability

4.1.1 Data

Based on the procedure of Haddad et al. [2020], step 1 of our approach starts with a set of 50 anomaly portfolios. For the portfolio construction, the universe of CRSP and Compustat stocks is being used, whereby the stocks were sorted into ten value-weighted portfolios for each of the 50 factors. Portfolios include all NYSE, AMEX, and NASDAQ firms; however, the breakpoints use only NYSE firms as in Fama and French [2016]. Nearly all the data used is provided by the authors on their website[5], which already contains the 10 constructed portfolios for each anomaly. Our sample consists of monthly returns from January 1974 to December 2019. In the replication, we use the original time-frame from January 1974 to December 2017. We define the time frame from January 1974 until December 1995 as our in-sample (IS) period

[4]https://github.com/luclet/QAM-Factor-Timing.git

[5]https://www.serhiykozak.com/data

Table 1: Percentage of variance explained by anomaly PCs

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
% var. explained	26.1	12.6	10.3	6.5	4.7	4.1	3.5	2.7	2.2	2.1
Cumulative	26.1	38.7	49.0	55.5	60.1	64.2	67.7	70.4	72.6	74.6

which is used to establish important parameters of our model and the time frame from January 1996 until December 2017 as our out-of-sample (OOS) period.

For each anomaly, the difference between its return on portfolio 10 and portfolio 1 is constructed resulting in long-short anomalies. Further, for each anomaly its corresponding book-to-market ratios of the underlying stocks are formed, representing an anomaly's measure of relative valuation. This measure, defined by bm, is built as the difference in log book-to-market ratios of portfolio 10 and portfolio 1.

Eventually, the data is market-adjusted and rescaled. In order to do so, the regression β for each anomaly with respect to aggregate market returns is calculated. Predictors and returns are then adjusted by subtracting $\beta * r_{mkt}$ from returns and $\beta * bm_{mkt}$ from the bm ratios. Next, the market-adjusted returns and bm ratios are rescaled such that they have equal variance across anomalies. To make sure that out-of-sample (OOS) statistics contain no look-ahead bias, the βs and variances used for these transformations are estimated using only the first half of the sample. The authors don't deliver the market return r_{mkt} and market bm. We employ the Fama-French market returns[6] and infer the bm_{mkt} from the authors data. The market bm-ratio consists of the average bm of the portfolio[7] with the highest absolute number of stocks making it the best approximation of the used stock market universe in our view. Next, we analyze the dominant components of our factors.

4.1.2 Dominant Components of the factors

In step 2 the set of factors is reduced to a few dominant components which are defined by the largest Principal Components (PC). Before doing so, anomalies containing

[6]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[7]Factor *age* was used by us.

NaN values[8] and anomaly portfolios not in the original set of portfolios[9] for the considered time period were dropped from the data set. Using the function *PCA* from scikit-learn we construct the PCs. Table 1 shows the percentage of variance explained by the first 10 PCs. Using Campbell and Thompson [2008] and their findings about monthly R^2 in market prediction, as well as their own derivation, the authors reason that we should include at most five PCs. Their approach to this result is established by using a monthly R^2 for predicting the market of 75 bps and a lose upper bound of a Sharpe ratio of 1 on an annual basis, or 8.3% on a monthly basis. Assuming that the PCs contribute equally to the R^2, the harmonic mean of the used dominant factors should be higher than 75bps/8.3%=9%. Taking these five components jointly into account, they explain nearly 60% of the total variation in returns.

Since we market-adjusted the portfolios, as done by the authors, we also include the aggregate market portfolio as pricing factor. Hence, we study $Z_{t+1} = (R_{mkt,t+1}, PC_{1,t+1} \cdots PC_{5,t+1})$ in the following step.

4.1.3 Predicting the large PCs of anomaly returns

Step 3 is to compute individual forecasts of the dominant components of factor returns. We use simple linear regressions to conduct this operation.

Predictors

Forecasts are computed on valuation ratios, as done before by the authors and various other related literature such as Shiller [1980], Fama and French [1988] or Campbell and Shiller [1988].

For each PC-portfolio we construct a single predictor namely its own net book-to-market ratio. Hence, to predict $PC_{i,t+1}$, we calculate its log book-to-market ratio $bm_{i,t}$ by combining the anomaly log book-to-market ratios according to portfolio weights given by the PCs eigenvector given in Q: $bm_{i,t} = q_i' bm_t^F$. Formally, as done by the authors Haddad et al. [2020], we apply the eigenvalue decomposition of

[8] 'invaci', 'ipo'

[9] 'exchsw', 'divg','divp'

Table 2: Predicting dominant equity components with BE/ME ratios

	MKT	PC1	PC2	PC3	PC4	PC5
Own BM	0.02	0.14	0.08	0.14	0.27	0.15
	(0.01)	(0.07)	(0.08)	(0.08)	(0.10)	(0.06)
p-value	0.15	0.04	0.29	0.07	0.01	0.02
R^2	0.01	0.02	0.00	0.01	0.03	0.02

anomaly excess returns, $cov(F_{t+1}) = Q\Lambda Q'$, where Q is the matrix of eigenvectors and Λ is the diagonal matrix of eigenvalues. The i-th PC portfolio is formed as $PC_{i,t+1} = q_i' F_{t+1}$ where q_i is the i-th column of Q[10] [Haddad et al., 2020]. To ensure no look-ahead bias in our later OOS results, we estimate Q and Λ using only the first half of the data. We compute the following regression

$$PC_{i,t+1} = \beta_0 + \beta_1 bm_{i,t} \tag{11}$$

where $PC_{i,t+1}$ is our principal component i in time $t+1$ and the independent variable is the log-book-market ratio multiplied with its portfolio weights q_i'. We run the regression for each of the five relevant PCs and also forecast the market using the log aggregate book-market ratio. Step 3 helps us to dramatically decrease the dimensionality and therefore pave the way for our next steps.

Predictability results

Similar to the approach in Haddad et al. [2020], we predict the the first five anomaly PC portfolios and the aggregate market with a monthly holding period. Table 2 displays our regression results. The first row shows the predictive coefficient, in the second row we indicate the standard deviation and in the third row the p-value of the regression. Lastly, we provide the full-sample R^2. While 4 of the 6 regressions are significant, the betas are small and only account for a small share of the variation in returns. We thus suspect that the explanatory power of the BM ratios will be insufficient to make accurate predictions of future returns in the following steps.

