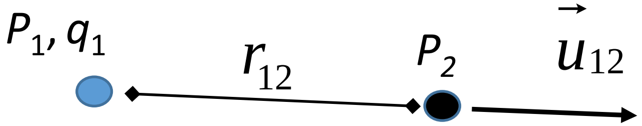
Campo coulombiano in forma vettoriale:

$$\vec{E}_{12} = k_0 \frac{q_1}{r_{12}^{\,2}} \vec{u}_{12}$$



$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$$

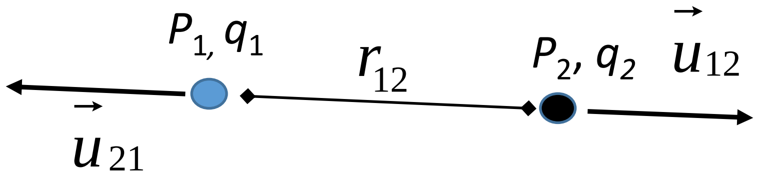
(vettore spostamento da P_1 a P_2)

$$k_0 = 8.99 \cdot 10^9 \frac{Nm^2}{C^2}$$

Forza di Coulomb

$$r_{12} = \left|\vec{r}_{12}\right| = \left|\vec{r}_2 - \vec{r}_1\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(distanza tra P_1 e P_2)



$$\vec{u}_{12} = \frac{\vec{r}_{12}}{r_{12}}$$ (versore corrispondente allo spostamento da P_1 a P_2)

$$\vec{F}_{12} = q_2 \vec{E}_{12} = k_0 \frac{q_1 q_2}{r_{12}^{\,2}} \vec{u}_{12}$$

$$\vec{u}_{21} = -\vec{u}_{12}$$ (versore corrispondente allo spostamento da P_2 a P_1)

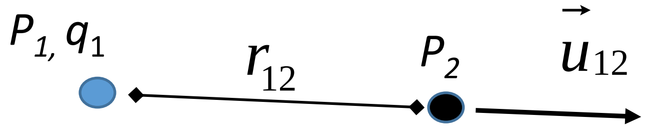
$$\vec{F}_{21} = q_1 \vec{E}_{21} = k_0 \frac{q_2 q_1}{r_{12}^{\,2}} \vec{u}_{21} = -\vec{F}_{12}$$

1. Tre cariche puntiformi Q_1 = 5 nC, Q_2 = -3 nC e Q_3 = 6 nC si trovano, rispettivamente, nei punti P1(0, 0), P2(0, -d_2), P3(d_3, 0), con d_2 = 10 cm e d_3 = 30 cm. Scrivere in forma vettoriale:

- il campo elettrico generato da Q_2 e Q_3 in P1, indicandone anche modulo e direzione (rispetto all'asse x);
- la forza agente sulla carica Q_1.

Campo coulombiano in forma vettoriale:

$$\vec{E}_{12} = k_0 \frac{q_1}{r_{12}^2} \vec{u}_{12}$$



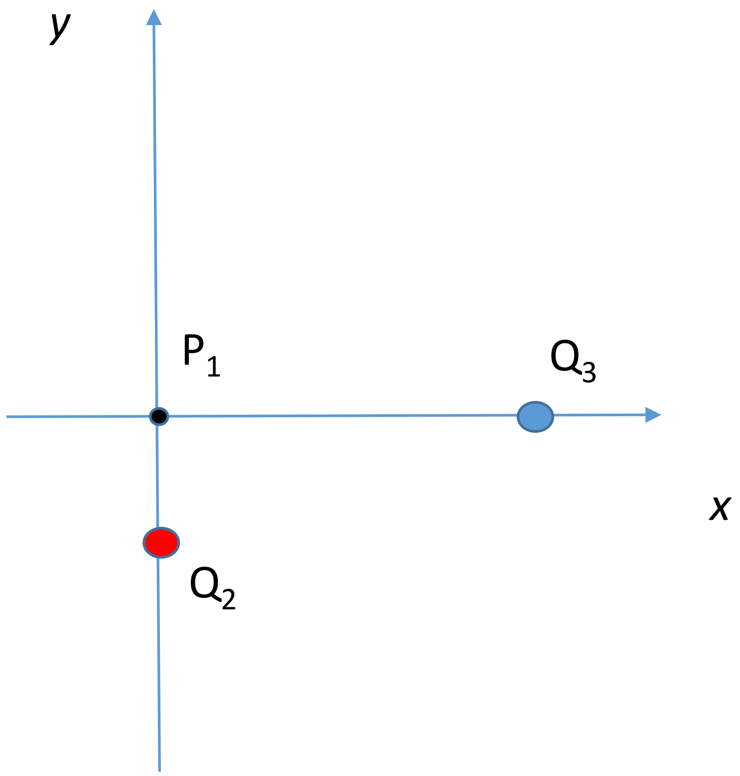
- il campo elettrico generato da Q_2 e Q_3 in P1, indicandone anche modulo e direzione (rispetto all'asse x);

Si tratta di calcolare:

$$\vec{E}_1 = \vec{E}_{21} + \vec{E}_{31}$$

$$\vec{E}_{21} = k_0 \frac{Q_2}{r_{21}^2} \vec{u}_{21}$$

$$\vec{E}_{31} = k_0 \frac{Q_3}{r_{31}^2} \vec{u}_{31}$$



$$\vec{E}_1 = \vec{E}_{21} + \vec{E}_{31}$$

P1(0, 0), P2(0, $-d_2$), P3(d_3, 0)

$d_2 > 0$, $d_3 > 0$, $Q_2 < 0$, $Q_3 > 0$

$$\vec{E}_{21} = k_0 \frac{Q_2}{r_{21}^2} \vec{u}_{21}$$

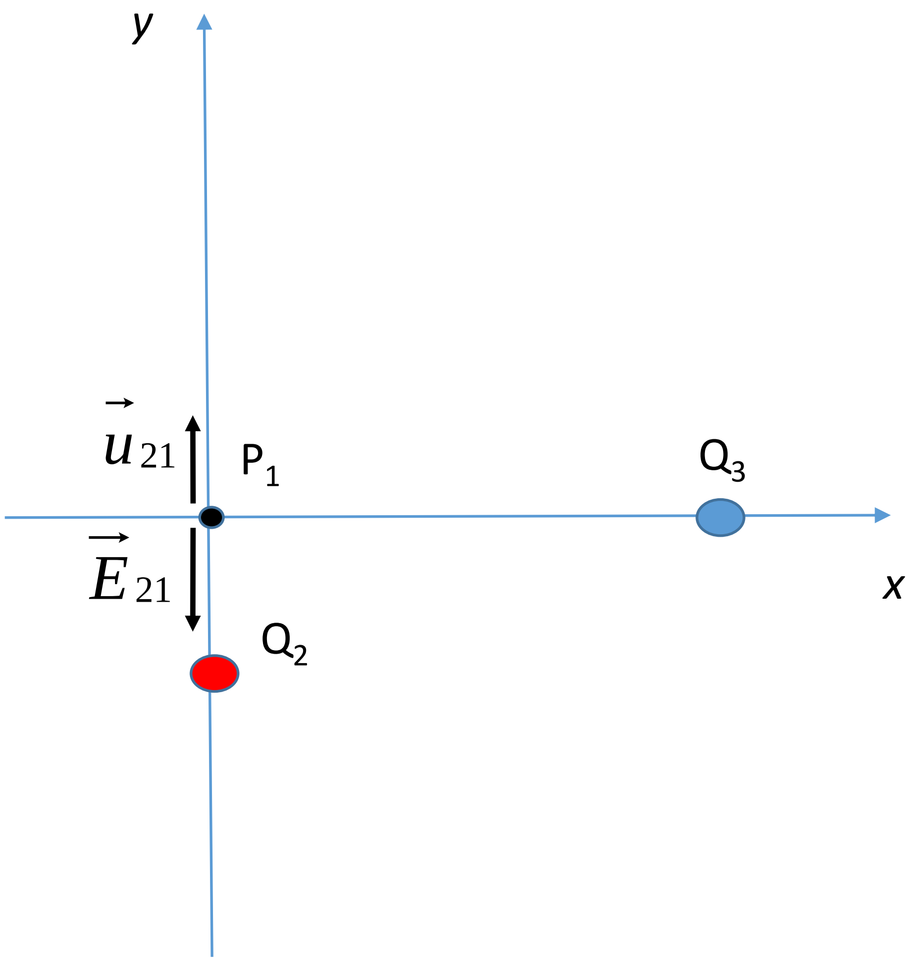
$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2 = (x_1 - x_2)\vec{i} + (y_1 - y_2)\vec{j}$$

