Advanced
Communication
Technology Lab

Performance Analysis of GLRT-Based SIMO-OFDM Demodulation System With Pilot-Aided Channel Estimation

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Outline

- Introduction
- Pilot-Aided SIMO-OFDM System Model
- Performance Analysis and Performance Results
  - A) Fixed Block Dispersive Channel
  - B) Random Block Dispersive Channel
- Conclusion

Introduction

OFDM-Based Pilot-Aided Channel Estimation

- Channel estimation is essential to OFDM systems since the receiver requires accurate channel state information (CSI) to perform coherent data demodulation or diversity combining when multiple-antenna techniques are deployed.

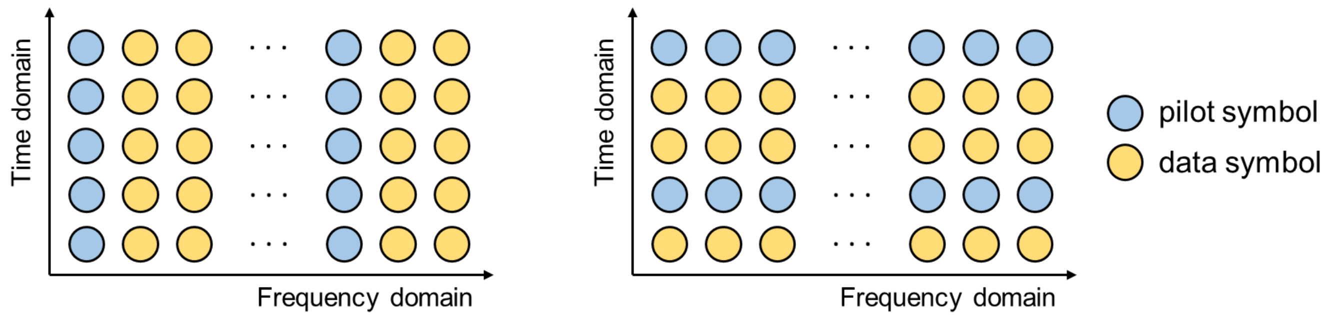
- **Classes of Methods**

	Blind Channel Estimation	Training-Based Channel Estimation
Methods	Perform channel estimation based on the statistical behavior of the received **data** signal.	Perform channel estimation based on known **pilot** or **preamble** symbols.
Advantages	Only relies on the received **data** symbols.	Reduces computational complexity, needs shorter receive signal records and more practical.
Disadvantages	Needs longer data records and entails higher complexity.	Acquires an overhead with training symbols.

- Pilot symbols and pilot placement place significant impact on channel estimation.

Pilot Placement

- In the OFDM systems, pilots symbols are scatter in the time and frequency domain to track time-variant and frequency-selective channel characteristics.
- In [1]-[3], **uniformly interleaved subcarriers** with **constant amplitude** pilots can achieve the minimum CRB $CRB_{\min} = \frac{L}{N\gamma_p}$ and enable high estimation accuracy for pilot-aided channel estimation on block dispersive channels.



A) Comb-type **B) Block-type**

[1] J.-W. Choi and Y.-H. Lee, "Optimum pilot pattern for channel estimation in OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2083-2088, Sep. 2005.
[2] S. Tong, B. M. Sadler, and M. Dong, "Optimal training and redundant precoding for block transmissions with application to wireless OFDM," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2113-2123, Dec. 2002.
[3] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consumer Electron.*, vol. 44, no. 3, pp. 1122-1128, Aug. 1998.

Channel Estimation Methods

- **Methods**

  received symbol $\widetilde{r}[k] = p[k]\widetilde{h}[k] + \widetilde{u}[k]$ for $k = 0, 1, ..., N-1$

  received vector $\widetilde{\mathbf{r}} = \mathbf{D}\widetilde{\mathbf{h}} + \widetilde{\mathbf{u}} = \mathbf{DF}_p\mathbf{h} + \widetilde{\mathbf{u}}$

	Least Square (LS)	Minimum Mean Square Error (MMSE)
CIR	$\widehat{\mathbf{h}}_{\text{LS}} = ((\mathbf{DF}_p)^h\mathbf{DF}_p)^{-1}(\mathbf{DF}_p)^h\widetilde{\mathbf{r}}$ direct LS	$\widehat{\mathbf{h}}_{\text{MMSE}} = \mathbf{R}_{\tilde{r}h}^h\mathbf{R}_{\tilde{r}\tilde{r}}\widetilde{\mathbf{r}}$ where $\mathbf{R}_{\tilde{r}h} = E\{\tilde{\mathbf{r}}\mathbf{h}^h\}$ $\mathbf{R}_{\tilde{r}\tilde{r}} = E\{\tilde{\mathbf{r}}\tilde{\mathbf{r}}^h\}$ direct MMSEE
CFR	Obtain LS estimate at **pilot** position.	Obtain MMSE estimate at **pilot** position.
	At **data** position, several interpolation methods (i.e., Linear, Polynomial, Sinc Interpolation etc.) are used to obtain channel response.	

- In [4], it shown that **Polynomial interpolation > Linear interpolation**
- In [5], it shown that the **direct MMSE > direct LS > Linear interpolation > Sinc interpolation.**

[4] M.-H. Hsieh and C.H. Wei, "Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels," *IEEE Trans. Consum. Electron.*, vol. 44, no. 1, pp. 217-225, Feb. 1998.
[5] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Sig. Process.*, vol. 49, no. 12, pp. 3065-3073, Dec. 2001.

Motivation (1/2)

1) Consider the **time-selective block dispersive channel** which is fixed only within one OFDM blocks but varies over distant blocks.
2) The block dispersive channel channel is **unknown** at the receiver.

Fixed Block Dispersive Channel

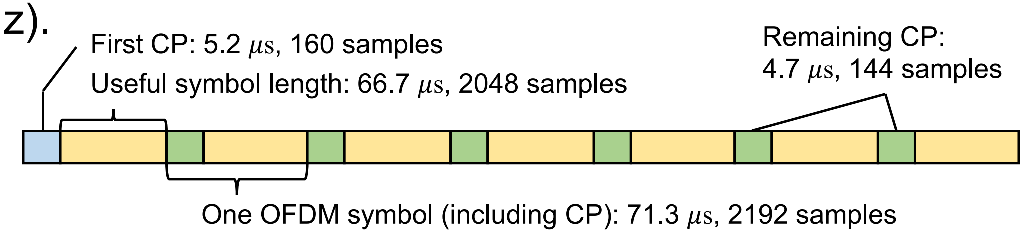
- The channel responses are the same among different OFDM blocks but the receiver only received a few OFDM blocks.

  e.g.) Recent IoT applications use short packets in the range of 512 bits up to 4096 bits [6].

Random Block Dispersive Channel

- The channel exhibits a short coherence time but not shorter than one OFDM block time.

  e.g.) LTE [7] is designed to operate in delay spreads up to ~5 μs and for speeds up to 350 km/h (1.2ms coherence time @ 2.6GHz).



[6] 3GPP TS 38.211 V15.4, *5G NR: Physical channels and modulation*, Dec. 2018.
[7] 3GPP TS 36.211 V12.3.0, *LTE; Evolved universal terrestrial radio access (E-UTRA): Physical channels and modulation*, Oct. 2014.

Related Works

- In [8], a tight lower bound on the **channel capacity** is derived for pilot-aided OFDM systems over i.i.d. Rayleigh fading channel and direct MMSE channel estimator is used at the receiver. And the optimal ratio of data to pilot power is also given.

  ⇒ direct MMSE is difficult to implement in practice under block dispersive channel

- In [9]-[12], the average **SEP** and **BEP characteristics** have been investigated.

ref	diversity system	component modulation	channel estimation	channel	detection techniques	comments
[9]	SISO	BPSK, QPSK and 16-ary QAM	LS/LMMSE + poly, **direct LS**, direct LMMSE	i.n.i.d Rayleigh fading paths	ZF-FDE (symbol)	Present a unified approach for analysis the BEP performance for various square QAM.
[10]	SISO	QPSK	LS + linear	correlated Rayleigh paths	coherent ML (symbol)	Easily find the optimal number of pilots under practical scenarios.
[11]	**SIMO**	M-ary PSK	**direct LS**, direct LMMSE	i.i.d Rayleigh paths	MRC rule (symbol)	Optimize the power allocation, pilot number and placement to minimize SER.
[12]	**SIMO**	16-ary QAM	LS + sinc/Wiener	i.i.d Rayleigh and Rician paths	MRC rule (symbol)	Prove the performance of Wiener interpolation is better than sinc interpolation.

