Numerical methods
HW #1
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1. a. $f(x) = X^5 + 2x - 1 = 0$; interval $(0, 1)$
$f(a_0) = f(0) = 0 + 0 - 1 = -1$
$f(b_0) = f(1) = 1 + 2 - 1 = 2$
i. $f(0) \cdot f(1) < 0$ therefore $f(x)$ has a root on interval $(0, 1)$

$P_1 = \frac{1}{2}(0 + 1) = 1/2$

$P_2 = \frac{1}{2}(a_1 + b_1)$

$f(P_1) = f(1/2) = \frac{1}{2}^5 + \frac{2}{2} - 1 = \frac{1}{32}$

$a_1 = a_0 = a$
$b_1 = P_1 = 1/2$

$P_2 = \frac{1}{2}(a + 1/2) = \frac{1}{4}$

$P_3 = \frac{1}{2}(a_2 + b_2)$
$f(P_2) = f(1/4) = \frac{1}{4}^5 + \frac{2}{4} - 1$
$= \frac{1}{1024} + \frac{1}{2} - 1 = \frac{1 - 512}{1024}$
$= \frac{-511}{1024}$

$a_2 = P_2 = 1/4$
$b_2 = b_1 = 1/2$
$P_3 = \frac{1}{2}\left(\frac{1}{4} + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{1 + 2}{4}\right) = \boxed{\frac{3}{8}}$
ii.

iii.
$f(P_3) = f\left(\frac{3}{8}\right) = \left(\frac{3}{8}\right)^5 + 2\left(\frac{3}{8}\right) - 1 = \frac{243}{32768} + \frac{6}{8} - 1$

$= \frac{243 + 24,576 - 32768}{32768} = \frac{-7,949}{32,768}$

$f(P_3) \cdot f(b_2) < 0$ so $a_3 = P_3 = \frac{3}{8}$
$f(P_3) \cdot f(a_2) > 0$ $b_3 = 1/2$

interval : $\boxed{(3/8, 1/2)}$

Q1 Q1.b

$f(x) = \cos x - x$ interval $(0,1)$

$1/2, 1$
$1/2, 3/4$
$5/8, 3/4$

$f(a_0) = f(0) = 1 - 0 = 1$

$f(b_0) = f(1) = -.460$

i. $f(a_0) \cdot f(b_0) < 0$ therefore $f(x)$ has root on interval $(0,1)$

$P_1 = \frac{1}{2}(a_0 + b_0) = \frac{1}{2}(0+1) = \frac{1}{2}$

$f(P_1) = .877 - \frac{1}{2} = .378$

$a_1 = \frac{1}{2}$ $b_1 = b_0 = 1$

$P_2 = \frac{1}{2}(a_1 + b_1) = \frac{1}{2}(\frac{1}{2} + 1) = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

$f(P_2) = f(\frac{3}{4}) = \cos(\frac{3}{4}) - \frac{3}{4} = -.018$

$a_2 = \frac{1}{2}$ $b_2 = \frac{3}{4}$

$P_3 = \frac{1}{2}(a_2 + b_2) = \frac{1}{2}(\frac{2+3}{4}) = \boxed{\frac{5}{8}}$

$f(P_3) = f(\frac{5}{8}) = .136$

$P_3 = a_3 = \frac{5}{8}$; $b_3 = \frac{3}{4}$ because $f(P_3) \cdot f(b_3) < 0$

Q.3.a

$$f(x) = x^5 + 2x - 1 = 0 \qquad a_0 = 0 \; ; \; b_0 = 1$$

$f(a_0) = f(0) = 0 + 0 - 1 = -1$

$f(b_0) = f(1) = 1 + 2 - 1 = 2$

$\Rightarrow f(a_0) \cdot f(b_0) < 0$ therefore $f(x)$ has a root on the interval

$$P_1 = \frac{a_0 f(b_0) - f(a_0) b_0}{f(b_0) - f(a_0)} = \frac{(0 \cdot 2) - (-1 \cdot 1)}{2 + 1} = \frac{1}{3}$$

b) is $f(P_1) = \frac{1}{3}^5 + 2\left(\frac{1}{3}\right) - 1 = \frac{1 + 162 - 243}{243}$

$$= \frac{-80}{243}$$

$a_1 = \frac{-80}{243} = P_1 = \frac{1}{3} \quad b_1 = b_0 = 1$

$$P_2 = \frac{a_1 f(b_1) - f(a_1) b_1}{f(b_1) - f(a_1)} = \frac{\left(\frac{1}{3} \cdot 2\right) - \left(\frac{-80}{243} \cdot 1\right)}{2 + \frac{80}{243}} = .428$$