$$= (0-0)\vec{i} + (0 - [-d_2])\vec{j} = d_2 \vec{j}$$

$$r_{21} = |\vec{r}_{21}| = \sqrt{0^2 + d_2^{\,2}} = \sqrt{d_2^2} = |d_2| = d_2$$

$$\vec{u}_{21} = \frac{\vec{r}_{21}}{r_{21}} = \frac{1}{d_2}(d_2 \vec{j}) = \vec{j}$$

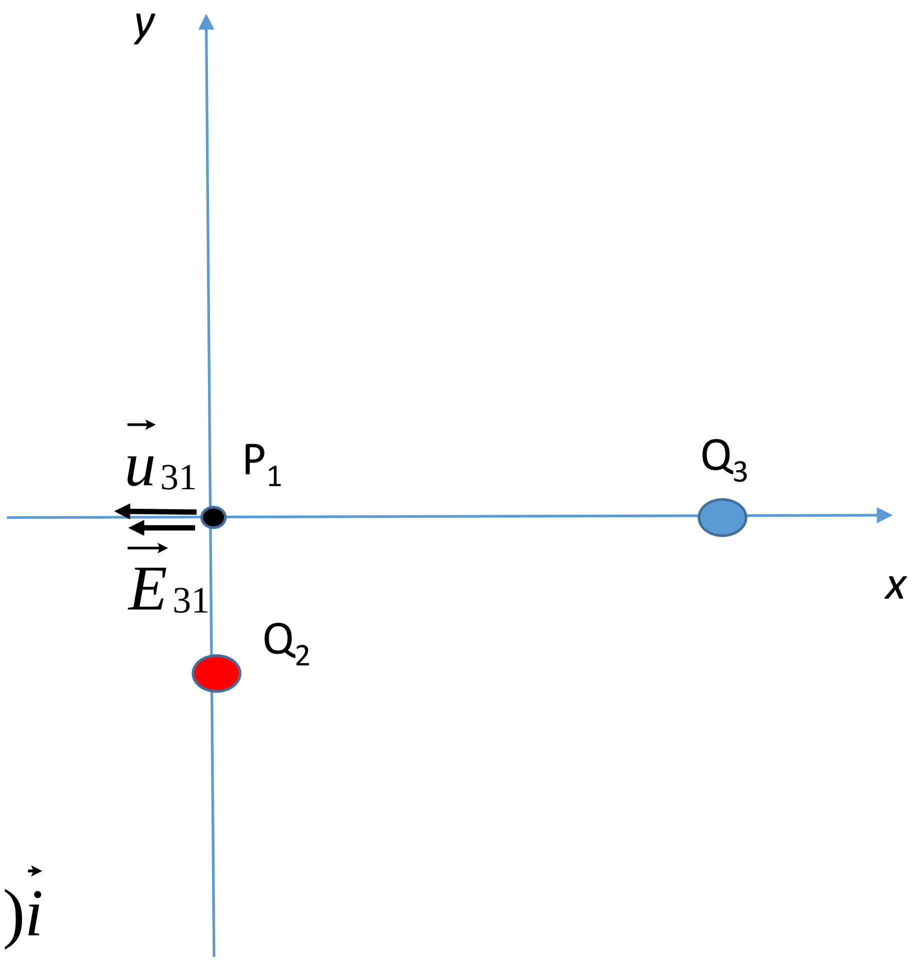
$$\vec{E}_{21} = k_0 \frac{(-3nC)}{(0.1m)^2} \vec{u}_{21} = k_0 \frac{(-3nC)}{(0.1m)^2} \vec{j} = (-2700 \frac{N}{C}) \vec{j}$$



$$\vec{E}_1 = \vec{E}_{21} + \vec{E}_{31}$$

P1(0, 0), P2(0, -d_2), P3(d_3, 0)

$d_2 > 0$, $d_3 > 0$, $Q_2 < 0$, $Q_3 > 0$



$$\vec{E}_{31} = k_0 \frac{Q_3}{r_{31}^{\;2}} \vec{u}_{31}$$

$$\vec{r}_{31} = \vec{r}_1 - \vec{r}_3 = (x_1 - x_3)\vec{i} + (y_1 - y_3)\vec{j}$$

$$= (0 - d_3)\vec{i} + (0 - 0)\vec{j} = -d_3\vec{i}$$

$$r_{31} = |\vec{r}_{31}| = \sqrt{(-d_3)^2 + 0^2} = \sqrt{d_3^2} = |d_3| = d_3$$

$$\vec{u}_{31} = \frac{\vec{r}_{31}}{r_{31}} = \frac{1}{d_3}(-d_3\vec{i}) = -\vec{i}$$

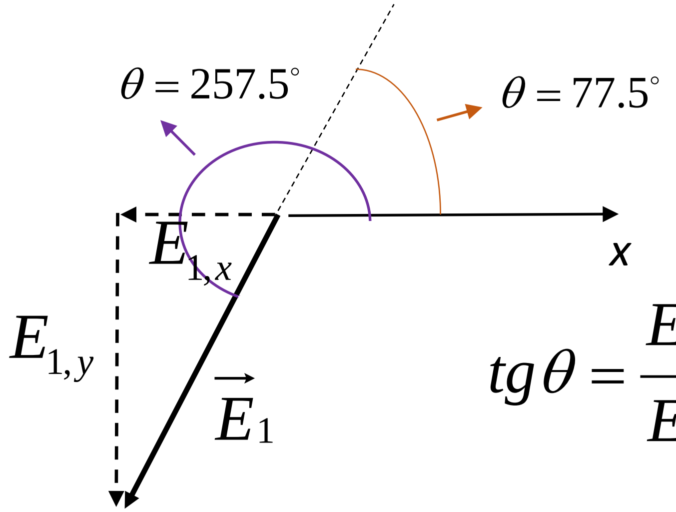
$$\vec{E}_{31} = k_0 \frac{(6nC)}{(0.3m)^2} \vec{u}_{31} = k_0 \frac{(6nC)}{(0.3m)^2} (-\vec{i}) = (-600 \frac{N}{C})\vec{i}$$

$$\vec{E}_{21} = (-2700\frac{N}{C})\vec{j}$$

$$\vec{E}_{31} = (-600\frac{N}{C})\vec{i}$$

$$\boxed{\vec{E}_1 = \vec{E}_{21} + \vec{E}_{31} = (-600\vec{i} - 2700\vec{j})\frac{N}{C}}$$

$$\boxed{\left|\vec{E}_1\right| = \sqrt{(-600\frac{N}{C})^2 + (-2700\frac{N}{C})^2} = 2766\frac{N}{C}}$$



