Sum-up of what I've done so far

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1 Problem statement

The aim of the study is to compute, preferably maximum final mass, round trip transfers from some parking orbit in the Earth-Moon system to some asteroid that has an orbit with characteristics close to the ones os the Earth. The set of candidate target asteroids as a size of 4258. The spacecraft has to spent at least 2-3 months with the asteroid before returning to the parking orbit. Also, the total round trip should not be longer than 5 years. During it's stay on the asteroid, the spacecraft will mine it (for water essentially) which will imply a very large return mass.

The mission should start after January 1$^{\text{st}}$ 2028 and end before January 1$^{\text{st}}$ 2048.

Here are the main assumptions tha will be used :

- The asteroid has a Keplerian orbit (elliptic) around the Sun.
- The Earth-Moon barycenter (EMB) has a Keplerian orbit (nearly circular) around the Sun. In particular it's orbital plane is in the plane of the ecliptic. Note that then, the Earth does not have a Keplerian motion around the Sun (if you look closely since the Earth-EMB distance is about 4000 km $<<$ 1AU).
- Consistently with the Earth-Moon Circular Restricted 3 Body Problem (CR3BP) assumptions, we assume that the Earth and Moon have a circular orbit around the EMB.
- When outside the Earth's Hill sphere (radius of 0.01 AU), we can approximate each of thr spacecraft maneuver as impulsive one (meaning jump in velocity $= \Delta V$). We can also neglect the Earth and Moon potentials $\Rightarrow$ it becomes a 2 body problem.

1.1 Univers parameters

Here are some useful constants :

- $\mu_0^{\text{Sun}} = 132712440018$ km^3/s^2 = Sun's gravitational parameter.
- $\mu_0^{\text{Earth}} = 398600.4418$ km^3/s^2 = Earth's gravitational parameter.
- $\mu_0^{\text{Moon}} = 4902.7779$ km^3/s^2 = Moon's gravitational parameter.
- $AU = 149597870.7$ km = Astronomical unit.
- $LD = 384400$ km = Earth-Moon constant distance (with our assumptions).
- 01/01/2028 at $0:00 = 61771$ in Modified Julian Date.

1.2 Objects specifications

Here is a list of the different objects that will be used in the sequel and their describing parameters :

1.2.1 The spacecraft

When *near* the EMB, the spacecraft propulsive system has a maximum thrust $T_{\max} = 50$ N and specific impulse $I_{\rm sp} = 375$ s. The initial mass of the spacecraft for the outbound trip (= when leaving the EMB system) is $m_0^{\rm out} = 5000$ kg. The initial mass of the spacecraft for teh return trip (= when leaving the asteroid) is $m_0^{\rm ret} = 100000$ kg (yes, 10^5). Note that during the impulsive maneuvers, we only need $I_{\rm sp}$ to compute the mass consumption (with $\delta m = (1 - e^{-\Delta V/(I_{\rm sp} g_0)})m_0$).

1.2.2 One Asteroid

Robert gave a collection of 4258 asteroids, which all have a Keplerian orbit around the Sun of the same kind as the one given here as example :

$$a = 1.021489281928 \text{ AU},\ e = 0.19544556093290,\ i = 1.398188196278°,$$

$$\Omega = 258.200691184349°,\ \omega = 10.190284437679°, \nu(01/01/2028) = 2.220003166300899 \text{ rad}$$

where $(a, e, i, \Omega, \omega, \nu)$ are the classical orbital elements, that is :

- a = semi-major axis, e = eccentricity, i = inclination w.r.t ecliptic plane
- Ω = Ascending Node Longitude, ω = argument of perihelion, $\nu(t)$ = true anomaly at time t

1.2.3 EMB, Earth and Moon

Using the Earth, Moon and Earth-Moon barycenter coordinates in the Heliocentric Ecliptic Inertial Frame (HEIF) from JPL's Horizons website (see `https://ssd.jpl.nasa.gov/horizons.cgi` with : *EPHEMERIS TYPE = VECTORS*, *COORDINATE ORIGIN = SUN (BODY CENTER) (500@10)*), to compute the average evolution of the EMB, Earth and Moon following parameters :

