Minimum fuel round trip from a L_2 Earth-Moon Halo orbit to Asteroid 2006 RH_{120}

M. Chyba and T. Haberkorn and R. Jedicke

Abstract The goal of this paper is to design a spacecraft round trip transfer from a parking orbit to asteroid 2006 RH_{120} during its geocentric capture while maximizing the final spacecraft mass or, equivalently, minimizing the delta-v. The spacecraft begins in a halo ”parking” orbit around the Earth-Moon L_2 libration point. The round-trip transfer is composed of three portions: the approach transfer from the parking orbit to 2006 RH_{120}, the rendezvous ”lock-in” portion with the spacecraft in proximity to and following the asteroid orbit, and finally the return transfer to L_2. An indirect method based on the maximum principle is used for our numerical calculations. To partially address the issue of local minima we restrict the control strategy to reflect an actuation corresponding to up to three thrust arcs during each portion of the transfer. Our model is formulated in the circular restricted four-body problem (CR4BP) with the Sun considered as a perturbation of the Earth-Moon circular restricted three body problem. A shooting method is applied to numerically optimize the round trip transfer, and the 2006 RH_{120} rendezvous and departure points are optimized using a time discretization of the 2006 RH_{120} trajectory.

Key words: Asteroid 2006 RH_{120}, Sun perturbed Earth-Moon Bicircular Restricted Four Body Problem, Geometric optimal control, Shooting method.

Monique Chyba
Department of Mathematics, University of Hawaii, Honolulu, HI, 96822, U.S.A. e-mail: chyba@hawaii.edu

Thomas Haberkorn
MAPMO-Fédération Denis Poisson, University of Orléans, 45067 Orléans, France, e-mail: thomas.haberkorn@univ-orleans.fr

Robert Jedicke
Institute for Astronomy, University of Hawaii, Honolulu, HI 96822, U.S.A. e-mail: jedicke@hawaii.edu

1 Introduction

A population of geocentric near Earth asteroids, Temporarily Captured Orbiters (TCOs), may be the lowest delta-v targets for spacecraft missions. The TCOs are defined by the following simultaneous conditions [1]:

1. negative geocentric Keplerian energy E_{planet};
2. geocentric distance less than three Earth Hill radii ($3R_{H,\oplus} \sim 0.03$ AU);
3. make at least one full revolution around the Earth in the Sun-Earth rotating frame.

They are often referred to as *minimoons* but we will use TCO here. In [1], the authors generated, pruned and integrated a very large random sample of ”test-particles” from the near Earth object (NEO) population to determine the steady-state orbit distribution of the TCO population. Of the 10 million integrated test-particles over 16,000 became TCOs. These capture statistics imply that at any moment there is a one meter diameter TCO orbiting Earth. The advantages presented by the TCOs for space missions have been discussed in several papers [1, 2, 3, 4] and we will not repeat the arguments here. In particular, their location within the Earth-Moon system is very attractive and their geocentric energy will reduce the thrust required to reach them relative to otherwise equivalent heliocentric objects passing through cis-lunar space.

The orbits of the TCOs presented in [1] exhibit a wide range of behaviors with capture duration ranging from a few weeks to a few months. In this paper we focus on designing a round trip minimum fuel transfer to the only known TCO, 2006 RH_{120}. It is a few meters diameter asteroid that was discovered by the Catalina Sky Survey in September 2006. Its orbit from June 1st 2006 to July 31st 2007 is represented on Figures 1 and 2. The period June 2006 to July 2007 was chosen to include the time during which the asteroid was energetically bound in the Earth-Moon system. 2006 RH_{120} approaches as close as 0.72 Earth-Moon distance from the Earth-Moon barycenter.

Motivated by the successful Artemis mission and prior numerical simulations on the approach transfer [5], while awaiting the discovery of a suitable TCO we assume the spacecraft is hibernating on a halo orbit around the Earth-Moon L_2 libration point with a z-excursion of 5000 km (Figure 3). This orbit is similar to those successfully used for the Artemis mission [6, 7]. The highest z-coordinate of the halo orbit is $q_{HaloL_2} \approx (1.119, 0, 0.013, 0, 0.180, 0)$ and its period is $t_{HaloL_2} \approx 3.413$ normalized time units or 14.84 days.

The round trip is composed of a approach transfer to bring the spacecraft to 2006 RH_{120}, followed by a lock-in phase where the spacecraft travels with the asteroid, and finally a return transfer to the hibernating orbit. Clearly, this optimization problem presents a very large set of variables including the departure time, the target rendezvous point, the lock-in duration on the asteroid and the return transfer duration. To simplify our approach we first decompose the round trip into a approach transfer and a return transfer that we address separately. We consider the global mission once these two optimization sub-problems are solved.

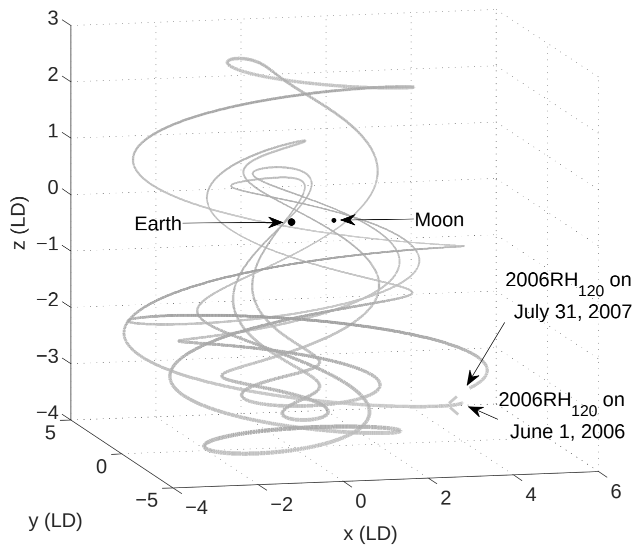

Fig. 1 *2006 RH_{120} trajectory between June 1^{st} 2006 and July 31^{st} 2007 in the Earth-Moon rotating frame. (Ephemerides generated using the Jet Propulsion Laboratory's HORIZONS database.)*

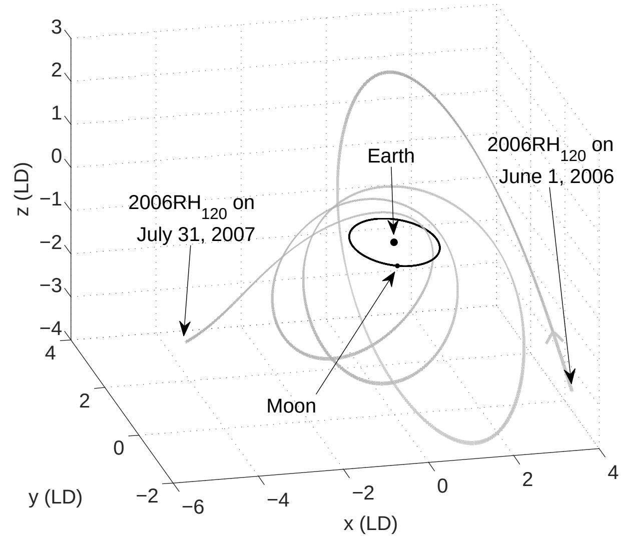

Fig. 2 *2006 RH_{120} trajectory between June 1^{st} 2006 and July 31^{st} 2007 in an Earth-centered inertial frame. (Ephemerides generated using the Jet Propulsion Laboratory's HORIZONS database.)*

The spacecraft's departure time from the hibernating orbit must occur after the TCO's detection time. We will vary the departure time or/and the detection time

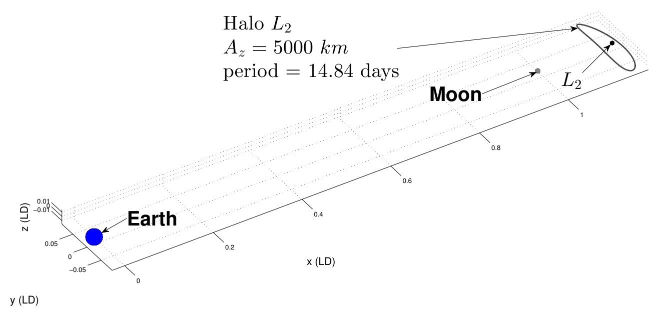

Fig. 3 *Parking halo orbit (in the EM rotating frame) around L_2 with a z-excursion of 5000 km.*

to analyze its impact on fuel consumption even though it is actually known for 2006 RH_{120}. In addition to the transfer duration discretization, we also discretize the TCO orbit to optimize the rendezvous point. The combination of discretizations results in more than 5000 optimization problems.

The return transfer optimizes the trajectory from 2006 RH_{120} to the Earth-Moon L_2 libration point rather than the halo orbit to reduce the number of calculations (it can be expanded easily if necessary). This problem will be solved with fixed transfer durations and we will study the influence of the departure point on the 2006 RH_{120} trajectory and of the transfer duration on the fuel consumption. As for the approach transfer, there are more than 5000 return transfers to be considered.

The global round trip will be analyzed based on the results of the rendezvous and return transfers. The best transfers in each case can be connected together to minimize the fuel consumption with the additional, obvious constraint that the return trip must begin after the rendezvous trip ends. The result provides us the optimal duration of the lock-in phase for mission planning purposes.