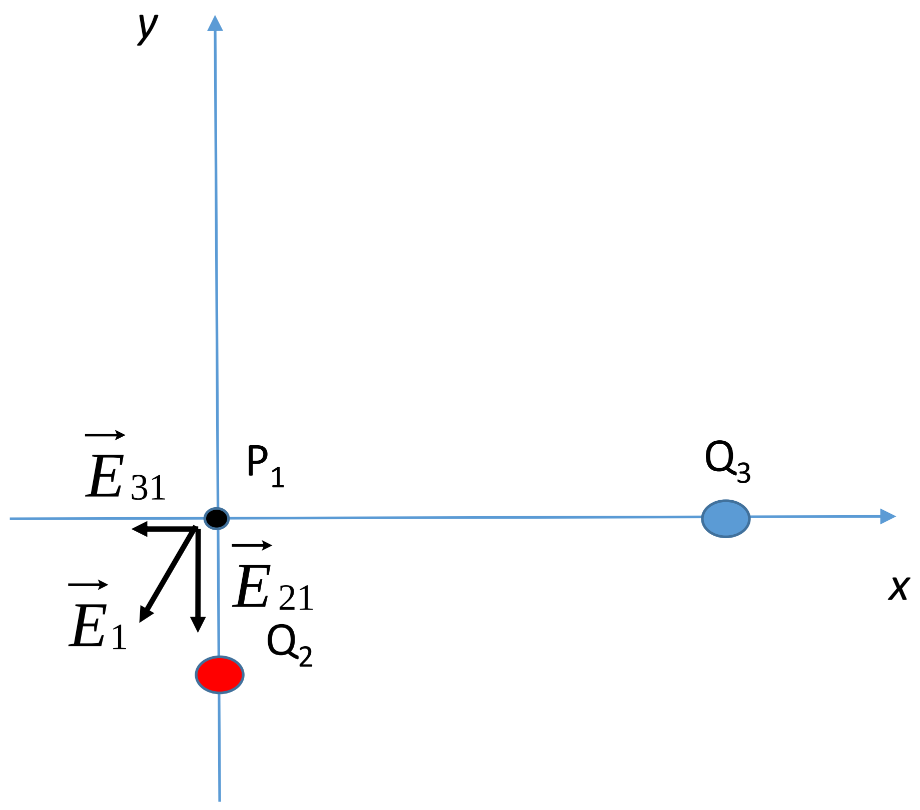
$$tg\theta = \frac{E_{1,y}}{E_{1,x}} = \frac{-2700}{-600} = 4.5$$

$$\theta = 1.35rad = 77.5^\circ$$

$$\boxed{\theta = (\pi + 1.35)rad = (180^\circ + 77.5^\circ) = 257.5^\circ}$$

P1(0, 0), P2(0, -d_2), P3(d_3, 0)

$d_2 > 0$, $d_3 > 0$, $Q_2 < 0$, $Q_3 > 0$

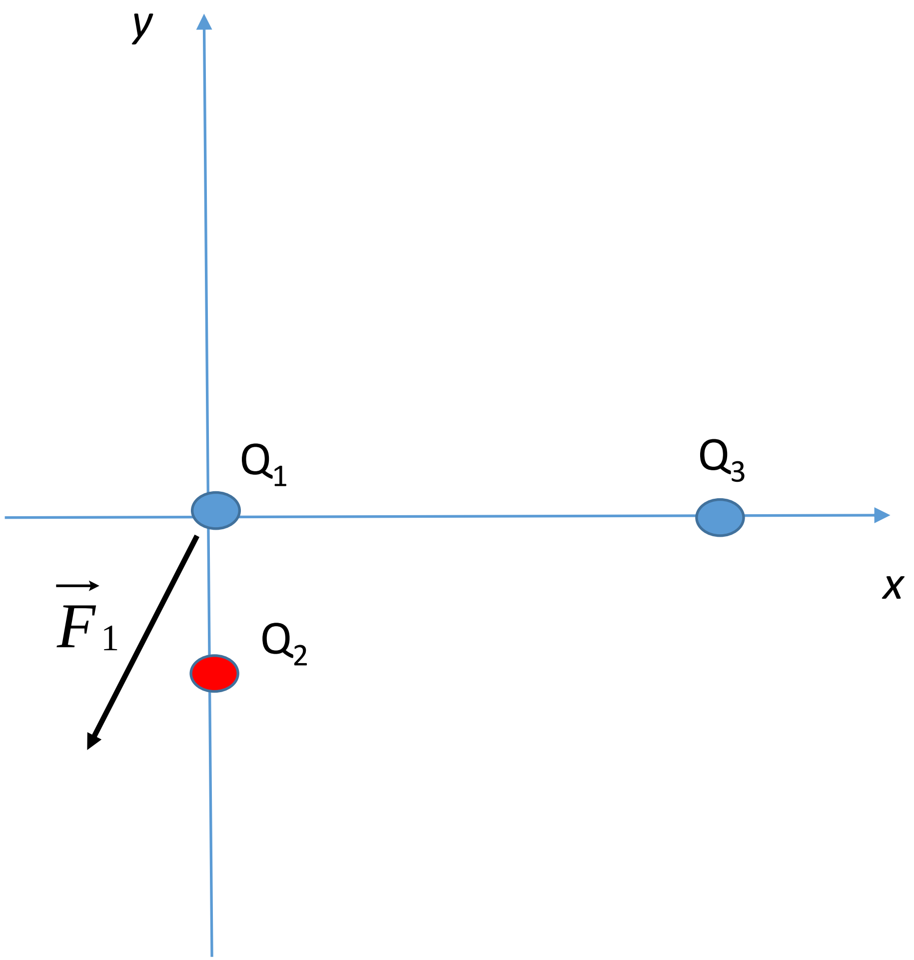


- la forza agente sulla carica Q_1.

$$\vec{E}_1 = (-600\vec{i} - 2700\vec{j})\frac{N}{C}$$

$$\vec{F}_1 = Q_1\vec{E}_1 = (5nC)(-600\vec{i} - 2700\vec{j})\frac{N}{C}$$

$$\boxed{= (-3\vec{i} - 13.5\vec{j})\cdot 10^{-6}\,N}$$



2. Tre cariche q_1, q_2 e q_3 sono disposte rispettivamente nei punti $P_1(-a,\ 0)$, $P_2(a,\ 0)$ e $P_3(0,\ b)$, con $a = 4$ m e $b = 5$ m (figura 1). I valori delle cariche sono $q_1 = q_2 = 1$ mC e $q_3 =$ -3 mC. Calcolare il campo elettrico nel punto $P(0,-a)$.