- The classical orbital elements of the EMB on 01/01/2028 at 0:00 are :

$$a_{\rm EMB} = 1.000004444118239 \text{ AU}, \quad e_{\rm EMB} = 0.016694896378168, \quad i_{\rm EMB} = 0,$$

$$\Omega_{\rm EMB} = 0, \quad \omega_{\rm EMB} = 1.798669025532871 \text{ rad}, \quad \nu_{\rm EMB} = 6.224355095006212 \text{ rad}$$

- In the EMB centered inertial frame **with the same axis as HEIF**, the Moon's circular orbit is assumed to satisfy :
  - Constant distance to EMB = $r_{\rm Moon} = (1-\mu)LD$,with $|mu = \frac{\mu_0^{\rm Moon}}{\mu_0^{\rm Earth}+\mu_0^{\rm Moon}} \approx 0.01215361914$.
  - Orbital period $T^{\rm Moon} = 2361000$ s ≈ 27.326 days.
  - Inclination with restpect to ecliptic - 5.15°.
  - Ascending node longitude of the inclined orbital planed is assumed to evolved linealry in time with the following relation :

$$\Omega^{\rm Moon}(t) = -0.992559541384275 - 0.000924945038810 \cdot t \text{ rad}$$

    where $t = 0$ is 01/01/2028 at 0:00 and t is expressed in days.
  - The angular position ($\approx$ true anomaly) of the Moon on 01/01/2028 is $\nu^0_{\rm Moon} = 0.407422649915963$ rad. Considering the circular orbit and the Moon's period, the angular position of the Moon at time t (if we use $t = 0$ for 01/01/2028) is :

$$\nu_{\rm Moon}(t) = \nu^0_{\rm Moon} + \frac{2\pi}{T^{\rm Moon}} t$$

So in this EMB centered reference frame, the Moon's position $R_{\text{Moon}}^{\text{EMB}}(t)$ is given by :

$$R_{\text{Moon}}^{\text{EMB}}(t) = R_z(\Omega_{\text{Moon}}(t)) \cdot R_x(i_{\text{Moon}}) \cdot r_{\text{Moon}} \begin{bmatrix} \cos(\nu_{\text{Moon}}(t)) \\ \sin(\nu_{\text{Moon}}(t)) \\ 0 \end{bmatrix}$$

where $R_z(\Omega_{\text{Moon}}(t))$ is the rotation matrix of angle $\Omega_{\text{Moon}}(t)$ around the z-axis, and $R_x(i_{\text{Moon}})$ is the rotation matrix of angle i_{Moon} around the x-axis. And the Moon's velocity $V_{\text{Moon}}^{\text{EMB}}(t)$ is simply the time derivative of $R_{\text{Moon}}^{\text{EMB}}$.
The Moon's position and velocity in the Heliocentric frame is given by

$$R_{\text{Moon}}(t) = R_{\text{Moon}}^{\text{EMB}}(t) + R_{\text{EMB}}(t), \quad V_{\text{Moon}}(t) = V_{\text{Moon}}^{\text{EMB}}(t) + V_{\text{EMB}}(t),$$

where $(R_{\text{EMB}}(t), V_{\text{EMB}}(t))$ is the position/velocity of the EMB in HEIF.

- The Earth's position/velocity in the EMB inertial frame are given as *opposite* the Moon's one, that is :

$$(R_{\text{Earth}}^{\text{EMB}}(t), V_{\text{Earth}}^{\text{EMB}}(t)) = -\frac{\mu}{1-\mu}(R_{\text{Moon}}^{\text{EMB}}(t), V_{\text{Moon}}^{\text{EMB}}(t))$$

  And we get the Earth's position/veocity in HEIF by simply adding the EMB's position/velocity in HEIF.