[10] Plots representing the eigenvectors (loadings) of the different PCs can be found in the Appendix.

We only report the aforementioned sections of the authors table, as the incomplete data and uncertainties of the exact methodology lead to questionable results. Two main issues arise while recreating Haddad et al. [2020] results. Since the authors don't deliver r_{mkt}, it is impossible to recreate the exact same βs of the papers step 1. This infers with the whole adjustment process of the factor returns and the book-market-ratios. Additionally, the authors don't deliver bm_{mkt}. This also sways our results from Table 2. Both issues combined lead to ambiguous results in the following steps.

Importance of restrictions

Overall in the paper, our baseline prediction includes just five PC-portfolios. This is derived according to the theoretical reasoning mentioned in step 2. Restricting our PC-portfolio universe that much can come across as quite extreme. In Haddad et al. [2020], the authors show that only large PCs can be predicted satisfactory by their bm. Therefore, in addition to being a theoretical optimal number of PCs, the limitation to five components is also justified empirically. According to Haddad et al. [2020] restricting the prediction to these dominant factors also yields robust predictability, without any inference of smaller PCs, which often give false predictions. Following the prediction of dominant components, we now turn to forecasting individual factor returns.

4.1.4 Predicting individual factors

Step 4 of the approach is to infer expected return forecasts for the individual factors using the forecasts of the dominant components. The estimates from Table 2 can be used to generate the forecast returns for each anomaly, since factors are known linear combinations of the PC portfolio, weighted by their loading in the respective PC.

More specifically, we calculate the percentage change in book-to-market ratio for each point in time in the train set and each anomaly[11].

$$bm\_change^F_{train,t} = \frac{bm^F_{train,t}}{bm^F_{train,t-1}} - 1 \tag{12}$$

Next, we multiply this change by the according coefficient for each anomaly. The coefficient for each anomaly can be constructed as a linear combination of all the estimates listed in Table 2, weighted by the respective loading the anomaly has in each PC according to the eigenvector matrix.

$$return\_change^F_{train,t+1} = bm\_change^F_{train,t} * \beta^F \tag{13}$$

This multiplication results in the forecasted change of returns for each anomaly at each point in time. We then add this relative change to the previous return, to calculate the absolute predicted return for period t+1.

$$return^F_{train,t+1} = return^F_{train,t} * (1 + return\_change^F_{train,t+1}) \tag{14}$$

Hence, one can note that each anomaly return is implicitly predicted by the whole cross-sections of book-to-market ratios.

To compute the predictive power of our forecast returns, we regress the predicted returns for t+1 on the true returns which results in the R^2 for each anomaly shown in Table 3. We differentiate between in- and out-of-sample regressions and R^2.

Our individual R^2 are insignificantly small compared to the results from Haddad et al. [2020], meaning that many of the anomalies are not meaningfully predictable by their own book-to-market ratio. However, even the values computed by Haddad et al. [2020] for the OOS period range from 4.5% to -4.7% - neither high values for a R^2. Describing their results as "highly predictable" or "substantial anomaly predictability" [Haddad et al., 2020] as done by the authors therefore seems inappropriate. We further note that at this step in our replication, our results naturally deviate from the ones in the reference paper since already the estimates in Table 2 deviate significantly from the ones Haddad et al. [2020] computed. Thus, comparisons to the values obtained by Haddad et al. [2020] have to be taken with a grain of salt.

[11] We calculate the percentage change because the computed regressions in step 3 are log-log regressions. Hence, a 1% change in *Own bm* results in a β% change of PC return.

Table 3: Predicting individual anomaly returns: R^2 (%)

	IS	OOS		IS	OOS
Size	0.6	1.4	Value-momentum	0.0	0.0
Value (A)	1.3	1.6	Value-momentum-prof	2.8	0.4
Gross profitability	0.0	0.0	Short interest	0.6	1.8
Value-profitablity	4.5	2.1	Momentum (12m)	1.4	0.2
F-score	1.1	0.8	Industry momentum	0.5	0.0
Debt issuance	1.7	0.2	Momentum-reversals	2.9	0.7
Share repurchases	0.6	0.9	Long run reversals	1.8	0.5
Net issuance (A)	3.7	2.5	Value (M)	0.3	2.5
Accruals	0.1	0.0	Net issuance (M)	1.0	1.3
Asset growth	0.6	0.1	Earnings surprises	0.7	0.0
Asset turnover	1.4	0.1	Return on book equity (Q)	2.7	4.5
Gross margins	0.0	0.5	Return on market equity	0.1	1.7
Earnings/price	0.8	1.6	Return on assets (Q)	0.3	4.6
Cash flows/price	0.4	0.1	Short-term reversals	0.1	0.8
Net operating assets	0.0	0.0	Idiosyncratic volatility	0.2	0.1
Investment/assets	3.4	0.1	Beta arbitrage	0.8	0.7
Investment/capital	0.0	0.6	Seasonality	0.5	1.1
Investment growth	1.8	0.0	Industry rel. Reversals.	0.6	0.7
Sales growth	4.7	0.2	Industry rel. Rev. (L.V.)	0.0	0.0
Leverage	0.9	1.0	Ind. mom-reversals	0.0	0.2
Return on assets (A)	0.7	1.1	Composite issuance	1.5	0.0
Return on book equity (A)	1.7	1.3	Price	0.4	1.6
Sales/price	0.9	3.6	Share volume	1.6	0.2
Growth in LTNOA	1.1	0.4	Duration	0.2	4.1
Momentum (6m)	0.0	0.0	Firm age	1.8	1.7

4.1.5 Optimal Factor Timing Portfolio

The final, fifth step consists of constructing an optimal factor timing portfolio, using our forecasts in returns. The main idea is to increase positions when expected returns are larger. Hence, we compute a weight vector $w_{F.T.,t}$ for each period t containing the optimal weights for each PC and the market.