We use indirect shooting methods [3, 4, 5, 8] to solve the optimization problems associated with our mission. The main difficulty with these methods is the initialization of the algorithm and the existence of numerous local minima. To partially reduce the number of local minima, we fix the control structure to be composed of at most three constant norm thrust arcs separated in time by 2 ballistic arcs. We have two motivations for imposing this control structure. First, preliminary calculations on a set of random TCOs with a free control structure for a similar control problem in [5] provided results mimicking transfers with at most three impulsive thrust maneuvers: more thrusting periods reduced the cost only rarely. Second, since the parking orbit is not a periodic orbit of the CR4BP we have to impose an initial impulse to leave the halo orbit. Indeed, starting the transfer with a ballistic arc is extremely unlikely to be efficient or even possible since the time duration of the transfer is fixed. This strongly suggests a strategy with one impulse to leave the hibernating orbit, a second one to redirect the spacecraft toward the rendezvous point, and a final one to match the position and the velocity of the asteroid at rendezvous.

Based on the GRAIL spacecraft's characteristics we consider a chemical propulsion spacecraft with maximum thrust $T_{\max} = 22\ N$, specific impulse $I_{sp} = 230s$, and initial mass $m_0 = 350\ kg$. Information about monopropellant engine types can be found in [9].

The novelty of this work is at least threefold. First, the target object is a TCO with a "crazy straw" geocentric trajectory quite different from the periodic elliptical orbits considered in the literature of minimum fuel transfers. Second, we consider synchronized transfers to produce a global round trip mission and add a practical detection constraint. The existence of efficient round trip transfers enables multiple rendezvous scenario with successive TCOs to maximize the spacecraft utility. Third, the calculation of all the possible approach transfers with respect to departure time and rendezvous point, and all possible return transfers with respect to the departure point on the TCO trajectory, makes this study comprehensive. The trade-off is our restricting the mission to a three-thrust strategy.

The techniques presented here can be applied to any TCO. Asteroid 2006 RH_{120} was chosen as a test-bed to illustrate the algorithm since it is currently the only known TCO. Future discoveries of TCOs should be common as [2] predict that the Large Synoptic Survey Telescope (LSST) could detect about 1.5 TCOs/lunation — more than a dozen per year. This would provide ample population candidates for a real asteroid space mission. Furthermore [10] have shown that LSST will efficiently detect the largest TCOs that are arguably the most interesting spacecraft targets.

The outline of the paper is as follows: Section 2 presents the equations of motion used as the dynamics of the optimal control problems. Section 3 gives the exact formulation of the two optimal control problems, the necessary conditions satisfied by their solutions, and the numerical method used for the calculations. Section 4 provides the numerical results for the two optimal transfers and discusses the complete round trip problem. We conclude with a discussion of future research opportunities.

2 Equations of motion

We expected that the spacecraft would remain within or near Earth's Hill sphere during its entire mission in which case the Earth-Moon circular restricted three body problem (CR3BP) would provide a good first-order approximation for the equations of motion (see [11]). In the CR3BP setting the spacecraft has negligible mass and responds to the gravitational fields of 2 primaries, P_1 and P_2, of respective masses M_1 and M_2 with $M_1 > M_2$. In addition, the two primaries follow circular orbits around their barycenter. A normalization to obtain a dimensionless system is introduced by setting the mass unit to $M_1 + M_2$, the unit of length as the constant distance between the two primaries, and the unit of time so that the period of the primaries around their barycenter is 2π. The only parameter in the model is then $\mu = M_1/(M_1 + M_2)$. Table 1 provides numerical values for some of the parameters of our CR3BP model.

CR3BP parameters		Sun Perturbed parameters	
μ	0.01215361914	μ_S	$3.289 \cdot 10^5$
1 norm. dist. (LD)	384400 km	r_S	$3.892 \cdot 10^2$
1 norm. time	104.379 h	ω_S	-0.925 rad/norm. time

Table 1 Numerical values for the CR3BP and Sun Perturbed CR3BP.

Finally, we introduce a rotating reference frame with origin at the center of mass with the x-axis oriented from P_1 to P_2. The z-axis is orthogonal to the orbital plane of the 2 primaries and the y-axis is orthogonal to both in a conventional right-hand coordinate system. In this reference frame the potential energy of the spacecraft with position and velocity $q = (x, y, z, \dot{x}, \dot{y}, \dot{z})$ is

$$\Omega_3(x,y,z) = \frac{x^2+y^2}{2} + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} + \frac{\mu(1-\mu)}{2},$$

with ρ_1 (resp. ρ_2) the distance from the spacecraft to the first (resp. second) primary, that is

$$\rho_1 = \sqrt{(x-\mu)^2+y^2+z^3}, \quad \rho_2 = \sqrt{(x-1+\mu)^2+y^2+z^2}.$$

The ballistic motion of the spacecraft in our 3-body CR3BP is then given by

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_3}{\partial x}, \qquad \ddot{y} + 2\dot{x} = \frac{\partial \Omega_3}{\partial y}, \qquad \ddot{z} = \frac{\partial \Omega_3}{\partial z}. \tag{1}$$

It is well known that there exists 5 equilibrium points in this system, the so called Lagrange points L_1, L_2, L_3, L_4 and L_5. The points $L_{1,2,3}$ are distributed along the x-axis of the frame while L_4 and L_5 form an equilateral triangle with the two primaries in the xy-plane. We will focus on L_2, motivated by the existence of periodic orbits around this point that can be used as hibernating orbits for a spacecraft awaiting detection of a TCO. We could also choose L_1 or L_3 but preliminary computation suggested that L_2 is a better choice. Halo orbits around L_2 are periodic orbits that are isomorphic to circles (see [11] for a proof of their existence and how to compute them).

Even though the TCO's orbit is in a vicinity of the Earth-Moon system during its capture it can be as far as 5 normalized distance units from the CR3BP origin. Our preliminary calculations showed that efficient transfers might require that the spacecraft range even further from the CR3BP origin to maximize the thrust impact on the spacecraft's motion. To increase the accuracy of our model in these cases we use an extended CR3BP as in [12] and employ a Sun-perturbed Earth-Moon CR3BP in which the Sun follows a circular orbit around the Earth-Moon center of mass without modifying their circular orbits. In this 4-body case the potential energy of the spacecraft is $\Omega_4 = \Omega_3 + \Omega_S$ with

$$\Omega_S(x,y,z,\theta) = \frac{\mu_S}{r_S} - \frac{\mu_S}{\rho_S^2}(x\cos\theta + y\sin\theta), \tag{2}$$

where θ is the time dependent angular position of the Sun in the rotating frame, r_S is the constant distance from the Sun to the center of the reference frame, ρ_S is the distance from the spacecraft to the Sun ($\rho_S = \sqrt{(x - r_S\cos\theta)^2 + (y - r_S\sin\theta)^2 + z^2}$) and μ_S is the Sun's normalized mass ($\mu_S = M_{\text{Sun}}/(M_1 + M_2)$). As the Sun is assumed to follow a circular orbit in the rotating frame its angular position is $\theta(t) = \theta_0 + t\omega_S$ with ω_S the angular speed of the circular orbit and θ_0 the angular position of the Sun at time 0. The equations of motion take the same form as in (1) but with the perturbed potential Ω_4 replacing Ω_3. The values of the new parameters are given in Table 1.

We assume the spacecraft thrusters can produce a thrust of at most $T_{\max}$ Newton in any direction of $\mathbb{R}^3$. With the thrust direction represented by $u = (u_1, u_2, u_3) \in \bar{B}(0,1) \subset \mathbb{R}^3$, and when $m(\cdot)$ is the mass of the spacecraft, the controlled equations of motion are an affine control system

$$\dot{q}(t) = F_0(t, q(t)) + \frac{\tilde{T}_{\max}}{m(t)} \sum_{i=1}^{3} F_i u_i(t) \tag{3}$$

where the drift is given by:

$$F_0(t,q) = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ 2\dot{y} + x - \frac{(1-\mu)(x+\mu)}{\rho_1^3} - \frac{\mu(x-1+\mu)}{\rho_2^3} - \frac{(x - r_S\cos\theta(t))\mu_S}{\rho_S^3(t)} - \frac{\mu_S\cos\theta(t)}{r_S^2} \\ -2\dot{x} + y - \frac{(1-\mu)y}{\rho_1^3} - \frac{\mu y}{\rho_2^3} - \frac{(y - r_S\sin\theta(t))\mu_S}{\rho_S^3(t)} - \frac{\mu_S\sin\theta(t)}{r_S^2} \\ -\frac{(1-\mu)z}{\rho_1^3} - \frac{\mu z}{\rho_2^3} - \frac{z\mu_S}{\rho_S^3(t)} \end{pmatrix}. \tag{4}$$

The vector field F_1 (resp. F_2 and F_3) is the vector of the canonical base e_4 (resp. e_5 and e_6) of $\mathbb{R}^6$ and $\tilde{T}_{\max}$ is the maximum thrust expressed in normalized units. To complete the model, the mass decreases proportionally to the delivered thrust

$$\dot{m}(t) = -\frac{\tilde{T}_{\max}}{I_{sp} g_0} \|u(t)\|, \tag{5}$$

where I_{sp}, the specific impulse, is thruster-dependent, and g_0 is the gravitational acceleration at Earth's sea level. For our numerical tests we use

$$T_{\max} = 22\ N,\ I_{sp} = 230\ s,\ g_0 = 9.80665\ m/s^2,\ m_0 = 350\ kg.$$

With our fixed control structure it is simple to change these parameters using a continuation to consider other thruster specifications *e.g.* a solar electric propulsion with smaller $T_{\max}$ but higher I_{sp}. Indeed, the main obstacle to a continuation that is based solely on the thruster parameters would be the possible change in the control structure since there is no smooth continuation path between a minimum fuel transfer with n switchings and another with $n \pm 1$ switchings.

