Real-time monophonic guitar pitch estimation using 12 tuned concurrent Fourier transforms

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May 12, 2022

Abstract

Short summary. Note that even though we focus on guitar pitch estimation, the methods described in this thesis are general to all instruments and even singing. The only guitar specific thing is the parameter optimization. We focus on guitar as it is not easily replaced by a MIDI variant as is possible with instruments such as a piano.

Contents

1 Introduction

Pitch estimation, which is also referred to as f0 estimation, is an important subtask within the field of Automatic Music Transcription (AMT). The goal of pitch estimation is to estimate the pitch or fundamental frequency f_0 of a given signal. In the context of AMT, pitch estimation is used to determine which notes are played in a given signal.

Real-time pitch estimation is a subproblem where we want to estimate the note associated with the measured pitch while the musician is playing it with minimal latency. This entails we have to use the latest received signal. In contrast to non-real-time methods, we have no knowledge of what may happen ahead of time (we cannot peak-ahead) and signal corresponding to previous notes is mostly irrelevant. This limits the methods we can use to solve this problem.

If pitch estimation can accurately be performed in real-time, it can be used to create a digital (MIDI) instrument from an acoustic instrument. This digital instrument can then be used as an input for audio synthesizers, allowing musicians to produce sounds from a wide variety of instruments. Furthermore, accurate real-time pitch estimation can be used to automatically correct detuned instruments by pitch shifting the original signal to the closest harmonious note.

The Fourier transform is often used for pitch estimation. The transform decomposes a signal into the frequencies that make up the signal. Predominant frequencies in the signal show up as spectral peaks in the frequency domain. These peaks are important

to human perception of melody [11]. Other popular methods used for pitch estimation include non-negative matrix factorization, autocorrelation, statistical model based estimation and hidden Markov model based estimation.

Our research focuses on monophonic pitch estimation. Here, we assume the signal contains at most one note. It is much easier to perform monophonic pitch estimation compared to polyphonic pitch estimation [12], especially when using Fourier transform based methods, as fundamental limits of the Fourier transform inhibit our ability to discern two low pitched notes [6]. Furthermore, hexaphonic guitar pickups are becoming more widespread, which allows us to view the guitar as six monophonic instruments instead of one six-way polyphonic instrument. State of the art commercial guitar synthesizer solutions also use these hexaphonic pick-ups, which indicates the infeasibility of accurate and responsive real-time polyphonic pitch estimation.

This thesis builds upon a preliminary research project [9]. In our research project, we found that Fourier transform based pitch estimation methods might not be well suited for real-time use due to fundamental limitations of the Fourier transform [7]. In this work, we will further research if Fourier transform based methods are viable, as real-time transcription research often relies Fourier transform based methods.

Researching pitch estimation is not accessible, as many other small problems have to be solved in order to produce a working prototype on which experiments can be performed. Furthermore, automated experimentation is a lot of work to implement and as a consequence, pitch estimation research often only includes some informal testing or no experiments at all. To combat these problems, we set out to create a modular pitch estimation framework in which every pitch estimation subproblem is implemented as a separate module which can be replaced. This allows the research of one specific subproblem and the effect of specific combinations of subproblems. In addition, it provides researchers with many tools which aid in developing and fine-tuning the solutions to the different subproblems.

TODO: Rewrite this paragraph. The goal of this thesis is to research the limits of Fourier transform based real-time pitch estimation. To correctly assess the limits, we develop a pitch estimation framework. This framework will focus on extensibility and the ability to perform automated tests. This is important, as much work in this field does not provide its associated source code. This limits the ability to build on other's work and hinders direct comparisons between different methods. Our framework can provide a common ground for the different methods and algorithms to be implemented and compared in. The framework is available at `www.github.com/lucmans/digistring`.

2 Related work

Much research has been performed on Fourier transform based real-time pitch estimation. All research we found relies on obtaining a high resolution frequency domain in which spectral peaks can be isolated and notes can be associated with. These methods are deemed infeasible by some due to low frequency resolution [7].This is especially problematic when adhering to a real-time constraint, as extra short signal frames have to be used. Some papers circumvent this problem by choosing a very high real-time constraint [14, 13], however, this inhibits the use for real-world applications. On top of the latency from the pitch estimation algorithm, conventional operating systems also have a latency when delivering audio samples to your program due to how audio drivers work [22].

A big problem with Fourier based pitch estimation is the occurrence of overtones [23]. Especially octaves are a problem, as the fundamental and all overtones of the higher note overlap with overtones of the lower note. This is referred to as the octave problem [25]. Overtones are periodic in nature, as they diminishingly repeat every multiple of the fundamental frequency. As a consequence, they could also be detected using a subsequent Fourier transform [19] on the frequency domain. However, this does not solve the octave problem.