[8] S. Adireddy, L. Tong, and H. Viswanathan, "Optimal placement of training for frequency-selective block-fading channels," *IEEE Trans. Inform. Theory*, vol. 48, no. 8, pp. 2338-2353, Aug. 2002.
[9] M.-X. Chang and Y. T. Su, "Performance analysis of equalized OFDM systems in Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 1, no. 4, pp. 721-732, Oct. 2002.
[10] W. Zhang, X.Xia, and P.-C. Ching, "Optimal training and pilot pattern design for OFDM systems in Rayleigh fading," *IEEE Trans. Broadcast.*, vol. 52, no. 4, pp. 505-514, Dec. 2006.
[11] X. Cai and G. B. Giannakis, "Error probability minimizing pilots for OFDM with M PSK modulation over Rayleigh-fading channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 1461-55, Jan. 2004.
[12] P. Tan and N. C. Beaulieu, "Effect of channel estimation error on bit error probability in OFDM systems over Rayleigh and Rician fading channels," *IEEE Trans. Commun.*, vol. 56, no. 4, pp. 675-685, Apr. 2008.

Motivation (2/2)

- Due to analytical difficulty, there are some restrictions on block dispersive channels and component modulation schemes in order to simplify the pilot-aided data detection performance analysis in existing studies.

Contributions

- Given the performance analysis of generalized likelihood ratio test based (GLRT-based) SIMO-OFDM systems in the presence of channel estimation errors over block dispersive channel.

> **Why SIMO?**
>
> Under the time varying dispersive channel, MIMO systems are unable to perform channel estimation within one OFDM symbol block time.

- Relax the restrictions on block dispersive channels and component modulation schemes to provide a more general analysis on the data detection performance for pilot- aided OFDM systems.
- The random dispersive channel is generally modeled to have independent but not necessarily identical distributed random paths over various antenna elements.

Pilot-Aided OFDM System Model

Transmitted Signal

Normalization

pilot symbols: $\frac{1}{N}\sum_{n\in\mathcal{Z}_N}|p[n]|^2=1$

data symbols: $E\{d_k[n]\}=1$

- OFDM block transmission with $N\phi$ subcarriers.
- In a block, complex-valued symbol $\rho_k^{(1/2)}d_k[n]$ modulated on the $(n\phi+k)$ -th subcarrier.

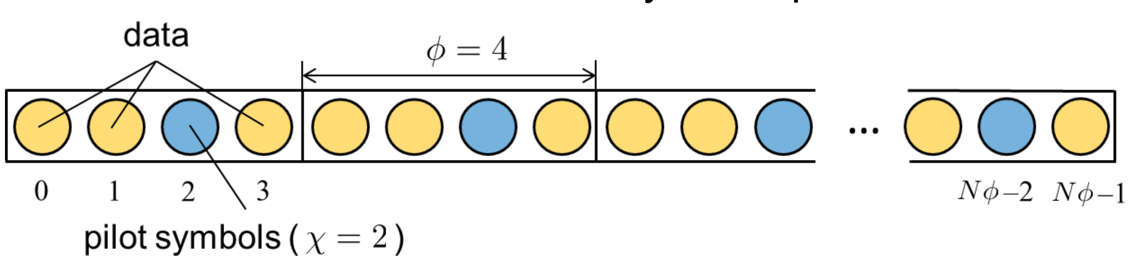
$$d_k[n]=\begin{cases}\text{pilot symbol with average power } \rho_k=\rho^{(p)}\text{, when } k=\chi\\ \text{data symbol with average power } \rho_k=\rho^{(d)}\text{, otherwise}\end{cases}$$

- **Baseband transmitted signal**

$$s[m]=\sum_{k\in\mathcal{Z}_\phi}\sum_{n\in\mathcal{Z}_N}\sqrt{\frac{\rho_k}{N\phi}}d_k[n]\omega_{\phi N}^{-m(n\phi+k)} \tag{1}$$

for $m\in\{-\alpha\phi N,-\alpha\phi N+1,\ldots,-1,0,1,\ldots,\phi N-1\}$

where $\alpha=\dfrac{\text{number of guard symbols}}{\text{number of useful symbols per block}}$ and $\omega_{\phi N}\triangleq\exp\{-j2\pi/\phi N\}$



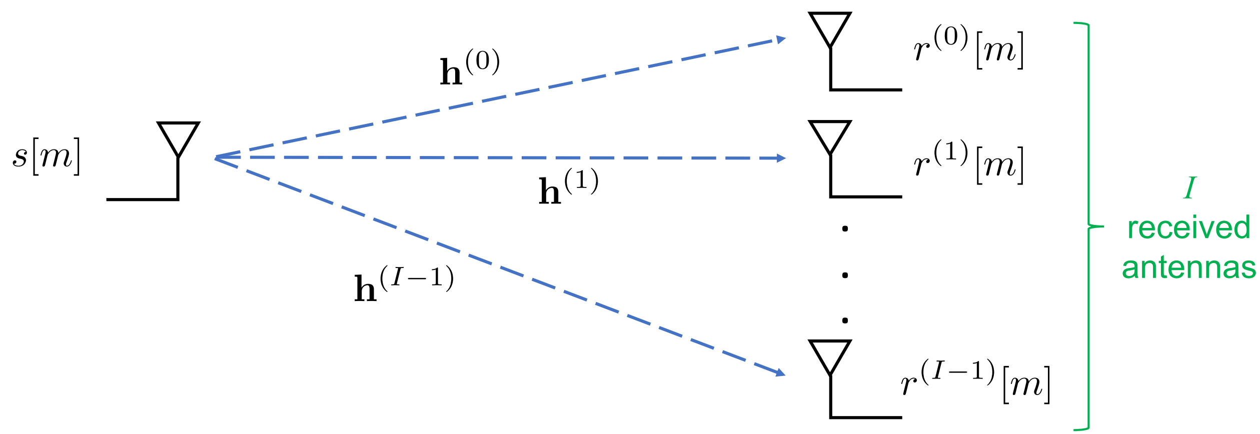
N: pilot length
ϕ: interleaving length

Channel Model

- Consider the time-selective block dispersive channel in a SIMO system with I received antennas
- Antenna elements are placed sufficiently apart that I received signals suffer independent channel effects.
- **Channel impulse response (CIR) vector**

$$\mathbf{h}^{(i)} \triangleq [h^{(i)}[l]; l \in \mathcal{Z}_L] \quad (2)$$

where L is the length of the channel with $L \leq \alpha\phi N$.



Received Signal

- **Time-domain received signal (delete guard portion)**

When i-th antenna path is aligned perfectly in time and frequency

$$r^{(i)}[m] = \sum_{l=0}^{L-1} h^{(i)}[l]s[m - l \bmod \phi N] + u^{(i)}[m], m \in \mathcal{Z}_{\phi N} \text{ and } i \in \mathcal{Z}_I \quad (3)$$

where $u^{(i)}[m]$'s are i.i.d circularly symmetric complex Gaussian (CSCG) noise samples with mean zero and variance $E\{|u^{(i)}[m]|^2\} = \sigma^2$.

- **Frequency-domain received samples (vector form)**

$$\widetilde{\mathbf{r}}_k^{(i)} = \rho_k^{\frac{1}{2}} \mathbf{D}_k \mathbf{W}_L \mathbf{\Omega}_k \mathbf{h}^{(i)} + \widetilde{\mathbf{u}}_k^{(i)}, k \in \mathcal{Z}_\phi \text{ and } i \in \mathcal{Z}_L \quad (4)$$

where $\widetilde{\mathbf{u}}_k^{(i)} \triangleq [\widetilde{u}^{(i)}[n\phi + k]; n \in \mathcal{Z}_N] \Rightarrow$ the DFT matrix of noise

$$\mathbf{D}_k \triangleq \mathrm{diag}([d_k[n]; n \in \mathcal{Z}_N])$$

Note

$\mathbf{D}_k$ carries the data vector $\mathbf{d}_k$ for $k \neq \chi$ and the pilot vector $\mathbf{p}$ otherwise.

$$\left.\begin{aligned} \mathbf{W}_L &\triangleq [\omega_N^{nl}; n \in \mathcal{Z}_N, l \in \mathcal{Z}_L] \\ \mathbf{\Omega}_k &\triangleq \mathrm{diag}([\omega_{\phi N}^{kl}; l \in \mathcal{Z}_L]) \end{aligned}\right\} \text{the DFT coefficient}$$

Generalized Likelihood Ratio Test (GLRT)

- The GLRT criterion [13] is used in the situations where no stochastic model for the unknow parameter (i.e. CIR's $\{\mathbf{h}^{(i)}\}$) is assumed at the receiver.
- Although GLRT does not always asymptotically optimal, the consider GLRT decision rule approach to the coherence ML decision rule when the pilot power approach to the infinity.

