VARIABILI ALEATORIE V.A. (R.V.)

V.A. DI BERNOULLI :

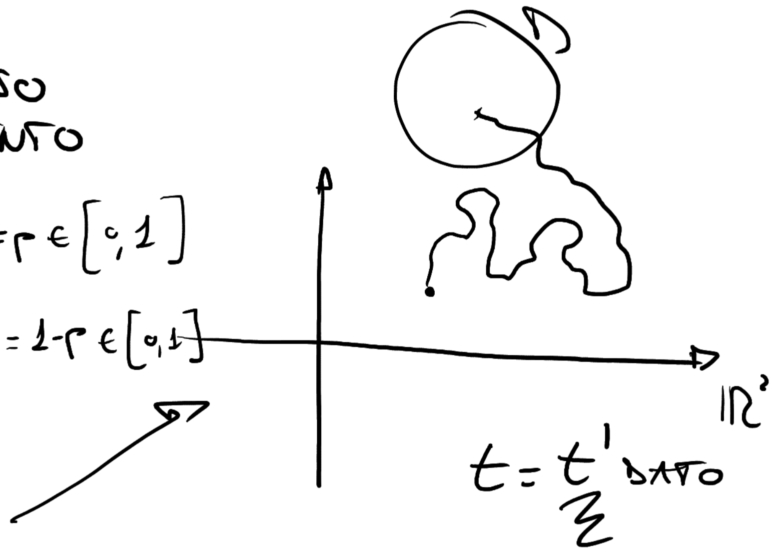
DICOTOMICI

- CONSIDERO ESPERIMENTI PROBABILISTICI CON ESITO DICOTOMICO. (CIOÈ 2 SOLE ETICHETTE)

SUCCESSO
FALLIMENTO

$$\Omega \ \{s, f\} \longrightarrow \begin{array}{l} \mathbb{P}(\{s\}) = p \in [0,1] \\ \mathbb{P}(\{f\}) = 1-p \in [0,1] \end{array}$$

$$X : \Omega \longrightarrow \mathbb{R}$$



$$X(s) = 1$$
$$X(f) = 0$$

$$I_m(X) = \{0, 1\}$$

PMF:

$$P_X(0) = \overset{q}{1-p}$$
$$P_X(1) = p$$

$$\underbrace{\mathbb{P}(\{s\})}_{\overset{\shortparallel}{p}} + \underbrace{\mathbb{P}(\{f\})}_{\overset{\shortparallel}{q}} = 1$$

$$\mathbb{P}(T) = \frac{1}{4}$$
$$\mathbb{P}(C) = \frac{3}{4}$$

X SI CHIAMA V.A. DI BERNOULLI.

PMF:

$$\underbrace{P_X(k) = p^k(1-p)^{1-k}}$$ $$k \in I_m(X)$$

$$\mathbb{P}(X=1) = \mathbb{P}(X^{-1}(1)) = \mathbb{P}(\{s\}) = p$$

$$\mathbb{P}(X=0) = \mathbb{P}(X^{-1}(0)) = \mathbb{P}(\{f\}) = (1-p)$$

$p \longrightarrow$ PARAMETRO

ES LANCIO 1 DADO EQUO

$$\{5\} = \{s\}$$

$$\{1,2,3,4,6\} = \{f\}$$

$$p = \frac{1}{6}$$

$$1-p = \frac{5}{6}$$

V.A. BINOMIALE

ESP. PROB DI TIPO DICOTOMICO.
RIPETO QUESTO ESPERIMENTO n VOLTE IN MODO INDIPENDENTE E IDENTICO. CONTO QUANTI SUCCESSI OTTENGO.

NATURALE FISSATO.

ES|| LANCIO 5 VOLTE UN DADO EQUO E CONTO QUANTI "6" HO OTTENUTO. ($n = 5$)

$$X : \Omega \longrightarrow \mathbb{R}$$

$$\omega \in \Omega, \quad \omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) \rightsquigarrow X(\omega) = \text{\# DI OCCORRENZE DI "6" NELLA STRINGA.}$$

$$X \sim \text{BINOMIALE}(\underbrace{n}_{\text{N° DI PROVE}}, \underbrace{p}_{\text{PROB DI SUCC. IN OGNI PROVA}})$$

PMF:

$$I_m(X) = \{0, 1, 2, \dots, n\}$$

$$\forall k \in I_m(X)$$

$$? \quad \mathbb{P}(X = k) = p_X(k)$$

K=0

$$p_X(0) = \mathbb{P}(X = 0) = \mathbb{P}\left(\left\{\omega \in \Omega \,/\, \omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5), \ \omega_i \in \{1,2,3,4,5\} \ i \in \{1, 2, \dots, 5\}\right\}\right)$$

$$= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \left(\frac{5}{6}\right)^5$$