Many different transform have been researched for pitch estimation, however, Fourier transform remains popular as it is broadly studied and its behavior is well known [21]. Lately, the CQT transform is gaining popularity as it may provide higher resolution in the frequency domain [29] at the cost of lower computational efficiency [26]. However, the CQT transform is efficiently implemented using Fourier transforms [8] and the main problem with Fourier is the fundamentally low frequency resolution on short frames, so we are left with the same problem. One big advantage is that the frequency bins can perfectly align with the notes of an instrument [10]. However, as described in Section 3.5, overtones are dissonant with respect to our notes and consequently, the CQT bins do not align with the overtones. If a note perfectly aligns with a Fourier bin, all overtones will also align. In order to cover every note, we could in-

stead perform 12 Fourier transform in parallel. This difference is especially important when performing polyphonic transcription.

TODO: Note on problems with other research, such as no experiments, no available source code, assuming normal continuous Fourier behavior instead of DFT, downsampling

TODO: Vergelijk materiaal

3 Preliminaries

In order to effectively research Fourier transform based real-time pitch estimation, it is important to have a thorough understanding of how computers deal with audio and how the Fourier transform is implemented in computers. Furthermore, in order to use the output of the Fourier transform, we need to understand the characteristics of the sound generated by instruments. Moreover, in order to interpret the results of our estimated pitch and reason about the performance of the used algorithms, some music theory is necessary. Lastly, we will discuss the concept of real-time in depth, as there are multiple definition for real-time which often leads to incorrect use of the concept.

3.1 Audio in computers

Audio in computers is represented through a series of equally spaced samples. A sample is the height of the audio wave at a specific point in time. The sample rate determines the number of samples per second used for representing the audio. The sample format determines how the height of the audio wave is represented in a sample. Often used formats are 8/16/24 bit integers and 32 bit IEEE-754 floating point numbers (which we will refer to as floats). The integer samples simply uniformly spread the range of the waveform over range of the integer [28]. Float samples typically take values in $[-1, 1]$. Samples with values outside this range are considered to be clipping. Because float samples have 24 bits precision (23 mantissa bits and a sign bit [5]), 24 bit integer samples can converted lossless to float samples. The 8 exponent bits can be used to scale the samples to a different order, which allows us to accurately describe very soft and loud audio. Float samples have many advantages over integer samples for digital signal processing [28], so we will always use float samples throughout this thesis.

When working with audio input/output in computers, a small latency is always introduced [1]. One part of the latency comes from the used audio hardware and is not configurable. The other part comes from the audio driver's buffers. Audio drivers work on buffer of samples instead of single samples for computational efficiency. A full buffer of data has to be gathered from the audio in, or a full buffer has to be send to the audio out, so the first or last sample respectively is one buffer length behind. The buffer length is calculated by dividing the number of samples in the buffer by the sample rate. In order to minimize latency, the number of samples per buffer has to be minimized and the sample rate has to be maximized. Since these latencies are outside of Digistring's control and can be mostly circumvented by running Digistring's algorithms on specialized hardware, we won't take them into account in this thesis.

3.2 Fourier transform

The Fourier transform is a mathematical transform which transforms a function of time to a complex valued function of frequency and phase. Here, the magnitude represents the amplitude and the argument represents the phase of the corresponding sine wave. The function which maps the frequencies to amplitudes is called the spectrum of the time dependent function. The Fourier transform works on continuous functions and assumes an infinite time interval. Concepts such as continuous and infinite cannot be represented by a computer. Consequently, the discrete Fourier transform (DFT) has to be used for Fourier analyses on computers. The DFT can efficiently be calculated using the fast Fourier transform (FFT) algorithm.

The DFT transforms a finite sequence of equally spaced samples, which we will refer to as a frame, into an equal number of complex values representing the amplitude and phase, which we refer to as bins. Technically, the bins do not form a spectrum as they are not continuous, however, within digital signal processing it is still referred to as the spectrum of the frame.When working with audio, the samples are real valued, and the DFT output is symmetrical. Because of this, we can discard the second half of the output. In the rest of this thesis, we will only consider the first half of the output. Each bin corresponds to a specific frequency. All other frequencies are spread out over multiple bins. This is called spectral leakage and will be discussed later. Given a frame F, the number of samples in the frame is $n_F = |F|$. Using n_F and sample rate f_{SR}, we can calculate the distance between bins in Hz:

$$\Delta f_{bin} = \frac{f_{\text{SR}}}{n_F}$$

This is also referred to as the frequency resolution.

Closely related to the frequency resolution is the frame length, which is calculated as follows:

$$t_F = \frac{n_F}{f_{\text{SR}}} = \Delta f_{bin}{}^{-1}$$

Given a bin number $i \in [0, \lfloor \frac{n_F}{2} \rfloor]$, the frequency of a bin can be calculated as:

$$f_{bin} = \Delta f_{bin} * i$$

The 0 Hz bin corresponds to the so called DC offset. This is the average amplitude of the signal. The frequency of the last bin is also called the Nyquist frequency and is equal to half the sample rate. The Nyquist frequency is important for two reasons [31]. The first relates to the sampling theorem, which states that if a continuous function is sampled at a rate of f_{SR} and only contains frequencies f for which $f \leq \frac{f_{\text{SR}}}{2}$, the samples will completely determine the original function. In other words, the samples perfectly describe the original waveform. Secondly, frequencies f for which $f > \frac{f_{\text{SR}}}{2}$ are spuriously moved into $[0, \frac{f_{\text{SR}}}{2}]$. This implies we cannot simply downsample the input signal for an easy performance gain, as it might introduce noise.