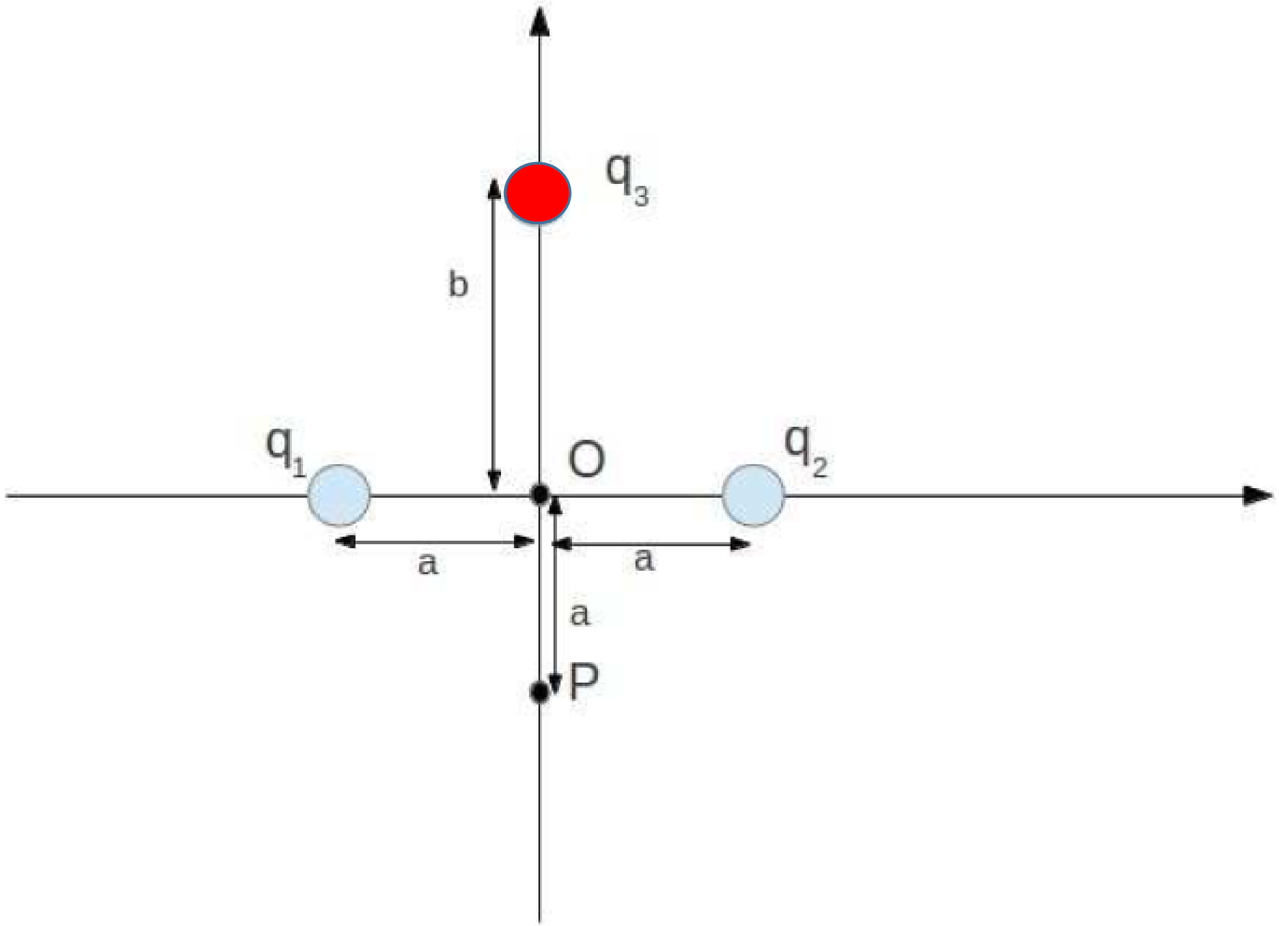
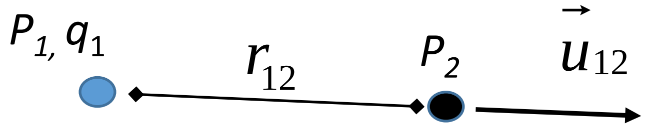


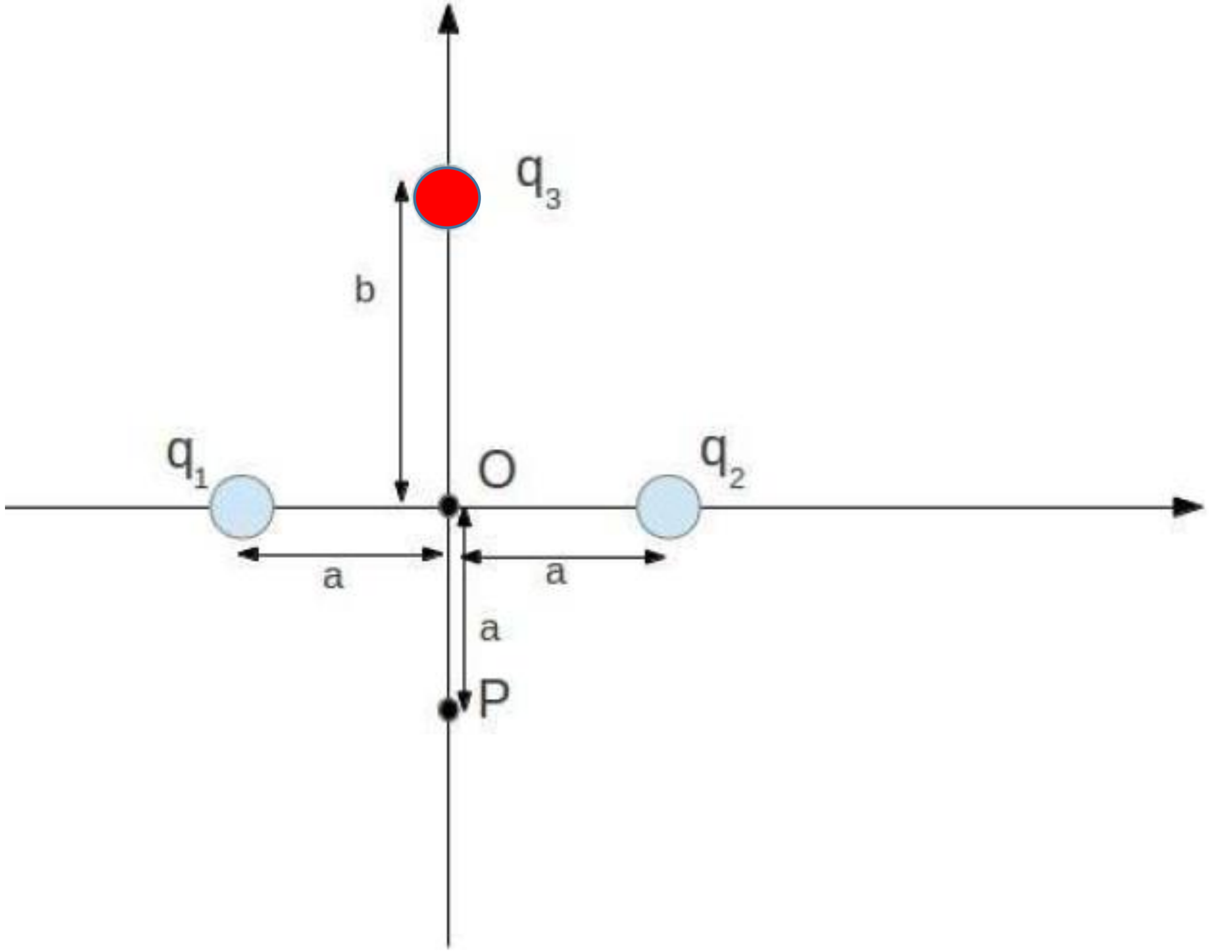
Figure 1: problema 2

Campo coulombiano in forma vettoriale:

$$\vec{E}_{12} = k_0 \frac{q_1}{r_{12}^2} \vec{u}_{12}$$



Calcolare il campo elettrico nel punto P(0,-a).



Si tratta di calcolare:

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$$

$$\vec{E}_{1P} = k_0 \frac{q_1}{r_{1P}^2} \vec{u}_{1P}$$

$$\vec{E}_{2P} = k_0 \frac{q_2}{r_{2P}^2} \vec{u}_{2P}$$

$$\vec{E}_{3P} = k_0 \frac{q_3}{r_{3P}^2} \vec{u}_{3P}$$

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$$

$P_1(-a,\ 0)$, $P_2(a,\ 0)$ e $P_3(0,\ b)$, $P(0,-a)$.