1.3 Equations of motion

1.3.1 EMB and asteroid

The EMB and asteroid's position/velocity can be computed using their orbital elements, a fast way to compute it is by using the mean eccentricity and mean anomaly (to be honest I didn't know this before and I computed the states by integrating the true anomaly dynamics). A pretty concise explanation is at this address : `https://downloads.rene-schwarz.com/download/M001-Keplerian_Orbit_Elements_to_Cartesian_State_Vectors.pdf`. And here is my Matlab code (the real part extraction is because with the impulse maneuvers sometimes I jump onto unwanted non elliptic orbits) :

```
function [q,nu] = Orb2Cart_dt(mu0,xOrb,dt)
    % Inputs :
    %   mu0 : gravitational parameters (same units as xOrb and dt)
    %   xOrb : classical orbital elements (a,e,i,Omega,argper,nu) at time 0
    %   dt : time duration at which cartesian coordinates have to be
    %       computed
    % Outputs :
    %   q : position/velocity at time dt
    %   nu : true anomaly at time dt
    a = xOrb(1); e = xOrb(2); %i = xOrb(3); Omega = xOrb(4);
    %argper = xOrb(5);
    nu0 = xOrb(6);

    % compute mean eccentricity at t0
    % N/B : the real part extraction is for when the orbit is not elliptic,
    % result is then not valid but does not provoke an error
    E0 = atan2(real(sqrt(1-e^2)*sin(nu0)),real(e+cos(nu0)));

    % mean anomaly at time 0
    M0 = E0-e*sin(E0);

    % mean anomaly at time dt
```

```
    M = M0 + dt*sqrt(mu0/a^3);

    % mean eccentricity at time dt
    E = M;
    tol = 1.e-12;
    j = 0;
    jMax = 100;
    while (abs(E-e*sin(E)-M) > tol)&&(j<jMax)
        E = E - (E-e*sin(E)-M)/(1-e*cos(E));
        j = j+1;
    end;
    if abs(E-e*sin(E)-M) > tol
        fprintf(1,'[Orb2Cart_dt] Could not compute mean eccentricity !\n');
    end;

    % true eccentricity at time dt,
    % N/B : the real part extraction is for when the orbit is not elliptic,
    % result is then not valid but does not provoke an error
    nu = 2*atan2(real(sqrt(1+e)*sin(E/2)),real(sqrt(1-e)*cos(E/2)));

    q = Gauss2Cart(mu0,Orb2Gauss([xOrb(1:5);nu]));

end
```

1.3.2 Spacecraft

Cartesian coordinates When not in the vicinity of EMB, the spacecraft will evolve only subjected to Sun's potential so orbital elements will do the job. However, when evolving close to EMB, the orit will not be Keplerian anymore and so, the simpliest set of coordinates to describe the spacecraft are the cartesian ones, whether they are expressed in HEIF of in the EMB centered inertial frame (which is only a translation away). This gives :

$$\left\{\begin{array}{ll} \dot{r}(t) & = v(t) \\ \dot{v}(t) & = -\mu_0^{\text{Sun}} \frac{(r(t)-R_{\text{Sun}}(t))}{(r(t)-R_{\text{Sun}}(t))^3} - \mu_0^{\text{Earth}}(t) \frac{(r(t)-R_{\text{Earth}}(t))}{(r(t)-R_{\text{Earth}}(t))^3} - \mu_0^{\text{Moon}} \frac{(r(t)-R_{\text{Moon}}(t))}{(r(t)-R_{\text{Moon}}(t))^3} \end{array}\right. \tag{1}$$

where if we are in HEIF, then the Sun's position is $R_{\text{Sun}}(t) = 0$ while if we are in the EMB centered frame then $R_{\text{Sun}}(t) = -R_{\text{EMB}}(t)$ (the EMB position in HEIF).