3 Problem statement

The aim of this paper is to design a minimum fuel round-trip transfer to 2006 RH_{120} from a hibernating orbit including a rendezvous period during which the spacecraft travels with the TCO. Since we want the round trip journey to account for various synchronization constraints, it becomes complex when written as a single optimal control problem. To avoid this issue and obtain more general results on each portion of the global transfer, we decouple the rendezvous and return transfers. After optimizing these two problems for departure time, rendezvous point, return time and transfer duration, it is straightforward to pair them with a lock-in phase between the spacecraft and the asteroid to complete the round trip. In this section we introduce the rendezvous and return transfers as optimal control problems and provide the necessary conditions for an optimal control strategy with its associated trajectory.

3.1 Approach transfer

The first component of the round trip journey is to enable the spacecraft to encounter 2006 RH_{120}. We fix the origin of our mission time frame, $t_c = 0$, as the asteroid's geocentric capture time, June 1^{st} 2006 and introduce the following assumptions.

Assumption 3.1 *At t_c the spacecraft is already hibernating on a CR3BP-periodic halo orbit around the Earth-Moon L_2 libration point with a z-excursion of* $5000\,km$*. We arbitrarily fix the position and velocity of the spacecraft at t_c to be $q_{HaloL_2} \approx$* $(1.119, 0, 0.013, 0, 0.180, 0)$ *which corresponds to the highest z point on the halo orbit. The spacecraft's starting location can be easily altered in future work.*

We denote by t_{start} the departing time of the spacecraft from the hibernating orbit. The position and velocity q_{start} of the spacecraft at t_{start} are determined as the result of the CR3BP uncontrolled dynamic, see eqs. (1). We integrate from q_{HaloL_2} at $t_c = 0$ to t_{start} to guarantee the spacecraft departs from its correct location on the L_2-halo periodic orbit. Our algorithm treats t_{start} as an optimization variable of the rendezvous problem and we will discretize the spacecraft's departure time to analyze its impact on the final mass. The detection time t_d^{RH} is a practical mission constraint and in section 4 we discuss how our results provide information regarding ideal windows of detection for 2006 RH_{120} corresponding to the best transfers. 2006 RH_{120} was actually discovered on September $14th$, 2006, about 3.5 months after it was captured, but we consider it as a parameter of the problem associated with the mission start time to gain insights on future studies with other TCOs.

The rendezvous point between the spacecraft and the asteroid is a position and velocity q_f^{rdv} on the 2006 RH_{120} orbit corresponding to a time $t_{\text{rdv}}^{\text{RH}} > t_{\text{start}}$. Our algorithm treats the rendezvous point as an optimization variable and includes a discretization of the 2006 RH_{120} orbit to analyze the impact of the rendezvous point on the fuel consumption. We also require that $t_{\text{rdv}}^{\text{RH}} \leq$ July 31^{st} 2007, *i.e.* that the rendezvous must take place before the asteroid leaves Earth's gravitational field.

To reduce the complexity of the optimization problem we fix the structure of the thrusts for the candidate trajectories to achieve optimality. Our choice is motivated by the desire to mimic a pseudo-realistic strategy with at most three impulsive boosts: one to depart from the halo orbit, a second to redirect the spacecraft to encounter 2006 RH_{120} on its orbit, and a third for the rendezvous to match the position and velocity of the spacecraft to the asteroid. In other words, we impose a control strategy with at most three thrust arcs and two ballistic arcs. Prior numerical calculations have shown that this strategy provides fuel efficient transfers. We employ the same thrust structure for the return portion of the journey. To summarize:

Assumption 3.2 *For our transfers, we restrict the thrust strategy* $u(\cdot):[t_{\text{start}},t_{\text{rdv}}^{\text{RH}}]\to \bar{B}(0,1)\subset\mathbb{R}^3$*, i.e. the control, to have a piecewise constant norm with at most three switchings:*

$$\|u(t)\| = \begin{cases} 1 & \textit{if } t\in[t_{\text{start}},t_1]\cup[t_2,t_3]\cup[t_4,t_{\text{rdv}}^{\text{RH}}] \\ 0 & \textit{if } t\in(t_1,t_2)\cup(t_3,t_4) \end{cases}, \tag{6}$$

where t_1,t_2,t_3,t_4 *are called the switching times and satisfy* $t_{\text{start}}<t_1<t_2\le t_3<t_4<t_{\text{rdv}}^{\text{RH}}$*. We denote by* $\mathscr{U}$ *the set of measurable functions* $u:[t_{\text{start}},t_{\text{rdv}}^{\text{RH}}]\to B^3(0,1)$ *satisfying* (6) *for some switching times* $(t_{\text{start}},t_1,t_2,t_3,t_4,t_{\text{rdv}}^{\text{RH}})$.

We denote by $\xi=(q,m)$ the state variables, and by $f(t,\xi,u)$ the controlled Sun perturbed CR3BP dynamics (3) including the mass evolution of the spacecraft (5). The optimal control problem can then be written as:

$$(OCP)_{t_{\text{start}},t_{\text{rdv}}^{\text{RH}}}^{\text{rdv}} \begin{cases} \min_{t_1,t_2,t_3,t_4,u(\cdot)} \int_{t_{\text{start}}}^{t_{\text{rdv}}^{\text{RH}}} \|u(t)\|dt \\ s.t.\ \dot{\xi}(t)=f(t,\xi(t),u(t)),\ a.e.\ t\in[t_{\text{start}},t_{\text{rdv}}^{\text{RH}}] \\ \xi(t_{\text{start}})=(q_{\text{start}},m_0) \\ q(t_{\text{rdv}}^{\text{RH}})=q_{\text{rdv}}^{\text{RH}} \\ t_{\text{start}}<t_1<t_2\le t_3<t_4<t_{\text{rdv}}^{\text{RH}} \\ u(.)\in\mathscr{U} \end{cases} \tag{7}$$

Since our optimization approach is variational, studying the impact of each of our variables on fuel consumption would produce a large number of local extrema in a direct optimization with respect to $(t_{\text{start}},t_{\text{rdv}}^{\text{RH}})$ and $(t_1,t_2,t_3,t_4,u(\cdot))$. To address this issue, we discretize the set of departure times and durations of the approach transfer $(t_{\text{start}},t_{\text{rdv}}^{\text{RH}})$ and solve $(OCP)_{t_{\text{start}},t_{\text{rdv}}^{\text{RH}}}^{\text{rdv}}$ for the finite number of discretized combinations. This produces an approximation of the optimal transfer with respect to $(t_{\text{start}},t_{\text{rdv}}^{\text{RH}})$ and $(t_1,t_2,t_3,t_4,u(\cdot))$ that becomes more accurate as the discretization on $(t_{\text{start}},t_{\text{rdv}}^{\text{RH}})$ is refined. We use a 15 day discretization of t_{start} from June 1$^{\text{st}}$ 2006 to 360 days later, and a one day discretization on $t_{\text{rdv}}^{\text{RH}}$ from June 1$^{\text{st}}$ 2006 to July 31$^{\text{st}}$ 2007, with the additional constraint that $t_{\text{rdv}}^{\text{RH}}>t_{\text{start}}$. Note that by fixing the departure time and rendezvous point $q_{\text{rdv}}^{\text{RH}}$, we fix the transfer duration. Finally, as it is unlikely that a short duration transfer would yield reasonable fuel consumption, we require that the transfer duration $t_{\text{rdv}}^{\text{RH}}-t_{\text{start}}$ be greater than 7 days. Our choice of discretization leads to 5975 different $(OCP)_{t_{\text{start}},t_{\text{rdv}}^{\text{RH}}}^{\text{rdv}}$ to be solved.

To summarize our optimization algorithm, let us consider how we would select the best approach transfer after 2006 RH_{120} was detected on September 14th 2006. First, the calculation of a high-accuracy TCO orbit sufficient for mission planning requires that the object be observed for many days, perhaps a few weeks, unless high-accuracy radar range and range-rate measurements can be obtained (in which case the orbit element accuracy collapses much faster). Thus, in practice, there is an additional constraint expressed as $t_{\text{start}} > t_d^{\text{RH}} + t_{calc}$ where t_{calc} is the time required to calculate a sufficiently accurate TCO orbit. However, preliminary orbits obtained soon after detection are useful for producing preliminary mission scenarios and should not be neglected. For this reason, our algorithm ignores t_{calc}. Indeed, we imagine that if a spacecraft is actually hibernating in a L_2 halo orbit awaiting discovery of a suitable TCO then every effort will be made to obtain a high accuracy orbit as rapidly as possible.