$a > 0,\ \ b > 0$

$q_1 > 0,\ q_2 > 0,\ q_3 < 0,\ q_1 = q_2$

$$\vec{E}_{1P} = k_0 \frac{q_1}{r_{1P}^{\ 2}} \vec{u}_{1P}$$

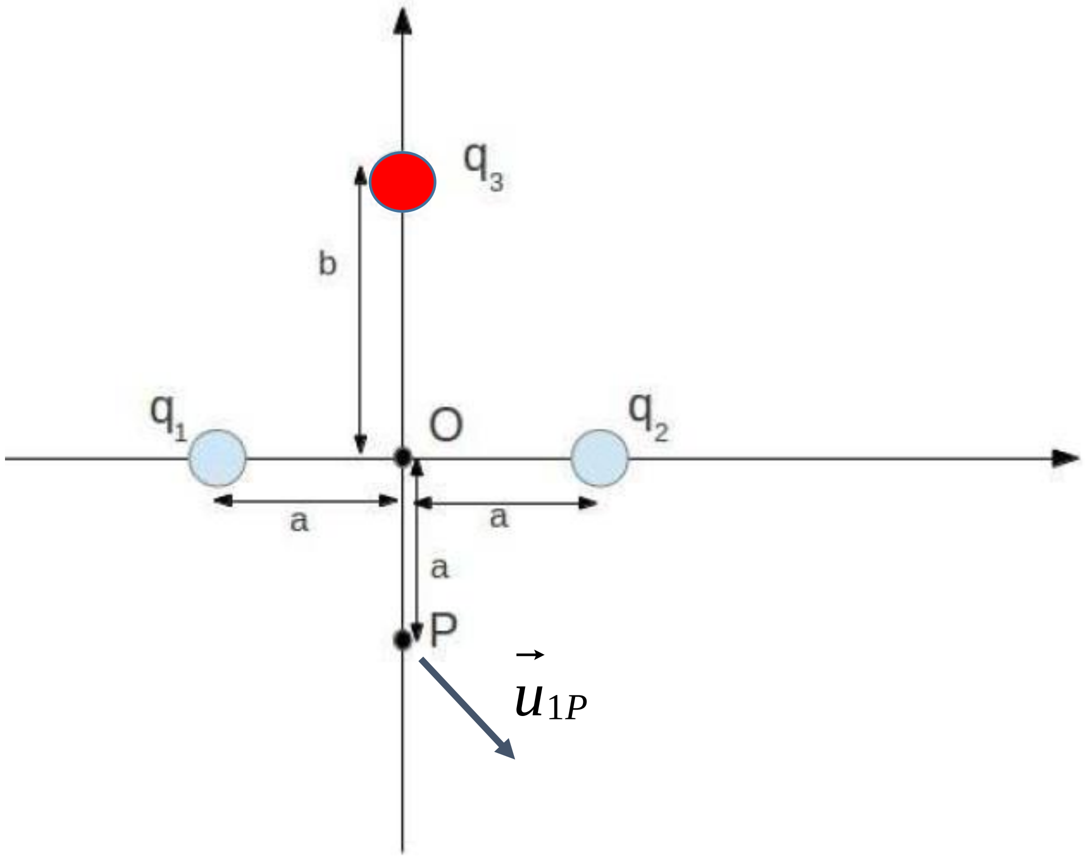
$$\vec{r}_{1P} = \vec{r}_P - \vec{r}_1 = (x_P - x_1)\vec{i} + (y_P - y_1)\vec{j}$$

$$= (0 - [-a])\vec{i} + (-a - 0)\vec{j} = a\vec{i} - a\vec{j}$$

$$r_{1P} = \left|\vec{r}_{1P}\right| = \sqrt{a^2 + (-a)^2} = \sqrt{2a^2} = \sqrt{2}|a| = \sqrt{2}a$$

$$\vec{u}_{1P} = \frac{\vec{r}_{1P}}{r_{1P}} = \frac{1}{\sqrt{2}a}(a\vec{i} - a\vec{j}) = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$$

$$\vec{E}_{1P} = k_0 \frac{q_1}{(\sqrt{2}a)^2}\vec{u}_{1P} = k_0 \frac{q_1}{2a^2}\left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$



$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$$

$P_1(-a,\ 0)$, $P_2(a,\ 0)$ e $P_3(0,\ b)$, $P(0,-a)$.

$a > 0,\ \ b > 0$

$q_1 > 0,\ q_2 > 0,\ q_3 < 0,\ q_1 = q_2$

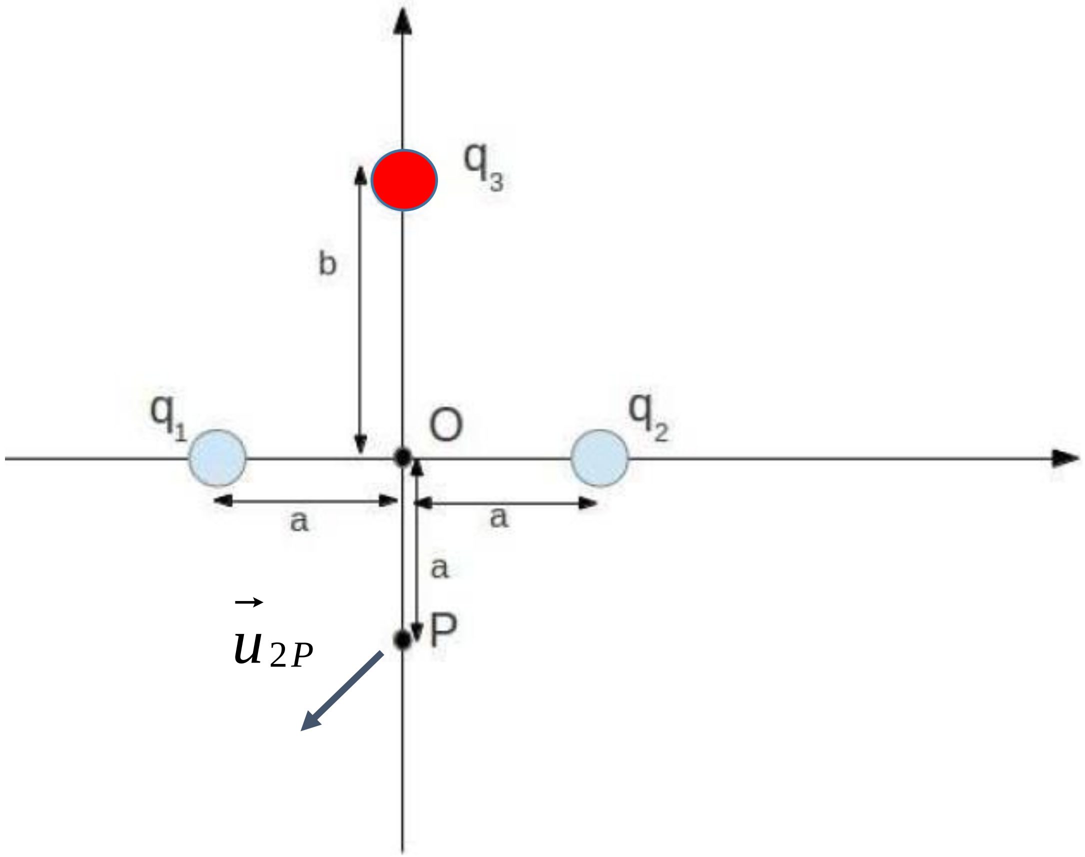
$$\vec{E}_{2P} = k_0 \frac{q_2}{r_{2P}^{\ 2}} \vec{u}_{2P}$$

$$\vec{r}_{2P} = \vec{r}_P - \vec{r}_2 = (x_P - x_2)\vec{i} + (y_P - y_2)\vec{j}$$
$$= (0-a)\vec{i} + (-a-0)\vec{j} = -a\vec{i} - a\vec{j}$$

$$r_{2P} = \left|\vec{r}_{2P}\right| = \sqrt{(-a)^2 + (-a)^2} = \sqrt{2a^2} = \sqrt{2}\left|a\right| = \sqrt{2}a$$

$$\vec{u}_{2P} = \frac{\vec{r}_{2P}}{r_{2P}} = \frac{1}{\sqrt{2}a}(-a\vec{i} - a\vec{j}) = -\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$$

$$\vec{E}_{2P} = k_0 \frac{q_2}{(\sqrt{2}a)^2}\vec{u}_{2P} = k_0 \frac{q_2}{2a^2}\left(-\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$



$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$$

$P_1(-a,\ 0)$, $P_2(a,\ 0)$ e $P_3(0,\ b)$, $P(0,-a)$.