In the sequel, we will need to model impulse maneuver. This is pretty easy since an impulse maneuver is a discontinuity on the velocity, so if we apply a $\delta V \in \mathbb{R}^3$ to the spacecraft at time $\bar{t}$ then we have the follwing junction conditions :

$$\big(R(\bar{t}^+), V(\bar{t}^+)\big) = \big(R(\bar{t}^-), V(\bar{t}^-) + \delta V\big)$$

CR3BP rotating frame Since we did quite a few continuous thrust minimum fuel consumption transfer to mini-moon in the CR3BP, it makes sense to use the CR3BP when we are close to EMB (this is not the only reason). I will not enter into the details of the CR3BP model derivation, but we at least need to explicit how one goes from the EMB centered inertial frame to the CR3BP rotating frame. So here it is :

(i) Let (R, V) be the position/velocity of an object in the EMB centered frame and the (R^M, V^M) be the Moon's position and velocity in this same frame. Assume they are expressed in AU and day (otherwise just adpat later).

(ii) Recall that in the CR3BP rotating frame the Moon is at rest and it's position is $(1-\mu, 0, 0)$ (with μ given before as the ration between Moon's mass and the mass of Earth and Moon). We thus introduce the inetrtial to rotating change of coordinates matrix :

$$M_{\text{inertial}}^{\text{rotating}} = {}^t[R^M,\ (R^M \wedge V^M) \wedge R^M,\ R^M \wedge V^M]$$

where each vector has been normalized.

(iii) Let $c_d = AU/LD$ and $c_v = c_d/(day/T^{\text{Moon}})$, the conversion coefficients for the distance and velocity. Then

$$R_{\text{CR3BP}} = c_d M_{\text{inertial}}^{\text{rotating}} \cdot R, \quad V_{\text{CR3BP}} = c_v M_{\text{inertial}}^{\text{rotating}} \cdot V - {}^t[0,0,1] \wedge R_{\text{CR3BP}}$$

are the object's coordinates in the CR3BP rotating frame.

Sun perturbed CR3BP Unfortunately, in our previous work in the Sun perturbed CR3BP, we assumed that the Sun-Earth-Moon Barycenter had a circular orbit around EMB while with our assumptions it is the Sun that has a nearly circular orbit around the EMB. This does not seem like much of a difference (the Sun-Earth-Moon barycenter is not far from the Sun's center) but the comparison of the two models gives a significant difference when just looking at a drift dynamic.

The model we used in our previous paper is given in [1]. If we assume that the Sun is rotating around the EMB (in the EMB centered inertial frame), then the Sun's potential is only (using [1]'s notations and without checking too much) :

$$\Omega_S(x,y,z,\theta) = -\frac{mu_S}{\rho_S^2}(x\cos\theta + y\sin\theta$$

And so this removes some part of the terms that are in the velocity dynamics. HERE THERE IS A STUDY TO DO ABOUT THE DIFFERENCE BETWEEN THE TWO MODELS BECAUSE THEY GIVE A FAR TOO LARGE DIFFERENCE WHEN I DO A NUMERICAL INTEGRATION OF A DRIFT. IN PARTICULAR THE MODELS THAT AGREE ARE (IF I REMOVE THE MOON'S INCLINATION) THE CARTESIAN MODEL (1) AND THE CR3BP MODEL WITH THE LATEST POTENTIAL.

Denoting by $q = (x, y, z, \dot{x}, \dot{y}, \dot{z})$ the state of the spacecraft in the rotating frame and by $u = (u_1, u_2, u_3)$ the control, the CR3BP dynamics with Sun's perturbation assuming the Sun rotates around EMB in the same orbital plane as Earth and Moon is as follows.

