$$X \sim N(\mu, \sigma^2) \qquad \mu \in \mathbb{R}, \ \sigma^2 \in \mathbb{R}_+$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad , x \in \mathbb{R}$$

$\mu = 0 \ , \sigma^2 = 1$

$$X \sim N(0, 1)$$ NORMALE STANDARD.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad , x \in \mathbb{R}$$

INVARIANZA PER TRASFORMAZIONI LINEARI:

$a, b \in \mathbb{R}$

$$Y = aX + b$$

$$Y \sim N(a\mu + b, a^2\sigma^2)$$

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## VALORE ATTESO / VARIANZA / MOMENTI DI UNA V.A. CONTINUA

**DEF** (VALORE ATTESO (O MEDIA) DI UNA V.A. CONTINUA)

LA MEDIA DI $X$ V.A. CONTINUA E' DEFINITA COSÍ:

$$\mathbb{E}X = \int_{-\infty}^{+\infty} t f_X(t)\, dt = \int_{\mathbb{R}} t f_X(t)\, dt$$

ES $X \sim \text{Exp}(\lambda)$

$$f_X(x) = \lambda e^{-\lambda x} \mathbb{1}_{\mathbb{R}_+}(x)$$

$$\mathbb{E}X = \int_{-\infty}^{+\infty} t f_X(t)\,dt$$

$$= \int_{-\infty}^{+\infty} t \lambda e^{-\lambda t} \mathbb{1}_{\mathbb{R}_+}(t)\,dt$$

$$= \int_{0}^{+\infty} t \lambda e^{-\lambda t}\,dt$$

DEF (VARIANZA DI V.A. CONTINUA)

$$\mathbb{V}\text{ar}\, X = \mathbb{E}\left[ (X - \mathbb{E}X)^2 \right]$$

$$= \int_{-\infty}^{+\infty} (t - \mathbb{E}X)^2 f_X(t)\,dt$$

DEF (MOMENTI DI V.A. CONTINUE)

$$\mu_k(X) = \mathbb{E}X^k \qquad k \in \mathbb{N}$$

$$= \int_{-\infty}^{+\infty} t^k f_X(t)\, dt$$

---

ES1 $X \sim Exp(\lambda)$

$$\mathbb{E}X = \frac{1}{\lambda}, \quad \mathbb{V}ar X = \frac{1}{\lambda^2}$$

ES2 $X \sim N(\mu, \sigma^2)$

$$\mathbb{E}X = \mu, \quad \mathbb{V}ar X = \sigma^2$$

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## FUNZIONE DI DISTRIBUZIONE CUMULATA

(ESISTE SIA PER V.A DISCRETE, SIA PER V.A CONTINUE!)

DEF: SIA $X$ V.A. (DISCRETA O CONTINUA). LA F.NE DI DISTRIBUZIONE CUMULATA (CDF) È DEFINITA COSÌ:

$$F_X : \mathbb{R} \to \mathbb{R}$$

$$x \longmapsto F_X(x) = \mathbb{P}(X \leq x)$$

SE $X$ DISCRETA

$$F_X(x) = \mathbb{P}(X \leq x) = \sum_{\substack{k \in I_m(X) \\ k \leq x}} \underbrace{p_X(k)}_{\mathbb{P}(X=k)}$$