$a > 0,\ \ b > 0$

$q_1 > 0$, $q_2 > 0$, $q_3 < 0$, $q_1 = q_2$

$$\vec{E}_{3P} = k_0 \frac{q_3}{r_{3P}^{\ 2}} \vec{u}_{3P}$$

$$\vec{r}_{3P} = \vec{r}_P - \vec{r}_3 = (x_P - x_3)\vec{i} + (y_P - y_3)\vec{j}$$

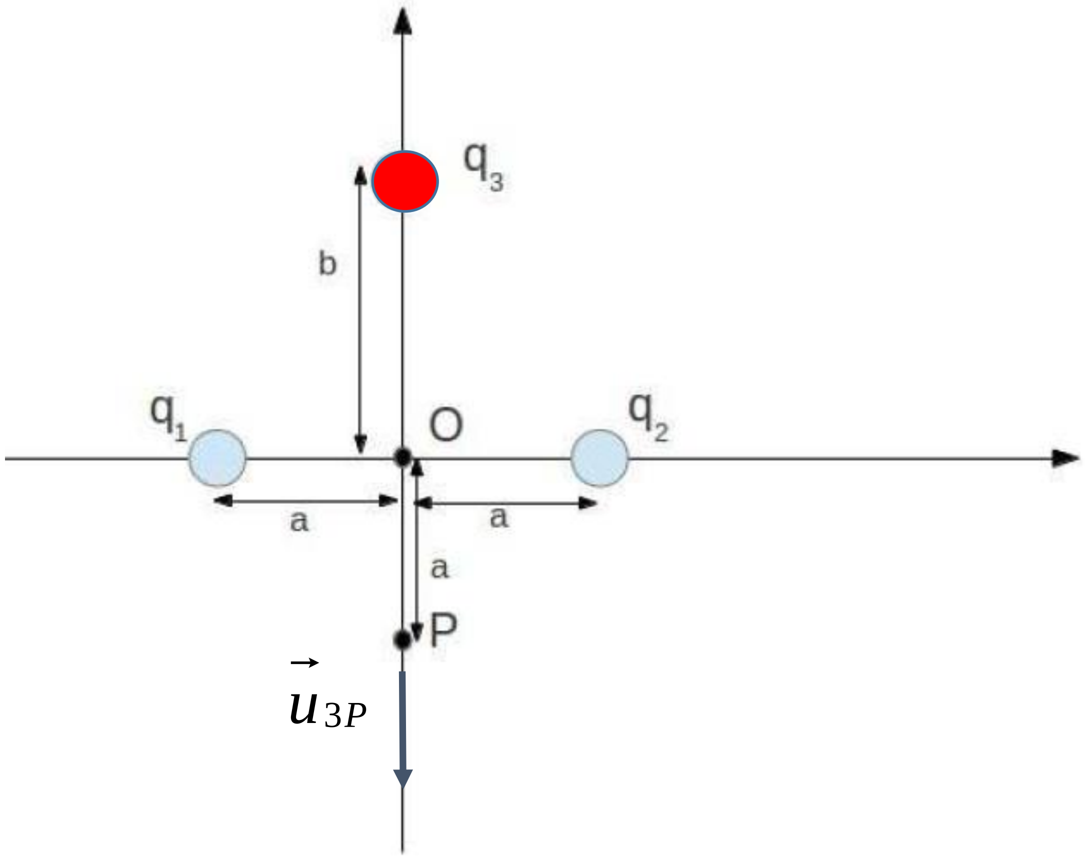
$$= (0-0)\vec{i} + (-a-b)\vec{j} = -(a+b)\vec{j}$$

$$r_{3P} = \left|\vec{r}_{3P}\right| = \sqrt{(0)^2 + (-a-b)^2} = \sqrt{(a+b)^2}$$

$$= |a+b| = a+b$$

$$\vec{u}_{3P} = \frac{\vec{r}_{3P}}{r_{3P}} = \frac{1}{a+b}[-(a+b)\vec{j}] = -\vec{j}$$

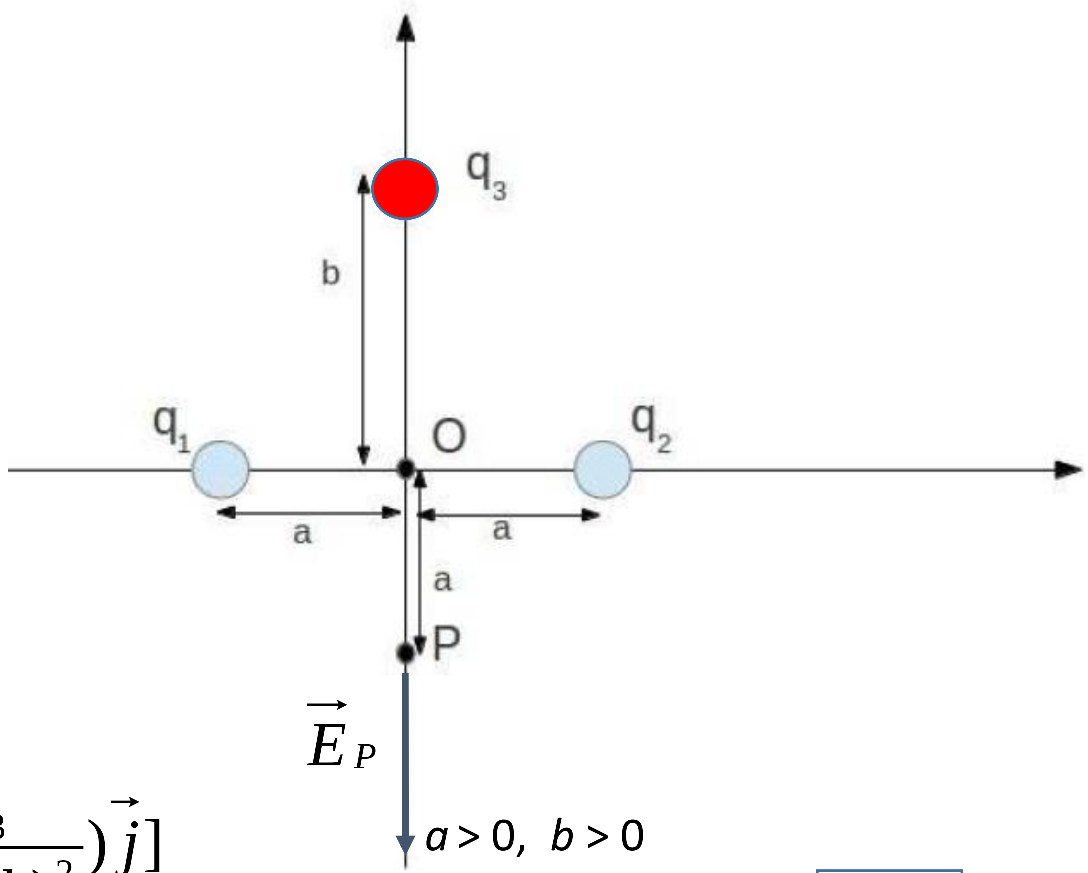
$$\vec{E}_{3P} = k_0 \frac{q_3}{(a+b)^2} \vec{u}_{3P} = k_0 \frac{q_3}{(a+b)^2}(-\vec{j})$$



$$\vec{E}_{1P} = k_0 \frac{q_1}{(\sqrt{2}a)^2}\vec{u}_{1P} = k_0 \frac{q_1}{2a^2}\left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$

$$\vec{E}_{2P} = k_0 \frac{q_2}{(\sqrt{2}a)^2}\vec{u}_{2P} = k_0 \frac{q_2}{2a^2}\left(-\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$

$$\vec{E}_{3P} = k_0 \frac{q_3}{(a+b)^2}\vec{u}_{3P} = k_0 \frac{q_3}{(a+b)^2}(-\vec{j})$$