Joint Channel Estimation and Block Data Detection

- Given $\mathbf{h}^{(i)}$ and $\mathbf{D}_k$, $\widetilde{\mathbf{r}}_k^{(i)}$'s are independent complex Gaussian random vectors with mean vectors $\rho_k^{\frac{1}{2}}\mathbf{D}_k\mathbf{W}_L\mathbf{\Omega}_k\mathbf{h}^{(i)}$ and covariance matrix $\sigma^2\mathbf{I}_N$.
- By maximizing the conditional joint density of $\tilde{\mathbf{r}}_k^{(i)}$'s given $\mathbf{h}^{(i)}$ and $\mathbf{D}_k$

$$\{\widehat{\mathbf{d}}_k\} = \arg\min_{\{\mathbf{d}_k\}}\min_{\{\mathbf{h}^{(i)}\}}\{\sum_{i\in\mathcal{Z}_I}\sum_{k\in\mathcal{Z}_\phi}\|\widetilde{\mathbf{r}}_k^{(i)} - \rho_k^{\frac{1}{2}}\mathbf{D}_k\mathbf{W}_L\mathbf{\Omega}_k\mathbf{h}^{(i)}\|^2\}$$

Complicated to realized when $N\phi$ is large.

$$= \arg\min_{\{\mathbf{d}_k\}}\{\sum_{i\in\mathcal{Z}_I}\sum_{k\in\mathcal{Z}_\phi}\|\widetilde{\mathbf{r}}_k^{(i)} - \rho_k^{\frac{1}{2}}\mathbf{D}_k\mathbf{W}_L\mathbf{\Omega}_k\widehat{\mathbf{h}}_1^{(i)}(\{\mathbf{d}_k\},\{\widetilde{\mathbf{r}}_k^{(i)}\})\|^2\} \quad (5)$$

[13] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*. John Wiley & Sons Inc., 1968.

Channel Estimation

- $\widetilde{\mathbf{r}}_{\chi}^{(i)}$'s carry known pilot $\mathbf{p}$ only and can be used to estimate the CIR $\mathbf{h}^{(i)}$'s without interference from data.

- Given $\mathbf{h}^{(i)}$ and $\mathbf{p}$, $\widetilde{\mathbf{r}}_{\chi}^{(i)}$'s are independent CSCG vector with mean $(\rho^{(p)})^{1/2}\mathbf{D}_{\chi}\mathbf{W}_{L}\mathbf{\Omega}_{k}\mathbf{h}^{(i)}$ and covariance matrix $\sigma^2\mathbf{I}_N$.

- Based on $\widetilde{\mathbf{r}}_{\chi}^{(i)}$, the ML estimate of $\mathbf{h}^{(i)}$ can be obtained by maximizing the likelihood density of $\widetilde{\mathbf{r}}_{\chi}^{(i)}$ given $\mathbf{h}^{(i)}$

$$\begin{aligned}\widehat{\mathbf{h}}_2^{(i)}(\widetilde{\mathbf{r}}_{\chi}^{(i)}) &= \arg\max_{\mathbf{h}^{(i)}} f(\widetilde{\mathbf{r}}_{\chi}^{(i)}|\mathbf{h}^{(i)}) \\ &= \arg\min_{\mathbf{h}^{(i)}}\{\|\widetilde{\mathbf{r}}_{\chi}^{(i)} - (\rho^{(p)})^{\frac{1}{2}}\mathbf{D}_{\chi}\mathbf{W}_{L}\mathbf{\Omega}_{\chi}\mathbf{h}^{(i)}\|^2\} \\ &= (\rho^{(p)})^{-\frac{1}{2}}((\mathbf{D}_{\chi}\mathbf{W}_{L}\mathbf{\Omega}_{\chi})^h\mathbf{D}_{\chi}\mathbf{W}_{L}\mathbf{\Omega}_{\chi})^{-1}(\mathbf{D}_{\chi}\mathbf{W}_{L}\mathbf{\Omega}_{\chi})^h\widetilde{\mathbf{r}}_{\chi}^{(i)}.\end{aligned} \tag{6}$$

Decision Rule for symbol $d_k[n]$

Note

$\mathbf{1}_n = [0 \ldots 010 \ldots 0]^t$

n^{th} position

- Using the CIR estimate $\widehat{\mathbf{h}}_2^{(i)}(\widetilde{\mathbf{r}}_\chi^{(i)})$, $d_k[n]$ can be decided through the generalized likelihood ratio test (GLRT) [13]

$$\widehat{d}_k[n] = \arg\min_d \{ \sum_{i \in \mathcal{Z}_I} |\widetilde{r}^{(i)}[n\phi + k] - (\rho^{(p)})^{\frac{1}{2}} d \mathbf{1}_n^t \mathbf{W}_L \mathbf{\Omega}_k \widehat{\mathbf{h}}_2^{(i)}(\widetilde{\mathbf{r}}_\chi^{(i)})|^2 \} \tag{7}$$

$$= \arg\min_d \sum_{i \in \mathcal{Z}_I} F_{k,n}^{(i)}(d) \tag{8}$$

for each $n \in \mathcal{Z}_N$ and $k \in \mathcal{Z}_\phi$, where we have conveniently defined

$$F_{k,n}^{(i)}(d) \triangleq |\widetilde{r}^{(i)}[n\phi + k] - \varphi^{\frac{1}{2}} d \mathbf{1}_n^t \mathbf{V}_k \widetilde{\mathbf{r}}_\chi^{(i)}|^2 \tag{9}$$

$$\mathbf{V}_k \triangleq \mathbf{W}_L \mathbf{\Omega}_k ((\mathbf{D}_\chi \mathbf{W}_L \mathbf{\Omega}_\chi)^h \mathbf{D}_\chi \mathbf{W}_L \mathbf{\Omega}_\chi)^{-1} (\mathbf{D}_\chi \mathbf{W}_L \mathbf{\Omega}_\chi)^h \tag{10}$$

where $\varphi \triangleq \rho^{(d)}/\rho^{(p)}$ represents the ratio of average data power to average pilot power.

[13] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*. John Wiley & Sons Inc., 1968.

Decision Rule for symbol $d_k[n]$

- Rewrite rule (7)

$$\widehat{d}_k[n] = \arg\max_d [2 \sum_{i \in \mathcal{Z}_I} \mathrm{Re}\{\widetilde{r}^{(i)}[n\phi + k](\widehat{\widetilde{h}}^{(i)}[n\phi + k])^* d^*\} - (\rho^{(d)})^{\frac{1}{2}} |d|^2 |\widehat{\widetilde{h}}^{(i)}[n\phi + k]|^2] \quad (11)$$

$$= \arg\min_d \left| \frac{\sum_{i \in \mathcal{Z}_I} \widetilde{r}^{(i)}[n\phi + k](\widehat{\widetilde{h}}^{(i)}[n\phi + k])^*}{\sum_{i' \in \mathcal{Z}_I} |\widehat{\widetilde{h}}^{(i)}[n\phi + k]|^2} - (\rho^{(d)})^{\frac{1}{2}} d \right|^2 \quad (12)$$

Maximum Ratio Combining (MRC) [11]-[12]

The decision statistic using MRC is given by

$$\widehat{d}_k[n] = \frac{\sum_{i=0}^{I-1} \tilde{r}^{(i)}[n\phi + k](\widehat{\widetilde{h}}^{(i)}[n\phi + k])^*}{\sum_{i'=0}^{I-1} |\widehat{\widetilde{h}}^{(i)}[n\phi + k]|^2}$$

where $\widehat{\widetilde{h}}^{(i)}[n\phi + k]$ denotes the estimated CFR at i-th received antenna.

[11] X. Cai and G. B. Giannakis, "Error probability minimizing pilots for OFDM with M PSK modulation over Rayleigh-fading channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 1461-55, Jan. 2004.
[12] P. Tan and N. C. Beaulieu, "Effect of channel estimation error on bit error probability in OFDM systems over Rayleigh and Rician fading channels," *IEEE Trans. Commun.*, vol. 56, no. 4, pp. 675-685, Apr. 2008.