Step 1: Solve $(OCP)^{\text{rdv}}_{t_{\text{start}}, t_{\text{rdv}}^{\text{RH}}}$ for all t_{start} and $t_{\text{rdv}}^{\text{RH}}$ satisfying:

(i) $t_{\text{start}} \in [\![t_c, t_c + 360 \text{ days}]\!]$ with 15 day steps
(ii) $t_{\text{rdv}}^{\text{RH}} \in [\![t_{\text{start}} + 7 \text{ days, July } 31^{\text{st}}\ 2007]\!]$ with one day steps.

Step 2: Select the $(OCP)^{\text{rdv}}_{t_{\text{start}}, t_{\text{rdv}}^{\text{RH}}}$ with the best final mass among those with $t_{\text{start}} \geq$ September 14th 2006.

The first step is performed without any consideration for the detection time and is only performed once. Step 2 is essentially instantaneous as it is only a simple analysis of the results of Step 1. Note that once Step 2 is performed it is always possible to locally refine the discretization of t_{start} and $t_{\text{rdv}}^{\text{RH}}$ around the selected values in order to improve the final mass. This however, assumes that the chosen discretization is fine enough to already capture the best $(t_{\text{start}}, t_{\text{rdv}}^{\text{RH}})$.

3.2 Return transfer

After the approach transfer, the spacecraft will travel with the asteroid in a lock-in configuration. The optimal duration of the lock-in phase in terms of fuel efficiency, not science, is determined by the start time of the return transfer. In this section we focus on the third component of the round trip journey that optimizes the spacecraft's return trip from 2006 RH_{120} to the Earth-Moon system. For simplicity, we select the final point of the journey as the Earth-Moon L_2 libration point rather than an L_2 halo orbit because the latter requires significantly higher number of calculations and the difference in fuel consumption would be minimal (but it should be a topic for further study). The position and velocity of the L_2 point are given by $q_{L_2} \approx (1.15569383, 0, 0, 0, 0, 0)$.

We denote by $q_{\text{start}}^{\text{RH}}$ the departure point from the 2006 RH_{120} orbit at time $t_{\text{start}}^{\text{RH}}$. The spacecraft can depart the asteroid only after its rendezvous so that $t_{\text{start}}^{\text{RH}} > t_{\text{rdv}}^{\text{RH}}$. On the return trip portion of the journey we also require the control $u(.)$ to be composed of three thrust arcs and two ballistic ones, i.e. $u(.) \in \mathscr{U}$. The final time of the return transfer is denoted by t_f and satisfies $t_f > t^{\mathrm{RH}}_{\mathrm{start}}$. The optimal control for the return transfer is now:

$$(OCP)^{\mathrm{return}}_{t^{\mathrm{RH}}_{\mathrm{start}},t_f} \begin{cases} \min_{t_1,t_2,t_3,t_4,u(\cdot)} \int_{t^{\mathrm{RH}}_{\mathrm{start}}}^{t_f} \|u(t)\|dt \\ s.t.\ \dot{\xi}(t) = f(t,\xi(t),u(t)),\ a.e.\ t \in [t^{\mathrm{RH}}_{\mathrm{start}}, t_f] \\ \xi(t^{\mathrm{RH}}_{\mathrm{start}}) = (q^{\mathrm{RH}}_{\mathrm{start}}, m^{\mathrm{RH}}_0) \\ q(t_f) = q_{L_2} \\ t_{\mathrm{start}} < t_1 < t_2 \le t_3 < t_4 < t_f \\ u(.) \in \mathscr{U} \end{cases} \tag{8}$$

Remark 3.1 *The initial mass for the return trip, m^{RH}_0, is not necessarily identical to the final mass of the approach transfer, m^{rdv}_f. Depending on mission specifics m^{RH}_0 may be equal to, less than (if the mission leaves some equipment or consumes some fuel), or greater than (if the mission brings back samples for example) m^{rdv}_f. We don't expect m^{RH}_0 to play a large role in the fuel consumption, therefore, for simplicity we chose to set it to 300 kg, which is 50 kg less than the mass m_0 of the spacecraft when it departed its L_2 halo orbit.*

As for the approach transfer, we discretize the optimization variables to study their impact on the fuel consumption. We also use a discretization of $(t^{\mathrm{RH}}_{\mathrm{start}}, t_f - t^{\mathrm{RH}}_{\mathrm{start}})$ and solve $(OCP)^{\mathrm{return}}_{t^{\mathrm{RH}}_{\mathrm{start}},t_f}$ for all the discretization pairs . The discretization on $t^{\mathrm{RH}}_{\mathrm{start}}$ is the same as on $t^{\mathrm{RH}}_{\mathrm{rdv}}$ with one day time steps from June 1st 2006 to July 31st 2007, while the discretization on the transfer duration $t_f - t^{\mathrm{RH}}_{\mathrm{start}}$ is in 30 day time steps from 30 to 360 days. This leads to 5112 different $(OCP)^{\mathrm{return}}_{t^{\mathrm{RH}}_{\mathrm{start}},t_f}$ combinations. In the final analysis, when constructing the complete round transfer, we will impose a constraint on the relation between the rendezvous and departure times but we treat them separately now to gain insight on the problem.

Figure 4 provides an overview of the round trip journey.

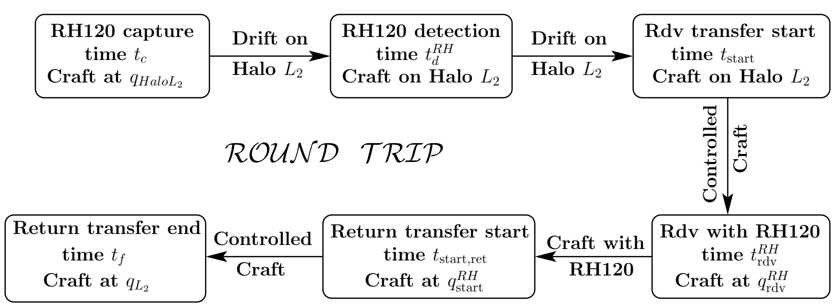

Fig. 4 *Chronology of the round trip journey.*

3.3 Necessary conditions for optimality

The maximum principle, see [13], provides first order necessary conditions for a control and associated trajectory to be optimal. In this section we apply the maximum principle to our optimization problems.

Let us first focus on the approach transfer. We denote by $\xi(t) = (q(t), m(t)) \in \mathbb{R}^6 \times \mathbb{R}_+$ the state of $(OCP)^{\text{rdv}}_{t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}}$, with $q(t) = (r(t), v(t))$ the position and velocity of the vehicle, and $m(t)$ its mass, at time t. For $(OCP)^{\text{rdv}}_{t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}}$, the maximum principle introduces an adjoint state $(p^0, p_\xi(\cdot))$ defined on $[t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}]$ and the Hamiltonian,

$$H(t, \xi(t), p^0, p_\xi(t), u(t)) = p^0 \|u(t)\| + \left\langle p_\xi(t), \dot{\xi}(t) \right\rangle, \text{ for a.e. } t \in [t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}], \quad (9)$$

where $\langle , \rangle$ is the standard inner product. One of the conditions of the maximum principle is that the optimal control maximizes the Hamiltonian. This maximization requires that the optimal control be a multiple of the vector $p_v(\cdot)$ which translates into the following condition:

$$u(t) = \|u(t)\| \frac{p_v(t)}{\|p_v(t)\|}, \text{ for a.e. } t.$$

Without any constraint on the structure of the control the maximization of the Hamiltonian leads to the definition of the switching function ψ

$$\psi(t) = p^0 + \tilde{T}_{\max} \left(\frac{\|p_v(t)\|}{m(t)} - \frac{1}{I_{\text{sp}} g_0} p_m(t) \right), \quad (10)$$

where $\|u(t)\| = 1$ if $\psi(t) > 0$ or $\|u(t)\| = 0$ is $\psi(t) < 0$. However, since in $(OCP)^{\text{rdv}}_{t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}}$ the control structure is constrained to have at most three thrust arcs, a rewriting of the optimal control problem following an approach similar to [14] implies that the switching function cancels at the constrained switching times but does not prescribe $\|u(t)\|$. The following theorem gives all the necessary conditions obtained from the maximum principle applied to $(OCP)^{\text{rdv}}_{t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}}$.

Theorem 3.1 *If $(q(\cdot), m(\cdot), u(\cdot)) : [t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}] \to \mathbb{R}^7 \times B^3(0,1)$ and $(t_1, t_2, t_3\ , t_4) \in \mathbb{R}^4_+$ is an optimal solution of $(OCP)^{\text{rdv}}_{t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}}$, then there exists an absolutely continuous adjoint state $(p^0, p_\xi(\cdot)) = (p^0, p_r(\cdot), p_v(\cdot), p_m(\cdot)) \in \mathbb{R}^- \times \mathbb{R}^7$ defined on $[t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}]$ and such that:*

(a) $(p^0, p_\xi(\cdot)) \neq 0$, $\forall t \in [t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}]$, and $p^0 \leq 0$ is constant.
(b) The state and adjoint state satisfy the Hamiltonian dynamics:

$$\begin{aligned} \dot{\xi}(t) &= \frac{\partial H}{\partial p_\xi}(t, \xi(t), p^0, p_\xi(t), u(t)), \textit{ for a.e. } t \in [t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}] \\ \dot{p}_\xi(t) &= -\frac{\partial H}{\partial \xi}(t, \xi(t), p^0, p_\xi(t), u(t)), \textit{ for a.e. } t \in [t_{\text{start}}, t^{\text{RH}}_{\text{rdv}}] \end{aligned} \quad (11)$$

(c)

$$u(t) = \frac{p_v(t)}{\|p_v(t)\|},\ \forall t \in [t_{\text{start}}, t_1] \cup [t_2, t_3] \cup [t_4, t_{\text{rdv}}^{\text{RH}}] \tag{12}$$

(d) $\psi(t_1) = \psi(t_2) = \psi(t_3) = \psi(t_4) = 0$
(e) $p_m(t_{\text{rdv}}^{\text{RH}}) = 0$

Condition (e) is the final transversality condition and comes from the fact that the final mass is free. In the case where we also allow a free initial time, t_{start}, we would obtain an initial transversality condition of the form

$$\left\langle p_q(t_{\text{start}}), F_0^{\text{CR3BP}}(q(t_{\text{start}})) \right\rangle = 0,$$

where $F_0^{\text{CR3BP}}(\cdot)$ is the uncontrolled dynamics of the vehicle in the CR3BP model (without the Sun perturbation).

