X V.A. DISCRETA $\quad \operatorname{Im}(X)$

$$\mathbb{E}X = \sum_{k \in \operatorname{Im}(X)} k \cdot \underbrace{p_X(k)}_{\text{PMF}}$$

$$\operatorname{Var} X = \mathbb{E}\Big[\underbrace{(X - \mathbb{E}X)^2}_{Z}\Big] = \sum_{k \in \operatorname{Im}(Z)} k \cdot p_Z(k)$$

$$\overset{\text{TH. VISTO}}{=} \sum_{k \in \operatorname{Im}(X)} (k - \mathbb{E}X)^2 \, p_X(k)$$

$$\mathbb{E}g(X) \overset{\text{TH}}{=} \sum_{k \in \operatorname{Im}(X)} g(k) \cdot p_X(k) \qquad g : \mathbb{R} \to \mathbb{R}$$

$$g(x) = x^n \qquad x \in \mathbb{R}, \; n \in \mathbb{N} = \{1, 2, \dots\}$$

MOMENTO PRIMO (CIOÈ $n = 1$)

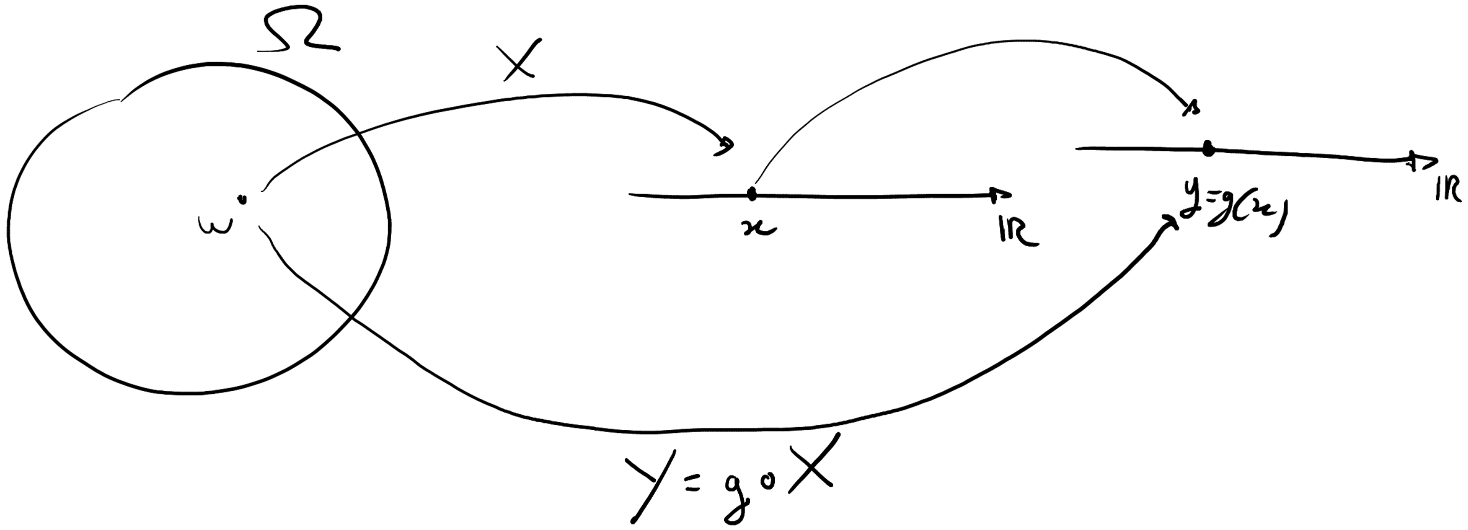
$$\mathbb{E}X^1 = \mathbb{E}X$$

SECONDO

$$\mathbb{E}X^2 = \sum_{k \in \operatorname{Im}(X)} k^2 p_X(k)$$

DEF: CHIAMO MOMENTO DI ORDINE $n \in \mathbb{N}$ DELLA V.A. X LA QUANTITÀ:

$$m_n(X) = \mathbb{E}X^n \underset{\text{TH.}}{=} \sum_{k \in I_m(X)} k^n \, p_X(k)$$