$f(P_2) = f(.428) = .428^5 + 2(.428) - 1$
$$= -.13$$

$a_2 = P_2 = .428 \qquad b_2 = 1$

$$P_3 = \frac{a_2 f(b_2) - f(a_2) b_2}{f(b_2) - f(a_2)} = \frac{(.428 \times 2) - (-.13 \times 1)}{2 - (-.13)} = \boxed{.463}$$

$f(P_3) = .463^5 + 2(.463) - 1 = -.053$

$a_3 = .463 \qquad b_3 = b_2 = 1$

Interval : $(.463, 1)$ //

Q.3.b

$f(x) = \cos x - x$ interval $(0,1)$

$f(a_0) = f(0) = 1$

$f(b_0) = f(1) = -.460$

$\Rightarrow f(a_0) \cdot f(b_0) < 0$ therefore $f(x)$ has a root on interval $(0,1)$

$P_1 = \dfrac{a_0 f(b_0) - f(a_0) b_0}{f(b_0) - f(a_0)} = \dfrac{(0 \cdot -.460) - (1 \cdot 1)}{-.460 - 1} = .685$

$f(P_1) = .089$

$a_1 = \dfrac{.089}{.685} \qquad b_1 = 1$

$P_2 = \dfrac{a_1 f(b_1) - f(a_1) b_1}{f(b_1) - f(a_1)} = \dfrac{(.685 \times -.46) - (.089 \times 1)}{-.46 - .089}$

$\qquad\qquad\qquad\qquad\qquad = .736$

$f(P_2) = .005$

$a_2 = P_2 = .736 \qquad b_2 = 1$

$P_3 = \dfrac{a_2 f(b_2) - f(a_2) b_2}{f(b_2) - f(a_2)} = \dfrac{(.736 \times \cancel{1} \overset{-.46}{}) - (.005 \times 1)}{-.46 - .005} = \boxed{.739}$

$f(P_3) = 4.12 \times 10^{-4}$

$a_3 = P_3 = .739 \qquad b_3 = b_2 = 1 \qquad$ because $f(P_3) \cdot f(b_2) < 0$

interval: $(.739, 1)$ //

Q2.a

```
%Luc Rulinda, ME 58100, HW 1 Q2
f = @(x) exp(x)+x^2-x-4;
%f = @(x) x^3-x^2-10*x+7;
a= 0;
b= 2;
fprintf('With a0: %d and b0: %d... \n', a,b);
disp('Iteration #    (a,f(a))           (b,f(b))        (p,f(p))')
chi= 1e-6;

fa= f(a);
fb= f(b);
imax= 100;
k=0;

while (i<=imax)
    p=0.5*(a+b);
    fp=f(p);
    fprintf('%6d    (%2.7f,%2.7f)    (%2.10f,%2.7f)    (%2.7f,%2.7f)\n', i, a, fa, b, fb, p, fp);

    if fp*fa<0
        b=p;
        fb=fp;
    else
        a=p;
        fa=fp;
    end
    I=i;
    if (abs(fp)<=chi)
        iteration=i;
        i=imax;
        k=1;
    end
    i=i+1;
end
fprintf('Final x value: %.7f \n', p);
fprintf('Total # of iterations: %5d \n', I);
```

```
With a0: 0 and b0: 2...
Iteration #    (a,f(a))              (b,f(b))            (p,f(p))
     0    (0.0000000,-3.0000000)    (2.0000000000,5.3890561)    (1.0000000,-1.2817182)
     1    (1.0000000,-1.2817182)    (2.0000000000,5.3890561)    (1.5000000,1.2316891)
     2    (1.0000000,-1.2817182)    (1.5000000000,1.2316891)    (1.2500000,-0.1971570)
     3    (1.2500000,-0.1971570)    (1.5000000000,1.2316891)    (1.3750000,0.4707017)
     4    (1.2500000,-0.1971570)    (1.3750000000,0.4707017)    (1.3125000,0.1256070)
     5    (1.2500000,-0.1971570)    (1.3125000000,0.1256070)    (1.2812500,-0.0385101)
     6    (1.2812500,-0.0385101)    (1.3125000000,0.1256070)    (1.2968750,0.0428578)
     7    (1.2812500,-0.0385101)    (1.2968750000,0.0428578)    (1.2890625,0.0020020)
     8    (1.2812500,-0.0385101)    (1.2890625000,0.0020020)    (1.2851563,-0.0182969)
     9    (1.2851563,-0.0182969)    (1.2890625000,0.0020020)    (1.2871094,-0.0081581)
    10    (1.2871094,-0.0081581)    (1.2890625000,0.0020020)    (1.2880859,-0.0030807)
    11    (1.2880859,-0.0030807)    (1.2890625000,0.0020020)    (1.2885742,-0.0005400)
    12    (1.2885742,-0.0005400)    (1.2890625000,0.0020020)    (1.2888184,0.0007308)
    13    (1.2885742,-0.0005400)    (1.2888183594,0.0007308)    (1.2886963,0.0000954)
    14    (1.2885742,-0.0005400)    (1.2886962891,0.0000954)    (1.2886353,-0.0002223)
    15    (1.2886353,-0.0002223)    (1.2886962891,0.0000954)    (1.2886658,-0.0000635)
    16    (1.2886658,-0.0000635)    (1.2886962891,0.0000954)    (1.2886810,0.0000159)
    17    (1.2886658,-0.0000635)    (1.2886810303,0.0000159)    (1.2886734,-0.0000238)
    18    (1.2886734,-0.0000238)    (1.2886810303,0.0000159)    (1.2886772,-0.0000039)
    19    (1.2886772,-0.0000039)    (1.2886810303,0.0000159)    (1.2886791,0.0000060)
    20    (1.2886772,-0.0000039)    (1.2886791229,0.0000060)    (1.2886782,0.0000011)
    21    (1.2886772,-0.0000039)    (1.2886781693,0.0000011)    (1.2886777,-0.0000014)
    22    (1.2886777,-0.0000014)    (1.2886781693,0.0000011)    (1.2886779,-0.0000002)
```

