PMF CONGIUNTE

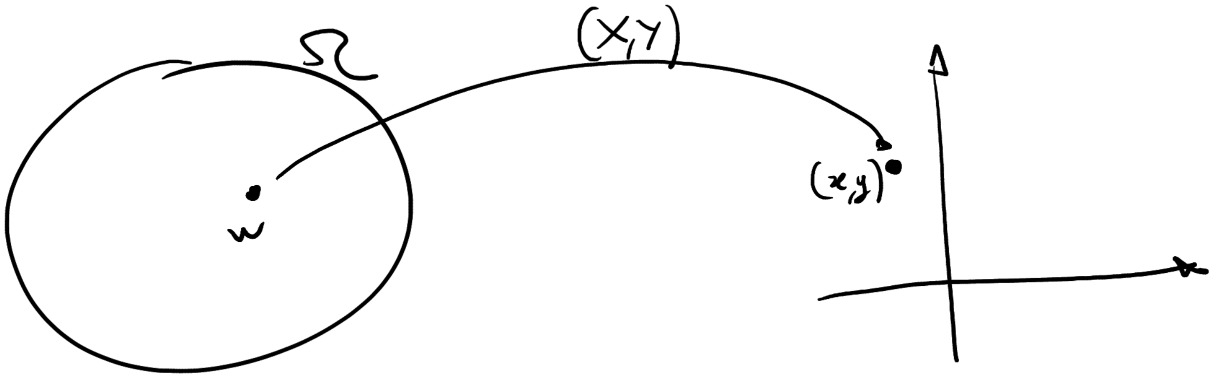
(X,Y) $\qquad$ X, Y V.A.

$$(X,Y) : \Omega \longrightarrow \mathbb{R}^2$$
$$\omega \in \Omega \rightsquigarrow (X,Y)(\omega) = (X(\omega), Y(\omega))$$

ω', ω''

$$I_m((X,Y)) \subseteq \mathbb{R}^2$$

$$(X,Y)(\omega) = (X(\omega), Y(\omega)) = (x,y)$$



ES: LANCIO UNA MONETA EQUA 5 VOLTE.

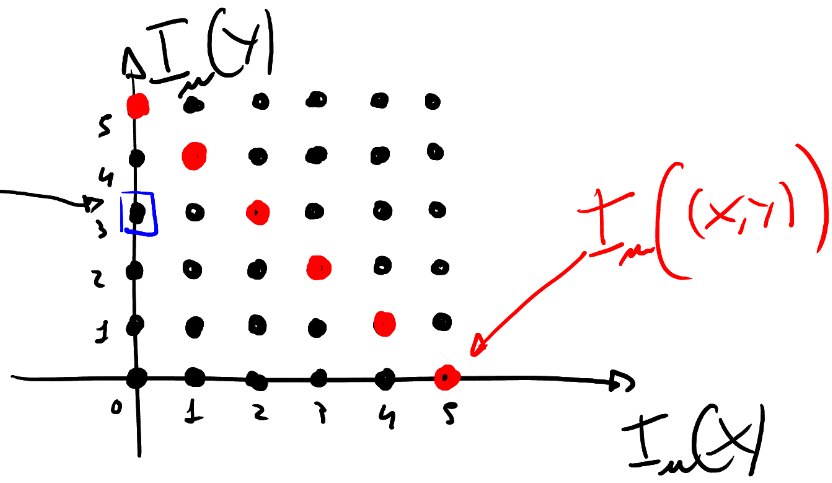
X : CONTA IL N° DI TESTE

Y : CONTA IL N° DI CROCI

(X,Y)

$$I_m(X) = \{0,1,2,3,4,5\}$$
$$I_m(Y) = \{0,1,2,3,4,5\}$$

$$I_m\big((X,Y)\big) \subseteq I_m(X) \times I_m(Y)$$



$$\mathbb{P}(X=0,\ Y=3) = 0$$

$$\parallel$$

$$\mathbb{P}\big((X,Y)=(0,3)\big) = P_{(X,Y)}(0,3)$$

DEFINIAMO LA PMF CONGIUNTA DI (X,Y):