ES. MOMENTO DI ORDINE 2 DI $X \sim Be(p)$

$$\mathbb{E}X = p \qquad \mathrm{Var}\, X = p(1-p)$$

$$m_2(X) = \mathbb{E}X^2 = \sum_{x \in I_m(X)} x^2 \, p_X(x)$$

$$= \cancel{0^2 (1-p)} + 1^2 p = p$$

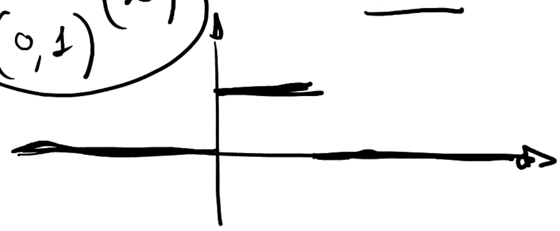
$$m_3(X) = \mathbb{E}X^3$$

(

$X \sim Be(p)$ $\qquad p = \mathbb{P}(\{succ\})$

$\downarrow$

$\mathbb{1}_{\{s\}}$ $\qquad \mathbb{1}_{(0,1)}(x) \quad x \in \mathbb{R}$



$$m_n(X) = \mathbb{E}X^n = p \qquad \forall\, n \in \mathbb{N}$$

PROPRIETÀ DI MEDIA E VARIANZA:

1) SIANO X, Y V.A , $a, b \in \mathbb{R}$. ALLORA

$$\mathbb{E}[aX + bY] = a\,\mathbb{E}X + b\,\mathbb{E}Y$$

[LINEARITÀ]

2) $$\mathbb{V}\mathrm{ar}\, X = \mathbb{E}X^2 - [\mathbb{E}X]^2$$

3) SIA X V.A E $a, b \in \mathbb{R}$. ALLORA

$$\mathbb{V}\mathrm{ar}(aX + b) = a^2\, \mathbb{V}\mathrm{ar}\, X$$

↳ OPERATORE QUADRATICO INVARIANTE PER TRASLAZIONI

MEDIA E VAR. DI ALCUNE V.A. NOTE:

$X \sim Be(p)$ $\quad \mathbb{E}X = p \quad , \mathbb{V}\text{ar}\, X = p(1-p)$

$X \sim Bin(n, p)$ $\quad \mathbb{E}X = np \quad , \mathbb{V}\text{ar}\, X = np(1-p)$

$X \sim Ge(p)$ $\quad \mathbb{E}X = 1/p \quad , \mathbb{V}\text{ar}\, X = \frac{(1-p)}{p^2}$

$X \sim Po(\lambda)$ $\quad \mathbb{E}X = \lambda \quad , \mathbb{V}\text{ar}\, X = \lambda$

ESERCIZIO

SE $\mathbb{E}X = 2 \quad , \quad \mathbb{E}X^2 = 8,$

CALCOLARE

ⓐ $\mathbb{E}\left[(2+4X)^2\right]$

ⓑ $\mathbb{E}\left[X^2 + (X+1)^2\right]$

ⓒ $\mathbb{V}\text{ar}\, X$

—

ⓐ $\mathbb{E}\left((2+4X)^2\right) = \mathbb{E}\left[4 + 16X^2 + 16X\right]$

LINEARITÀ

$= \mathbb{E}[4] + 16\,\mathbb{E}X^2 + 16\,\mathbb{E}X$

$= 4 + 16 \cdot 8 + 16 \cdot 2 = \boxed{164}$

(b) $$\mathbb{E}\left[X^2+(X+1)^2\right]=\mathbb{E}\left[X^2+X^2+2X+1\right]$$

$$=\mathbb{E}\left[2X^2+2X+1\right]\underset{\text{LINEARITÀ}}{=}2\,\mathbb{E}X^2+2\,\mathbb{E}X+1$$

$$=2\cdot 8+2\cdot 2+1=\boxed{21}$$

(c) $$\mathbb{V}\mathrm{ar}\,X=\mathbb{E}\left[(X-\mathbb{E}X)^2\right]\qquad\left(=\mathbb{E}X^2-(\mathbb{E}X)^2\right)$$

$$=\mathbb{E}\left[X^2+(\mathbb{E}X)^2-2X\,\mathbb{E}X\right]$$

$$\underset{\text{LINEARITÀ}}{=}\mathbb{E}X^2+(\mathbb{E}X)^2-2\,\mathbb{E}X\,\mathbb{E}X$$

$$=\mathbb{E}X^2+(\mathbb{E}X)^2-2(\mathbb{E}X)^2$$

$$=\mathbb{E}X^2-(\mathbb{E}X)^2=8-2^2=8-4=\boxed{4}$$

ESERCIZIO

SIA X UNA V.A. CON PMF

$$p_X(x)=\begin{cases}1/4, & x=1,2\\ 1/3, & x=3\\ 1/6, & x=4\end{cases}$$

$$I_m(X) = \{1, 2, 3, 4\}$$

CALCOLARE $\mathbb{E}X$ e $\mathbb{V}\text{ar}X$.