Q2.b

```
>> numMeth_hw1_q2
With a0: 0 and b0: 2...
Iteration #    (a,f(a))              (b,f(b))            (p,f(p))
     0    (0.0000000,7.0000000)    (2.0000000000,-9.0000000)    (1.0000000,-3.0000000)
     1    (0.0000000,7.0000000)    (1.0000000000,-3.0000000)    (0.5000000,1.8750000)
     2    (0.5000000,1.8750000)    (1.0000000000,-3.0000000)    (0.7500000,-0.6406250)
     3    (0.5000000,1.8750000)    (0.7500000000,-0.6406250)    (0.6250000,0.6035156)
     4    (0.6250000,0.6035156)    (0.7500000000,-0.6406250)    (0.6875000,-0.0227051)
     5    (0.6250000,0.6035156)    (0.6875000000,-0.0227051)    (0.6562500,0.2894592)
     6    (0.6562500,0.2894592)    (0.6875000000,-0.0227051)    (0.6718750,0.1331291)
     7    (0.6718750,0.1331291)    (0.6875000000,-0.0227051)    (0.6796875,0.0551486)
     8    (0.6796875,0.0551486)    (0.6875000000,-0.0227051)    (0.6835938,0.0162057)
     9    (0.6835938,0.0162057)    (0.6875000000,-0.0227051)    (0.6855469,-0.0032537)
    10    (0.6835938,0.0162057)    (0.6855468750,-0.0032537)    (0.6845703,0.0064750)
    11    (0.6845703,0.0064750)    (0.6855468750,-0.0032537)    (0.6850586,0.0016104)
    12    (0.6850586,0.0016104)    (0.6855468750,-0.0032537)    (0.6853027,-0.0008217)
    13    (0.6850586,0.0016104)    (0.6853027344,-0.0008217)    (0.6851807,0.0003943)
    14    (0.6851807,0.0003943)    (0.6853027344,-0.0008217)    (0.6852417,-0.0002137)
    15    (0.6851807,0.0003943)    (0.6852416992,-0.0002137)    (0.6852112,0.0000903)
    16    (0.6852112,0.0000903)    (0.6852416992,-0.0002137)    (0.6852264,-0.0000617)
    17    (0.6852112,0.0000903)    (0.6852264404,-0.0000617)    (0.6852188,0.0000143)
    18    (0.6852188,0.0000143)    (0.6852264404,-0.0000617)    (0.6852226,-0.0000237)
    19    (0.6852188,0.0000143)    (0.6852226257,-0.0000237)    (0.6852207,-0.0000047)
    20    (0.6852188,0.0000143)    (0.6852207184,-0.0000047)    (0.6852198,0.0000048)
    21    (0.6852198,0.0000048)    (0.6852207184,-0.0000047)    (0.6852202,0.0000001)
Final x value: 0.6852202
Total # of iterations:    21
```