Performance Analysis

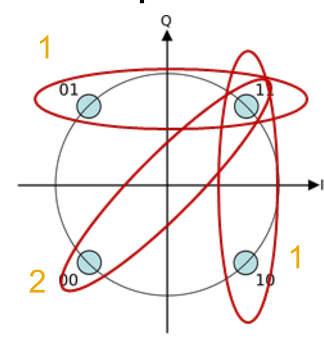
A) Conditional BEP Upper Bound

A) Conditional BEP Upper Bound

- Using the union bound argument [14], the BEP of rule (6) is bounded for given CIR's $\{\mathbf{h}^{(i)}\}$ by

$$P_b(\{\mathbf{h}^{(i)}\}) \leq \frac{1}{N(\phi-1)} \sum_{k\in\mathcal{Z}_\phi} \sum_{n\in\mathcal{Z}_N} \sum_{\substack{d_k[n],\hat{d}_k[n]\in\mathcal{S} \\ \hat{d}_k[n]\neq d_k[n]}} \frac{D_H(d_k[n],\widehat{d}_k[n])}{M\log_2 M}$$
$$\times \Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}_k[n]) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d_k[n]) \mid d_k[n],\{\mathbf{h}^{(i)}\}\} \qquad (13)$$

where $D_H(d_k[n],\widehat{d}_k[n])$ is the Hamming distance between $d_k[n]$ and $\widehat{d}_k[n]$. $\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}_k[n]) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d_k[n]) \mid d_k[n],\{\mathbf{h}^{(i)}\}\}$ is the pairwise error probability.



[14] M. K. Simon and M.S. Alouini, *Digital Communication over Fading Channels*, John Wiley & Sons, Inc., 2005.

Pairwise Error Probability

Notation

$\widehat{d}_k[n] = \hat{d}$ and $d_k[n] = d$

- Given $\mathbf{h}^{(i)}$'s, $\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d)$ and $\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d})$ are correlated noncentral chi-square random variables with $2I$ degrees of freedom.
- Due to the independence among received antennas, $\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}_k[n]) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d_k[n]) \mid d_k[n], \{\mathbf{h}^{(i)}\}\}$ a special case of quadratic sum in complex Gaussian random variable [15]

Only β depends on CIR's $\{\mathbf{h}^{(i)}\}$, while the other parameters do not.

$$
\begin{aligned}
&\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d)|d,\{\mathbf{h}^{(i)}\}\} \\
&=[1-Q_1(\beta^{1/2}\varepsilon_2,\beta^{1/2}\varepsilon_1)][1-\frac{\sum_{a=0}^{I-1}\binom{2I-1}{a}(\frac{\nu_2}{\nu_1})^a}{(1+\nu_2/\nu_1)^{2I-1}}]+Q_1(\beta^{1/2}\varepsilon_1,\beta^{1/2}\varepsilon_2)\frac{\sum_{a=0}^{I-1}\binom{2I-1}{a}(\frac{\nu_2}{\nu_1})^a}{(1+\nu_2/\nu_1)^{2I-1}} \\
&\quad+\frac{1}{(1+\nu_2/\nu_1)^{2I-1}}\Big[\sum_{a=2}^{I}\binom{2I-1}{I-a}(\frac{\nu_2}{\nu_1})^{I-a}\times[Q_a(\beta^{1/2}\varepsilon_1,\beta^{1/2}\varepsilon_2)-Q_1(\beta^{1/2}\varepsilon_1,\beta^{1/2}\varepsilon_2)]\Big] \\
&\quad-\frac{1}{(1+\nu_2/\nu_1)^{2I-1}}\Big[\sum_{a=2}^{I}\binom{2I-1}{I-a}(\frac{\nu_2}{\nu_1})^{I-1+a}\times[Q_a(\beta^{1/2}\varepsilon_2,\beta^{1/2}\varepsilon_1)-Q_1(\beta^{1/2}\varepsilon_2,\beta^{1/2}\varepsilon_1)]\Big]
\end{aligned}
\tag{14}
$$

where $Q_a(x,y)$ is the generalized Marcum Q-function of order a and specifically $Q_1(a,b)$ is the first-order Marcum function.

[15] J. G. Proakis and M. Salehi, *Digital Communications, 5th ed.* New York: McGraw-Hill, 2008.

Some Parameters in PEP

- Definition of some parameters used in PEP [15]

$$\varepsilon_1 \triangleq \left[\frac{1-\lambda^2}{2}\gamma_d |d-\widehat{d}|^2 \left(\vartheta\zeta - \nu_1\right)\right]^{\frac{1}{2}} \quad (15)$$

$$\varepsilon_2 \triangleq \left[\frac{1-\lambda^2}{2}\gamma_d |d-\widehat{d}|^2 \left(\vartheta\zeta + \nu_2\right)\right]^{\frac{1}{2}} \quad (16)$$

$$\beta \triangleq \sum_{i\in\mathcal{Z}_I} \left|\mathbf{1}_n^t \mathbf{W}_L \mathbf{\Omega}_k \mathbf{h}^{(i)}\right|^2 \quad (17)$$

$$\nu_1 = \left(\xi^2+\zeta\right)^{\frac{1}{2}} - \xi \quad (18)$$

$$\nu_2 = \left(\xi^2+\zeta\right)^{\frac{1}{2}} + \xi \quad (19)$$

where ξ, ζ, λ and ϑ are defined as in ***Appendix A***, and $\gamma_d \triangleq \rho^{(d)}/\sigma^2$ represents the ratio of transmitted data symbol power to noise power (symbol DNR).

Note

β is the squared CFR magnitudes

[15] J. G. Proakis and M. Salehi, *Digital Communications, 5th ed*. New York: McGraw-Hill, 2008.

φ : the received DPR

γ_d : the average received symbol DNR

γ_p : the average received symbol PNR

γ_{avg} : the average received symbol SNR

Performance Results

A) BEP Characteristics for Fixed Block Dispersive Channel

Pilot Sequences

Parameters	Values
χ	2

- **ZC sequence**

  Consider the ZC sequences which adopted for pilot aided channel estimation in LTE [7] and 5G NR [6].

$$p^{(ZC)}[n] = \exp\{-j\frac{\pi\nu n^2}{N}\} \quad \text{(when } N \text{ is even)}$$

$$p^{(ZC)}[n] = \exp\{-j\frac{\pi\nu n(n+1)}{N}\} \quad \text{(when } N \text{ is odd)}$$

  for $n \in \mathcal{Z}_N$, where ν is a positive integer relatively prime to N and $\|\mathbf{p}\|^2 = N$.

- All ZC sequences $\mathbf{p}^{(ZC)}$ consist of constant-amplitude symbols and thus provide optimum channel estimation for the fixed dispersive channel [17]-[18].

[7] 3GPP TS 36.211 V12.3.0, *LTE; Evolved universal terrestrial radio access (E-UTRA): Physical channels and modulation*, Oct. 2014.
[6] 3GPP TS 38.211 V15.4, *5G NR: Physical channels and modulation*, Dec. 2018.
[16] W.-C. Chen and C.-D. Chung, "Spectrally efficient OFDM pilot waveform for channel estimation," *IEEE Trans. Commun.*, vol. 65, no. 1, pp. 387-402, Jan. 2017.
[17] C.-D. Chung and W.-C. Chen ,"Preamble sequence design for spectral compactness and initial synchronization in OFDM," *IEEE Trans. Veh. Technol.*, vol. 67, no. 2, pp. 1428-1443, Feb. 2018.

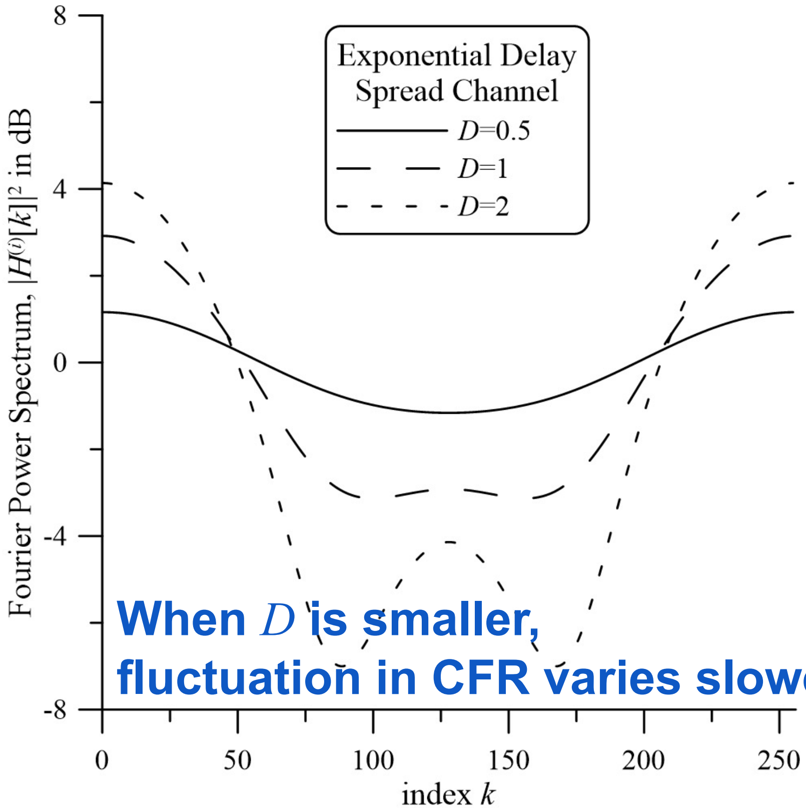
Fixed Block Dispersive Char

Note

Coefficient D represents **the ratio of sampling time $T_b/N\phi$ to root-mean-square delay spread value** and the constant amplitudes are chosen to normalize $\|\mathbf{h}^{(i)}\| = 1$.

