$2^5 - 1 = 31$

2.14 Construct a table for $GF(2^5)$ based on the primitive polynomial $p(X) = 1 + X^2 + X^5$. Let α be a primitive element of $GF(2^5)$. Find the minimal polynomials of α^3 and α^7.

$p(\alpha) = 1 + \alpha^2 + \alpha^5 \Rightarrow \alpha^5 = 1 + \alpha^2$

Power Prescirtation	Polynomial Presentation	5-Tuple	
0	0	00000	
1	1	10000	
α	α	01000	
α^2	α^2	00100	
α^3	α^3	00010	
α^4	α^4	00001	
α^5	$1+\alpha^2$	10100	
α^6	$\alpha+\alpha^3$	01010	$\alpha\cdot\alpha^5$
α^7	$\alpha^2+\alpha^4$	00101	$\alpha\cdot\alpha^6$
α^8	$1+\alpha^2+\alpha^3$	10110	$\alpha\cdot\alpha^7 = \alpha^3+\alpha^5$ ($1+\alpha^2$)
α^9	$\alpha+\alpha^3+\alpha^4$	01011	$\alpha\cdot\alpha^8$
α^{10}	$1+\alpha^4$	10001	$\alpha\cdot\alpha^9 = \alpha^2+\alpha^4+\alpha^5$
α^{11}	$1+\alpha+\alpha^2$	11100	$= 1+2\alpha^2+\alpha^4$
α^{12}	$\alpha+\alpha^2+\alpha^3$	01110	
α^{13}	$\alpha^2+\alpha^3+\alpha^4$	00111	
α^{14}	$1+\alpha^2+\alpha^3+\alpha^4$	10111	
α^{15}	$1+\alpha+\alpha^2+\alpha^3+\alpha^4$	11111	
α^{16}	$1+\alpha+\alpha^3+\alpha^4$	11011	
α^{17}	$1+\alpha+\alpha^4$	11001	
α^{18}	$1+\alpha$	11000	
α^{19}	$\alpha+\alpha^2$	01100	
α^{20}	$\alpha^2+\alpha^3$	00110	
α^{21}	$\alpha^3+\alpha^4$	00011	
α^{22}	$1+\alpha^2+\alpha^4$	10101	$\alpha+\alpha^3+\alpha^5$ ($1+\alpha^2$)
α^{23}	$1+\alpha+\alpha^2+\alpha^3$	11110	
α^{24}	$\alpha+\alpha^2+\alpha^3+\alpha^4$	01111	
α^{25}	$1+\alpha^3+\alpha^4$	10011	
α^{26}	$1+\alpha+\alpha^2+\alpha^4$	11101	
α^{27}	$1+\alpha+\alpha^3$	11010	
α^{28}	$\alpha+\alpha^2+\alpha^4$	01101	
α^{29}	$1+\alpha^3$	10010	
α^{30}	$\alpha+\alpha^4$	01001	