SE $X$ CONTINUA $\quad X \in (-\infty, x]$

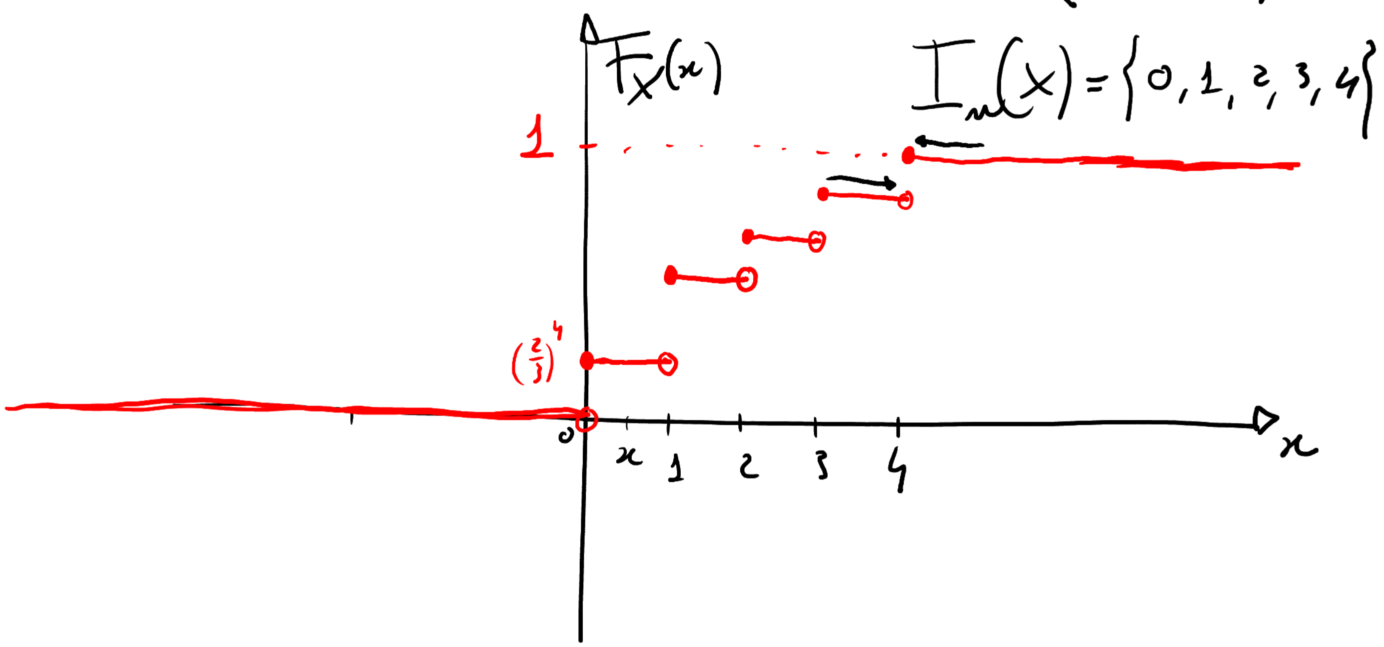
$$F_X(x) = \mathbb{P}(X \leq x) = \int_{(-\infty, x]} f_X(t)\, dt$$

$$= \int_{-\infty}^{x} f_X(t)\, dt$$

---

<u>CDF NEL CASO DI $X$ DISCRETA</u>

ES $\quad X \sim \mathrm{BIN}\left(4, \frac{1}{3}\right)$




$$F_X(x) = \mathbb{P}(X \leq x) = 0 \qquad \forall\, x \in (-\infty, 0)$$

$$F_X(0) = \mathbb{P}(X \leq 0) = p_X(0) = \binom{4}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{4-0} \qquad x = 0$$

$$= \left(\frac{2}{3}\right)^4$$

$$F_X(x) = \mathbb{P}(\underline{X \le x}) \qquad \underline{\forall\, x \in (0,1)}$$