The DFT assumes the frame to be periodic. In other words, the frame is regarded as infinitely repeating. This may distort the waveform if the beginning and end of a frame do not align and leads to artifacts in the spectrum. For example, in Figure 1, we take a frame shown by the red lines. The frame is not aligned to a period of the waveform and causes a distortion when repeated as seen in the second graph. Note that only sine waves with frequencies which are a multiple of Δf_{bin} would exactly fit the frame. In our example, the distortion introduces a new high frequency component from suddenly going up and down around the frame border. It also introduces a low frequency component, as the distance between two peaks within a frame is shorter can the distance of two peaks between frames. The introduction of these new frequencies is a form of spectral leakage, as the peak in the continuous spectrum corresponding to the sine frequency leaks into multiple bins. We can remove the distortion around frame borders by forcing the beginning and end of a frame to align. This is commonly done by applying a window function, which forces the frame ends to zero.

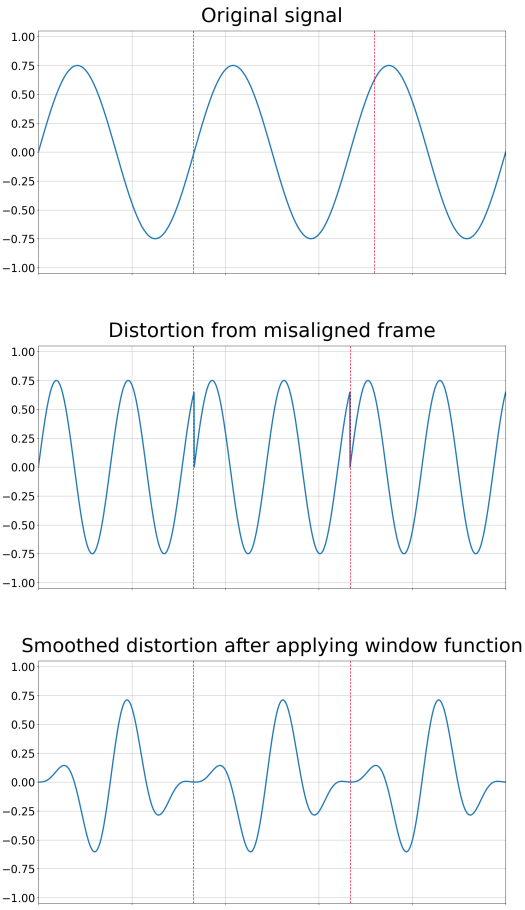

Figure 1: Distortion from misaligned framing

Given signal $s(n)$ and window function $w(n)$, we get the resulting signal $\text{res}(n)$ using:

$$\text{res}(n) = s(n) * w(n)$$

Figure 2 shows the working of a window function on a frame graphically.