Remark 3.2 *Notice that transversality conditions at the rendezvous with asteroid 2006 RH_{120} cannot be used because we do not have an analytic expression for its orbit. In case there is an analytic expression for the rendezvous orbit it would imply that the Hamiltonian must be zero at the rendezvous, and that $p_q(t_{\text{rdv}}^{\text{RH}})$ should be orthogonal to $\dot{q}_{\text{rdv}}^{\text{RH}}$.*

Remark 3.3 *A state, control, and adjoint state $(\xi(\cdot), u(\cdot), p^0, p_\xi(\cdot))$ satisfying the conditions of Theorem 3.1 is called an extremal of $(OCP)_{t_{\text{start}}, t_{\text{rdv}}^{\text{RH}}}^{\text{rdv}}$. We assume that the extremals of $(OCP)_{t_{\text{start}}, t_{\text{rdv}}^{\text{RH}}}^{\text{rdv}}$ are normal, that is $p^0 \neq 0$.*

For the return transfer, the maximum principle applied to $(OCP)_{t_{\text{start}}^{\text{RH}}, t_f}^{\text{return}}$ gives the same necessary conditions as in Theorem 3.1, with t_{start} and $t_{\text{rdv}}^{\text{RH}}$ replaced by $t_{\text{start}}^{\text{RH}}$ and t_f.

3.4 Numerical method

For our numerical calculations we assume that the extremals are normal, i.e. $p^0 \neq 0$ so we can normalize it to -1. A study of the existence of abnormal extremals is out of the scope of this paper.

Both optimal control problems are solved using a shooting method based on the necessary conditions. The shooting method consists in rewriting the necessary conditions of the maximum principle as the zero of a nonlinear function, namely the shooting function. Using the necessary conditions, in particular the Hamiltonian dynamics (11) and the maximization of the control (12), $(\xi(t), p_\xi(t))$ is completely defined by its initial value $(\xi_0, p_{\xi,0})$ at times t_{start} (respectively $t_{\text{start}}^{\text{RH}}$ for the return transfer) and by the switching times (t_1, t_2, t_3, t_4). Then, fixing ξ_0, we denote by S the shooting function for the approach transfer:

$$S(p_{\xi,0},t_1,t_2,t_3,t_4) = \begin{cases} q(t_{\mathrm{rdv}}^{\mathrm{RH}}) - q^{\mathrm{rdv}} \\ \psi(t_i),\ i=1,2,3,4 \\ p_m(t_{\mathrm{rdv}}^{\mathrm{RH}}) \end{cases} \qquad S \in \mathbb{R}^{11}. \tag{13}$$

For the return transfer we replace $t_{\mathrm{rdv}}^{\mathrm{RH}}$ by t_f, and q^{rdv} by q_{L_2}. It follows that if we find $(p_{\xi,0},t_1,t_2,t_3,t_4)$ such that $S(p_{\xi,0},t_1,t_2,t_3,t_4) = 0 \in \mathbb{R}^{11}$, then the associated $(\xi(\cdot),u(\cdot),-1,p_\xi(\cdot))$ satisfies the necessary conditions of Theorem 3.1.

We computed the shooting function using the adaptative step integrator DOP853, see [15]. To find a zero of S we used the quasi-Newton solver *HYBRD* of the Fortran *minpack* package. Since $S(\cdot)$ is nonlinear, Newton's method are very sensitive to the initial guess and seldom converge. To address the initialization sensitivity we used two initialization techniques as described below.

The first initialization technique is a direct approach, see [16], consisting of discretizing the state ξ and control u in order to rewrite the optimal control problem as a nonlinear parametric optimization problem (NLP). In (NLP) the dynamic is discretized using a fixed step fourth order Runge-Kutta scheme. The size of the (NLP) depends on the size of the discretization. The (NLP) is solved using the modeling language *Ampl*, see [17], and the optimization solver *IpOpt*, see [18]. Once a solution of the (NLP) is obtained we use the value of the Lagrange multipliers associated with the discretization of the dynamic at the initial time as our initial guess for $p_{\xi,0}$. The other unknowns are directly transcribable from (NLP). This approach cannot be used to solve our optimal control problems because, in order to have a sufficiently accurate solution, each (NLP) should be solved with a high-resolution time discretization which would yield execution times of a few hours. We thus use this initialization technique when the second one fails.

The second initialization technique is a continuation from the known solution of one optimal control problem to another. For instance, if we know the solution of a $(OCP)^{\mathrm{rdv}}_{t_{\mathrm{start}},t_{\mathrm{rdv}}^{\mathrm{RH}}}$, then we can reasonably hope that in some, if not most, of the cases this solution is *connected* to the solution of a nearby $(OCP)^{\mathrm{rdv}}_{t_{\mathrm{start}},t_{\mathrm{rdv}}^{\mathrm{RH}}+\delta t}$. To follow the connection between these two problems we could use elaborate continuation methods like in [19] or [8]. Here, we chose to use a linear prediction continuation which doesn't require the computation of the sensitivity of the shooting function but is nevertheless enough for our purpose. A solving with the continuation method usually takes a few seconds on a standard laptop.

Typically, the direct approach is used on one case and the continuation method enables us to solve tens or hundreds of other close cases. When the continuation method fails we then use the direct approach to once again initiate the continuation. To limit the number of local minima we add direct approach solvings and continuations from other neighbors to try to improve the solutions in terms of final mass. This *local minima trimming* is based on two heuristics. The first one is a selection of locally optimal cases with the assumption that the evolution of the final mass should be more or less continuous with respect to the 2006 RH_{120} rendezvous point (for the rendezvous trip) or to the 2006 RH_{120} departure point (for the return trip). For instance, for the forward trip, if two transfers with a comparable duration and neighbor rendezvous points exhibit a large final mass difference (say more than 10 kg), we employ a direct approach with the rendezvous point corresponding to the lower final mass. The second heuristic is a random selection of transfers and is used sparsely. This second round of calculations is essential and allowed us to greatly improve the solutions computed on the first solving round. The large number of optimal control problems we need to solve and the trimming of local minima requires several days of computation.

4 Numerical results

In this section, we provide the best rendezvous and return transfers and the evolution of the criterion with respect to the discretization of the initial and final times. We then discuss how the results can be combined to design a global round trip mission.

Since our numerical approach is variational it is not possible to guarantee that the proposed trajectories are optimal despite the restriction of the control structure. Even the use of a (second order) sufficient condition, see [20], would only yield a proof of local optimality. It is thus likely that among the 5945 rendezvous trips and the 5112 return trips there are local minima that can be improved by finding other better local or, ideally, global minima.

4.1 Approach transfer

The best approach transfer under the imposed restricted thrust structure is represented on Figures 5 and 6. Table 2 summarizes the main features of this transfer.

Best rendezvous to 2006 RH_{120}

Parameter	Symbol	Value
Departure date	t_{start}	06/16/2006
Arrival date	$t_{\text{rdv}}^{\text{RH}}$	10/27/2006
Final position		$(-1.958, 0.401, -3.992)$
Final velocity		$(0.224, 1.728, -0.029)$
Final Mass	m_f	280.855 kg
Delta-V	ΔV	496.43 m/s
Max dist. to Earth		1714 Mm (4.46 LD)
Min dist. to Earth		366080 km (0.95 LD)

Table 2 *Table summarizing the best approach transfer to 2006 RH_{120}.*

The departure time from the hibernating orbit is $t_{\text{start}} = 15$ *days*, which implies that the detection of the asteroid should occur before capture time to allow for an

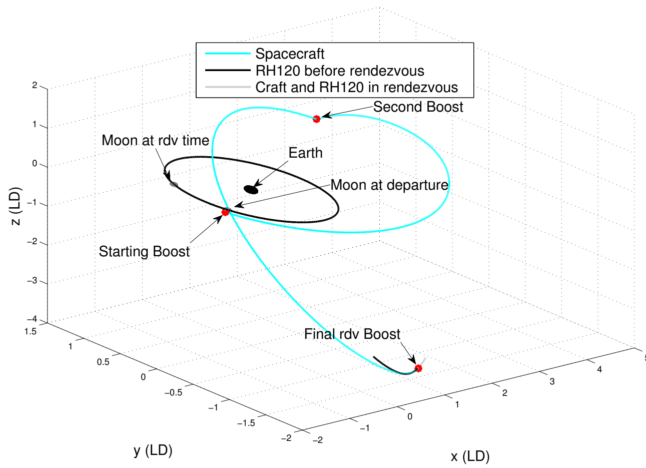

Fig. 5 *Best approach transfer to 2006 RH*$_{120}$ *in a geocentric inertial frame.*

