# Three Body Equation

Let $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ , $\boldsymbol{r}_3 \in \mathbb{R}^3$ be the positions of the of the three bodies, with masses $m_1$, $m_2$, $m_3$ where $m_1 \geq m_2 \geq m_3$.

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$$\ddot{\boldsymbol{r}}_1 = G\left(\frac{m_2}{|\boldsymbol{r}_{12}|^3}\boldsymbol{r}_{12} + \frac{m_3}{|\boldsymbol{r}_{13}|^3}\boldsymbol{r}_{13}\right)$$

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$$\ddot{\boldsymbol{p}} = -R_\omega(t)\ddot{R}_\omega^{-1}(t)\boldsymbol{p} - 2R_\omega(t)\dot{R}_\omega^{-1}(t)\dot{\boldsymbol{p}} + G\left(\frac{m_1}{|\boldsymbol{p}_{31}|^3}\boldsymbol{p}_{31} + \frac{m_2}{|\boldsymbol{p}_{32}|^3}\boldsymbol{p}_{32}\right)$$

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- $\boldsymbol{p}_{31} = \boldsymbol{p}_1 - \boldsymbol{p}_3 = \begin{bmatrix} -x_1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} x_1 + x \\ y \\ z \end{bmatrix}$

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$$\ddot{\boldsymbol{p}} = -\omega^2 \boldsymbol{I}_{3\times 3}\boldsymbol{p} + 2\begin{pmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\dot{\boldsymbol{p}} + G\left(\frac{m_1}{|\boldsymbol{p}_{31}|^3}\boldsymbol{p}_{31} + \frac{m_2}{|\boldsymbol{p}_{32}|^3}\boldsymbol{p}_{32}\right)$$

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$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \omega^2 x + 2\omega\dot{y} - G\left(\frac{m_1}{|\boldsymbol{p}_{31}|^3}(x + x_1) + \frac{m_2}{|\boldsymbol{p}_{32}|^3}(x - x_2)\right) \\ \omega^2 y - 2\omega\dot{x} - yG\left(\frac{m_1}{|\boldsymbol{p}_{31}|^3} + \frac{m_2}{|\boldsymbol{p}_{32}|^3}\right) \\ -zG\left(\frac{m_1}{|\boldsymbol{p}_{31}|^3} + \frac{m_2}{|\boldsymbol{p}_{32}|^3}\right) \end{bmatrix}$$

$$|\boldsymbol{p}_{31}| = \sqrt{(x + x_1)^2 + y^2 + z^2},\ |\boldsymbol{p}_{32}| = \sqrt{(x - x_2)^2 + y^2 + z^2}$$

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$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix} x + 2\dot{y} - \frac{1-\mu}{|\boldsymbol{r}_{31}|^3}(x+\mu) - \frac{\mu}{|\boldsymbol{r}_{32}|^3}[x-(1-\mu)] \\ y - 2\dot{x} - y\left(\frac{1-\mu}{|\boldsymbol{r}_{31}|^3} + \frac{\mu}{|\boldsymbol{r}_{32}|^3}\right) \\ -z\left(\frac{1-\mu}{|\boldsymbol{r}_{31}|^3} + \frac{\mu}{|\boldsymbol{r}_{32}|^3}\right)\end{bmatrix}$$

$$|\boldsymbol{r}_{31}| = \sqrt{(x+\mu)^2 + y^2 + z^2},\ |\boldsymbol{r}_{32}| = \sqrt{(x-(1-\mu))^2 + y^2 + z^2}$$

# Clean Up

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- $U(x, y, z) := \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{|\boldsymbol{r}_{31}|} + \frac{\mu}{|\boldsymbol{r}_{32}|}$

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- $\frac{1}{2}\frac{d}{dt}\Big(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\Big) = \ddot{x}\dot{x} + \ddot{y}\dot{y} + \ddot{z}\dot{z}$

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- $\frac{1}{2}\Big(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\Big) = U(x, y, z) - \frac{C}{2}$
- $x^2 + y^2 + 2\frac{1-\mu}{|r_{31}|} + 2\frac{\mu}{|r_{32}|} - \big(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\big) = C$