$$\mathbb{E}X = \sum_{x \in I_m(X)} x \cdot p_X(x)$$

$$= 1 \cdot \mathbb{P}(X=1) + 2 \cdot \mathbb{P}(X=2) + 3 \cdot \mathbb{P}(X=3) + 4 \cdot \mathbb{P}(X=4)$$

$$= 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{3} + 4 \frac{1}{6}$$

$$= \frac{1}{4} + \frac{1}{2} + 1 + \frac{2}{3} = \frac{3 + 6 + 12 + 8}{12} = \boxed{\frac{29}{12}}$$

$$\mathbb{V}\text{ar}X = \sum_{x \in I_m(X)} (x - \mathbb{E}X)^2 p_X(x)$$

$$= \left(1 - \frac{29}{12}\right)^2 \frac{1}{4} + \left(2 - \frac{29}{12}\right)^2 \frac{1}{4} + \left(3 - \frac{29}{12}\right)^2 \frac{1}{3} + \left(4 - \frac{29}{12}\right)^2 \frac{1}{6}$$

$$= 1.0764$$

---

es. $X \sim \text{Bin}(n, p)$ $\quad \mathbb{E}X = 7$, $\mathbb{V}\text{ar}X = 2.1$

CALCOLARE $\mathbb{P}(X=4)$

$\mathbb{P}(X > 12)$

$$\mathbb{E}X = np \qquad \mathrm{Var}\,X = np(1-p)$$

$$\begin{cases} np = 7 \\ np(1-p) = 2.1 \end{cases}$$

$$7(1-p) = 2.1$$

$$1-p = \frac{2.1}{7} = \frac{\cancel{21}^{3}}{10} \cdot \frac{1}{\cancel{7}}$$

$$= \frac{3}{10}$$

$$p = 1 - \frac{3}{10} = \frac{7}{10}$$

$$n \frac{7}{10} = 7$$

$$\boxed{n = 10}$$

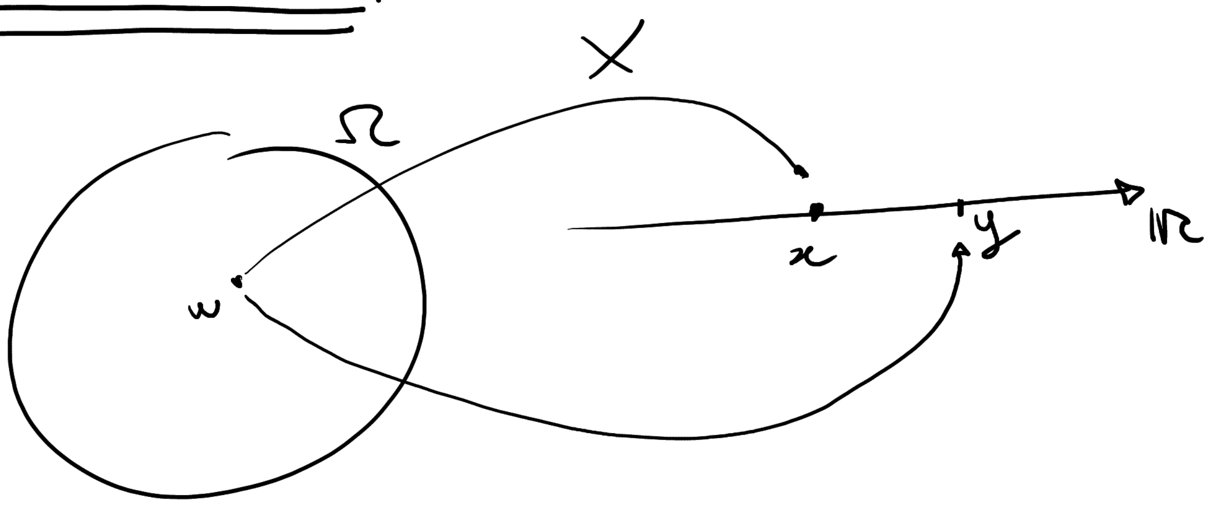
cioè $\quad X \sim \mathrm{Bin}\left(10, \frac{7}{10}\right)$

$$\mathbb{P}(X=4) = p_X(4) = \binom{10}{4}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^6$$