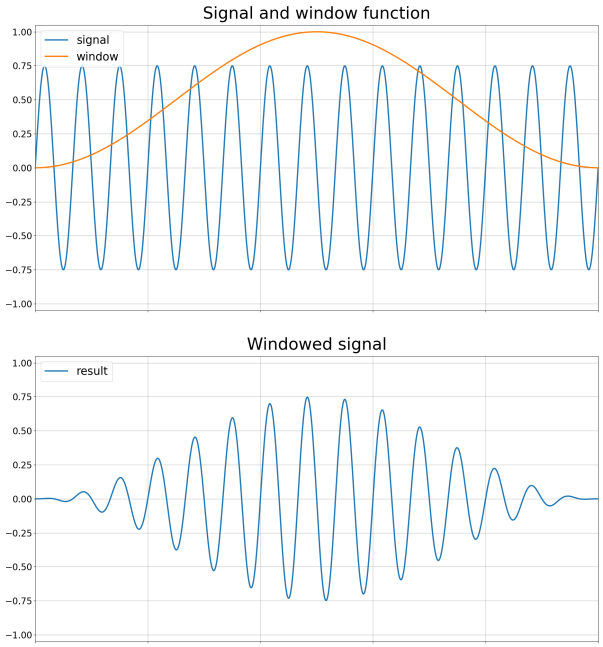

Figure 2: An example of using a window function on a signal

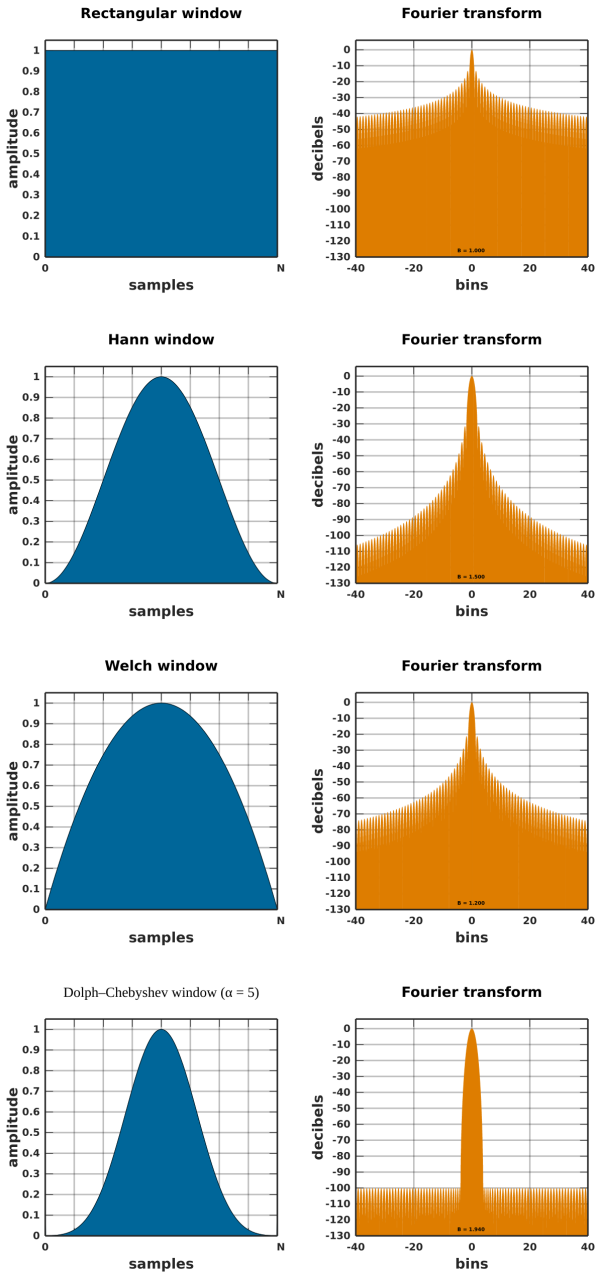

Figure 3: Examples of the spectral leakage of some window functions

TODO: Spectral leakage from windowing falls into three categories: Scalloping loss, Main lobe width (of a sine wave) and Sidelobes max level/decrease. The different windows functions trade of between a narrow center lobe and little overall leakage [15], see Figure 3 for some examples. Using the rectangular window could be considered as using no window function. It simply only takes n_F samples from the wave without any alteration. The Hann window is an all round performing window and as a result is often used. The Welch window is a window with a very narrow center lobe. The Dolph–Chebyshev window has little and very evenly spread overall leakage.

3.3 Real-time

Real-time is a difficult concept, as it has multiple definitions based on what field of research it is used in. As the formal definitions relate to very different concepts, it usually does not cause any problems. Problems arise when the term is informally used, as the vernacular definitions often miss an important aspect of the formal definitions of real-time, which causes statements made about systems which adhere

to these vernacular definitions to be unreliable or useless. Here are a few examples of different definitions (vernacular definitions are marked red):

1. Being synced with actual clock time (or wall time). This is for instance relevant when playing media such as audio and video. When such media is played at an incorrect speed, it could be considered distorted. The hardware which keeps track of the clock time is called a real-time clock.

2. A system must response within a specified time constraint, which is called the real-time constraint or deadline. This constraint is usually a relatively short time. This definition comes from real-time computing and is relevant when making car airbags or airplane control systems. Failing to response within the real-time constraint leads to failures of the system. Real-time systems are often classified into hard, firm and soft real-time based on the impact of missing the deadline [20].

3. A system which can provide a result or feedback with no noticeable delay after receiving some input. Examples of such systems include graphical user interfaces or instant chatting/calling. There are no hard deadlines which the system has to respond within and the system does not fail if some delay does occur. Only user experience is slightly impacted. In the field of real-time computing, this is often referred to as near real-time.

4. A system which can process data faster than it acquires data. This is technically not real-time, however, it is often used as such in academic literature. It is important for real-time systems to process data faster than it acquires data so it does not lag behind after some time, however, this is an implicit deadline. Not having this deadline explicit may lead to non-sensible expectations of the system.

Even though the first definition is very relevant when working with audio, it is not relevant for us. The audio drivers of operating systems handle all timing for us. We simply have to wait for samples to be recorded and made available to our program. We only have to keep the sampling rate in mind when working with the samples as shown in the previous section.

In order to allow guitarists to use their guitar as a MIDI instrument, our system has to respond within a small time frame. On top of that, if the system fails to respond quickly enough, the usefulness of the result degrades, as timing is very important when playing an instrument. These restrictions would classify our system as a soft real-time system. We choose a real-time of constraint of TODO milliseconds. We elaborate on this choice in Section 4.1.