More specifically, we apply the prediction method used in step 4 to forecast PC returns $E_t[Z_{t+1}]$. Namely, we construct the book-to-market ratio of each PC by multiplying the book-to-market ratios of each anomaly in the train set with its respective loading in the according PC for each period. Additionally, we include the market book-to-market ratio time-series. We then calculate the percentage change in the book-to-market ratio for each PC and the market in each period, multiply it with the estimates constructed in Table 2 and again add the relative change of return to the actual return in the previous period to compute the actual predicted PC return for t+1, $E_t[Z_{t+1}]$.

Next, we construct forecast errors and compute an estimate of the conditional covariance matrix of the market and PC returns, $\Sigma_{Z,t}$. Further, as done by Haddad et al. [2020] we assume it to be homoscedastic for estimation means. In a next step, we combine these estimates to calculate the portfolio weights $\omega_t = \Sigma_{Z,t}^{-1} E_t[Z_{t+1}] = \Sigma_Z^{-1} E_t[Z_{t+1}]$. Although the factor timing strategy is expressed in terms of principal components (predicted returns contained in vector Z_{1+1}), implicitly the underlying factors are traded. We finally scale the weights across the PCs and market such that our portfolio weights sum up to 100%. Finally, we multiply the weights of the principal components with the predicted principal component returns for each period to obtain the strategy's return. The following equation summarizes the strategy:

$$\omega_{\text{F.T. },t} = \Sigma_Z^{-1}\left[E_t\left(R_{mkt,t+1}\right), E_t\left(PC_{1,t+1}\right), \ldots, E_t\left(PC_{5,t+1}\right)\right]' \tag{15}$$

Table 4 reports sharpe ratio and information ratio of this portfolio, both for the in- and out-of-sample time frame. Both IS and OOS sharpe ratios are small, pointing towards the simple observation, that the strategy does not outperform a buy-and-hold strategy. A real world example shows, it underperforms the sharpe

Table 4: Factor Timing Performance

d

	Factor Timing
IS Sharpe ratio	0.19
OOS Sharpe ratio	0.000047
IS Inf. ratio	-1'606'579'285'377
OOS Inf. ratio	-3'187'683'649'222

ratio of holding the S&P 500 Index significantly. We suspect that the underlying reason for our low sharpe ratio are the low betas and R^2 of our predictive regressions from Table 2. Thus, with the data at our availability, the book to market ratio is not suited to single handedly explain and thus neither to predict a sufficiently large share of the variation on factor returns. Please find a visual representation of the cumulative in sample, out of sample and market returns attached in the appendix. The Information ratio was computed using the market as benchmark. As a consequence of distorted predictive returns, the in- and out-of-sample Information ratio yields implausible values as well, which make it difficult to infer a meaningful statement about this performance estimate. However, having a negative value we can say, that the factor timing strategy underperformed the market.

4.2 Extensions

As seen in the previous chapters, our results do not coincide with the ones from Haddad et al. [2020]. Hence, our results do not support the predictability power stated by Haddad et al. [2020]. However, we extended their research by modifying some of their methods and compare it to our replication results. The added extensions are discussed below.

4.2.1 Adjusting Time-horizon

As a first extension, we extend the time-horizon to include the data up until December 2019 and compare the OOS R^2 from 1995-2017 with the ones from 2017 to 2019. In Table 5 we report the R^2 for the individual predictions using the normal

Table 5: Predicting individual anomaly returns: R^2 (%), time-horizon until 31.12.2017 and 31.12.2019

	2017		2019			2017		2019	
	IS	OOS	IS	OOS		IS	OOS	IS	OOS
Size	0.6	1.4	0.6	5.4	Value-momentum	0.0	0.0	0.0	2.6
Value (A)	1.3	1.6	1.3	2.0	Value-momentum-prof	2.8	0.4	2.8	5.1
Gross profitability	0.0	0.0	0.0	0.0	Short interest	0.6	1.8	0.6	7.3
Value-profitablity	4.5	2.1	4.5	7.6	Momentum (12m)	1.4	0.2	1.4	7.2
F-score	1.1	0.8	1.1	37.3	Industry momentum	0.5	0.0	0.5	1.5
Debt issuance	1.7	0.2	1.7	0.1	Momentum-reversals	2.9	0.7	2.9	0.0
Share repurchases	0.6	0.9	0.6	0.9	Long run reversals	1.8	0.5	1.8	0.3
Net issuance (A)	3.7	2.5	3.7	3.5	Value (M)	0.3	2.5	0.3	9.6
Accruals	0.1	0.0	0.1	1.5	Net issuance (M)	1.0	1.3	1.0	0.1
Asset growth	0.6	0.1	0.6	3.3	Earnings surprises	0.7	0.0	0.7	25.7
Asset turnover	1.4	0.1	1.4	5.4	Return on book equity (Q)	2.7	4.5	2.7	1.1
Gross margins	0.0	0.5	0.0	1.4	Return on market equity	0.1	1.7	0.1	1.5
Earnings/price	0.8	1.6	0.8	9.3	Return on assets (Q)	0.3	4.6	0.3	0.2
Cash flows/price	0.4	0.1	0.4	1.1	Short-term reversals	0.1	0.8	0.1	0.1
Net operating assets	0.0	0.0	0.0	3.5	Idiosyncratic volatility	0.2	0.1	0.2	3.4
Investment/assets	3.4	0.1	3.4	6.7	Beta arbitrage	0.8	0.7	0.8	0.0
Investment/capital	0.0	0.6	0.0	10.7	Seasonality	0.5	1.1	0.5	3.8
Investment growth	1.8	0.0	1.8	1.3	Industry rel. Reversals.	0.6	0.7	0.6	2.5
Sales growth	4.7	0.2	4.7	0.2	Industry rel. Rev. (L.V.)	0.0	0.0	0.0	7.8
Leverage	0.9	1.0	0.9	10.2	Ind. mom-reversals	0.0	0.2	0.0	0.2
Return on assets (A)	0.7	1.1	0.7	1.7	Composite issuance	1.5	0.0	1.5	6.0
Return on book equity (A)	1.7	1.3	1.7	3.4	Price	0.4	1.6	0.4	0.2
Sales/price	0.9	3.6	0.9	13.3	Share volume	1.6	0.2	1.6	0.5
Growth in LTNOA	1.1	0.4	1.1	1.1	Duration	0.2	4.1	0.2	0.5
Momentum (6m)	0.0	0.0	0.0	0.6	Firm age	1.8	1.7	1.8	1.2

and the extended data set to compare. The in-sample results remain unchanged, since the time-frame extension did not affect the in-sample period. The majority of the out-of-sample R^2 increased. A potential explanation for the increased R^2 is a lower variation of the BM ratios in that time period without great outlier changes.