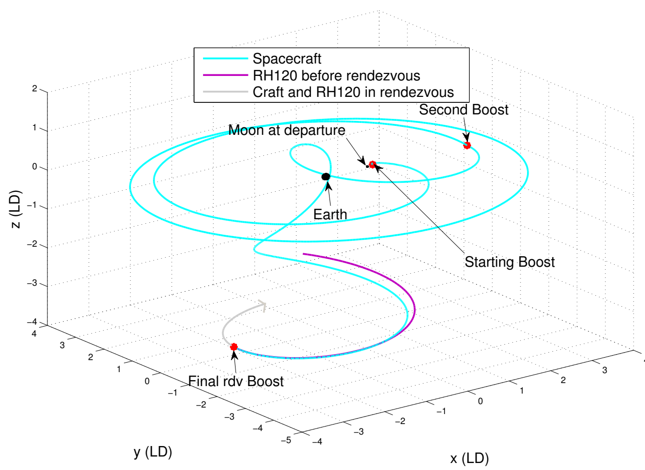

Fig. 6 *Best approach transfer to 2006 RH*$_{120}$ *in the CR3BP rotating frame.*

accurate calculation of 2006 RH$_{120}$'s orbit before the mission. The rendezvous between the spacecraft and 2006 RH$_{120}$ occurs 133 days later on October 27th 2006, 148 days after capture, i.e. $t_{\text{rdv}}^{\text{RH}} = 148$ *days*. The point on 2006 RH$_{120}$'s orbit at which rendezvous occurs is $q_f^{\text{rdv}} \approx (-1.958, 0.401, -3.992, 0.224, 1.728, -0.029)$. The best transfer rendezvous point is far from the Earth-Moon orbital plane and 5.08 LD from the L_2 libration point. A possible explanation is that the thrusters have a larger impact on the motion of the vehicle as the spacecraft's distance from the two primaries increases. It is also important to note that this rendezvous point is not simply the closest one in terms of distance or energy (see figure 7). There is first a rapid increase in the final mass for the rendezvous with 2006 RH_{120} that occurs near capture time, the reason being that these transfers correspond to approach transfers with a short duration. The best transfers are obtained between 120 and 170 days, after which the final mass is roughly constant with a mean of 252.85 kg and a standard deviation of 8.21 kg. This behavior is good in terms of the design of a real mission as it provides flexibility for the spacecraft's departure time from its hibernating orbit.

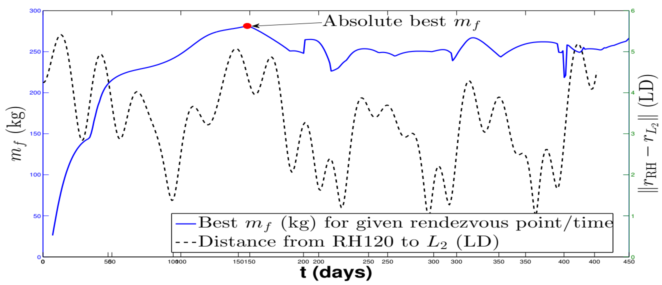

Fig. 7 *Best final mass (kg) among all t_{start} with respect to the rendezvous time (left y-axis) and distance from the rendezvous point to L_2 (LD, right y-axis).*

The final mass of the spaceraft for the best overall rendezvous is $m_f \approx 280.855\ kg$ corresponding to a $\Delta V \approx 496.43\ m/s$ where we compute ΔV such that $m_f = m_0 e^{-\frac{\Delta V}{I_{\text{sp}} g_0}}$. Note that the spacecraft performs only one revolution around the Earth in the inertial reference frame. We obtained, in the CR3BP model, a better transfer in [5] with a ΔV of 203.6 m/s but in this case the rendezvous would take place on June 26th 2006 and the duration would be about 415 days. This would require the unrealistic scenario that 2006 RH_{120} be detected and spacecraft launched about 14 months before June 1st 2006.

Finally, the norm of the control is shown on Figure 8 with three thrust arcs lasting respectively 16.44 min, 1.62 hours and 4.23 min and two ballistic arcs lasting 68.70 and 64.25 days. The second thrust takes place approximately in the middle of the transfer but this is typically not the case (see the best return transfer below). The position and velocity of the spacecraft at the beginning of the second thrust arc is $q(t_2) = (3.286, -0.141, -0.012, -0.476, -3.185, 0.012)$, at a EM barycenter distance of 3.29 LD or 1.26 million km.

Figure 9 gives the evolution of the final mass with respect to $t_{\text{rdv}}^{\text{RH}}$ and t_{start}. Notice that the scale for t_{start} needs to be multiplied by 15 (the discretization rate) to justify the void region for which no approach transfer are associated. It also reflects the fact that $t_{\text{rdv}}^{\text{RH}} \geq t_{\text{start}} + 7\ days$.

Figure 10 is a selection of the evolution of the final mass and ΔV with respect to the rendezvous time $t_{\text{rdv}}^{\text{RH}}$ for various starting dates t_{start}, i.e. it provides cross sections through the 3D Figure 9.

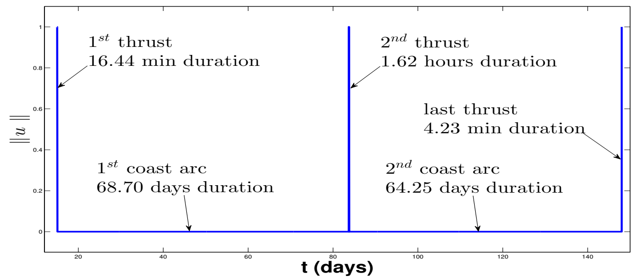

Fig. 8 *Norm of control for the best approach transfer to 2006 RH_{120}.*

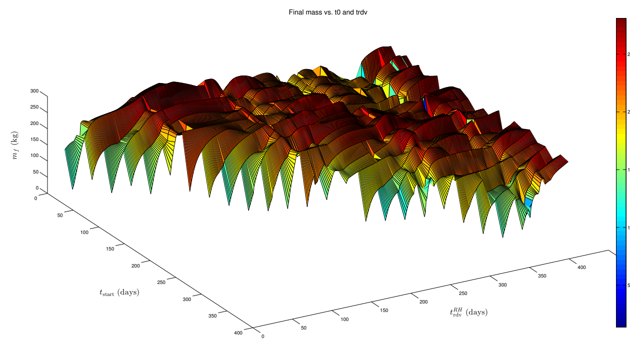

Fig. 9 *Evolution of final mass for $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{\text{RH}}_{\text{rdv}}}$ with respect to the rendezvous and starting dates.*

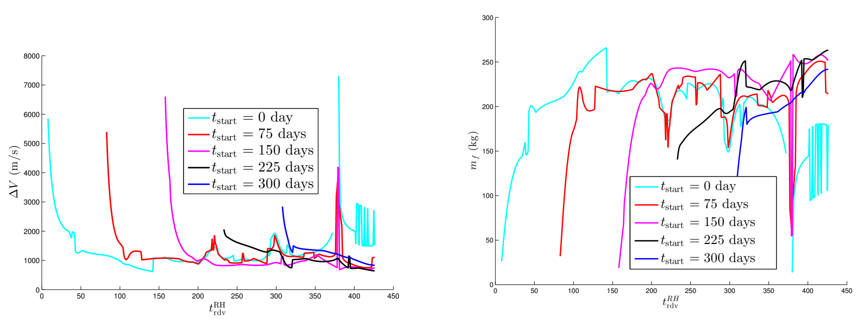

Fig. 10 *Evolution of ΔV (left) and final mass (right) for $(OCP)^{\text{rdv}}_{t_{\text{start}},t^{\text{RH}}_{\text{rdv}}}$ with respect to the rendezvous date and various starting dates $t_{\text{start}} \in \{0, 75, 150, 225, 300\}$ days.*

Figures 9 and 10 show that there is first a gradual increase of the final mass with respect to the transfer duration $t^{\text{RH}}_{\text{rdv}} - t_{\text{start}}$ which then stops after 30 to 120 days, depending on the starting date. This suggests that after a period of about 2 months the final mass is less sensitive to an increase in transfer duration and depends more heavily on the rendezvous point.

Figure 11 provides a histogram of the number of approach transfers corresponding to a given final mass range. 68.8% of transfers provide a final mass above 200kg. This is promising for the mission design because it provides flexibility with respect to the departure time and transfer duration and because the less fuel that is used per mission the more additional TCOs that can be targeted.