Other work in real-time pitch estimation often uses the forth definition of real-time. This is problematic when using Fourier transform based methods, as many papers choose large frame sizes to get a high resolution in the frequency domain. For instance, in order to discern the two lowest notes on a guitar which are 4.9 Hz apart, we would need a frame length of 204 milliseconds. This implicit deadline is well over our real-time constraint and would be unplayable for any musician. Other papers we found which do explicitly set a real-time constraint, choose very high constraints from 140 ms [14] up to 360 ms [13]. These constraints were likely chosen with the inherent limits of their solutions in mind. It is very important to set a real-time constraint solely based on the expectation of the systems from an outside perspective. Real-time constraints chosen with the inner working of the system in mind are merely a measure of performance that is hoped to be achieved and claiming a system is real-time based on such constrains is considered fraudulent. We have found no papers Fourier transform based pitch estimation papers which choose a sensible real-time constraint.

3.4 Music theory and notation

In modern western music, we use the twelve-tone equal temperament (12-TET) music system. This system divides an octave, which is the interval between a pitch and another pitch with double the frequency, into twelve equally spaced semitones on the logarithmic scale. The logarithmic scale is used such that the perceived interval between two adjacent notes is constant [24]. As a result, the ratio between two frequencies in an n-semitone interval is $\sqrt[12]{2}^n$ or $2^{\frac{n}{12}}$, invariant to pitch. A semitone can be further divided into 100 logarithmically scaled cents.

Using scientific pitch notation, every note can be uniquely identified by combining the traditional note names A to G (with accidentals such as ♯ and ♭) with an octave number (e.g. $\mathrm{E}_3^\flat$). An octave starts at C, which means the octave number increases between B and C. This counter intuitively implies A_3 is higher than C_3. Note that in 12-TET, $\mathrm{C}^\sharp$ and $\mathrm{D}^\flat$ are enharmonically equivalent. In this thesis, we will always refer to the sharp (♯) note instead of the enharmonically equivalent flat note (♭). The range of a typical electric guitar in standard tuning is from E_2 up to E_6.

The 12-TET music system only describes the relation between two notes in an interval. In order to play with other musicians in harmony, an arbitrary note has to be tuned to a specific frequency. Per ISO 16, the standard tuning frequency of the A_4 is 440 Hz within an accuracy of 0.5 Hz [4]. In this thesis, we will always assume a 12-TET music system with a 440 Hz tuning note.

Using the above information, we can translate frequencies into scientific note names and vice versa. In order to numerically work with note names, we assign a value to each note as shown in Table 1.

name	number	name	number
C	0	$F^\sharp$	6
$C^\sharp$	1	G	7
D	2	$G^\sharp$	8
$D^\sharp$	3	A	9
E	4	$A^\sharp$	10
F	5	B	11

Table 1: The number corresponding to each note name

In order to make calculations easier, we use C_0 as a tuning note instead of A_4. We can calculate the frequency of C_0 using the fact that C_0 is 57 semitones lower than A_4:

$$\begin{aligned} f_{C_0} = f_{A_4} * 2^{\frac{-57}{12}} &= 440 * 2^{\frac{-57}{12}} \\ &\approx 16.352 \text{ Hz} \end{aligned}$$

We can calculate the frequency f_{N_O}, where N is the note name which is represented by a numerical value given by Table 1 and O is the octave number using:

$$\begin{aligned} f_{N_O} &= f_{C_0} * 2^O * 2^{\frac{N}{12}} \\ &= f_{C_0} * 2^{O+\frac{N}{12}} \end{aligned}$$

To calculate the closest note N_O corresponding to a frequency f, we first calculate the number of semitones n_s between the tuning note f_{C_0} and f:

$$n_s = \left\lfloor 12 * {}^2\log \frac{f}{f_{C_0}} \right\rceil$$

Here, $\lfloor \ldots \rceil$ denotes rounding to the nearest integer. By rounding, we find the closest note to f. Now we can calculate N and O as follows:

$$\begin{aligned} N &= n_s \mod 12 \\ O &= \left\lfloor \frac{n_s}{12} \right\rfloor \end{aligned}$$

Note that we assume $a \mod b$ always return a number c for which $0 \leq c < b$. Some programming languages allow the modulo operator to return a value c for which $-b < c < b$, resulting in $-13 \mod 10 = -3$ instead of $-13 \mod 10 = 7$. Furthermore, when using a conversion to an integer instead of a floor, the octave number is rounded up when the note distance is negative.

In order to calculate the error e (in cents) between the given frequency to the closest tuned note, we first calculate the tuned frequency f_t of the closest note:

$$f_t = f_{C_0} * 2^{\frac{n_s}{12}}$$

Then the error e can be calculated using:

$$e = 1200 * {}^2\log \frac{f}{f_t}$$

In digital music processing, notes are often represented through MIDI note numbers. A MIDI note number can take a value from 0 to 127. The MIDI standard defines C_4 to be MIDI note number 60. This makes the standard tuning frequency A_4 number 69 and C_0 number 12.