4.2.2 Changing amount of Principle Components

As also analysed by Haddad et al. [2020], we change the number of principal components and compare the output. The autors postulate that the PCs should account for 60% of cumulative explained, however this is an assumption. Further they argue that for a PC to account for an disproportionally large amount of the variation, it should explain 9% or more of the entire variation. We find that the 6th PC is insignificant and doesn't add much value. Using only four PCs does not only decrease the cumulative explained variance, there is also no obvious benefit in doing so when

Table 6: Predicting dominant equity components with BE/ME ratios - with extensions

	MKT	PC1	PC2	PC3	PC4	PC5
Own BM	0.02	0.01	0.00	0.01	0.00	0.00
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
p-value	0.15	0.00	0.00	0.09	0.08	0.02
R^2	0.01	0.05	0.03	0.01	0.01	0.02

looking at the results. Weighing off the benefits of an increased share of explained variance and dimensionality reduction, we agree with the author´s hoice of using five PCs.

4.2.3 Using non-adjusted Book/Market-ratios and long-only portfolios

When replicating the paper we asked ourselves why Haddad et al. [2020] market-adjusted the book-to-market ratios. Adjusting the returns seems plausible, since we want to have factor returns controlled for the market effect. However, there is no such reasoning for the book-to-market ratio.

Further, as discussed in Chapter 4.1.1, Haddad et al. [2020] create long-short portfolios by subtracting the value-weighted portfolio 1 from portfolio 10. According to Cazalet and Roncalli [2014], long-only strategies are more present in practice. Also, implementing long-short portfolios in practice might be problematic because of terms and regulations. Hence, we decided to test the method using only the long portfolio 10 (without subtraction of portfolio 1), which is given in the data set from Haddad et al. [2020]. To summarize, we analyse how the results change when implementing the long-only portfolio instead of the long-short and we further don't adjust the book-to-market ratio. We expect to get more insignificant results because of the usage of only the long portfolio.

As Table 6 shows, the predictability of *Own BM* decreases to nearly 0 when not adjusting the book/market ratio and using only the 10th portfolio making the strategy insignificant, since there is no sufficient predictive power in the book-to-market ratio. This makes sense as the strength of factor signals is naturally reduced

Table 7: Predicting individual anomaly returns: R^2 (%), time-horizon until 31.12.2017, with extensions

	Without extensions		With extensions			Without extensions		With extensions	
	IS	OOS	IS	OOS		IS	OOS	IS	OOS
Size	0.6	1.4	1.0	3.8	Value-momentum	0.0	0.0	0.3	0.3
Value (A)	1.3	1.6	1.9	2.2	Value-momentum-prof	2.8	0.4	0.7	0.0
Gross profitability	0.0	0.0	2.6	0.4	Short interest	0.6	1.8	0.0	0.7
Value-profitablity	4.5	2.1	3.7	1.9	Momentum (12m)	1.4	0.2	1.7	0.0
F-score	1.1	0.8	1.4	0.3	Industry momentum	0.5	0.0	1.0	0.2
Debt issuance	1.7	0.2	3.3	0.5	Momentum-reversals	2.9	0.7	4.4	0.9
Share repurchases	0.6	0.9	2.2	0.4	Long run reversals	1.8	0.5	2.3	0.5
Net issuance (A)	3.7	2.5	4.5	0.3	Value (M)	0.3	2.5	0.4	3.9
Accruals	0.1	0.0	1.2	0.2	Net issuance (M)	1.0	1.3	0.8	0.2
Asset growth	0.6	0.1	2.6	0.9	Earnings surprises	0.7	0.0	0.2	0.3
Asset turnover	1.4	0.1	1.1	0.1	Return on book equity (Q)	2.7	4.5	3.0	0.0
Gross margins	0.0	0.5	0.0	0.0	Return on market equity	0.1	1.7	0.4	0.3
Earnings/price	0.8	1.6	0.4	0.6	Return on assets (Q)	0.3	4.6	0.8	0.3
Cash flows/price	0.4	0.1	0.4	1.7	Short-term reversals	0.1	0.8	0.3	1.4
Net operating assets	0.0	0.0	2.6	0.1	Idiosyncratic volatility	0.2	0.1	0.6	0.1
Investment/assets	3.4	0.1	1.6	0.4	Beta arbitrage	0.8	0.7	0.0	0.2
Investment/capital	0.0	0.6	0.0	1.4	Seasonality	0.5	1.1	0.4	0.1
Investment growth	1.8	0.0	0.2	0.4	Industry rel. Reversals.	0.6	0.7	0.0	0.4
Sales growth	4.7	0.2	1.0	0.8	Industry rel. Rev. (L.V.)	0.0	0.0	0.2	0.4
Leverage	0.9	1.0	1.0	2.2	Ind. mom-reversals	0.0	0.2	0.5	0.1
Return on assets (A)	0.7	1.1	2.4	0.0	Composite issuance	1.5	0.0	1.1	2.9
Return on book equity (A)	1.7	1.3	4.4	0.0	Price	0.4	1.6	0.2	0.3
Sales/price	0.9	3.6	1.4	2.8	Share volume	1.6	0.2	4.1	1.2
Growth in LTNOA	1.1	0.4	1.8	0.1	Duration	0.2	4.1	0.2	2.5
Momentum (6m)	0.0	0.0	1.3	0.2	Firm age	1.8	1.7	2.3	1.9

Table 8: Factor Timing Performance - with extensions

	Factor Timing	
	Without extensions	With extensions
IS Sharpe ratio	0.19	-0.00016
OOS Sharpe ratio	0.000047	0.0000013
IS Inf. ratio	-1'606'579'285'377	-67.69
OOS Inf. ratio	-3'187'683'649'222	-1'225

when using long-only portfolios. Table 7 shows the R^2 using this extension compared to to the setting without extensions. For the majority of factors the R^2 increases, meaning that these extensions lead to an increase of explained variance by the predictor. The calculated sharpe ratios in Table 8 exhibit a decrease compared to the starting setting, showing that this strategy would underperform the original variant. As discussed before, we cannot make meaningful statements about the information ratio because of the tremendous negative values we obtain in the replication setting. However, we can see that using the extensions leads to an increase of the information ratio.

