# All Paths for a Sum (medium)

We'll cover the following

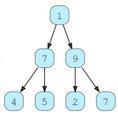


## Problem Statement

Given a binary tree and a number 'S', find all paths from root-to-leaf such that the sum of all the node values of each path equals 'S'.

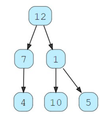
**Example 1:**

S: 12
Output: [[1, 7, 4], [1, 9, 2]]
Explanation: Here are the two paths with sum '12':
1 -> 7 -> 4 and 1 -> 9 -> 2




**Example 2:**

S: 23
Output: [[12, 7, 4], [12, 1, 10]]
Explanation: Here are the two paths with sum '23':
12 -> 7 -> 4 and 12 -> 1 -> 10




## Try it yourself

Try solving this question here:

Java | Python3 | JS | C++

```python
class TreeNode:
  def __init__(self, val, left=None, right=None):
    self.val = val
    self.left = left
    self.right = right


def find_paths(root, sum):
  allPaths = []
  # TODO: Write your code here
  return allPaths


def main():

  root = TreeNode(12)
  root.left = TreeNode(7)
  root.right = TreeNode(1)
  root.left.left = TreeNode(4)
  root.right.left = TreeNode(10)
  root.right.right = TreeNode(5)
  sum = 23
  print("Tree paths with sum " + str(sum) +
        ": " + str(find_paths(root, sum)))

main()
```

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## Solution

This problem follows the Binary Tree Path Sum pattern. We can follow the same **DFS** approach. There will be two differences:

1. Every time we find a root-to-leaf path, we will store it in a list.
2. We will traverse all paths and will not stop processing after finding the first path.

## Code

Here is what our algorithm will look like:

Java | Python3 | C++ | JS

```python
class TreeNode:
  def __init__(self, val, left=None, right=None):
    self.val = val
    self.left = left
    self.right = right


def find_paths(root, required_sum):
  allPaths = []
  find_paths_recursive(root, required_sum, [], allPaths)
  return allPaths

def find_paths_recursive(currentNode, required_sum, currentPath, allPaths):
  if currentNode is None:
    return

  # add the current node to the path
  currentPath.append(currentNode.val)

  # if the current node is a leaf and its value is equal to required_sum, save the current path
  if currentNode.val == required_sum and currentNode.left is None and currentNode.right is None:
    allPaths.append(list(currentPath))
  else:
    # traverse the left sub-tree
    find_paths_recursive(currentNode.left, required_sum -
                         currentNode.val, currentPath, allPaths)
    # traverse the right sub-tree
    find_paths_recursive(currentNode.right, required_sum -
                         currentNode.val, currentPath, allPaths)
```

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## Time complexity

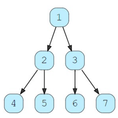
The time complexity of the above algorithm is $O(N^2)$, where '$N$' is the total number of nodes in the tree. This is due to the fact that we traverse each node once (which will take $O(N)$), and for every leaf node, we might have to store its path (by making a copy of the current path) which will take $O(N)$.

We can calculate a tighter time complexity of $O(NlogN)$ from the space complexity discussion below.

## Space complexity

If we ignore the space required for the `allPaths` list, the space complexity of the above algorithm will be $O(N)$ in the worst case. This space will be used to store the recursion stack. The worst-case will happen when the given tree is a linked list (i.e., every node has only one child).

How can we estimate the space used for the `allPaths` array? Take the example of the following balanced tree:




Here we have seven nodes (i.e., `N = 7`). Since, for binary trees, there exists only one path to reach any leaf node, we can easily say that total root-to-leaf paths in a binary tree can't be more than the number of leaves. As we know that there can't be more than $(N+1)/2$ leaves in a binary tree, therefore the maximum number of elements in `allPaths` will be $O((N+1)/2) = O(N)$. Now, each of these paths can have many nodes in them. For a balanced binary tree (like above), each leaf node will be at maximum depth. As we know that the depth (or height) of a balanced binary tree is $O(logN)$ we can say that, at the most, each path can have $logN$ nodes in it. This means that the total size of the allPaths list will be $O(N * logN)$. If the tree is not balanced, we will still have the same worst-case space complexity.

From the above discussion, we can conclude that our algorithm's overall space complexity is $O(N * logN)$.

Also, from the above discussion, since for each leaf node, in the worst case, we have to copy $log(N)$ nodes to store its path; therefore, the time complexity of our algorithm will also be $O(N * logN)$.

## Similar Problems

**Problem 1:** Given a binary tree, return all root-to-leaf paths.

*Solution:* We can follow a similar approach. We just need to remove the "check for the path sum."

**Problem 2:** Given a binary tree, find the root-to-leaf path with the maximum sum.

*Solution:* We need to find the path with the maximum sum. As we traverse all paths, we can keep track of the path with the maximum sum.