The MIDI note number m corresponding to the note closest to frequency f can be calculated using the semitone distance from a frequency with a known MIDI note number. Let $m(N_O)$ denote the MIDI note number of N_O:

$$m = \left\lfloor 12 * {}^2\log \frac{f}{f_{N_O}} \right\rceil + m(N_O)$$

Conversely, the frequency f of the note corresponding to MIDI number m can be calculated as follows:

$$f = f_{N_O} * 2^{(m - m(N_O))/12}$$

3.5 Physical properties of sound

The perceived loudness of a note over time can be described using an ADSR envelope. The ADSR envelope of a played note is the convex hull of the waveform of the signal, see Figure 4 for an example. This convex hull can be divided into four parts: Attack, Decay, Sustain and Release. When a note is strummed on the guitar, a percussive sound is generated which causes a loud and sharp attack along with the note. This percussive sound quickly decays and only the actually fretted note will sustain. Finally, when the note is released, it dies out quickly.

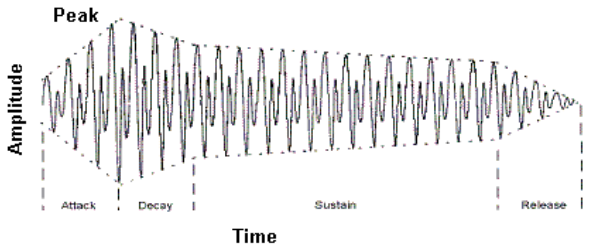

Figure 4: Example of an ADSR envelope (TODO: Better image)

The percussive sound generated when strumming a note is called a transient. Transients contain a high degree of non-periodic components. Because of this, transients appear very chaotically in the frequency domain and are often considered noise. As a transient is of high amplitude, it overshadows the note which will eventually sustain. Consequently, we cannot use the samples from a transient for Fourier based pitch estimation. This in turn increases our minimum latency, as we have to wait for samples which do not contain the transient anymore.

When playing a note on an instrument, many sine waves are generated. The most notable frequency is called the fundamental frequency and determines what note is actually played. Integer multiples of the fundamental frequency can resonate and give rise to harmonic overtones [27]. In practice, these overtones are not exact integer multiples due to non-linear effects.

Many other frequencies are generated along with the fundamental and its overtones. The instrument specific pattern of these frequencies, along with the characteristics of the overtones, is called the timbre of the instrument [17]. The timbre is what differentiates the sound of the same note played on two different instruments [24]. Generally, the amplitude of the timbre frequencies is low compared to the fundamental frequency and can be disregarded as noise in the frequency domain. Figure 5 shows the effect of timbre on a waveform.

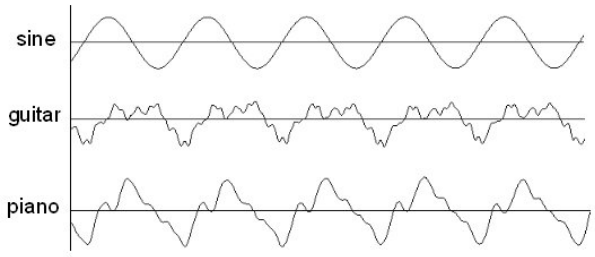

Figure 5: Example of difference in timbre between instruments compared to a sine wave.

In Section 2, we mentioned overtones are dissonant with respect to notes in 12-TET. This is true for all overtones, except for octaves, which are all overtones numbers equal to $2^n - 1$ for every n. In Table 2, we show an example for the overtones of C_4. Note that the series of errors is always the same, regardless of what the starting note is.

n	f_{overtone}	closest note	f_{note}	error
0	261.626	C_4	261.626	-
1	523.251	C_5	523.251	0
2	784.877	G_5	783.991	1.955
3	1046.502	C_6	1046.502	0
4	1308.128	E_6	1318.510	-13.686
5	1569.753	G_6	1567.982	1.955
6	1831.379	$A_6^{\#}$	1864.655	-31.174

Table 2: Example of overtone series from C_4 and the errors compared to the closest note

TODO: Rewrite. As mentioned in Section 2, when using the CQT transform, none of the non-octave overtones coincides with a CQT bin as the CQT bins are exponentially spaced like the notes in a scale. This causes the frequencies of the overtones to spread out over bins, resulting in more noise in the frequency domain. Furthermore, overtones are important for discerning fundamentals from noise generated by transients. By using a Fourier transform tuned to a specific note, all its overtones are also coincide with Fourier bins. By performing a Fourier tuned to every note in the 12 tone scale, we can measure every note and its overtones.

