Solution Review: Problem Challenge 3

Count of Structurally Unique Binary Search Trees (hard)

Given a number 'n', write a function to return the count of structurally unique Binary Search Trees (BST) that can store values 1 to 'n'.

Example 1:

```
Input: 2
Output: 2
Explanation: As we saw in the previous problem, there are 2 unique BSTs storing numbers from 1-2.
```

Example 2:

```
Input: 3
Output: 5
Explanation: There will be 5 unique BSTs that can store numbers from 1 to 3.
```

Solution

This problem is similar to Structurally Unique Binary Search Trees. Following a similar approach, we can iterate from 1 to 'n' and consider each number as the root of a tree and make two recursive calls to count the number of left and right sub-trees.

Code

Here is what our algorithm will look like:

Java | Python3 | C++ | JS

```python

class TreeNode:
  def __init__(self, val):
    self.val = val
    self.left = None
    self.right = None


def count_trees(n):
  if n <= 1:
    return 1
  count = 0
  for i in range(1, n+1):
    # making 'i' root of the tree
    countOfLeftSubtrees = count_trees(i - 1)
    countOfRightSubtrees = count_trees(n - i)
    count += (countOfLeftSubtrees * countOfRightSubtrees)

  return count


def main():
  print("Total trees: " + str(count_trees(2)))
  print("Total trees: " + str(count_trees(3)))


main()
```

Run Save Reset

Time complexity

The time complexity of this algorithm will be exponential and will be similar to Balanced Parentheses. Estimated time complexity will be $O(n * 2^n)$ but the actual time complexity ($O(4^n/\sqrt{n})$) is bounded by the Catalan number and is beyond the scope of a coding interview. See more details here.

Space complexity

The space complexity of this algorithm will be exponential too, estimated $O(2^n)$ but the actual will be ($O(4^n/\sqrt{n})$.

Memoized version

Our algorithm has overlapping subproblems as our recursive call will be evaluating the same sub-expression multiple times. To resolve this, we can use memoization and store the intermediate results in a **HashMap**. In each function call, we can check our map to see if we have already evaluated this sub-expression before. Here is the memoized version of our algorithm, please see highlighted changes:

Java | Python3 | C++ | JS

```python
class TreeNode:
  def __init__(self, val):
    self.val = val
    self.left = None
    self.right = None


def count_trees(n):
  return count_trees_rec({}, n)


def count_trees_rec(map, n):
  if n in map:
    return map[n]

  if n <= 1:
    return 1
  count = 0
  for i in range(1, n+1):
    # making 'i' the root of the tree
    countOfLeftSubtrees = count_trees_rec(map, i - 1)
    countOfRightSubtrees = count_trees_rec(map, n - i)
    count += (countOfLeftSubtrees * countOfRightSubtrees)

  map[n] = count
  return count


def main():
  print("Total trees: " + str(count_trees(2)))
  print("Total trees: " + str(count_trees(3)))
```

Run Save Reset

The time complexity of the memoized algorithm will be $O(n^2)$, since we are iterating from '1' to 'n' and ensuring that each sub-problem is evaluated only once. The space complexity will be $O(n)$ for the memoization map.