$$= P_X(0)$$

$$F_X(1) = \mathbb{P}(X \le 1) = P_X(0) + P_X(1) \qquad \underline{x=1}$$

$$F_X(x) = P_X(0) + P_X(1) + P_X(2) + P_X(3) + P_X(4) \qquad \forall x > 4$$

$$= 1$$

<u>PROPRIETÀ</u>

① $\lim_{x \to -\infty} F_X(x) = 0$

② $\lim_{x \to +\infty} F_X(x) = 1$

③ COSTANTE A TRATTI E CONTINUA A DESTRA

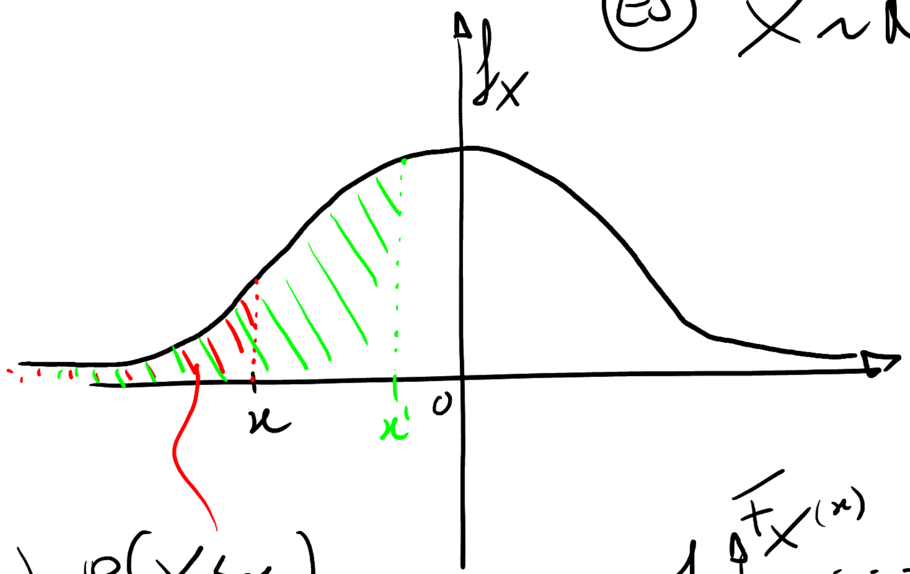
④ È NON DECRESCENTE

---

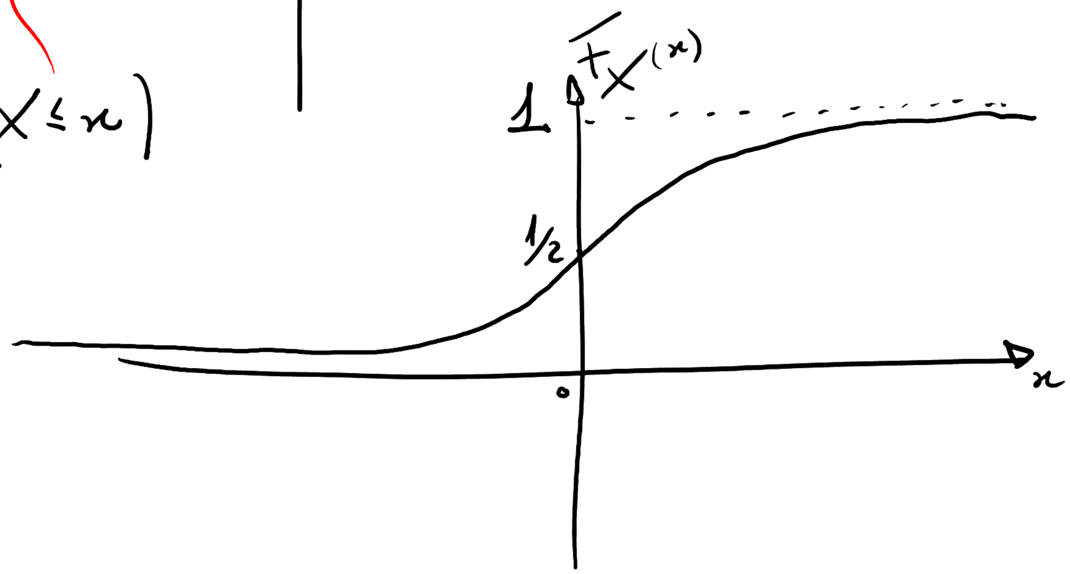
<u>~~SE~~ X V.A. CONTINUA</u>

$$F_X(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\, dt$$

ES $X \sim N(0,1)$




$F_X(x) = \mathbb{P}(X \leq x)$




PROPRIETÀ

1) $\lim_{x \to -\infty} F_X(x) = 0$

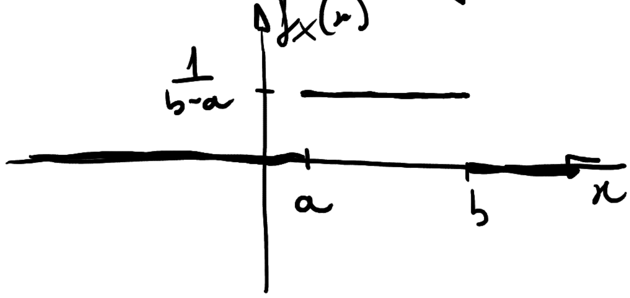
2) $\lim_{x \to +\infty} F_X(x) = 1$

3) CONTINUA

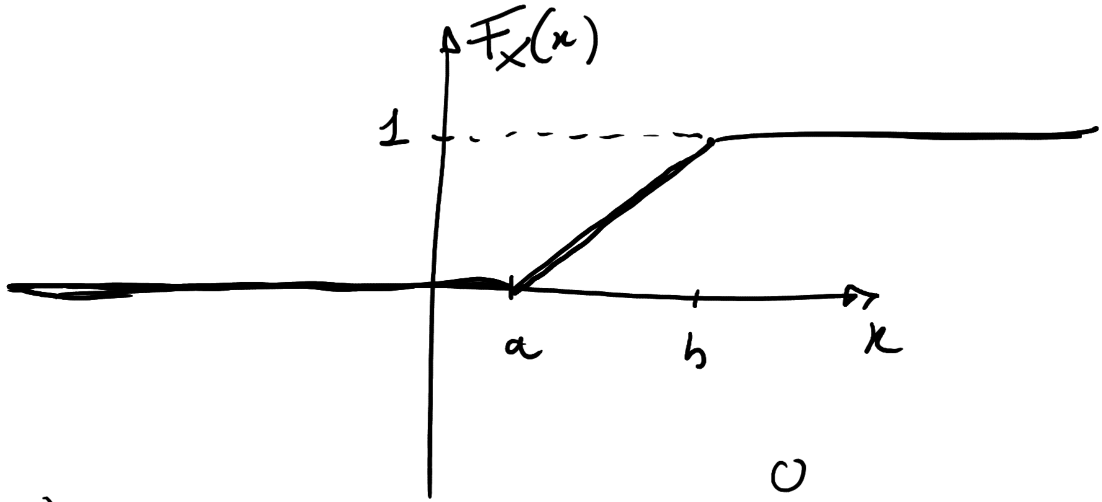
4) NON DECRESCENTE

---

ES $X \sim$ UNIFORME$([a,b])$ $\quad a < b.$



$F_X(x)$




$\forall\, x \in (a,b)$

$$\mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt = \cancel{\int_{-\infty}^{a} f_X(t)\,dt}^{\,0} + \int_{a}^{x} f_X(t)\,dt$$