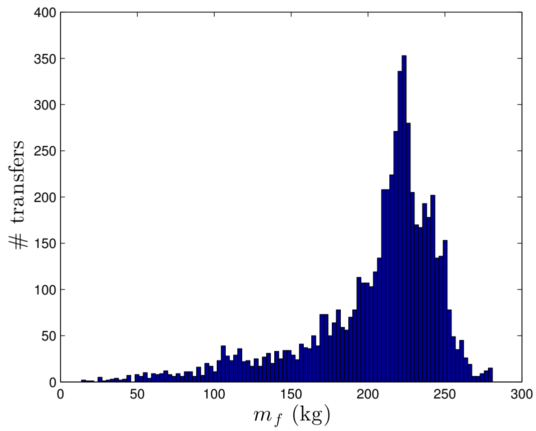

Fig. 11 *Number of approach transfers per final mass range. Most approach transfers provide a final mass above 200kg with a peak at around 230kg.*

Table 3 provides the best approach transfers for each of 25 departure times for the spacecraft from its hibernating location. Officially, 2006 RH_{120} was discovered on September 14^{th} 2006, 105 days after its capture by Earth's gravity, so that $t_d^{\mathrm{RH}} = 105$. Table 3, suggests that in this case the best departure time satisfying $t_{\text{start}} \geq t_d^{\mathrm{RH}}$ is $t_{\text{start}} = 180$ days after June 1^{st} 2006. In this scenario the 75 days between the detection time and the departure of the spacecraft for the rendezvous mission ensure that the celestial mechanics computations required to predict 2006 RH_{120}'s orbit with enough accuracy can be completed. This approach transfer provides a final mass of 267.037 kg or, equivalently $\Delta V = 610.224$ m/s, and a rendezvous date 312 days after capture, on April 9th 2007. In particular, we will see that this approach transfer can be combined with the best return transfer provided in the following section. If practical considerations delay the spacecraft departure then Table 3 shows that it will have minimal impact on the mission's fuel consumption. For instance, for $t_{\text{start}} = 285$ days the final mass is 266.525 kg, not even a one kilo difference from a departure 180 days after capture. However, this late rendezvous time might seriously compromise the efficiency of the return transfer. Clearly, an early detection of the TCO or timely departure of the spacecraft once the asteroid orbit has been determined is preferable for a fuel efficient round trip transfer.

t_{start} (d.)	$t_{\text{rdv}}^{\text{RH}}$ (d.)	m_f (kg.)	t_{start} (d.)	$t_{\text{rdv}}^{\text{RH}}$ (d.)	m_f (kg.)
0	141	265.831	195	392	255.172
15	148	280.855	210	363	262.025
30	111	234.909	225	425	263.304
45	138	251.379	240	359	240.076
60	146	231.103	255	425	260.92
75	414	250.772	270	416	256.618
90	273	250.608	285	425	266.525
105	414	250.02	300	425	241.721
120	290	252.547	315	425	246.111
135	390	245.707	330	407	254.773
150	380	258.222	345	425	251.158
165	314	244.521	360	425	245.369
180	312	267.037			
Avg	-	253.091	σ	-	11.136

Table 3 *Best rendezvous dates and final mass for the 25 different t_{start}. The average final mass is* 253 *kg with a standard deviation of* 11 *kg.*

4.2 Return transfer from 2006 RH$_{120}$ to L_2

To get a global idea of the impact of the choice of departure time from asteroid 2006 RH$_{120}$ and duration of the transfer we first study the return transfer as a completely decoupled problem from the approach transfer. In an unrealistic way we assume the spacecraft can depart 2006 RH$_{120}$ as soon as June 1$^{\text{st}}$ 2006 and we use a 1 day discretization of the 2006 RH$_{120}$ orbit. However, to keep the number of calculations under control we use a 30 days discretization for the transfer duration, $t_f - t_{\text{start}}^{\text{RH}}$, from 30 to 360 days, yielding a total of 5112 combinations for $(t_{\text{start}}^{\text{RH}}, t_f - t_{\text{start}}^{\text{RH}})$. Since the mass at departure from 2006 RH$_{120}$ is unknown beforehand, and in order to compare all the return trips, we choose arbitrarily to set the initial mass of the return trip to 300 kg, exactly 50 kg less than the initial mass of the approach transfer.

The best return transfer to L_2 under our thrust restrictions is shown in Figure 12 and Table 4 summarizes its main features.

Best return trip from 2006 RH$_{120}$

Parameter	**Symbol**	**Value**
Departure date	$t_{\text{start}}^{\text{RH}}$	06/01/2007
Arrival date	t_f	01/27/2008
Initial position		$(0.238, -0.598, -2.228)$
Initial velocity		$(-0.947, -0.477, 0.496)$
Final Mass	m_f	250.712 kg
Delta-V	ΔV	404.815 m/s
Max dist. to Earth		2031 Mm (5.28 LD)
Min dist. to Earth		265520 km (0.69 LD)

Table 4 *Table summarizing the best return transfer from 2006 RH$_{120}$ to L_2.*

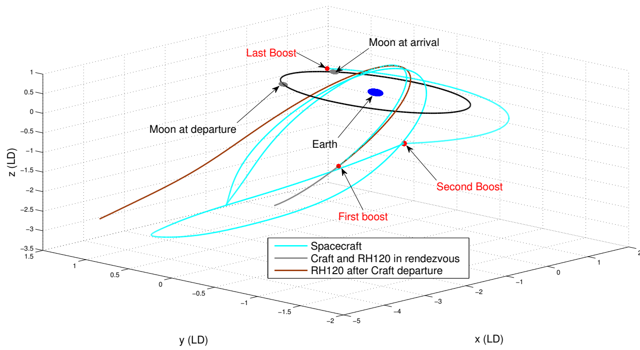

Fig. 12 *Best return transfer from 2006 RH_{120} to Earth-Moon L_2 in a geocentric inertial frame.*

The starting date of the best return transfer is 365 days after June 1st 2006, June 1st 2007, and the transfer duration is 240 days. This departure date occurs shortly before 2006 RH_{120} escapes Earth's gravity and after the best approach transfer which makes it an ideal candidate for a complete round trip. The final mass for this transfer is $m_f \approx 250.712$ kg, corresponding to $\Delta V \approx 404.815$ m/s, slightly better than the ΔV for the best approach transfer.

Figure 13 provides the norm of the control associated with the best return transfer. This thrust strategy has three thrust arcs lasting 2.15 min, 1.32 hours and 3.06 min with two intervening ballistic arcs lasting 213.79 and 26.15 days respectively. Contrary to the best approach transfer, the second thrust arc does not occur in the middle of the transfer but rather near the end. However, the second thrust arc is the longest one as was the case for the approach transfer .

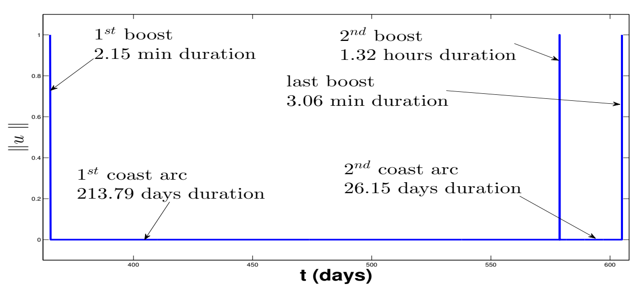

Fig. 13 *Norm of control for the best return transfer from 2006 RH_{120}.*

Figure 14 illustrates the evolution of the final mass with respect to $t_{\text{rdv}}^{\text{RH}}$ for various choices of $t_{\text{start}}^{\text{RH}}$. For the return trip there is not a large variation in the final mass with respect to either the starting date or the transfer times.

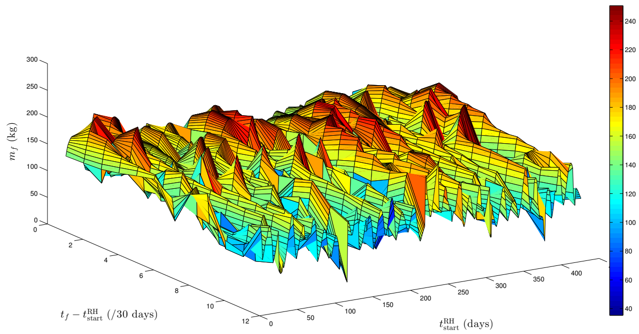

Fig. 14 *Evolution of final mass for* $(OCP)^{\text{return}}_{t^{\text{RH}}_{\text{start}},t_f}$ *with respect to the starting date and transfer time.*

Figure 15 provides a selection of the evolution of the final mass and ΔV with respect to the departure date $t^{\text{RH}}_{\text{start}}$ for various transfer durations $t_f - t^{\text{RH}}_{\text{start}}$, i.e. it presents several cross sectional views through 3D Figure 14.

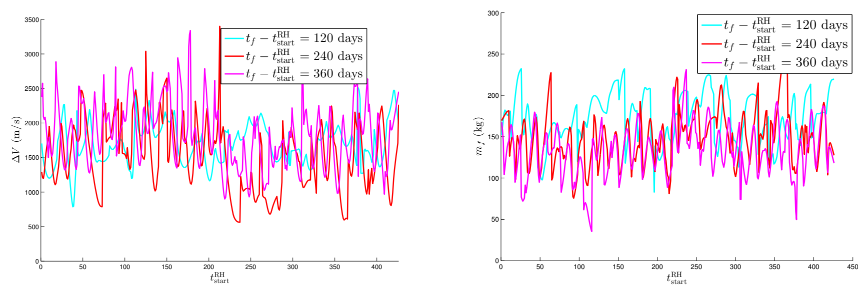

Fig. 15 *Evolution of* ΔV *(left) and final mass (right) for* $(OCP)^{\text{return}}_{t^{\text{RH}}_{\text{start}},t_f}$ *with respect to the starting date and various transfer durations* $t_f - t^{\text{RH}}_{\text{start}} \in \{120, 240, 360\}$ *days.*

Based on the evolution of the final mass with respect to the transfer duration, allowing more time for the transfer does not always provide a more efficient return transfer. However, it is possible that the optimal control problem from a fixed duration to one with a maximum allowed duration would give better results. Indeed, for the return transfer, it would make sense to be more lax with respect to the transfer duration than for the synchronized approach transfer. This remark is partially illustrated by the results from [5] where the transfer duration was free, albeit those results are applicable only to a rendezvous type transfer and the CR3BP model.