4.2.4 Changing predictor to 12 month return volatility

The main assumption made by Haddad et al. [2020] is that the book-to-market ratio is a good predictor for factor returns. To get an overview, they calculate the total OOS R^2 for various forecasting methods, showing that their choice results in the highest R^2. What they didn't try is using volatility as a predictor. Hence, we implemented another extension, using the annualized volatility for every period as predictor for factor returns. Computations were made twice, once for the long-short factor portfolios and once, as described before, for long-only factor portfolios. We expect a negative correlation between volatility and factor returns.

Neither for the long-short nor for the long-only do the results look significant. However, as we have seen before, our results are also distorted in the case without extensions. We thus establish that past volatility appear to have a lower explanatory power as the change in book to market ratios. Our expectation of a negative correlation materializes only in few components in Table 9 and 10, however, the estimates are all rather insignificant. The R^2 computed in Table 11 shows that using one of the two extensions rather improves the R^2. A significant difference between long-short and long-only is not perceivable. If we look at the Sharpe ratio and Information ratio in Table 12, we can see that for both extensions the Sharpe ratio decreases. The Information ratio on the other hand increases. However, the Information ratios without extensions are definitely implausible which is why we

Table 9: Predicting dominant equity components with 12 month volatility (long-short)

	MKT	PC1	PC2	PC3	PC4	PC5
12-m Vola.	0.01	0.01	-0.01	-0.01	-0.00	0.01
	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)
p-value	0.11	0.10	0.02	0.74	0.54	0.15
R^2	0.01	0.03	0.02	0.01	0.00	0.04

Table 10: Predicting dominant equity components with 12 month annualized volatility (long-only)

	MKT	PC1	PC2	PC3	PC4	PC5
12-m Vola.	0.01	0.01	-0.00	-0.00	0.00	-0.00
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
p-value	0.11	0.00	0.59	0.96	0.17	0.22
R^2	0.01	0.04	0.00	0.00	0.01	0.01

cannot make any meaningful statements about these results.

Summing up, we do not see any significant improvement by using the extensions provided. However, since our replication results do not coincide with the ones from Haddad et al. [2020], any results building on the predcitive regressions suffer from the same endogenous bias as in our initial replication attempt. We thus are not able to finally conclude, whether the suggested extensions constitute an improvement to the approach outlined by Haddad et al. [2020] or not.

Table 11: Predicting individual anomaly returns: R^2 (%), time-horizon until 31.12.2017, with extensions, using 12 month volatility as predictor

	Without Extensions		With Extensions Long-Short		With Extensions Long-Only	
	IS	OOS	IS	OOS	IS	OOS
Size	0.6	1.4	0.9	1.4	1.3	3.8
Value (A)	1.3	1.6	2.5	1.6	3.7	2.2
Gross profitability	0.0	0.0	0.2	0.0	3.9	0.4
Value-profitablity	4.5	2.1	5.8	2.1	5.0	1.9
F-score	1.1	0.8	1.4	0.8	1.3	0.3
Debt issuance	1.7	0.2	1.7	0.2	3.4	0.5
Share repurchases	0.6	0.9	0.9	0.9	3.5	0.4
Net issuance (A)	3.7	2.5	3.2	2.2	4.6	0.3
Accruals	0.1	0.0	0.0	0.0	1.0	0.2
Asset growth	0.6	0.1	2.8	0.1	2.5	0.9
Asset turnover	1.4	0.1	0.6	0.1	1.3	0.1
Gross margins	0.0	0.5	0.2	0.5	0.4	0.0
Earnings/price	0.8	1.6	0.8	1.9	0.5	0.6
Cash flows/price	0.4	0.1	1.0	0.1	1.2	1.7
Net operating assets	0.0	0.0	0.1	0.1	3.4	0.1
Investment/assets	3.4	0.1	3.7	0.1	2.3	0.4
Investment/capital	0.0	0.6	0.1	0.6	0.0	1.4
Investment growth	1.8	0.0	1.6	0.0	0.6	0.4
Sales growth	4.7	0.2	4.7	0.2	1.5	0.8
Leverage	0.9	1.0	1.4	1.0	1.5	2.2
Return on assets (A)	0.7	1.1	0.8	1.1	2.4	0.0
Return on book equity (A)	1.7	1.3	2.6	1.3	6.0	0.0
Sales/price	0.9	3.6	2.2	3.6	2.4	2.7
Growth in LTNOA	1.1	0.4	1.3	0.4	1.4	0.1
Momentum (6m)	0.0	0.0	1.0	0.1	1.9	0.2
Value-momentum	0.0	0.0	0.4	0.0	1.1	0.3
Value-momentum-prof	2.8	0.4	2.9	0.2	1.5	0.0
Short interest	0.6	1.8	1.2	0.1	0.0	0.7
Momentum (12m)	1.4	0.2	1.9	0.2	1.8	0.0
Industry momentum	0.5	0.0	0.5	0.0	1.2	0.2
Momentum-reversals	2.9	0.7	3.8	0.7	6.7	0.9
Long run reversals	1.8	0.5	4.1	0.5	4.1	0.5
Value (M)	0.3	2.5	0.9	2.4	1.5	3.9
Net issuance (M)	1.0	1.3	1.0	1.3	1.5	0.2
Earnings surprises	0.7	0.0	1.3	0.0	0.5	0.3
Return on book equity (Q)	2.7	4.5	2.9	4.5	3.7	0.0
Return on market equity	0.1	1.7	0.3	1.8	0.5	0.3
Return on assets (Q)	0.3	4.6	0.3	4.6	1.2	0.3
Short-term reversals	0.1	0.8	0.2	1.6	0.4	1.3
Idiosyncratic volatility	0.2	0.1	0.5	0.6	0.6	0.1
Beta arbitrage	0.8	0.7	1.1	0.7	0.2	0.2
Seasonality	0.5	1.1	0.1	1.1	0.1	0.1
Industry rel. Reversals.	0.6	0.7	0.3	1.5	0.1	0.4
Industry rel. Rev. (L.V.)	0.0	0.0	0.1	0.1	0.2	0.3
Ind. mom-reversals	0.0	0.2	0.7	0.3	0.6	0.1
Composite issuance	1.5	0.0	1.6	0.0	1.0	2.9
Price	0.4	1.6	0.7	1.6	0.4	0.3
Share volume	1.6	0.2	1.8	0.2	4.4	1.2
Duration	0.2	4.1	0.6	4.1	0.5	2.5
Firm age	1.8	1.7	2.1	1.7	2.6	1.9

