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CMSC 426 HW 1

1) Find equation of plane: $P_1 = (-1, 2, 2)$, $P_2 = (0, 1, 3)$, $P_3 = (-1, 1, 2)$

Let $\vec{v} = \overrightarrow{P_1P_2} = P_2 - P_1 = \langle 1, -1, 1 \rangle$

Let $\vec{u} = \overrightarrow{P_1P_3} = P_3 - P_1 = \langle 0, -1, 0 \rangle$

$$\hat{n} = \vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} i & j \\ 1 & -1 \\ 0 & -1 \end{matrix}$$

$$= -k + i$$

$$= i - k$$

$$\hat{n} \cdot ((x, y, z) - P_1) = \langle 1, 0, -1 \rangle \cdot \langle x+1, y-2, z-2 \rangle = 0$$

$$= (x+1) - (z-2) = 0$$

$$= x - z + 3 = 0$$

Equation of plane: $x - z + 3 = 0$ #

2) Find the line intersection between:

$3x + y + 4z - 4 = 0$

$2x + 2y - z + 2 = 0$

$3x + y + 4z = 4$ $\qquad$ $2x + 2y - z = -2$

$\hat{n}_1 = \langle 3, 1, 4 \rangle$ $\qquad$ $\hat{n}_2 = \langle 2, 2, -1 \rangle$

$$\vec{d} = \hat{n}_1 \times \hat{n}_2 = \begin{vmatrix} i & j & k \\ 3 & 1 & 4 \\ 2 & 2 & -1 \end{vmatrix} \begin{matrix} i & j \\ 3 & 1 \\ 2 & 2 \end{matrix}$$

$$= -i + 8j + 6k - 2k - 8i + 3j$$

$$= -9i + 11j + 4k$$

Let $z=0$,

$3x+2y=4 \quad -①$ $\qquad 2x+2y=-2 \quad -②$

$x=-1-y \quad -③$

Sub ③ into ①,

$3(-1-y)+2y=4$ $\qquad x=-1+7$

$-3-3y+2y=4$ $\qquad =6$

$-y=7$

$y=-7$ $\qquad P_1(6,-7,0)$ is a point in both planes.

Line of intersection:

$\langle x,y,z\rangle = \langle 6,-7,0\rangle + t\langle -9,11,4\rangle$

$(x,y,z) = (6-9t, -7+11t, 4t)$ #

3) Find distance between $p_0=(1,4,14)$, $3x+4y+5z-4=0$?

$3x+4y+5z=4$

$\hat{n}=\langle 3,4,5\rangle$

$P_1=(0,1,0)$ is a point on the plane

$\vec{v}=\overrightarrow{P_1P_0} = P_0-P_1 = \langle 1,3,14\rangle$

$d=|\vec{v}|\cos\theta$

$=\dfrac{|\vec{v}||\hat{n}|\cos\theta}{|\hat{n}|}$

$=\dfrac{|\vec{v}\cdot\hat{n}|}{|\hat{n}|}$

$=\dfrac{3+12+14(5)}{\sqrt{9+16+25}}$

$=17\sqrt{2}/2$ #

5) $\tilde{x} \cdot \tilde{l}$ is a line, $ax+by+c=0$
$\tilde{x}=(x,y,1)$ $\quad \tilde{l}=(a,b,c)$

a) $\tilde{l}=\tilde{x}_1 \times \tilde{x}_2$

Let $\tilde{x}_1=(x_1,y_1,1)$ $\quad \tilde{x}_2=(x_2,y_2,1)$

$$\tilde{x}_1 \times \tilde{x}_2 = \left| \begin{array}{ccc|cc} i & j & k & i & j \\ x_1 & y_1 & 1 & x_1 & y_1 \\ x_2 & y_2 & 1 & x_2 & y_2 \end{array} \right.$$

$= (y_1-y_2)i + (x_2-x_1)j + (x_1y_2-x_2y_1)k$

$= \langle y_1-y_2,\ x_2-x_1,\ x_1y_2-x_2y_1 \rangle$

$\tilde{x} \cdot \tilde{l} = \tilde{x} \cdot (\tilde{x}_1 \times \tilde{x}_2)$

$= (x,y,1) \cdot \langle y_1-y_2,\ x_2-x_1,\ x_1y_2-x_2y_1 \rangle$

$= (y_1-y_2)x + (x_2-x_1)y + x_1y_2 - x_2y_1$

$=0$

Let $\tilde{x}=(x_1,y_1,1)$,

$\tilde{x} \cdot \tilde{l} = (y_1-y_2)x_1 + (x_2-x_1)y_1 + x_1y_2 - x_2y_1$

$= \cancel{y_1x_1} - \cancel{y_2x_1} + \cancel{x_2y_1} - \cancel{x_1y_1} + \cancel{x_1y_2} - \cancel{x_2y_1}$

$=0$.

Let $\tilde{x}=(x_2,y_2,1)$

$\tilde{x} \cdot \tilde{l} = (y_1-y_2)x_2 + (x_2-x_1)y_2 + x_1y_2 - x_2y_1$

$= \cancel{y_1x_2} - \cancel{y_2x_2} + \cancel{x_2y_2} - \cancel{x_1y_2} + \cancel{x_1y_2} - \cancel{x_2y_1}$

$=0$

Therefore, the line $\tilde{l}$ passes through both $\tilde{x}_1$ and $\tilde{x}_2$. #

b) $\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$

Let $\tilde{l}_1 = (a_1, b_1, c_1)$, $\tilde{l}_2 = (a_2, b_2, c_2)$

$$\tilde{l}_1 \times \tilde{l}_2 = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \begin{matrix} i & j \\ a_1 & b_1 \\ a_2 & b_2 \end{matrix}$$

$$= (b_1c_2 - b_2c_1)i + (a_2c_1 - a_1c_2)j + (a_1b_2 - a_2b_1)k$$

$$= \langle b_1c_2 - b_2c_1 , a_2c_1 - a_1c_2 , a_1b_2 - a_2b_1 \rangle$$

$$\tilde{x} = \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} , \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} , 1 \right)$$

Assume that $\tilde{l}_1$ intersect $\tilde{l}_2$,

$\therefore \quad a_1x + b_1y + c_1 = a_2x + b_2y + c_2$

$(a_1 - a_2)x + (b_1 - b_2)y + c_1 - c_2 = 0$

We know that, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = \dfrac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

$$\therefore \quad (a_1 - a_2)\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right) + (b_1 - b_2)\left(\frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right) + c_1 - c_2$$

$$= \frac{1}{a_1b_2 - a_2b_1}\left[\cancel{a_1b_1c_2} - a_1b_2c_1 - a_2b_1c_2 + \cancel{a_2b_2c_1} + b_1a_2c_1 - \cancel{b_1a_1c_2} - \cancel{b_2a_2c_1} + b_2a_1c_2\right]$$

$$+ c_1 - c_2$$

$$= \frac{1}{a_1b_2 - a_2b_1}\left[a_2b_1c_1 + a_1b_2c_2 - a_1b_2c_1 - a_2b_1c_2\right] + c_1 - c_2$$

$$= (c_1 - c_2)\left(\frac{a_2b_1 - a_1b_2}{a_1b_2 - a_2b_1} + 1\right) = (c_1 - c_2).0 = 0_{\#}$$

$\therefore \tilde{x}$ is the intersection of $\tilde{l}_1$ and $\tilde{l}_2$.

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