$$\left\{\begin{array}{lcl} \dot{q}_{1,2,3} & = & \dot{q}_{4,5,6} \\ \dot{q}_4 & = & 2q_5 + q_1 - (1-\mu)\frac{(q_1+\mu)}{r_1^3} - \mu\frac{(q_1-1+\mu)}{r_2^3} - \mu_{Sun}\frac{(q_1-\rho_S\cos(\theta_S))}{r_S^3} + u_1 \\ \dot{q}_5 & = & -2q_4 + q_2 - (1-\mu)\frac{q_2}{r_1^3} - \mu\frac{q_2}{r_2^3} - \mu_{\text{Sun}}\frac{(q_2-\rho_S\sin(\theta_S))}{r_S^3} + u_2 \\ \dot{q}_6 & = & -(1-\mu)\frac{q_3}{r_1^3} - \mu\frac{q_3}{r_2^3} - q_3\frac{\mu_{\text{Sun}}}{r_S^3} + u_3 \end{array}\right. \tag{2}$$

Now, if we assume that it is the Sun-Earth-Moon barycenter that revolves aorund EMB (still in the same plane as Earth and Moon), the equations of motion are :

$$\left\{\begin{array}{lcl} \dot{q}_{1,2,3} & = & \dot{q}_{4,5,6} \\ \dot{q}_4 & = & 2q_5 + q_1 - (1-\mu)\frac{(q_1+\mu)}{r_1^3} - \mu\frac{(q_1-1+\mu)}{r_2^3} - \mu_{Sun}\frac{(q_1-\rho_S\cos(\theta_S))}{r_S^3} - \mu_{\text{Sun}}\frac{\cos(\theta_S)}{\rho_S^2} + u_1 \\ \dot{q}_5 & = & -2q_4 + q_2 - (1-\mu)\frac{q_2}{r_1^3} - \mu\frac{q_2}{r_2^3} - \mu_{\text{Sun}}\frac{(q_2-\rho_S\sin(\theta_S))}{r_S^3} - \mu_{\text{Sun}}\frac{\sin(\theta_S)}{\rho_S^2} + u_2 \\ \dot{q}_6 & = & -(1-\mu)\frac{q_3}{r_1^3} - \mu\frac{q_3}{r_2^3} - q_3\frac{\mu_{\text{Sun}}}{r_S^3} + u_3 \end{array}\right. \tag{3}$$

2 Impulse rendez-vous in Heliocentric

The thing to remember is that we are not doing an orbit transfer but a rendez-vous transfer so the time constraint (between 2028 and 2048) will play a significant role.

I choose to use 3 impulses to perform the rendez-vous (3 for outbound, 3 for return).

To go straight to the point, I used the following formulation of the problem to find rendez-vous solutions for the 4258 return trips (from asteroid to Earth). The criterion to minimize is

$$J^{\text{return}}(t_0, \delta V_1, \delta t_1, \delta V_2, \delta t_2, \delta V_3) = \|\delta V_1\| + \|\delta V_2\| + 0.5 \max(0, \delta V_3 - v_0).$$

And the constraints to be satisfied are

$$q(q^{\text{asteroid}}(t_0), \delta V_1, \delta t_1, \delta V_2, \delta t_2, \delta V_3) = q^{\text{EMB}}(t_0 + \delta t_1 + \delta t_2),$$

$$q(q^{\text{asteroid}}(t_0), \delta V_1, \delta t_1, \delta V_2, \delta t_2) \in \text{Moon's orbital plane},$$

and

$$t_0 \in [2030, 2047] \text{ years}, \quad (\delta t_1, \delta t_2) \in [1, 360]^2 \text{ days}$$

where :

- the threshold velocity $v_0 = 1.73$ km/s is the velocity gap below which we assume that a lunar gravity assist will perform the cis-lunar capture for a zero cost (see Robert's paper for a more detailed explanation). Note that I minimize a smoothed max by simply replacing it by $\sqrt{\max^2 + \epsilon}$.
- $q^{\text{asteroid}}(t_0)$ (resp. $q^{\text{EMB}}(t_0 + \delta t_1 + \delta t_2)$) is the asteroid's state at time t_0 (resp. EMB's state at time $t_0 + \delta t_1 + \delta t_2$)
- $q(q^{\text{asteroid}}(t_0), \delta V_1, \delta t_1, \delta V_2, \delta t_2, \delta V_3)$ is the spacecraft state (position/velocity) at time $t_0+\delta t_1+\delta t_2$ is it start at t_0 from $q^{\text{asteroid}}(t_0)$ direclty followed by a velocity jump of δV_1, then drift for δt_1, the impulse δV_2, then drift for δt_2 then impulse δV_3. All this in HEIF with only the Sun's potential.
- The state of the spacecraft before the last impulse δV_3 should be in the Moon's orbital plane, that is it's velocity is orthogonal to the normal to the Moon's orbital plane since it's position is already in it (it is on the EMB position).