Table 12: Factor Timing Performance - with extensions, using 12 month volatility as predictor

	Without Extensions	With Extensions Long-Short	With Extensions Long-Only
IS Sharpe ratio	0.19	0.000099	-0.000014
OOS Sharpe ratio	0.000047	-0.0000053	-0.000025
IS Inf. ratio	-1’606’579’285’377	2’516’270	-0.89
OOS Inf. ratio	-3’187’683’649’222	9’402’923	-18.16

5 Critical Analysis

In Haddad et al. [2020] the authors extend the field of factor timing, by applying a principal component analysis to identify the dominant sources of variation within a cross-section of returns. The results of the PCA are used to construct a forecast of different factor returns. With these forecasts the authors later create optimal factor portfolios.

As already hinted in previous sections, we were not able to recreate the approach suggested by the authors. The results reflect this statement as they allow no sensible discussion and interpretation. However, we can showcase the largest issues we faced with the paper and our approach to solving them.

The paper generally relies heavily on R^2 for an estimation of the effectiveness of predictive regressions. Since many portfolios are being formed and forecast, the authors define and derive a OOS total R^2 and OOS factor R^2. This measure is however only derived for OOS and there is no comment of how IS R^2 is derived. These results are later used to interpret and justify the results as valid. The lack of clarity concerning R^2 is detrimental to the overall result of the portfolio as it is not completely clear what the R^2 should show.

As the paper suggests a relevance to practitioners, we assumed that this would be further assessed with relevant restrictions, which are commonly being used. The authors however, fail to deliver a long-only version of their strategy. Furthermore, the authors do not account for a general transaction cost measure in their approach. After an inclusion of transaction cost, directly implemented equity-strategies often fail to deliver convincing results [Asness et al., 2013]. This is particularly important as the authors do not try to adjust rebalancing to minimize transaction cost. These arguments not only curtail the interpretation of the authors results, but also undermine their claim of practical relevance.

Another factor that reduces the practical implementation of such an approach is the broad universe and additional structuring of very individualized portfolios. Both aspects have different implications. Testing a strategy on many components is generally a good idea. Diversification-effects completely support this approach. In this context, however, the authors chose an universe that might include too many different stocks to actually invest into them. Most importantly, the additional structuring of the portfolio adds layers of complexity to the strategy. No existing financial product covers the portfolios suggested by the authors. An investor would have to first structure these portfolios and then adjust the weights between the portfolios and trade the stocks accordingly, every month. This seems to incur unaccounted implementation costs and might not yield enough additional return to justify using the strategy. Cazalet and Roncalli [2014] show impressively, how the factors change after a practical restriction of academic portfolios. For long-only portfolios the tracking-error increases substantially, resulting in a devastating under-performance compared to the academic long-short portfolios.

Combining our findings and the aforementioned critical analysis, one has to ask the questions of overall practical relevance of this approach. First, it seems like the approach has a questionable data foundation (factor zoo; Harvey and Liu [2019]), that is seemingly unpractical to implement. Secondly, the authors conduct a complicate analysis to predict returns of factors, some of which are not even considered to be completely robust - all to find optimal portfolios which would be to costly to implement and too complex to maintain.

Summing up our critical analysis, we find that even though the authors contribute to a very active field of academic research, the paper itself yields limited practical implications.

6 Conclusion

In this paper, we reviewed the work of Haddad et al. [2020]. We replicated and extended the paper which examines the intersection of factor investing and the timing of investments using a cross-section of 50 equity factor portfolios, where the multi-dimensionality is reduced to five Principal Components capturing most of the variability of anomaly returns.

We found that a one-to-one replication of their work is not possible due to the unavailability of data and ambiguous explanation. Furthermore, their work fails to account for important restrictions faced by practitioners, like limitations to a broad universe, complexity of the strategy, leverage and long-short factor portfolios, but also drawbacks stemming from transaction costs.

The replication of Haddad et al. [2020] results in strongly deviating results, which is why comparisons of the applied extensions to their results can only be made with caution. Applying their method to an extended data set until 2019, or a non-adjustment of the book-to-market ratio and the usage of a long-only factors, increases the R^2 of the individual anomaly return prediction. However, estimates resulting from a predictive regression using the PCs own book-to-market ratio to forecast PCs return, decrease significantly. The same applies to the Sharpe ratios. Eventually, changing the predictor to the 12 month return volatility presents similar outcomes as the other extensions.

Further researchers could build on this approach and focus on the practical implementation of such a factor timing strategy. In doing so the aforementioned limitations can serve as a natural starting point.

References

Ferhat Akbas, Will J Armstrong, Sorin Sorescu, and Avanidhar Subrahmanyam. Smart money, dumb money, and capital market anomalies. *Journal of Financial Economics*, 118(2):355–382, 2015.

Robert D Arnott, Noah Beck, and Vitali Kalesnik. Timing'smart beta'strategies? of course! buy low, sell high! *Of Course*, 2016.

Clifford Asness, Swati Chandra, Antti Ilmanen, and Ronen Israel. Contrarian factor timing is deceptively difficult. *The Journal of Portfolio Management*, 43(5):72–87, 2017.

Clifford S Asness, Tobias J Moskowitz, and Lasse Heje Pedersen. Value and momentum everywhere. *The Journal of Finance*, 68(3):929–985, 2013.

John Y Campbell and Robert J Shiller. The dividend-price ratio and expectations of future dividends and discount factors. *The Review of Financial Studies*, 1(3): 195–228, 1988.