$$= \frac{1}{b-a} \int_{a}^{x} dt$$

$$= \frac{1}{b-a} \Big[ t \Big]_{a}^{x}$$

$$= \frac{1}{b-a} (x - a)$$

$$= \frac{x-a}{b-a}$$

---

ES $X \sim N(\mu = 10,\ \sigma^2 = 36)$

1. $\mathbb{P}(X > 5)$
2. $\mathbb{P}(4 < X < 16)$
3. $\mathbb{P}(X \le 8)$

④ $\mathbb{P}(X < 20)$

⑤ $\mathbb{P}(X \geq 16)$

$\mathbb{E}X = 10$

$\mathbb{V}\text{AR}\, X = 36$

$\text{STDEV}\, X = 6 = \sqrt{36}$

$X \in (5, +\infty)$

$$\mathbb{P}(X > 5) = \int_5^{+\infty} f_X(t)\, dt$$

$$= 1 - \mathbb{P}(X \leq 5) = 1 - F_X(5)$$

$$= 1 - \text{PNORM}(\underset{x}{5}, \underset{\mu}{10}, \underset{\sigma}{6})$$

$\sigma \to$ DEV. STANDARD.

$$= 0.797671 6 \ldots$$

$$= 0.797\underline{7}$$

$$\mathbb{P}(4 < X < 16) = \mathbb{P}(X \leq 16) - \mathbb{P}(X \leq 4)$$

$$= \int_{-\infty}^{16} f_X(t)\, dt - \int_{-\infty}^{4} f_X(t)\, dt$$

$= \text{PNORM}(16, 10, 6) - \text{PNORM}(4, 10, 6)$

$= 0.6826895....$

$= 0,6827$

GLI ALTRI LI FATE VOI....

NB $\quad X$ DISCRETA

$$\mathbb{P}(X > 5) = 1 - \mathbb{P}(X \leq 5)$$