$a > 0,\ b > 0$

$q_1 > 0,\ q_2 > 0,\ q_3 < 0,\ \boxed{q_1 = q_2}$

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P}$$

$$= k_0\left[\left(\frac{q_1}{2\sqrt{2}a^2} - \frac{q_2}{2\sqrt{2}a^2}\right)\vec{i} + \left(-\frac{q_1}{2\sqrt{2}a^2} - \frac{q_2}{2\sqrt{2}a^2} - \frac{q_3}{(a+b)^2}\right)\vec{j}\right]$$

$$= k_0\left[\left(\frac{q_1}{2\sqrt{2}a^2} - \frac{q_1}{2\sqrt{2}a^2}\right)\vec{i} + \left(-\frac{q_1}{\sqrt{2}a^2} - \frac{q_3}{(a+b)^2}\right)\vec{j}\right] =$$

$$k_0\left[-\frac{1mC}{\sqrt{2}(4m)^2} - \frac{(-3mC)}{(9m)^2}\right]\vec{j} = \boxed{\left(-6.4\cdot 10^4\,\frac{N}{C}\right)\vec{j}}$$

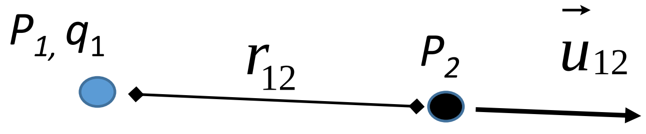
sull'asse x

3. Due cariche q_1 e q_2 si trovano, rispettivamente nelle posizioni x $= 0$ e x $= d$ $(d > 0)$.

- Scrivere l'espressione E(x) del campo elettrico in un punto generico sull'asse x.
- Se $q_1 = 1\ \mu$C, $q_2 = 3\ \mu$C e $d = 10$ cm calcolare il valore di x, diverso dall'infinito, per cui il campo elettrico si annulla.

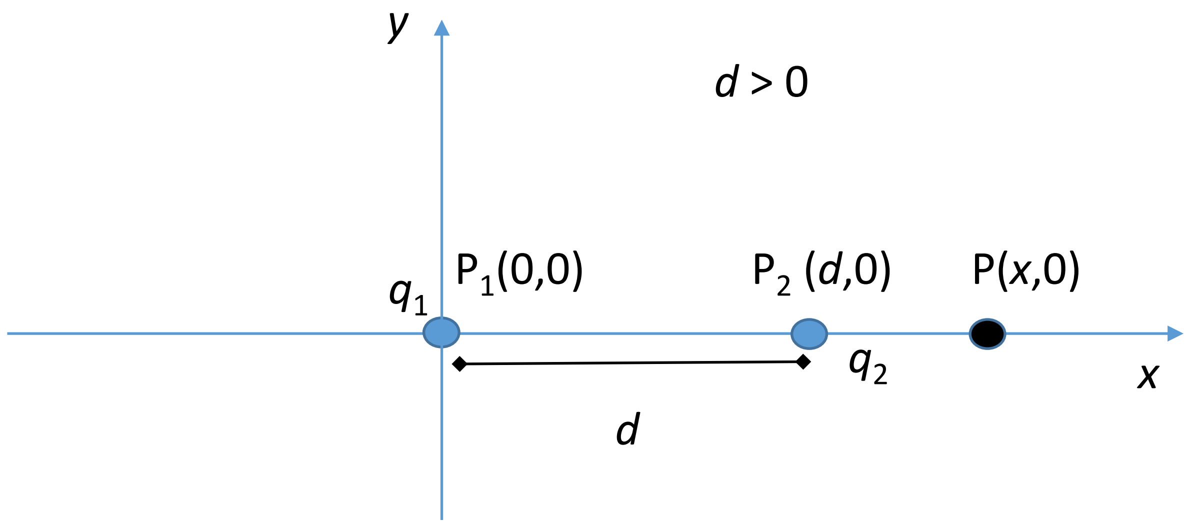
Campo coulombiano in forma vettoriale:

$$\vec{E}_{12} = k_0 \frac{q_1}{r_{12}^2} \vec{u}_{12}$$



- Scrivere l'espressione E(x) del campo elettrico in un punto generico sull'asse x.

Si tratta di calcolare $\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P}$



I tre punti giacciono sull'asse x
→ problema unidimensionale:

$$\vec{E}_P = E(x)\vec{i}$$

Nota:
x può assumere qualunque valore, positivo o negativo.
Bisognerà tenerne conto nello svolgere i calcoli.

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P}$$

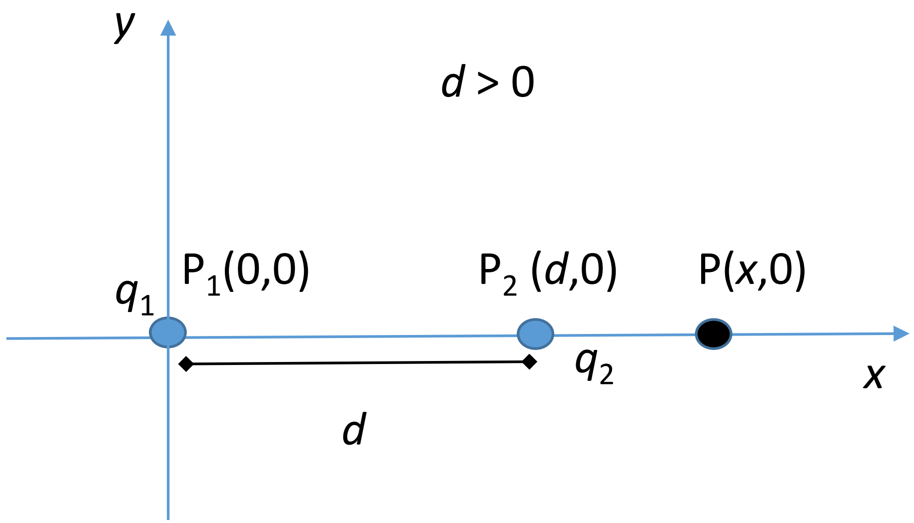
$$\vec{E}_{1P} = k_0 \frac{q_1}{r_{1P}^{\,2}} \vec{u}_{1P}$$

$$\vec{r}_{1P} = \vec{r}_P - \vec{r}_1 = (x_P - x_1)\vec{i} = (x-0)\vec{i} = x\vec{i}$$

$$r_{1P} = \left|\vec{r}_{1P}\right| = \sqrt{x^2} = |x|$$

$$\vec{u}_{1P} = \frac{\vec{r}_{1P}}{r_{1P}} = \frac{x}{|x|}\vec{i}$$

$$\vec{E}_{1P} = k_0 \frac{q_1}{(|x|)^2} \vec{u}_{1P} = k_0 \frac{q_1}{x^2} \frac{x}{|x|} \vec{i} = k_0 \frac{q_1}{x|x|} \vec{i}$$



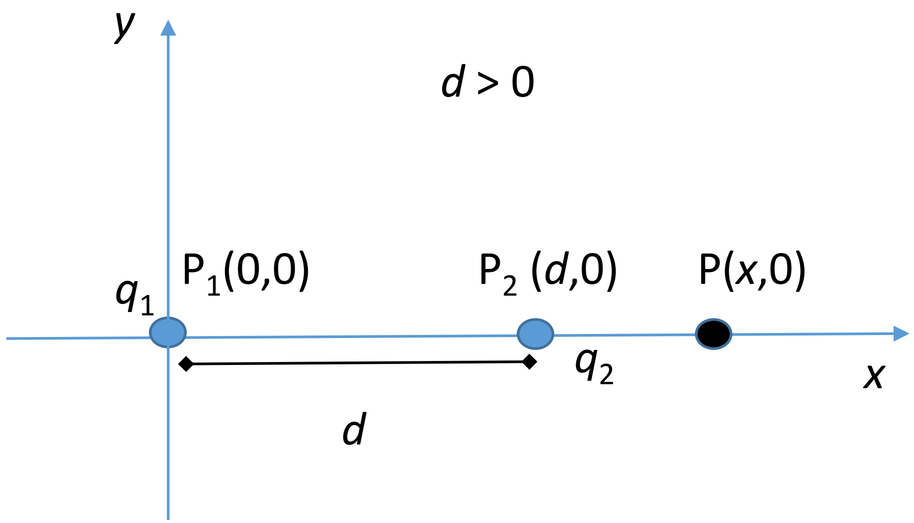
$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P}$$

