$-\frac{1}{2}+1$

$\frac{-1+2}{2}$ $\boxed{\frac{1}{2}}$

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Prova 01

1. $a_n = \dfrac{1}{n\sqrt{\ln(n)}}$; $b_n = \dfrac{(n!)^2}{(2n)!}$

a) Como $\lim\limits_{n\to\infty} \sqrt{\ln(n)} = \infty$ e $\lim\limits_{n\to\infty} n = \infty$, então

$$\lim_{n\to\infty} \frac{1}{n\sqrt{\ln n}} = 0$$

b) T. Integral $\rightarrow$ $f: [2,\infty)$

$$\int_2^{\infty} \frac{1}{x.(\ln x)^{\frac{1}{2}}}\,dx$$

$$\hookrightarrow \int \frac{1}{x.(\ln x)^{\frac{1}{2}}}\,dx = \int \frac{1}{u^{\frac{1}{2}}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{(\ln x)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2(\ln x)^{\frac{1}{2}} + C$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\hookrightarrow \int_2^{M} \frac{1}{x.(\ln x)^{\frac{1}{2}}}\,dx = 2(\ln 2)^{\frac{1}{2}} - 2(\ln M)^{\frac{1}{2}}$$

$$\hookrightarrow \lim_{M\to\infty} 2(\ln 2)^{\frac{1}{2}} - \underbrace{2(\ln M)^{\frac{1}{2}}}_{+\infty} = -\infty$$

$\therefore$ a série é DIVERGENTE

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c) T. Razão

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)(\ln(n+1))^{\frac{1}{2}}} \cdot \frac{n(\ln(n))^{\frac{1}{2}}}{x^n}\right| = \frac{n}{n+1} \cdot \left(\frac{\ln n}{\ln(n+1)}\right)^{\frac{1}{2}} \cdot |x|$$

$$\rightarrow \lim_{n\to\infty} \frac{n}{n+1} = \boxed{1}$$

L'Hopital

$$\rightarrow \lim_{n\to\infty} \left(\frac{\ln n}{\ln(n+1)}\right)^{\frac{1}{2}} = \left(\lim_{x\to\infty} \frac{\ln x}{\ln(x+1)}\right)^{\frac{1}{2}} = \boxed{1}$$

logo, $\lim_{n\to\infty} \frac{n}{n+1} \cdot \left(\frac{\ln n}{\ln(n+1)}\right)^{\frac{1}{2}} \cdot |x| = |x|$

portanto, se $|x| < 1$: convergente

se $|x| > 1$: divergente

- $x = 1$: $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^{\frac{1}{2}}}$ → divergente
- $x = -1$: $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^{\frac{1}{2}}}$ → convergente (T. Série Alternada)

∴ Raio de convergência = 1

Int. de convergência $I = [-1, 1)$

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d) pelo T. Termo Geral, como a série é convergente, logo, $\lim\limits_{n \to \infty} \frac{(n!)^2}{(2n)!} = 0$

e) T. Razão

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{((n+1)!)^2}{(2\cdot(n+1))!} \cdot \frac{(2n)!}{(n!)^2} \right| = \frac{(n+1)^2 \cdot \cancel{(n!)^2}}{(2n+2)\cdot(2n+1)\cdot\cancel{(2n)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{(n!)^2}}$$

$$= \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{(\cancel{n+1})^2}{2(2n+1)(\cancel{n+1})} = \frac{(n+1)}{2(2n+1)}$$

$$\rightarrow \lim_{n \to \infty} \frac{(n+1)}{2(2n+1)} \overset{LH}{=} \frac{1}{4} \longrightarrow \text{Convergente}$$

f) T. Razão

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)!^2 \cdot x^{n+1}}{(2(n+1))!} \cdot \frac{(2n)!}{(n!)^2 \cdot x^n} \right| = \frac{(n+1)\cdot|x|}{2(2n+1)}$$

$$\lim_{n \to \infty} \frac{(n+1)}{2(2n+1)} = \frac{1}{4}, \quad \text{logo} \lim_{n \to \infty} \frac{(n+1)\cdot|x|}{2(2n+1)} = \frac{1}{4}\cdot|x|$$

$\therefore$ se $|x| < 4 \rightarrow$ convergente

se $|x| > 4 \rightarrow$ divergente

Raio de convergência = 4