DEF: CHIAMO PMF CONGIUNTA DI (X,Y) [OPPURE DELLE V.A. X E Y] LA FUNZIONE

$$P_{(X,Y)} : I_m\big((X,Y)\big) \longrightarrow \mathbb{R}$$

$$(x,y) \longmapsto P_{(X,Y)}(x,y) = \mathbb{P}\big((X,Y)=(x,y)\big)$$

$$= \mathbb{P}(X=x,\ Y=y)$$

NOTAZIONE

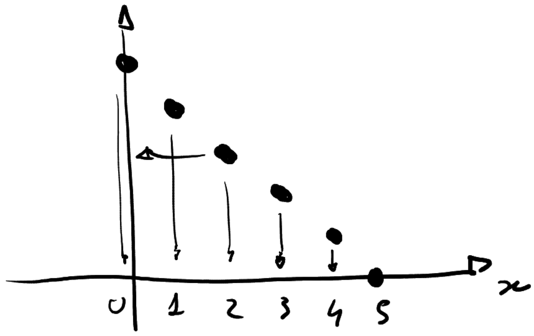
E

MI RENDO SUBITO CONTO CHE

PER DEDURRE LA PMF CONGIUNTA NON BASTANO LE PMF MARGINALI DELLE SINGOLE COMPONENTI.

IL VICEVERSA È INVECE POSSIBILE. SE ABBIAMO LA PMF CONGIUNTA POSSIAMO OTTENERE LE PMF MARGINALI:

$$P_{(X,Y)}(x,y) \qquad \forall\ (x,y) \in I_m((X,Y))$$



$$I_m(X) = \{0,1,2,3,4,5\}$$
$$I_m(Y) = \{0,1,2,3,4,5\}$$

$$P_X(x) = \mathbb{P}(\underbrace{X = x}) \qquad \forall\ x \in I_m(X)$$

$$\{X=x\} = \{X=x\} \cap \Omega$$

$$= \{X=x\} \cap \bigcup_{y \in I_m(Y)} \{Y=y\}$$

NB SIANO $y', y'' \in I_m(Y)$ ALLORA $\{Y=y'\} \cap \{Y=y''\} = \emptyset$

$$= \bigcup_{y \in I_m(Y)} \left(\underbrace{\{X=x\} \cap \{Y=y\}} \right)$$

NB SONO DISGIUNTI!

$$\{X=x\}\cap\{Y=y'\}$$
$$\cap$$
$$\{X=x\}\cap\{Y=y''\}$$
$$=$$
$$\emptyset$$

QUINDI

$$\boxed{\mathbb{P}(X=x)} = \mathbb{P}\left(\bigcup_{y\in I_m(Y)}\left(\{X=x\}\cap\{Y=y\}\right)\right)$$

$\mathbb{P}(X=x) = P_X(x)$

$$\overset{\text{ADD.}}{=} \sum_{y\in I_m(Y)} \mathbb{P}\left(\{X=x\}\cap\{Y=y\}\right)$$

$$= \sum_{y\in I_m(Y)} \mathbb{P}(X=x, Y=y) = \boxed{\sum_{y\in I_m(Y)} P_{(X,Y)}(x,y)}$$

ANALOGAMENTE:

$$P_Y(y) = \sum_{x\in I_m(X)} P_{(X,Y)}(x,y)$$

ESERCIZIO: SI CONSIDERINO LE V.A. X, Y CON PMF CONGIUNTA FATTA COSÍ:

$Y \backslash X$	1	2	3	4
5	$1/20$	$1/20$	$1/20$	0
6	$1/20$	$2/20$	$3/20$	$1/20$
7	$1/20$	$2/20$	$3/20$	$1/20$
8	0	$1/20$	$1/20$	$1/20$

$$P_{(X,Y)}(4,6) = \mathbb{P}(X=4, Y=6)$$

CON $I_m(X) = \{1,2,3,4\}$

$I_m(Y) = \{5,6,7,8\}$

ⓐ CALCOLARE LA PMF MARGINALE DI X E DI Y.