- **Channel model**

  CIR's $\{\mathbf{h}^{(i)}\}$ are fixed over consecutive OFDM blocks and modeled as the exponential delay spread (EDS) channel [18].



When D is smaller, fluctuation in CFR varies slower.

CIR: $$h^{(i)}[l] = C_h \exp\{j\theta_l^{(i)}\} \exp\{-\frac{l}{D}\},\quad l \in \mathcal{Z}_L \text{ and } i \in \mathcal{Z}_I$$

$i=0$

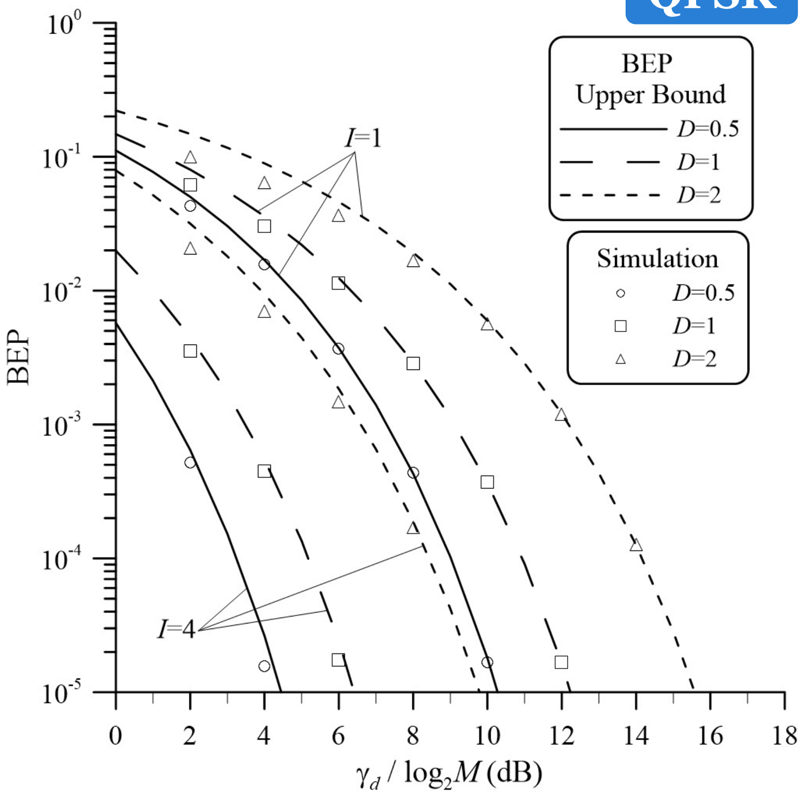
CFR: $$H^{(i)}[k] = \sum_{l=0}^{L-1} h^{(i)}[l] \exp\{-j2\pi lk/(N\phi)\}$$

Parameters	Values
N	64
ϕ	4
L	3
I	1
D	0.5, 1 and 2

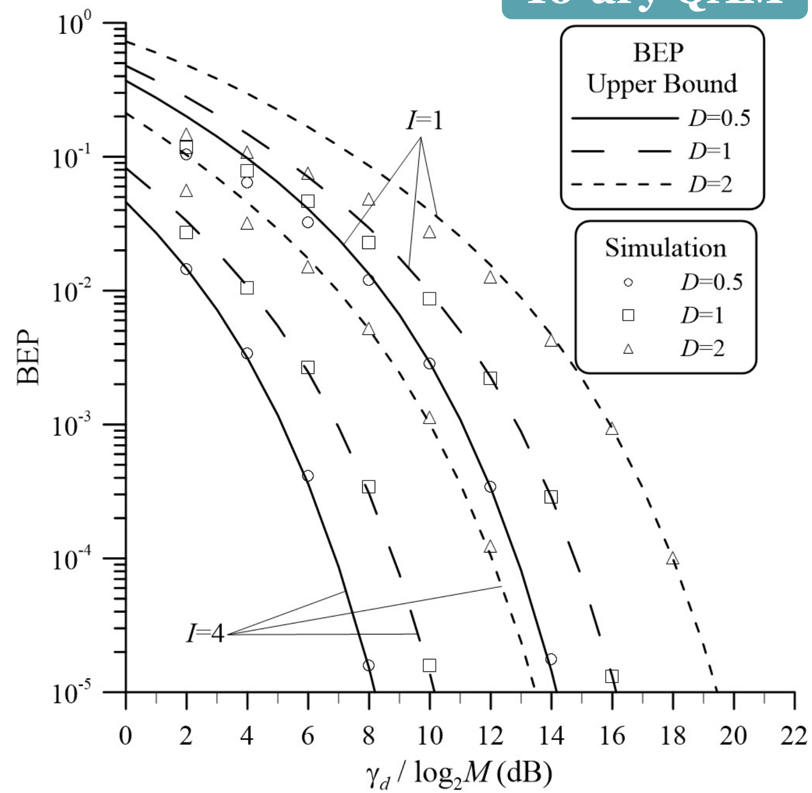
[18] H. Arslan and T.Yucek,"Delay spread estimation for wireless communication systems," *in Proc. IEEE 8th Int. Symp. Comput. and Commun.*, pp. 282287, Kemer-Antalya, Turkey, Jul. 2003.