John Y Campbell and Samuel B Thompson. Predicting excess stock returns out of sample: Can anything beat the historical average? *The Review of Financial Studies*, 21(4):1509–1531, 2008.

Zélia Cazalet and Thierry Roncalli. Facts and fantasies about factor investing. *Available at SSRN 2524547*, 2014.

Randolph B Cohen, Christopher Polk, and Tuomo Vuolteenaho. The value spread. *The Journal of Finance*, 58(2):609–641, 2003.

Michael J Cooper, Roberto C Gutierrez Jr, and Allaudeen Hameed. Market states and momentum. *The journal of Finance*, 59(3):1345–1365, 2004.

Eugene Fama and Kenneth R. French. Dissecting anomalies with a five-factor model. *Review ofFinancial Studies*, 29:69—-103, 2016.

Eugene F Fama and Kenneth R French. Dividend yields and expected stock returns. *Journal of financial economics*, 22(1):3–25, 1988.

Eugene F Fama and Kenneth R French. Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1):3–56, 1993.

Joachim Freyberger, Andreas Neuhierl, and Michael Weber. Dissecting characteristics nonparametrically. *The Review of Financial Studies*, 33(5):2326–2377, 2020.

Valentin Haddad, Serhiy Kozak, and Shrihari Santosh. Factor timing. *The Review of Financial Studies*, 33(5):1980–2018, 2020.

L. P. Hansen and R. Jagannathan. Implications of security market data for models of dynamic economies. *Journal of Political Economy*, 99:225—-62, 1991.

Campbell R Harvey and Yan Liu. A census of the factor zoo. *Available at SSRN 3341728*, 2019.

Serhiy Kozak. Kernel trick for the cross-section. *Available at SSRN 3307895*, 2020.

Serhiy Kozak, Stefan Nagel, and Shrihari Santosh. Interpreting factor models. *The Journal of Finance*, 73(3):1183–1223, 2018.

Serhiy Kozak, Stefan Nagel, and Shrihari Santosh. Shrinking the cross-section. *Journal of Financial Economics*, 135(2):271–292, 2020.

Lars A Lochstoer and Paul C Tetlock. What drives anomaly returns? *The Journal of Finance*, 75(3):1417–1455, 2020.

Robert J Shiller. Do stock prices move too much to be justified by subsequent changes in dividends?, 1980.