$P_X(x) \qquad \forall x \in I_m(X) = \{1,2,3,4\}$

$$P_X(1) = P_{(X,Y)}(1,5) + P_{(X,Y)}(1,6) + P_{(X,Y)}(1,7) + P_{(X,Y)}(1,8)$$

$$= \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + 0 = \frac{3}{20}$$

$$P_X(2) = P_{(X,Y)}(2,5) + P_{(X,Y)}(2,6) + P_{(X,Y)}(2,7) + P_{(X,Y)}(2,8)$$

$$= \frac{1}{20} + \frac{2}{20} + \frac{2}{20} + \frac{1}{20} = \frac{6}{20}$$

$$P_X(3) = \frac{8}{20}$$

$$P_X(4) = \frac{3}{20}$$

NB

$$\left(\frac{3}{20} + \frac{6}{20} + \frac{8}{20} + \frac{3}{20} = 1 \right)$$

$\underline{P_Y(y)}$?

$$\forall y \in I_m(Y) = \{5, 6, 7, 8\}$$

$$P_Y(5) = \sum_{x \in I_m(X)} P_{(X,Y)}(x, 5) = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + 0 = \frac{3}{20}$$

$$P_Y(6) = \frac{7}{20}$$

$$P_Y(7) = \frac{7}{20}$$

$$P_Y(8) = \frac{3}{20}$$

5 CALCOLARE

$$\mathbb{P}(X=2, Y=6) = P_{(X,Y)}(2,6) = \frac{2}{20}$$

$$\mathbb{P}(X>2, Y=6) = \mathbb{P}(X=3, Y=6) + \mathbb{P}(X=4, Y=6) = \frac{3}{20} + \frac{1}{20} = \frac{4}{20}$$

$$\mathbb{P}(X<2, Y>6) = \mathbb{P}(X=1, Y=7) + \mathbb{P}(X=1, Y=8)$$

$$= \frac{1}{20} + 0 .$$

INDIPENDENZA DI V.A.

NB A, B EVENTI.

A E B SONO INDIP. SE

1. $\mathbb{P}(A|B) = \mathbb{P}(A)$
2. $\mathbb{P}(B|A) = \mathbb{P}(B)$
3. $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

DEF: LE V.A. DISCRETE X E Y SONO INDIPENDENTI SE GLI EVENTI

$$\{X = x\} \quad \text{e} \quad \{Y = y\}$$

SONO INDIPENDENTI $\forall \; x \in I_m(X)$ E $\forall y \in I_m(Y)$

CIOÈ

$$\mathbb{P}(\{X = x\} \cap \{Y = y\}) = \mathbb{P}(\{X = x\}) \, \mathbb{P}(\{Y = y\})$$

$$\parallel$$

$$P_{(X,Y)}(x, y) = P_X(x) \cdot P_Y(y)$$

OSS

SE X, Y INDIP. ALLORA $I_m((X,Y)) = I_m(X) \cdot I_m(Y)$

INDIPENDENZA, MEDIA, VARIANZA:

SIANO X, Y DUE V.A. INDIPENDENTI.

UN VETTORE ALEATORIO (X,Y) A COMPONENTI INDIPENDENTI

PROP: $$\mathbb{E}[X \cdot Y] = \mathbb{E}X \cdot \mathbb{E}Y$$

$$\mathbb{E}\, g(X,Y) \quad , \quad g: \mathbb{R}^2 \to \mathbb{R}$$

PROP: $$\mathbb{V}\mathrm{ar}(X+Y) = \mathbb{V}\mathrm{ar}\,X + \mathbb{V}\mathrm{ar}\,Y$$