BEP Characteristic vs. $\gamma_d / \log_2 M$

QPSK



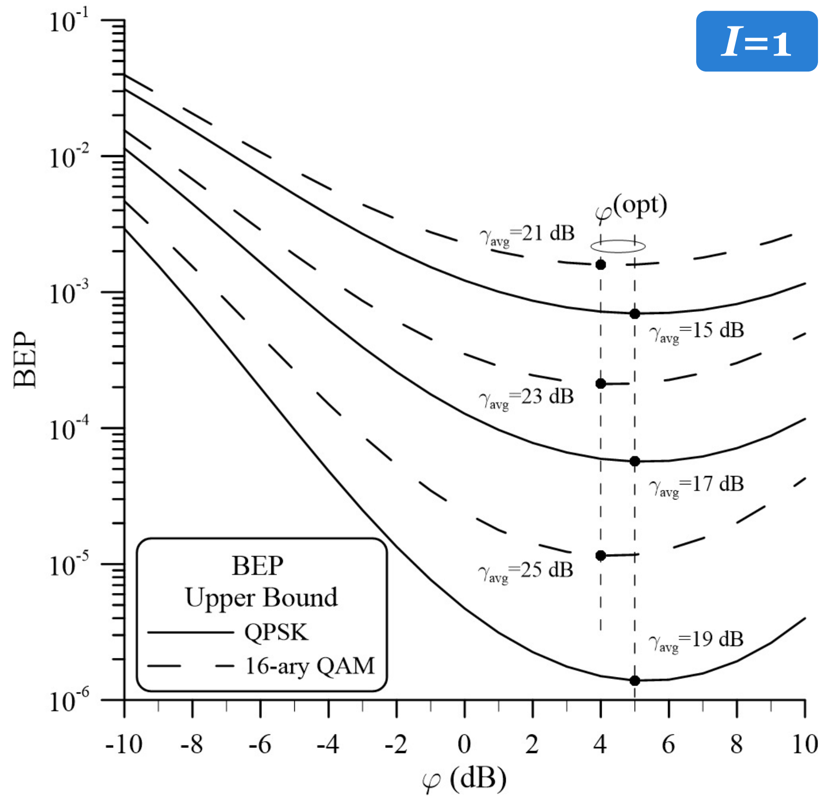
16-ary QAM

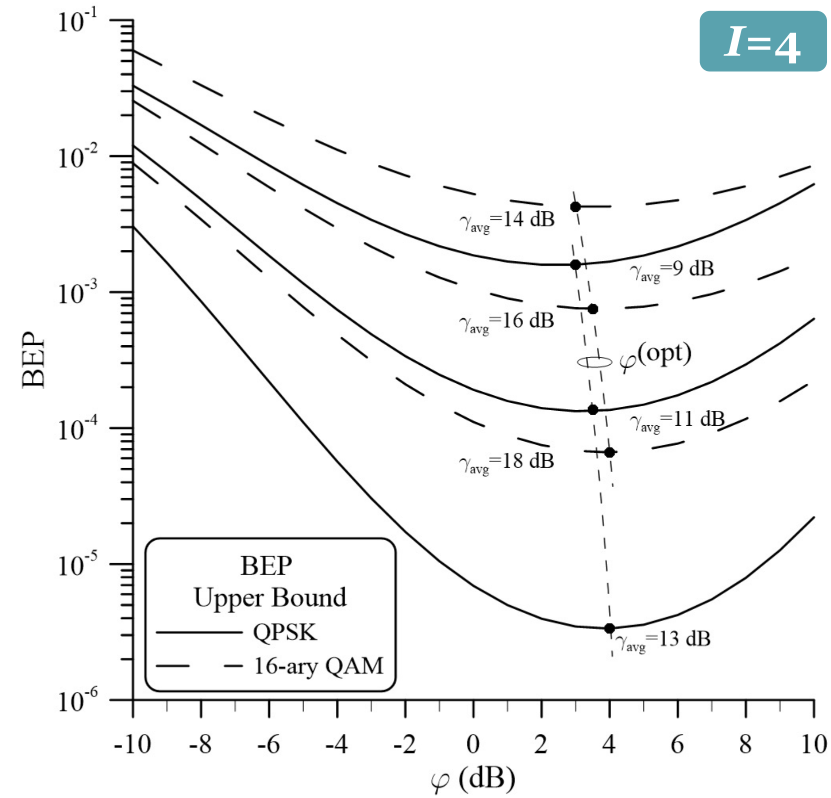


- BEP upper bound agrees with simulation when the BEP is below 10^{-2}.
- BEP performance improves when more antenna paths are adopted and when the channel is less fluctuated.

Parameters	Values
φ	1
I	1 and 4
D	05, 1 and 2

BEP Characteristic vs. φ





- The optimum DPR $\varphi^{(\mathrm{opt})}$ is relatively insensitive to average received SNR γ_{avg} and closed for different component modulation types (approximately, $\varphi^{(\mathrm{opt})} \in [3, 5]$)
- The optimum DPR $\varphi^{(\mathrm{opt})}$ is also insensitive to the number of received antennas I.

Parameters	Values
φ	1
I	1 and 4
D	2

Curve Fitting for $\varphi^{(\mathrm{opt})}$

- In order to extract the mathematical relationship between $\varphi^{(\mathrm{opt})}$ and γ_{avg} , the curve fitting methods are used [38].
- The polynomial function defines a propriate curve to fit $\varphi^{(\mathrm{opt})}$ and further analysis the relationship between the variables.
- The goodness of a fitting curve (degrees of the polynomial) is evaluated by root mean squared error (RMSE).

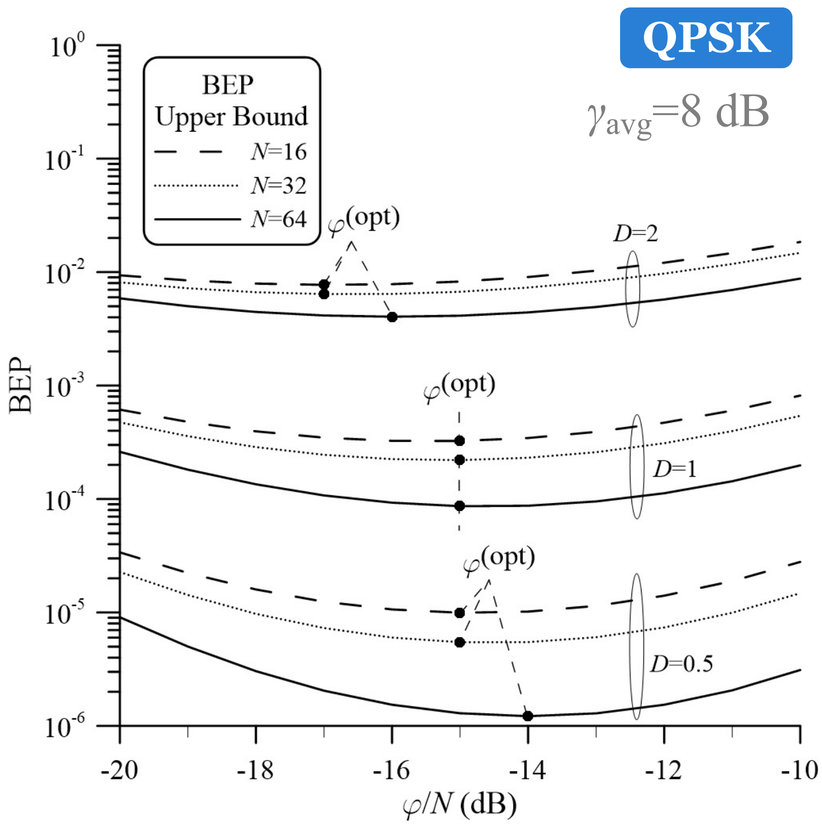
The number of $\varphi^{(\mathrm{opt})}$ between BEP 10^{-1}~ 10^{-6}

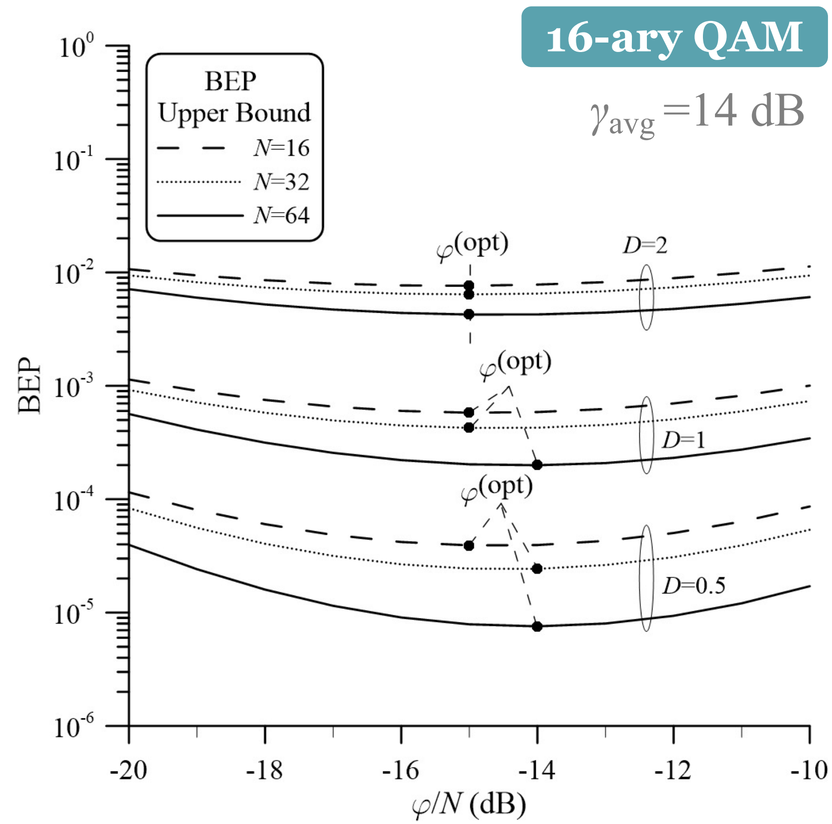
$$RMSE = \sqrt{MSE} = \frac{1}{A}\sum_{n=1}^{A}\left[\varphi^{(\mathrm{opt})}\left(\gamma_{\mathrm{avg},a}\right) - \varphi_a^{(\mathrm{opt})}\right]^2 \tag{20}$$

component modulation	I	curve-fitted polynomial function in dB
$M=4$ (QPSK)	1	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 5$ dB for $\gamma_{\mathrm{avg}} \in [6,19]$ dB
	2	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 0.0001702\gamma_{\mathrm{avg}}^6 - 0.0113\gamma_{\mathrm{avg}}^5 + 0.3034\gamma_{\mathrm{avg}}^4$ $-4.208\gamma_{\mathrm{avg}}^3 - 31.68\gamma_{\mathrm{avg}}^2 - 112.2\gamma_{\mathrm{avg}} + 190.9$ dB for $\gamma_{\mathrm{avg}} \in [5,16]$ dB
	4	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = -0.01506\gamma_{\mathrm{avg}}^2 + 0.6617\gamma_{\mathrm{avg}} - 2.099$ dB for $\gamma_{\mathrm{avg}} \in [4,13]$ dB
$M=16$ (16-ary QAM)	1	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 4$ dB for $\gamma_{\mathrm{avg}} \in [14,26]$ dB
	2	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 4$ dB for $\gamma_{\mathrm{avg}} \in [11,23]$ dB
	4	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = -0.0006416\gamma_{\mathrm{avg}}^6 + 0.03376\gamma_{\mathrm{avg}}^5 - 0.9691\gamma_{\mathrm{avg}}^4$ $+16.37\gamma_{\mathrm{avg}}^3 - 162.6\gamma_{\mathrm{avg}}^2 + 878.5\gamma_{\mathrm{avg}} - 1987$ dB for $\gamma_{\mathrm{avg}} \in [8,20]$ dB

[19] J.H. Mathews and K.K. Fink, *Numerical Methods Using Matlab, 4th edition.* Prentice Hall Inc. New Jersey, USA, 2004.