TODO: This thesis mainly focuses on monophonic transcription, as it is much easier to perform. But we can still see how well our strategy works in the polyphonic case. TODO: A big problem in monophonic pitch estimation is the octave problem. Octaves are difficult to discern as the fundamental frequency and overtones of the higher note coincide with the overtones of the lower note. A similar problem arises TODO: The main problem in polyphonic pitch estimation comes from the occurrence of overtones. As mentioned before, notes in octave tend overshadow each other. Furthermore, as shown in Table 3, the overtones of a note can coincide with the... TODO: Overtone overlap and polyphonic difficulty.

n	$f_0^{C_3} * n$
1	130.813
2	261.626
3	392.438
4	523.251
5	654.064

n	$f_0^{E_3} * n$
1	164.814
2	329.628
3	494.441
4	659.255
5	824.069

n	$f_0^{G_3} * n$
1	195.998
2	391.995
3	587.993
4	783.991
5	979.989

Table 3: Overtones of C, E and G

Note	f	Δf
$f_2^{C_3}$	392.438	0.443
$f_1^{G_3}$	391.995	
$f_4^{C_3}$	654.064	5.191
$f_3^{E_3}$	659.255	

Table 4: Colliding overtones

4 Real-time Fourier transform based monophonic pitch estimation

At the heart of our research lies a pitch estimation algorithm. The task of a pitch estimation algorithm is to convert a frame of samples to some representation of the notes contained in the frame. In this thesis, we will focus on spectral analysis methods of pitch estimation. Specifically, we focus on pitch estimation using the spectra obtained from frames using the Fourier transform.

4.1 Real-time constraint

Before we start constructing our pitch estimation system, let us first choose a real-time constraint. TODO maybe note unaccounted latencies and only Digistring latency here. The goal of our pitch estimation system is creating a digital representation of the played notes so it can for example be used for software synthesis. Consequently, the real-time constraint should reflect the critical latency. This is the latency for which delay between playing a note and receiving the feedback, such as hearing back the synthesized note, becomes problematic. The critical latency is highly subjective and also depends on the speed at which notes are played. Therefore, we will deduce a reasonable real-time constraint from a few different arguments.

Even though the critical latency is subjective, it is the factor determining if a real-time pitch estimation system is usable in musical context. In order to determine the critical latency empirically, we created a tool called *delayed playback*, which plays back recorded audio with an arbitrary latency. Using *delayed playback*, we found a latency of TODO milliseconds is a reasonable latency at which we can still comfortably play many songs. *Delayed playback* can also be used to verify if a real-time constraint chosen by someone else is reasonable by your own standards. The tool is described in depth in Section 5.1.

TODO: Rewrite paragraph, as playing faster is easier due to relying more on muscle memory instead of listening to yourself. As mentioned before, the critical latency depends, among other things, on the speed with which notes are played. For instance, let's assume a latency of 50 milliseconds. When playing a slow song consisting of quarter notes at 120 BPM, resulting in 2 notes per second, the latency is only a tenth of the note duration. However, when playing a fast solo consisting of sixteenth notes at 210 BPM, resulting in 12 notes per second, the latency is 70% of the note duration. This means you hear more of the previous note than the current note for the duration with which the current note is played. This tricks the brain in such a way that it prevents you from staying on beat. For non-musicians, the same effect can be simulated by listening to yourself speaking/singing in real-time using *delayed playback*. Given a certain playback latency, you'll notice it is much harder to speak/sing fast than slow.

There is a limit on how fast we can estimate the pitch of a played note. For instance, when we consider transients noise, we have to wait for the transient to pass before we can gather samples containing the note. Then, we need enough samples such that the lowest bin of the DFT doesn't exceed the frequency of the lowest note. In the case of E_2, which is 82.407 Hz, we would need a minimum frame length of $\frac{1}{82.407} * 1000 = 12.135$ milliseconds. Due to spectral leakage, using such small frame lengths is not feasible. However, as outlined in Section 8.1, we can translate this idea to the time domain. In practice, this would result in a minimum latency of 14 to 15 milliseconds.

In order to assess what we can reasonably expect from our system, we measured the latency of a commercial guitar synthesizer solution. This is described in depth in Appendix A. Here, we found the Axon AX 100 MKII at worst has a latency of 15 milliseconds when it is able to guess the note "the first try". Otherwise, the latency may reach up to 40 milliseconds.

Note that we only considered pitch estimation latency. If we want to do anything with the pitch estimation results in real-time, such as synthesizing audio, additional latency will be introduced. For this reason, we shouldn't put our real-time constraint

right at the critical latency, but leave some headroom for the program using our estimated pitches.

TODO: Note on unaccounted latencies (audio driver/hardware latencies). A careful reader may have noticed we actually discussed two different latencies. We mostly reason about the latency of our pitch estimation system. However, when reasoning about the critical latency, latencies from the audio driver and hardware as discussed in Section 3.1 do matter. TODO: Estimate on these latencies. Estimate of latency when running Digistring's algorithms on specialized hardware.

TODO: Our real-time constraint choice.