$$\vec{E}_{2P} = k_0 \frac{q_2}{{r_{2P}}^2} \vec{u}_{2P}$$

$$\vec{r}_{2P} = \vec{r}_P - \vec{r}_2 = (x_P - x_2)\vec{i} = (x-d)\vec{i}$$

$$r_{2P} = \left|\vec{r}_{2P}\right| = \sqrt{(x-d)^2} = |x-d|$$

$$\vec{u}_{2P} = \frac{\vec{r}_{2P}}{r_{2P}} = \frac{x-d}{|x-d|}\vec{i}$$



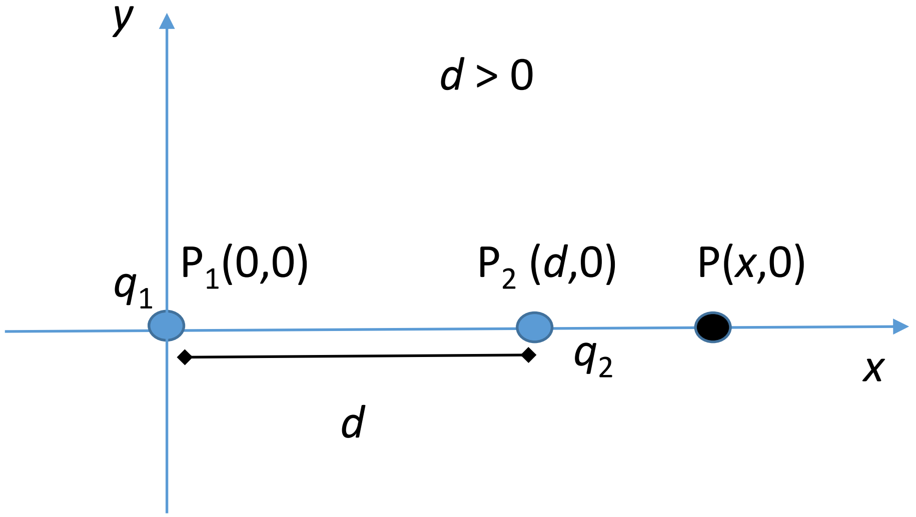
$$\vec{E}_{2P} = k_0 \frac{q_2}{(|x-d|)^2} \vec{u}_{2P} = k_0 \frac{q_2}{(x-d)^2} \frac{x-d}{|x-d|}\vec{i} = k_0 \frac{q_2}{(x-d)|x-d|}\vec{i}$$

$$\vec{E}_{1P} = k_0 \frac{q_1}{x|x|}\vec{i}$$

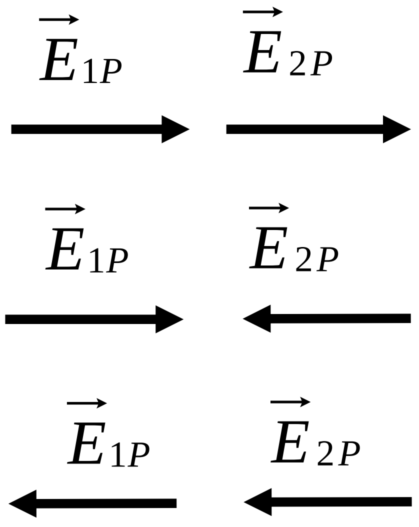
$$\vec{E}_{2P} = k_0 \frac{q_2}{(x-d)|x-d|}\vec{i}$$

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} = k_0[(\frac{q_1}{x|x|} + \frac{q_2}{(x-d)|x-d|})\vec{i}]$$

$$\rightarrow \boxed{E(x) = k_0(\frac{q_1}{x|x|} + \frac{q_2}{(x-d)|x-d|})}$$



Versi dei due campi se q_1, $q_2 > 0$



Si possono eliminare i moduli ricordando che

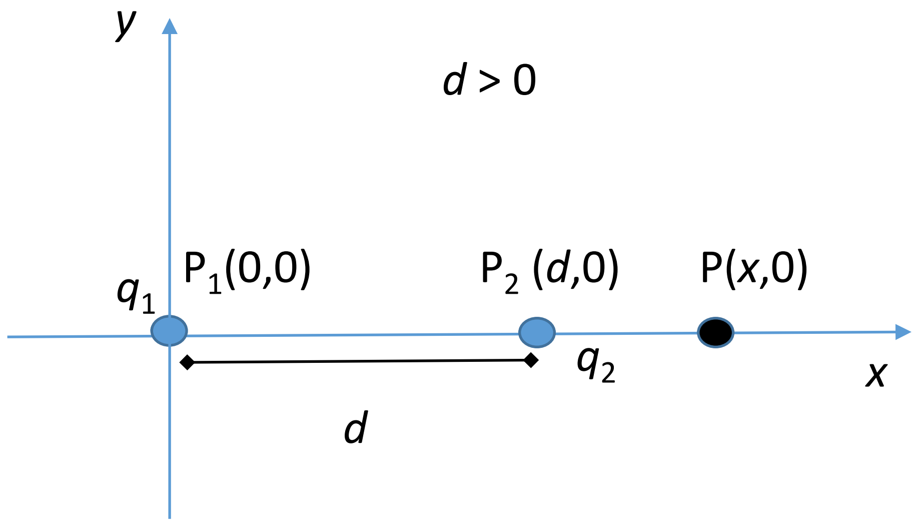
$|\alpha| = \alpha$ se $\alpha > 0$

$|\alpha| = -\alpha$ se $\alpha < 0$

$$E(x) = \begin{cases} k_0(\frac{q_1}{x^2} + \frac{q_2}{(x-d)^2}) & x > d \\ k_0(\frac{q_1}{x^2} - \frac{q_2}{(x-d)^2}) & 0 < x < d \\ k_0(-\frac{q_1}{x^2} - \frac{q_2}{(x-d)^2}) & x < 0 \end{cases}$$

- Se $q_1 = 1\ \mu\mathrm{C}$, $q_2 = 3\ \mu\mathrm{C}$ e $d = 10$ cm calcolare il valore di x, diverso dall'infinito, per cui il campo elettrico si annulla.

La soluzione va cercata nell'intervallo $0 < x < d$, l'unico in cui i due campi hanno verso opposto.



Per $0 < x < d$, si ha $E(x) = k_0\left(\frac{q_1}{x^2} - \frac{q_2}{(x-d)^2}\right)$ **(1)**

Risolviamo l'equazione

$$E(x) = k_0\left(\frac{q_1}{x^2} - \frac{q_2}{(x-d)^2}\right) = 0$$

$$\rightarrow q_1(x-d)^2 = q_2 x^2$$

$$\rightarrow (q_2 - q_1)x^2 + 2q_1 dx - q_1 d^2 = 0$$

$q_1 = 1\ \mu\mathrm{C}$, $q_2 = 3\ \mu\mathrm{C}$

$$x_{1,2} = \frac{-2q_1 d \pm \sqrt{4q_1^2 d^2 + 4(q_2 - q_1)q_1 d^2}}{2(q_2 - q_1)} = \frac{-2q_1 d \pm \sqrt{4q_2 q_1 d^2}}{2(q_2 - q_1)} = \frac{-q_1 \pm \sqrt{q_2 q_1}}{(q_2 - q_1)} d = \frac{(-1 \pm \sqrt{3})\cancel{\mu C}}{2\cancel{\mu C}} d$$

$$x_1 = \frac{-1+\sqrt{3}}{2} d = 0.37d \qquad x_2 = \frac{-1-\sqrt{3}}{2} d = -1.37d$$

Ma l'equazione **(1)** è valida solo per per $0 < x < d$, quindi l'unica soluzione accettabile è

$$\boxed{x_1 = 0.37d = 3.7cm}$$