BEP Characteristic vs. φ/N





- The sensitivity of optimum $\varphi^{(\mathrm{opt})}/N$ is relatively higher to channel fluctuation than than the sequence length, but still whin a small variation.

Parameters	Values
φ	1
I	4
D	05, 1 and 2

Performance Analysis

B) Average BEP Upper Bound

B) Average BEP Upper Bound

Notation

$\widehat{d}_k[n] = \hat{d}$ and $d_k[n] = d$

- Average the conditional BEP upper bound over the joint density of CIRs to yield the average BEP upper bound $E\{P_b(\{\mathbf{h}^{(i)}\})\}$.
- **BEP upper bound:** $$P_b(\{\mathbf{h}^{(i)}\}) \le \frac{1}{N(\phi-1)} \sum_{k\in\mathcal{Z}_\phi}\sum_{n\in\mathcal{Z}_N}\sum_{\substack{d_k[n],\hat{d}_k[n]\in\mathcal{S}\\ \hat{d}_k[n]\neq d_k[n]}} \frac{D_H(d_k[n],\widehat{d}_k[n])}{M\log_2 M} \times \underbrace{\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}_k[n]) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d_k[n]) \mid d_k[n],\{\mathbf{h}^{(i)}\}\}}_{\text{PEP}}$$

$$\beta \triangleq \sum_{i\in\mathcal{Z}_I} \left|\mathbf{1}_n^t \mathbf{W}_L \mathbf{\Omega}_k \mathbf{h}^{(i)}\right|^2$$

- **PEP:** $\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d)|d,\{\mathbf{h}^{(i)}\}\}$

$$= [1 - Q_1(\beta^{1/2}\varepsilon_2, \beta^{1/2}\varepsilon_1)][1 - \frac{\sum_{a=0}^{I-1}\binom{2I-1}{a}(\frac{\nu_2}{\nu_1})^a}{(1+\nu_2/\nu_1)^{2I-1}}] + Q_1(\beta^{1/2}\varepsilon_1, \beta^{1/2}\varepsilon_2)\frac{\sum_{a=0}^{I-1}\binom{2I-1}{a}(\frac{\nu_2}{\nu_1})^a}{(1+\nu_2/\nu_1)^{2I-1}}$$

$$+ \frac{1}{(1+\nu_2/\nu_1)^{2I-1}}\left[\sum_{a=2}^{I}\binom{2I-1}{I-a}(\frac{\nu_2}{\nu_1})^{I-a} \times [Q_a(\beta^{1/2}\varepsilon_1, \beta^{1/2}\varepsilon_2) - Q_1(\beta^{1/2}\varepsilon_1, \beta^{1/2}\varepsilon_2)]\right]$$

$$- \frac{1}{(1+\nu_2/\nu_1)^{2I-1}}\left[\sum_{a=2}^{I}\binom{2I-1}{I-a}(\frac{\nu_2}{\nu_1})^{I-1+a} \times [Q_a(\beta^{1/2}\varepsilon_2, \beta^{1/2}\varepsilon_1) - Q_1(\beta^{1/2}\varepsilon_2, \beta^{1/2}\varepsilon_1)]\right]$$

B) Average BEP Upper Bound

- Only β in the generalized Marcum Q-function depends on $\{\mathbf{h}^{(i)}\}$.
- Average the conditional BEP upper bound over the joint density of β to yield the average BEP upper bound $E\{P_b(\{\mathbf{h}^{(i)}\})\}$.
  1) Adopt the integral representation of the generalized Marcum Q-function in [20]

$$Q_a\left(\beta^{\frac{1}{2}}\varepsilon_1, \beta^{\frac{1}{2}}\varepsilon_2\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\tau^{-a+1}\left\{\cos\left((a-1)\left(\theta+\frac{\pi}{2}\right)\right)-\tau\cos\left(a\left(\theta+\frac{\pi}{2}\right)\right)\right\}}{(1+2\tau\sin\theta+\tau^2)} \times \exp\left\{-\frac{\beta\varepsilon_2^2}{2}\left(1+2\tau\sin\theta+\tau^2\right)\right\} d\theta \quad (21)$$

$$Q_a\left(\beta^{\frac{1}{2}}\varepsilon_2, \beta^{\frac{1}{2}}\varepsilon_1\right) = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\tau^{a}\left\{\cos\left(a\left(\theta+\frac{\pi}{2}\right)\right)-\tau\cos\left((a-1)\left(\theta+\frac{\pi}{2}\right)\right)\right\}}{(1+2\tau\sin\theta+\tau^2)} \times \exp\left\{-\frac{\beta\varepsilon_2^2}{2}\left(1+2\tau\sin\theta+\tau^2\right)\right\} d\theta \quad (22)$$

$$Q_1\left(\beta^{\frac{1}{2}}\varepsilon_1, \beta^{\frac{1}{2}}\varepsilon_2\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1+\tau\sin\theta}{(1+2\tau\sin\theta+\tau^2)} \exp\left\{-\frac{\beta\varepsilon_2^2}{2}\left(1+2\tau\sin\theta+\tau^2\right)\right\} d\theta \quad (23)$$

$$Q_1\left(\beta^{\frac{1}{2}}\varepsilon_2, \beta^{\frac{1}{2}}\varepsilon_1\right) = 1 + \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\tau^2+\tau\sin\theta}{(1+2\tau\sin\theta+\tau^2)} \exp\left\{-\frac{\beta\varepsilon_2^2}{2}\left(1+2\tau\sin\theta+\tau^2\right)\right\} d\theta \quad (24)$$

  2) Derive the moment generating function (MGF) of β , $\Phi_\beta(s) \triangleq E\{\exp\{s\beta\}\}$

⇒ achieve a single integral representation of the upper bound to BEP

[20] M. K. Simon and M. Alouini, "A unified approach to the probability of error for noncoherent and differentially coherent modulations over generalized fading channels," *IEEE Trans. Commun.*, vol. 46, no. 12, pp. 16251638, Dec. 1998.

Pairwise Error Probability

1) Therefore, PEP is a single integral with an integrand in which the sole channel-dependent parameter β resides only in $\exp\{-\frac{\beta\varepsilon_2^2}{2}(1+2\sin\theta+\tau^2)\}$

$$\Pr\{\sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(\widehat{d}) < \sum_{i\in\mathcal{Z}_I} F_{k,n}^{(i)}(d) \mid d, \{\mathbf{h}^{(i)}\}\}$$
$$= \frac{\varrho^I}{2\pi(1+\varrho)^{2I-1}} \int_{-\pi}^{\pi} \frac{\Psi(\theta; I, \tau, \varrho)}{1+2\tau\sin\theta+\tau^2} \times \exp\{-\frac{\beta\varepsilon_2^2}{2}(1+2\tau\sin\theta+\tau^2)\} d\theta \quad (25)$$

2) The average BEP upper bound $E\{P_b(\{\mathbf{h}^{(i)}\})\}$

$$E\{P_b(\{\mathbf{h}^{(i)}\})\} \leq \frac{1}{N(\phi-1)} \sum_{k\in\mathcal{Z}_\phi} \sum_{n\in\mathcal{Z}_N} \sum_{\substack{d_k[n],\widehat{d}_k[n]\in\mathcal{S} \\ \widehat{d}_k[n]\neq d_k[n]}} \frac{D_H(d_k[n], \widehat{d}_k[n])}{M\log_2 M}$$