Q4.a

```matlab
%Luc Rulinda, ME 58100, HW 1 Q4a
f = @(x) exp(x)+x^2-x-4;
%f =@(x) x^3-x^2-10*x+7;
a= 0;
b= 2;
fprintf('With a0: %d and b0: %d... \n', a,b);
disp('Iteration #    (a,f(a))          (b,f(b))        (p,f(p))')
chi= 1e-6;

fa= f(a);
fb= f(b);
imax= 100;
k=0;

while (i<=imax)
    p=(a*fb-b*fa)/(fb-fa);
    fp=f(p);
    fprintf('%6d    (%2.7f,%2.7f)    (%2.10f,%2.7f)    (%2.7f,%2.7f)\n', i, a, fa, b, fb, p, fp);
    if fp*fa<0
        b=p;
        fb=f(b);
    else
        a=p;
        fa=f(a);
    end
    I=i;
    if (abs(fp)<=chi)
        iteration=i;
        i=imax;
        k=1;
    end
    i=i+1;
end
fprintf('Final x value: %.7f \n', p);
fprintf('Total # of iterations: %5d \n', I);

if i==imax+1 && k==0
fprintf ('No root has been found for the number of iterations used imax= %3.0f \n',imax);
end
```

```
>> numMeth_hw1_q4
With a0: 0 and b0: 2...
Iteration #    (a,f(a))              (b,f(b))              (p,f(p))
      0    (0.0000000,-3.0000000)    (2.0000000000,5.3890561)    (0.7152175,-2.1590500)
      1    (0.7152175,-2.1590500)    (2.0000000000,5.3890561)    (1.0827150,-0.9577582)
      2    (1.0827150,-0.9577582)    (2.0000000000,5.3890561)    (1.2211367,-0.3389216)
      3    (1.2211367,-0.3389216)    (2.0000000000,5.3890561)    (1.2672217,-0.1103978)
      4    (1.2672217,-0.1103978)    (2.0000000000,5.3890561)    (1.2819317,-0.0349887)
      5    (1.2819317,-0.0349887)    (2.0000000000,5.3890561)    (1.2865637,-0.0109928)
      6    (1.2865637,-0.0109928)    (2.0000000000,5.3890561)    (1.2880161,-0.0034442)
      7    (1.2880161,-0.0034442)    (2.0000000000,5.3890561)    (1.2884708,-0.0010782)
      8    (1.2884708,-0.0010782)    (2.0000000000,5.3890561)    (1.2886131,-0.0003374)
      9    (1.2886131,-0.0003374)    (2.0000000000,5.3890561)    (1.2886577,-0.0001056)
     10    (1.2886577,-0.0001056)    (2.0000000000,5.3890561)    (1.2886716,-0.0000330)
     11    (1.2886716,-0.0000330)    (2.0000000000,5.3890561)    (1.2886760,-0.0000103)
     12    (1.2886760,-0.0000103)    (2.0000000000,5.3890561)    (1.2886773,-0.0000032)
     13    (1.2886773,-0.0000032)    (2.0000000000,5.3890561)    (1.2886778,-0.0000010)
     14    (1.2886778,-0.0000010)    (2.0000000000,5.3890561)    (1.2886779,-0.0000003)
Final x value: 1.2886779
Total # of iterations:    14
```

Q4.b

```matlab
%Luc Rulinda, ME 58100, HW 1 Q4b
%f = @(x) exp(x)+x^2-x-4;
f =@(x) x^3-x^2-10*x+7;
a= 0;
b= 2;
fprintf('With a0: %d and b0: %d... \n', a,b);
disp('Iteration #    (a,f(a))              (b,f(b))         (p,f(p))')
chi= 1e-6;

fa= f(a);
fb= f(b);
imax= 100;
k=0;

while (i<=imax)
    p=(a*fb-b*fa)/(fb-fa);
    fp=f(p);
    fprintf('%6d    (%2.7f,%2.7f)    (%2.10f,%2.7f)    (%2.7f,%2.7f)\n', i, a, fa, b, fb, p, fp);
    if fp*fa<0
        b=p;
        fb=f(b);
    else
        a=p;
        fa=f(a);
    end
    I=i;
    if (abs(fp)<=chi)
        iteration=i;
        i=imax;
        k=1;
    end
    i=i+1;
end
fprintf('Final x value: %.7f \n', p);
fprintf('Total # of iterations: %5d \n', I);

if i==imax+1 && k==0
fprintf ('No root has been found for the number of iterations used imax= %3.0f \n! imax).
```

```
>> numMeth_hw1_q4
With a0: 0 and b0: 2...
Iteration #    (a,f(a))              (b,f(b))             (p,f(p))
     0    (0.0000000,7.0000000)    (2.0000000000,-9.0000000)    (0.8750000,-1.8457031)
     1    (0.0000000,7.0000000)    (0.8750000000,-1.8457031)    (0.6924266,-0.0717333)
     2    (0.0000000,7.0000000)    (0.6924265842,-0.0717333)    (0.6854028,-0.0018189)
     3    (0.0000000,7.0000000)    (0.6854028380,-0.0018189)    (0.6852248,-0.0000452)
     4    (0.0000000,7.0000000)    (0.6852247866,-0.0000452)    (0.6852204,-0.0000011)
     5    (0.0000000,7.0000000)    (0.6852203602,-0.0000011)    (0.6852203,-0.0000000)
Final x value: 0.6852203
Total # of iterations:     5
```