Balanced Parentheses (hard)

We'll cover the following

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Problem Statement

For a given number 'N', write a function to generate all combination of 'N' pairs of balanced parentheses.

Example 1:

```
Input: N=2
Output: (()), ()()
```

Example 2:

```
Input: N=3
Output: ((())), (()()), (())(), ()(()), ()()()
```

Try it yourself

Try solving this question here:

Java | Python3 | JS | C++

```python
def generate_valid_parentheses(num):
  result = []
  # TODO: Write your code here
  return result


def main():
  print('All combinations of balanced parentheses are: ' +
        str(generate_valid_parentheses(2)))
  print('All combinations of balanced parentheses are: ' +
        str(generate_valid_parentheses(3)))


main()
```

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Solution

This problem follows the Subsets pattern and can be mapped to Permutations. We can follow a similar BFS approach.

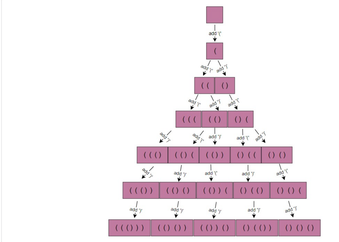
Let's take Example-2 mentioned above to generate all the combinations of balanced parentheses. Following a BFS approach, we will keep adding open parentheses `(` or close parentheses `)`. At each step we need to keep two things in mind:

- We can't add more than 'N' open parenthesis.
- To keep the parentheses balanced, we can add a close parenthesis `)` only when we have already added enough open parenthesis `(`. For this, we can keep a count of open and close parenthesis with every combination.

Following this guideline, let's generate parentheses for N=3:

1. Start with an empty combination: ""
2. At every step, let's take all combinations of the previous step and add `(` or `)` keeping the above-mentioned two rules in mind.
3. For the empty combination, we can add `(` since the count of open parenthesis will be less than 'N'. We can't add `)` as we don't have an equivalent open parenthesis, so our list of combinations will now be: "("
4. For the next iteration, let's take all combinations of the previous set. For "(" we can add another `(` to it since the count of open parenthesis will be less than 'N'. We can also add `)` as we do have an equivalent open parenthesis, so our list of combinations will be: "((", "()"
5. In the next iteration, for the first combination "((", we can add another `(` to it as the count of open parenthesis will be less than 'N', we can also add `)` as we do have an equivalent open parenthesis. This gives us two new combinations: "(((" and "(()". For the second combination "()", we can add another `(` to it since the count of open parenthesis will be less than 'N'. We can't add `)` as we don't have an equivalent open parenthesis, so our list of combinations will be: "(()", "()("
6. Following the same approach, next we will get the following list of combinations: "((())", "(()()", "(())(", "()(()", "()()("
7. Next we will get: "((()))", "(()())", "(())()", "()(())", "()()()"
8. Finally, we will have the following combinations of balanced parentheses: "((()))", "(()())", "(())()", "()(())", "()()()"
9. We can't add more parentheses to any of the combinations, so we stop here.

Here is the visual representation of this algorithm:



Code

Here is what our algorithm will look like:

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```python
from collections import deque


class ParenthesesString:
  def __init__(self, str, openCount, closeCount):
    self.str = str
    self.openCount = openCount
    self.closeCount = closeCount


def generate_valid_parentheses(num):
  result = []
  queue = deque()
  queue.append(ParenthesesString("", 0, 0))
  while queue:
    ps = queue.popleft()
    # if we've reached the maximum number of open and close parentheses, add to the result
    if ps.openCount == num and ps.closeCount == num:
      result.append(ps.str)
    else:
      if ps.openCount < num:  # if we can add an open parentheses, add it
        queue.append(ParenthesesString(
          ps.str + "(", ps.openCount + 1, ps.closeCount))

      if ps.openCount > ps.closeCount:  # if we can add a close parentheses, add it
        queue.append(ParenthesesString(ps.str + ")",
                                       ps.openCount, ps.closeCount + 1))

  return result
```

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Time complexity

Let's try to estimate how many combinations we can have for 'N' pairs of balanced parentheses. If we don't care for the ordering - *that `)` can only come after `(`* - then we have two options for every position, i.e., either put open parentheses or close parentheses. This means we can have a maximum of 2^N combinations. Because of the ordering, the actual number will be less than 2^N.

If you see the visual representation of Example-2 closely you will realize that, in the worst case, it is equivalent to a binary tree, where each node will have two children. This means that we will have 2^N leaf nodes and $2^N - 1$ intermediate nodes. So the total number of elements pushed to the queue will be $2^N + 2^N - 1$, which is asymptotically equivalent to $O(2^N)$. While processing each element, we do need to concatenate the current string with `(` or `)`. This operation will take $O(N)$, so the overall time complexity of our algorithm will be $O(N * 2^N)$. This is not completely accurate but reasonable enough to be presented in the interview.

The actual time complexity ($O(4^n / \sqrt{n})$) is bounded by the Catalan number and is beyond the scope of a coding interview. See more details here.

Space complexity

All the additional space used by our algorithm is for the output list. Since we can't have more than $O(2^N)$ combinations, the space complexity of our algorithm is $O(N * 2^N)$.

Recursive Solution

Here is the recursive algorithm following a similar approach:

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```python
def generate_valid_parentheses(num):
  result = []
  parenthesesString = [0 for x in range(2*num)]
  generate_valid_parentheses_rec(num, 0, 0, parenthesesString, 0, result)
  return result


def generate_valid_parentheses_rec(num, openCount, closeCount, parenthesesString, index, result):

  # if we've reached the maximum number of open and close parentheses, add to the result
  if openCount == num and closeCount == num:
    result.append(''.join(parenthesesString))
  else:
    if openCount < num:  # if we can add an open parentheses, add it
      parenthesesString[index] = '('
      generate_valid_parentheses_rec(
        num, openCount + 1, closeCount, parenthesesString, index + 1, result)

    if openCount > closeCount:  # if we can add a close parentheses, add it
      parenthesesString[index] = ')'
      generate_valid_parentheses_rec(
        num, openCount, closeCount + 1, parenthesesString, index + 1, result)


def main():
  print('All combinations of balanced parentheses are: ' +
        str(generate_valid_parentheses(2)))
  print('All combinations of balanced parentheses are: ' +
        str(generate_valid_parentheses(3)))
```

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