MATH2715 Workshop 6 Questions

QA. Let $X_i \sim \mathrm{N}(3,\,4)$ and suppose that we are going to observe $\underline{\mathrm{X}} = \{X_1, X_2, X_3, X_4, X_5, X_6\}$. Calculate the sample mean and variance for $\underline{\mathrm{x}} = \{4.3, 5.8, 4.8, 9.8, -0.4, 7.2\}$. What is the probability of getting a sample mean at least as extreme (that is, greater than) as what we have here? Is the probability of getting a sample variance at least as extreme as the one we have calculated less than 0.1?

QB. Let $X \sim \mathrm{N}(\mu, 9)$, and we are going to observe $\underline{\mathrm{X}} = \{X_1, X_2, \cdots X_n\}$. Give a 95% confidence interval for μ. What is the width of that confidence interval? How large should n be in order to get a confidence interval that has a width of at most 0.2?

QC. The compression force of a certain type of concrete is modeled by a Gaussian random variable with expectation μ and variance σ^2 The measurement unit is the psi (pound per square inch). In questions 1. to 4., it will be supposed that the variance σ^2 is known and equal to 1000. An empirical mean of 3250 psi has been observed from a sample of 12 measurements.

1. Give a 95% confidence interval for μ .

2. Give a 99% confidence interval for μ. Compare its width with that of the interval in the previous question.

3. If using the same sample, a confidence interval of width 30 psi were given, what would its confidence level be?

4. What minimal number of trials would be necessary to estimate μ with a precision of ± 15 psi, at confidence level 0.95?

5. From now on the theoretical variance is supposed to be unknown. For the 12 trials mentioned above:

   $$\sum_{i=1}^{12} x^2 = 126761700$$

   Give a 95% confidence interval for μ and compare it with that of question 1. Repeat the calculation for a 99% confidence interval and compare it with that of question 2.

6. Give a 95% confidence interval for the variance, and for the standard deviation.