Of course, I didn't brutally try to solve this problem (even if after solving it I guess that it could have worked) but I first computed solution of the 3 impulse orbit transfer, then looked at the EMB and asteroid positions realizing this optimal orbit transfer. Then I choose to initialize the rendez-vous problem with the positions (that is actually the time t_0) that are the closest to the ideal orbit transfer ones but in my given time frame.

I did not try to optimize the implementation since I was at first focus on the continuous part which I had to abandon because of a lack of time (and I realize the difference in the Sun perturbed models to late). So the solving method is in *Matlab* using the interior-point algorithm of *fmincon*. One solving is pretty fast but the problem has a lot of local minima so to improve the obtained solutions it takes a lot os brutal computing and thus a lot of time (because there is 4258 asteroids).

I first optimized the return trip because it is the most important since the fuel consumption will be very large due to the large initial mass (the fuel is what is mined on the asteroid).

For the outbound trip, I first solved the same kind of problem as the return trip, except that the criterion is

$$J^{\text{outbound}}(t_0, \delta V_1, \delta t_1, \delta V_2, \delta t_2, \delta V_3) = 0.5 \max(0, \delta V_1 - v_0) + \|\delta V_2\| + \|\delta V_3\|$$

and the constraints are

$$q(q^{\text{EMB}}(t_0), \delta V_1, \delta t_1, \delta V_2, \delta t_2, \delta V_3) = q^{\text{asteroid}}(t_0 + \delta t_1 + \delta t_2),$$

$$\delta V_1 \in \text{Moon's orbital plane},$$

and

$$t_0 + \delta t_1 + \delta t_2 \in t_0^{\text{return}} - [360, 90] \text{ days.}$$

Here t_0^{return} is the departure time of the optimal return trip (which is different for each asteroid). This last constraint means that the spacecraft has to be with the asteroid no less that 90 days and no more than 360 days (my months all last 30 days).

What to do with the impulse rendez-vous? The idea is that apart for the δV that is near the EMB (caution: it can also be δV_2 if δt_1 for the outbound and δt_2 for the return can be as small a 1 day), the other impulses are more or less realistic. Here as say more or less because with a maximum thrust of 50 N and an I_{sp} of 375 s, the computed δV can be equivalent to something like 30 days of continuous thrust !

The idea is to replace the unrealistic by a continuous thrust control by solving an optimal control problem (OCP) that, for the return trip, has an initial condition that comes from the impulse rendez-vous. More precisely, I take the rendez-vous transfer and propagate it till is reaches a given distance to EMB (Earth's Hill Sphere for example). This gives me a position/velocity that should lead me near EMB if I let it drift more. The (OCP) is then to find teh continuous control that will lead me to some parking orbit.

3 Continuous thrust in cis-lunar = Earth-Moon Barycentric

Here we used the the same optimal control problem formulation as in [1] with a fixed number of boosts (that can be 2, 3, $\cdots$, N). I could find solutions in the CR3BP with Sun perturbations when going to or coming from Earth's Hill sphere but I didn't manage to embed them in the cartesian coordinates frame (since the CR3BP has some simplifications that I don't want in the final solution).

References

[1] M. Chyba, T. Haberkorn, R. Jedicke, *Minimum fuel round trip from a L_2 Earth-Moon Halo orbit to Asteroid* 2006 RH_{120}. Celestial and Space Mechanics, Eds. B. Bonnard et M. Chyba, Springer (2015).