7 Appendix

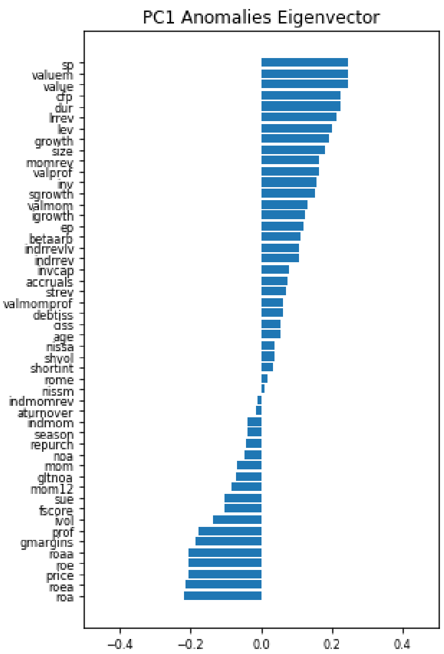

Figure 1: PC1 Eigenvector

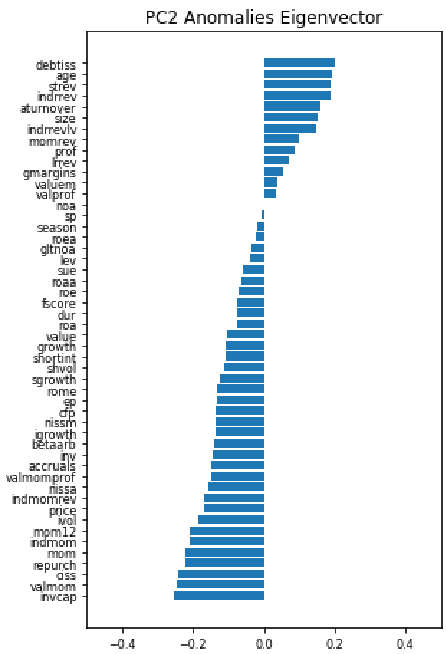

Figure 2: PC2 Eigenvector

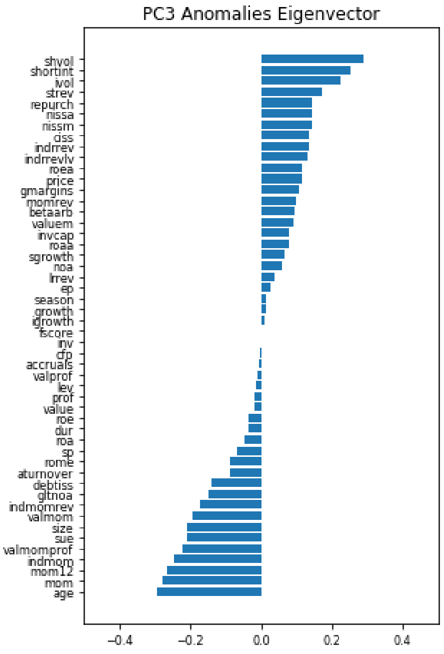

Figure 3: PC3 Eigenvector

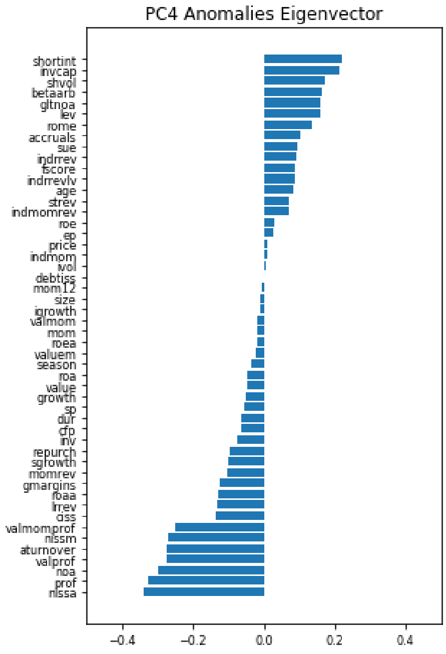

Figure 4: PC4 Eigenvector

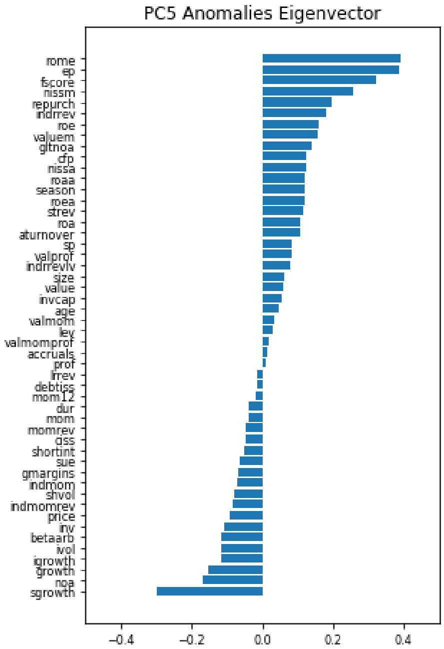

Figure 5: PC5 Eigenvector

Figure 6: Cumulative in- and out-of sample strategy return vs market return

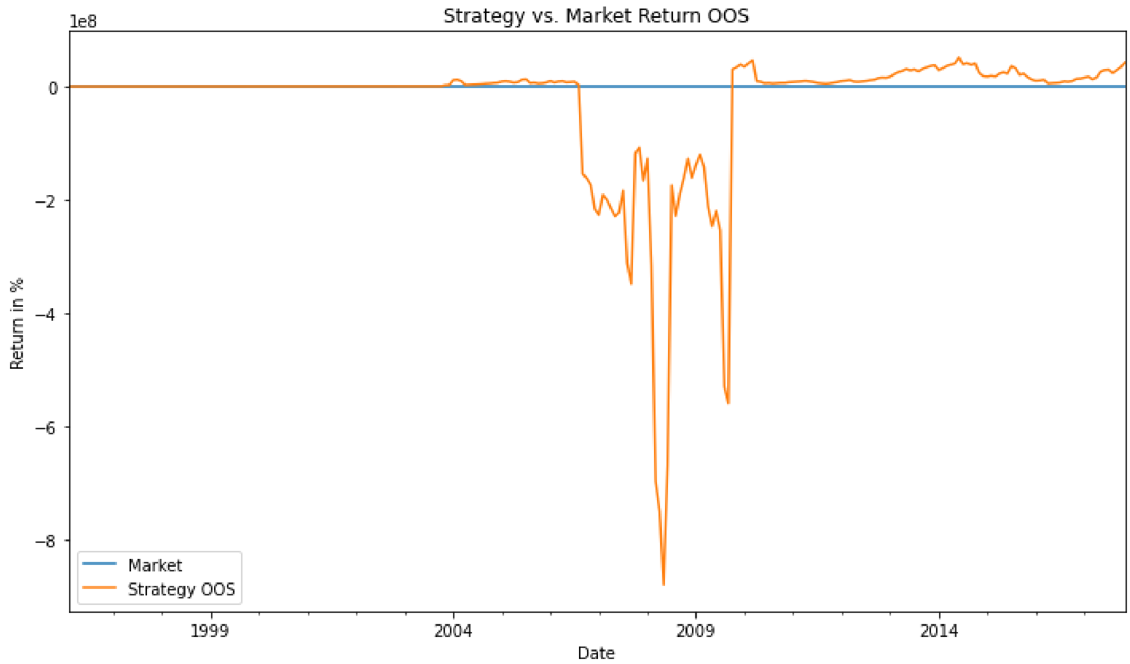

Figure 7: Cumulative out-of-sample strategy returns vs. market return

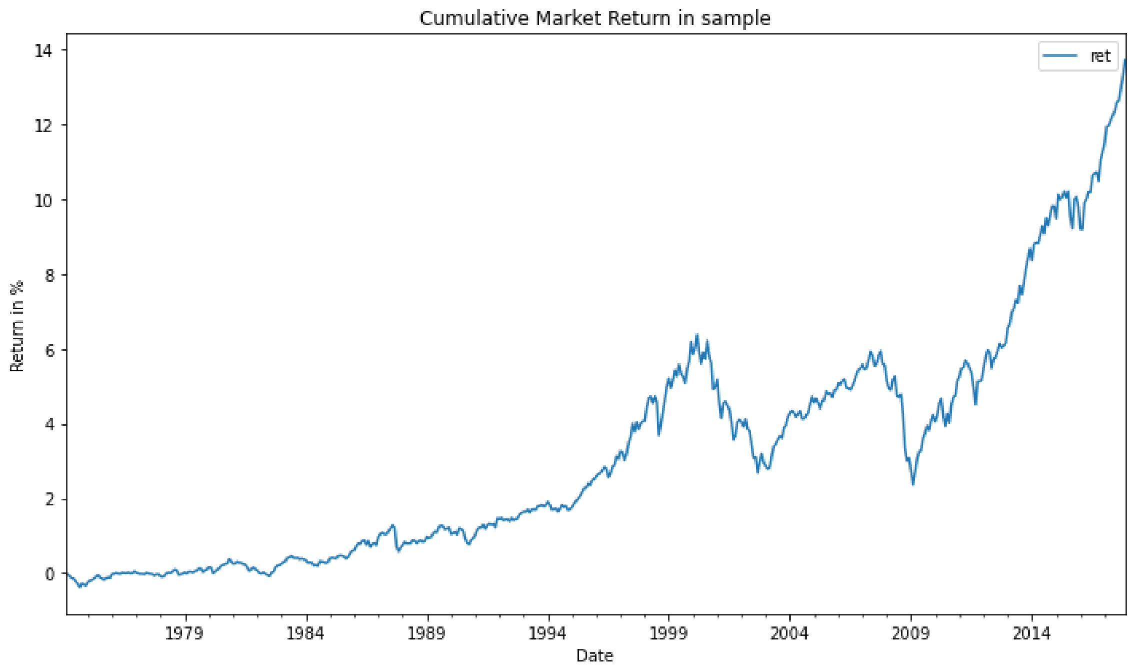

Figure 8: Cumulative market return