NB

$$\left[(1-p)^n\right]$$

$\underline{k=1}$

$$P_X(1) = \mathbb{P}(X=1)$$

$$= \binom{5}{1} \frac{1}{6} \left(\frac{5}{6}\right)^4$$

$$\mathbb{P}\left(\left(\overset{\{6\}}{s}, \overset{\omega_i \in \{1,2,3,4,5\}}{f}, f, f, f\right)\right)$$

$$= \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{1}{6}\left(\frac{5}{6}\right)^4$$

$$\mathbb{P}\left(\left(f, f, s, f, f\right)\right)$$

$$= \frac{5}{6} \frac{5}{6} \frac{1}{6} \frac{5}{6} \frac{5}{6} = \frac{1}{6}\left(\frac{5}{6}\right)^4$$

DI QUESTE SEQUENZE NE HO $\binom{5}{1}$

$$k \in I_m(X)$$

$$P_X(k) = \mathbb{P}(X=k)$$

PENSO AD UNA SINGOLA SEQUENZA CON <u>K SUCCESSI</u> E <u>(n-k) FALLIMENTI</u>

$$p^k (1-p)^{n-k}$$

$$= \binom{n}{k} p^k (1-p)^{n-k}$$

$$1 = \mathbb{P}(\Omega)$$

$$= \mathbb{P}(X=0) + \mathbb{P}(X=1)$$

$$+ \dots + \mathbb{P}(X=n)$$

$$= \sum_{k=0}^{n} \mathbb{P}(X = k)$$

$$= \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

BINOMIO DI NEWTON

$$= \left(p + (1-p)\right)^n = 1$$

V.A. GEOMETRICA

ESPERIMENTO DICOTOMICO.

RIPETO PROVE BERNOULLIANE INDIPENDENTI E IDENTICHE FINO A CHE OTTENGO IL PRIMO SUCCESSO. CONTO QUANTE PROVE HO FATTO.

$$\overbrace{(f, f, s)}^{3} \qquad \overbrace{(s)}^{1}$$

$$\underbrace{(f, s)}_{2} \qquad \underbrace{(f, f, f, f, f, f, s)}_{7}$$

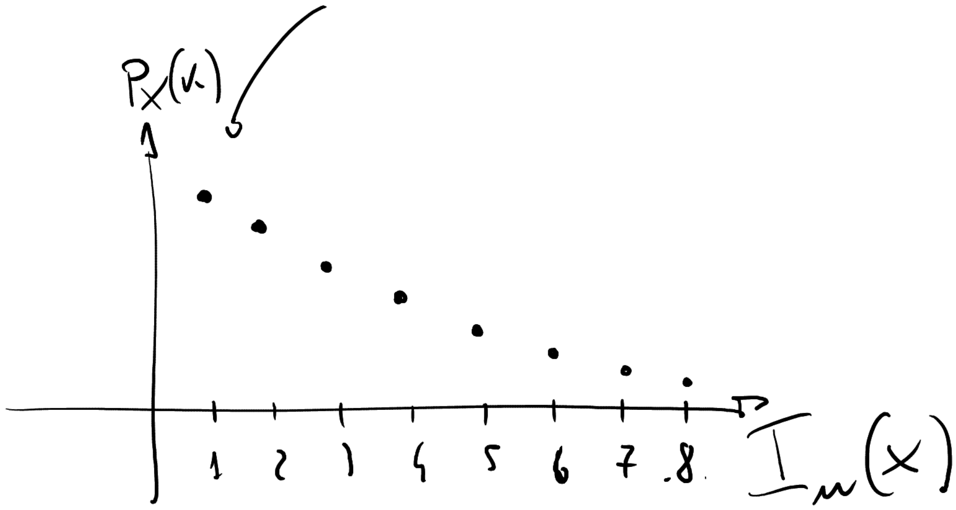
$$I_m(X) = \{1, 2, 3, \ldots\}$$

$$p_X(k) = \mathbb{P}(X = k) = p(1-p)^{k-1} \qquad k \in I_m(X)$$

$X \sim$ GEOMETRICA(p)

$$\boxed{(f, f, \ldots, f, s)}$$

K-MA PROVA



R

ANALISI STATISTICHE

~ MATLAB.

OPEN SOURCE GPL.

V.A. POISSON