$$= 0{,}0368$$

$$\mathbb{P}(X > 12) = 0$$

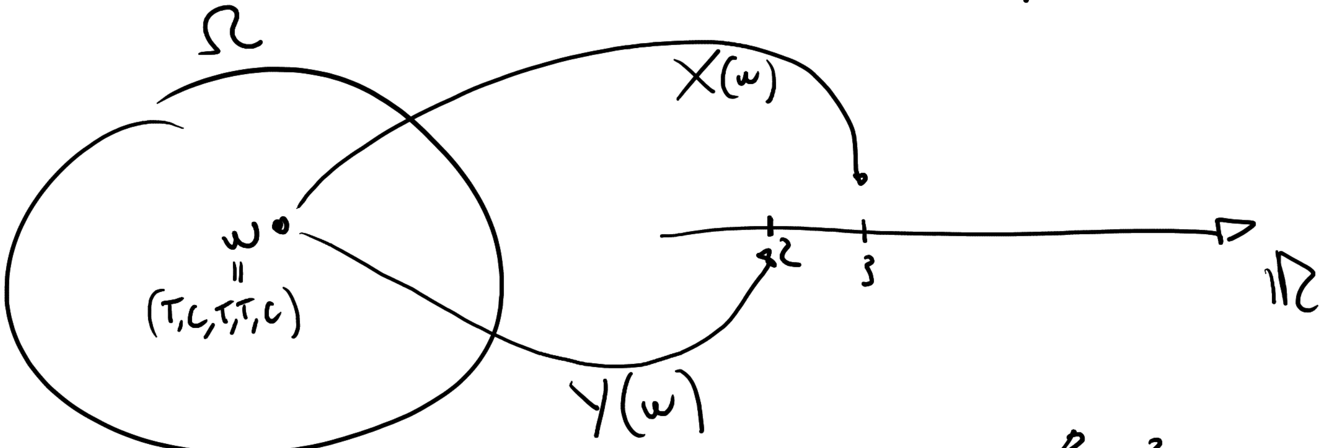
PMF CONGIUNTE:



ESEMPIO: LANCIO 5 VOLTE UNA MONETA EQUA.

X : N° DI TESTE $\qquad X \sim Bin(5, 1/2)$

Y : N° DI CROCI $\qquad Y \sim Bin(5, 1/2)$



$$\left\{ \mathbb{P}(X=3) = \mathbb{P}(Y=2) = \binom{5}{2}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \binom{5}{3}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 \right\}$$

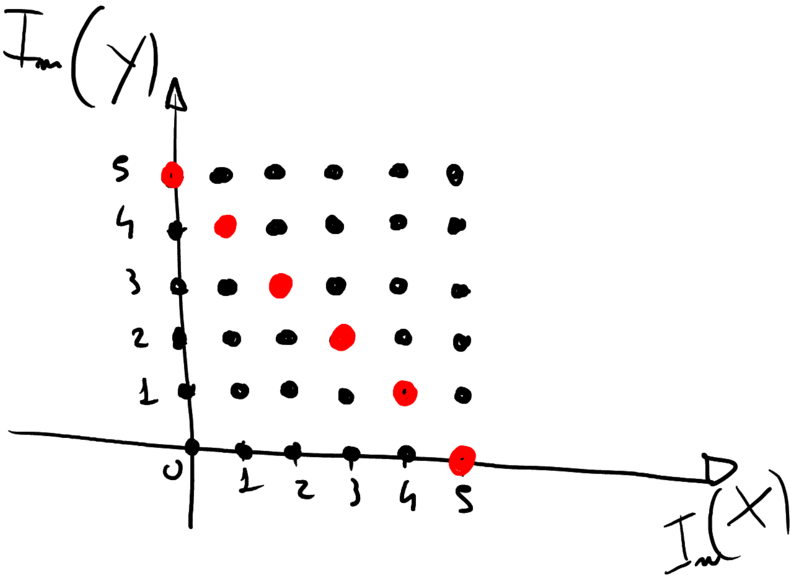
$$\mathbb{P}(\{X=3\} \cap \{Y=2\}) = 0$$

$$\mathbb{P}(X=3, Y=2)$$

NOTAZIONE

$$(X,Y): \Omega \longrightarrow \mathbb{R}^2$$

$$\omega \in \Omega \longmapsto (X,Y)(\omega) = (X(\omega), Y(\omega)) \in \mathbb{R}^2$$



(X,Y)

$$\mathrm{Im}((X,Y))$$

$(3,0)$