4.2 Basic algorithm for pitch estimation

Let us start with constructing the most basic Fourier based monophonic pitch estimation algorithm. Here, we simply return the note closest to the bin with the highest magnitude. Every frame, we try to read a full frame of samples from the audio driver. To minimize our latency, we want to choose our frame size as small as possible while still being able to discern the two closest notes in frequency a typical guitar can produce. Due to the exponentiation nature of notes, the lowest two notes are always the closest two. For a typical guitar, these notes are E_2 and F_2, which have a corresponding frequency of 82.407 Hz and 87.307 Hz respectively. This means we need a frequency resolution of at least $87.307 - 82.407 = 4.9$ Hz. This equates to a frame length of $4.9^{-1} = 0.204$ seconds, or 204 milliseconds. The bin centers do not have to exactly align with the notes to be able to discern them, so in practice we could get away with slightly shorter frames. Still, such large frame lengths are problematic for multiple reasons. A frame will contain multiple played notes when a guitarist plays at a moderate tempo (playing eighth notes in 150 BPM corresponds to 200 millisecond notes). Furthermore, if a frame contains the start of note, the whole frame might be useless due to the transient. Lastly, since we have to wait until the audio driver has enough samples ready for us to fill a frame, the first samples from that frame will be 200 milliseconds old. The average latency of processing a sample is 100 milliseconds due to the frame length alone. This is well over our real-time constraint.

4.3 Overlapping frames

Overlap frames for more context. Small caveat; if Estimator is unaware of overlapping, it may find notes completely in the past which weren't found when those samples were the present.

Real-time sample gathering and overlapping (non-block).

4.4 Zero padding

The number of output bins is determined by the number of samples that is transformed. We can artificially increase the number of samples in a transform by appending zeros to the frame. This technique is called zero padding. As only silence is added to the frame, it does not alter the spectrum. It is important to note that it does not increase the resolution of the DFT, it merely interpolates the coarse spectrum to become more smooth [3]. Two frequencies closer than Δf_{bin} together form one big peak in the smoothed spectrum [2]. This form of interpolation is relatively compute intensive [30].

4.5 Lagrange parabolic interpolation

TODO: dB scale is only perfect under a Gaussian window. Use better sources and better explanation (lagrange parabolic interpolation is called MQIFFT; doing it in log scale is better (LQIFFT), but exponential even better with correct tuning of exponential parameter (XQIFFT)). A less compute intensive method of interpolation is quadratic interpolation. It interpolates the actual location of a spectral peak between bins by fitting a quadratic function through the bins on a decibel scale [18, 16]. We calculate a value $p \in [-\frac{1}{2}, \frac{1}{2}]$, which is the offset in bins of the interpolated peak with respect to the peak bin. Using the magnitude of the peak $y(0)$ and the magnitude of the neighboring bins $y(-1)$ and $y(1)$, we define:

$$\alpha = 20 * {}^{10}\log y(-1)$$
$$\beta = 20 * {}^{10}\log y(0)$$
$$\gamma = 20 * {}^{10}\log y(1)$$

Then, we can calculate p as follows:

$$p = \frac{1}{2} \cdot \frac{\alpha - \gamma}{\alpha - 2\beta + \gamma}$$

The amplitude in dB a_i^{dB} corresponding to the interpolated peak is:

$$a_i^{\mathrm{dB}} = \beta - \frac{(\alpha - \gamma) * p}{4}$$

The non-dB interpolated amplitude is:

$$a_i = 10^{a_i^{\mathrm{dB}}/20}$$

Given the bin number b of the spectral peak location, the frequency f_i corresponding to the interpolated peak is:

$$f_i = \Delta f_{bin} * (b + p)$$

Quadratic interpolation can be combined with zero-padding for better results [30].

4.6 Peak picking

Gaussian peak picking. Min-max peak picking. Peak filtering.

5 Digistring

Details about the program I've written. Usage instructions, code choices, code structure, screenshots, expandability TODO!!! Our framework provides scientist willing to research real-time pitch estimation an efficient implementation of many basic functions (e.g. efficient overlapping buffers, debug wave form generation, note synthesis based on estimation).

General Digistring structure

Subsections with details of implementations. E.g. sample getting (sample format conversion in real time, sleeping, etc.), overlapping frames, audio synthesis (no plopping techniques), Estimators+EstimatorGraphics Lastly, experimentation is difficult... Many ways to report note estimations and all datasets have different annotation standard (solved using intermediate representation). Automatically calculates commonly used performance measures. Ambiguity of performance measure (all are both good and bad).

5.1 Playback latency

Details on how we made the playback latency tool.

6 Experiments

Note that the parameters were empirically optimized with informal experiments. Datasets. Actual experiments.

7 Conclusions

What we did in this thesis. Reflection on the performance of the system. Final reference to the source code.

8 Future work

What could still be improved/further researched.

8.1 Waveform packets

Low frequency tones from a guitar come in "packets". By detecting the distance between two packets, we can estimate the frequency.

A Measuring the latency of the AXON AX 100 mkII

Lekker meten en weten.

B Effect of different window functions

Plots met effect van verschillende window functions.

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