Figure 16 provides the number of approach transfers corresponding to a given final mass range. Contrary to the approach transfers, it resembles a normal distribution with an average final mass of about 160 kg. This distribution does not provide as much flexibility as the approach transfers because in the latter case there are many more transfers with a final mass close to the best one.

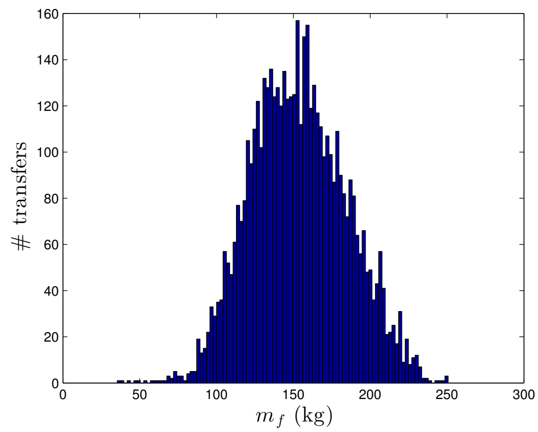

Fig. 16 *Number of return transfers per final mass range. Notice the shape resembles a normal distribution with a maximum at ∼170 kg.*

Table 5 gives a quick overview of the best return trips for each transfer duration $\Delta t^f = t^f - t^{\mathrm{RH}}_{\mathrm{start}}$. The best transfers require durations between 120 and 240 days and extending the duration beyond 240 days does not provide more fuel efficiency. Except for the return transfers lasting less than 150 days, they all depart 2006 RH_{120} at a late date. This is desirable because it is unrealistic to expect that the approach transfer arrives at 2006 RH_{120}early.

Δt^f (d.)	$t^{\mathrm{RH}}_{\mathrm{start}}$ (d.)	m_f (kg.)	Δt^f (d.)	$t^{\mathrm{RH}}_{\mathrm{start}}$ (d.)	m_f (kg.)
30	37	211.681	210	271	231.035
60	18	225.091	240	365	250.712
90	149	220.053	270	221	205.216
120	25	232.328	300	271	207.765
150	154	236.009	330	218	201.274
180	236	233.768	360	236	231.093

Table 5 *Best starting dates for the return trip for 12 different transfer durations $\Delta t_f = t^f - t^{\mathrm{RH}}_{\mathrm{start}}$. Mean value of final mass is 224 kg and standard deviation is 15 kg.*

4.3 Complete round trip mission

In this section we combine a approach transfer with a return transfer in a realistic way to design a round trip mission to 2006 RH_{120}. To do so, we need to account for some practical constraints such as the fact that the return transfer must start after the completion of the approach transfer, that is $t^{\mathrm{RH}}_{\mathrm{rdv}} < t^{\mathrm{RH}}_{\mathrm{start}}$. This means that the spacecraft stays with 2006 RH_{120} for $t^{\mathrm{RH}}_{\mathrm{start}} - t^{\mathrm{RH}}_{\mathrm{rdv}}$ days. We prefer to think of the lock-in duration between the spacecraft and 2006 RH_{120} to be a consequence of our calculation rather than a fixed value by the user. Our calculations determine what the ideal lock-in duration should be and the only remaining constraint is to ensure that it corresponds to a realistic duration for the science component of the mission.

From our prior calculations, the best rendezvous and return transfers satisfy the time constraint so we only need to modify the initial mass of the return transfer to match our desired scenario. We chose to simply impose that the mass at the end of the approach transfer equals the mass at the departure of the return transfer, in other words there is no loss or addition of mass during the lock-in phase. This is an arbitrary choice, and we could for instance have decided that some equipment was left on the asteroid or some material collected form the asteroid that would alter the departure mass. Based on our choice, the return transfer must start with an initial mass of 280.855 kg instead of the $300kg$ prescribed previously. This modification is addressed easily through a continuation on the previous best return transfer. It provides a return transfer that is nearly the same as the one with the higher mass. Table 6 provides the main features of the modified return trip while Table 2 remains applicable to the approach transfer. Figure 17 shows the entire round trip transfer in a geocentric inertial reference frame.

Best return trip from 2006 RH_{120} for the round trip mission

Parameter	Symbol	Value
Stay on 2006 RH_{120}	$t^{\mathrm{RH}}_{\mathrm{start}} - t^{\mathrm{RH}}_{\mathrm{rdv}}$	217 days
Departure date	$t^{\mathrm{RH}}_{\mathrm{start}}$	06/01/2007
Arrival date	t_f	01/27/2008
Initial position		$(0.238, -0.598, -2.228)$
Initial velocity		$(-0.947, -0.477, 0.496)$
Final Mass	m_f	234.713 kg
Delta-V	ΔV	404.814 m/s
Max dist. to Earth		2031 Mm (5.28 LD)
Min dist. to Earth		265519 km (0.69 LD)

Table 6 *Parameters of the best return transfer from 2006 RH_{120} to L_2 after pairing with the approach transfer (so $m_0 = 280.855$ kg).*

As mentioned in section 4.1, the best round trip transfer requires that 2006 RH_{120} be detected at, or almost immediately after, capture which is not an ideal scenario especially given the fact that 2006 RH_{120} was actually detected 105 days after June 1$^{\mathrm{st}}$ 2006. This suggests that additional scenarios should be considered. Moreover, the round trip transfer should also allow the spacecraft ample time to perform its science

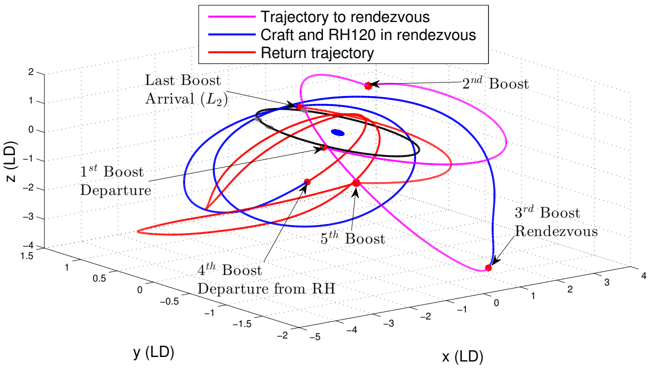

Fig. 17 *Best round trip transfer from Earth-Moon L_2 to 2006 RH_{120} in a geocentric inertial frame.*

mission at the TCO. If we denote by $\delta t_{\text{mission}}$ the minimum time the spacecraft needs to remain at 2006 RH_{120} this constraint can be expressed as $t^{\text{RH}}_{\text{start}} \geq t^{\text{RH}}_{\text{rdv}} + \delta t_{\text{mission}}$. Table 7 gives a sample of the best round trip transfers for various t^{RH}_d and $\delta t_{\text{mission}}$. Since the best return transfer departs one year after June 1st 2006 it can be used in almost all scenarios except the last one when the rendezvous portion ends 395 days after June 1st 2006. For instance, if we assume that the detection occurs 210 days after June 1st 2006, and that we need only 30 days for the lock-in phase, we can select 102 day duration approach transfer to combine with the best overall return transfer. However, if we impose a 60 day lock-in constraint for the spacecraft and the asteroid we need to select a approach transfer duration of 95 days to reach 2006 RH_{120}. In general, the longer the lock-in phase the more expensive the round trip transfer. Another way to look at our calculations would be to design efficient round trip transfers and deduce from these data the ideal windows for detection and lock-in phases. This would provide additional information for the overall design of a TCO mission.

t_d^{RH}	$\delta t_{\text{mission}}$	approach transfer			return transfer			round trip	
		t_{start}	$t^{\text{RH}}_{\text{rdv}}$	ΔV (m/s)	$t^{\text{RH}}_{\text{start}}$	$t_f - t^{\text{RH}}_{\text{start}}$	ΔV (m/s)	Total δV (m/s)	Duration
0	30	15	148	496.43	365	240	404.82	901.25	373
30	30	180	312	610.22	365	240	404.82	1.01504	372
30	60	180	305	684.66	365	240	404.82	1089.48	365
210	30	210	312	732.23	365	240	404.82	1137.05	342
210	60	210	305	809.61	365	240	404.82	1214.43	335
240	30	255	319	892.82	365	240	404.82	1297.63	304
240	60	240	305	1010.88	365	240	404.82	1415.70	305
270	30	270	335	1034.27	365	240	404.82	1439.09	305
300	30	330	395	936.80	425	120	704.06	1640.85	185

Table 7 *The best round trips with detection and mission duration constraints, $m_0 = 350$ kg, and $m_0^{\text{RH}} = 300$ kg. All times expressed in days.*

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