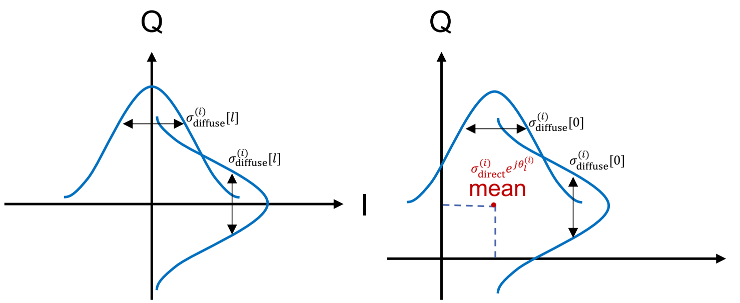
Moment generating function

$$\times \frac{\varrho^I}{2\pi(1+\varrho)^{2I-1}} \int_{-\pi}^{\pi} \frac{\Psi(\theta; I, \tau, \varrho) \boxed{\Phi_\beta\left(-\frac{\varepsilon_2^2}{2}(1+2\tau\sin\theta+\tau^2)\right)}}{1+2\tau\sin\theta+\tau^2} d\theta \quad (26)$$

where $\Phi_\beta(s) \triangleq E\{\exp\{s\beta\}\}$

Multipath Rician Fading Channel

- Similar to the multipath Rician fading in [21], the CIR $\mathbf{h}^{(i)}$ on the i-th antenna path can be modeled as



Direct path

1) Mean: $E\{h^{(i)}[0]\} = \sigma_{\text{direct}}^{(i)} \exp\{j\theta_l^{(i)}\}$
2) Power: $\underbrace{(\sigma_{\text{direct}}^{(i)}[0])^2}_{\text{direct power}} + \underbrace{(\sigma_{\text{diffuse}}^{(i)}[0])^2}_{\text{diffuse power}}$
3) Independent of all diffuse path responses

Diffuse paths

$h^{(i)}[1], h^{(i)}[2], \ldots, h^{(i)}[L-1]$ are independent CSCGs with common mean zero and respective diffuse path power $E\{|h^{(i)}[l]|^2\} = (\sigma_{\text{diffuse}}^{(i)}[l])^2$.

$$\sigma_{\text{diffuse}}^2[l] = C_h \exp\{-l/D\} \qquad \text{for } \; l \in \mathcal{Z}_L$$

$$C_h = \frac{1 - \exp\{-1/D\}}{(K+1)(1 - \exp\{-L/D\})}$$

[21] R.-Y. Yen, H. Liu and W.-K. Tsai, "QAM symbol error rate in OFDM systems over frequency-selective fast Ricean-fading channels," *IEEE Trans. Veh. Technol.*, vol. 57, no. 2, pp. 1322-1325, Mar. 2008.

The Statistic of β

- First, rewrite β as

$$\beta \triangleq \sum_{i=0}^{I-1} |\beta^{(i)}|^2 \tag{27}$$

where $\beta^{(i)} \triangleq \mathbf{1}_n^t \mathbf{W}_L \mathbf{\Omega}_k \mathbf{h}^{(i)}$ is the CFR $\widetilde{h}^{(i)}[n\phi + k]$ at the $(n\phi + k)$-th subcarriers on the i-th antenna.

- $\mathbf{h}^{(i)}$'s are mutually independent over different antenna paths, $\beta^{(i)}$'s are independent complex-valued Gaussian random variables with

$$E\{\beta^{(i)}\} = \sigma_{\text{direct}}^{(i)} \exp\{j\theta_l^{(i)}\}$$
$$E\{(\beta^{(i)} - E\{\beta^{(i)}\})^2\} = 0$$
$$E\{|\beta^{(i)} - E\{\beta^{(i)}\}|^2\} = \sum_{l \in \mathcal{Z}_L} (\sigma_{\text{diffuse}}^{(i)})^2$$

- Therefore, β follows a <u>noncentral chi-square distribution</u> with $2I$ degrees of freedom.

Note

When the direct power is zero (i.e., no line of sight), the distribution of $|\beta^{(i)}|^2$ reduces to a central chi-square distribution.

Moment Generating Function of β

The MGF of a noncentral χ^2 with one degree of freedom [22]

Let Z have a standard normal distribution, with mean 0 and variance σ^2, then $(Z+\mu)^2$ has a noncentral chi-squared distribution with one degree of freedom.
The moment generating function of $(Z+\mu)^2$ is derived as

$$E\{e^{s(Z+\mu)^2}\} = \exp\left\{\frac{\mu^2 s}{1-2\sigma^2 s}\right\}(1-2\sigma^2 \mathrm{s})^{-\frac{1}{2}}.$$

- The noncentral chi-square distributed $|\beta^{(i)}|^2$ has noncentral parameter $(\sigma^{(i)}_{\text{direct}})^2$, and variance $\sum_{l\in\mathcal{Z}_L}(\sigma^{(i)}_{\text{diffuse}})^2$.
- **MGF:** $\Phi_\beta(s) = \prod_{i=0}^{I-1}\Phi_{|\beta^{(i)}|^2}(s)$

$$= \prod_{i=0}^{I-1}(1-s\sum_{l\in\mathcal{Z}_L}(\sigma^{(i)}_{\text{diffuse}}[l])^2)^{-1}\times\exp\{\frac{(\sigma^{(i)}_{\text{direct}})^2 s}{1-s\sum_{l\in\mathcal{Z}_L}(\sigma^{(i)}_{\text{diffuse}}[l])^2}\} \quad (28)$$

[22] R. J. Muirhead, *Aspects of Multivariate Statical Theory.* John Wiley & Sons Inc., 2005.

Identical Distributed for All Antenna Paths

- Consider the l-th paths in all antenna paths have identical distribution, and thus the SIMO block multipath Rician fading model is specialized to exhibit identical power.
  $\Rightarrow$ $h^{(i)}[l]$ for all $i \in \mathcal{Z}_I$ have the same power

$$\sigma_{\text{direct}}^{(i)} = \sigma_{\text{direct}}$$
$$\sigma_{\text{diffuse}}^{(i)}[l] = \sigma_{\text{diffuse}}[l]$$

- The MGF $\Phi_\beta(s)$ simplifies to

$$\Phi_\beta(s) = \exp\{\frac{Is\sigma_{\text{direct}}^2}{1 - s\sum_{l\in\mathcal{Z}_L}\sigma_{\text{diffuse}}^2\,[l]}\}(1 - s\sum_{l\in\mathcal{Z}_L}\sigma_{\text{diffuse}}^2\,[l])^{-I} \qquad (29)$$

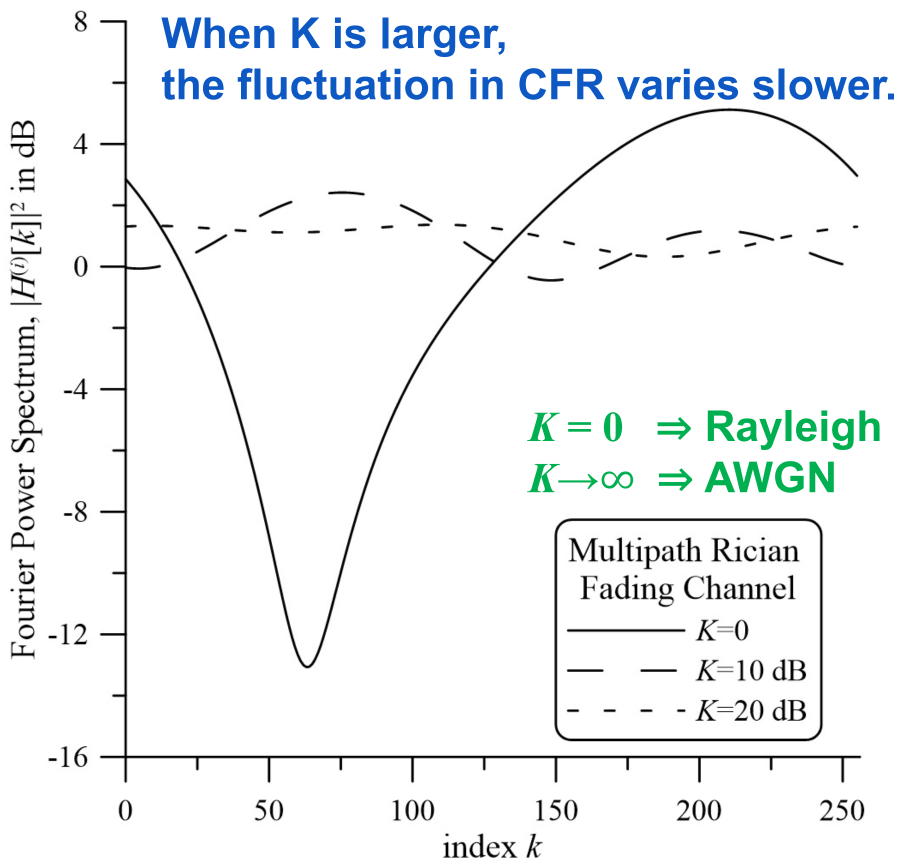
Performance Results

B) Average BEP Characteristics for Random Block Dispersive Channel

Random Block Dispersive Channel

- **Channel model**

  CIR's $\{\mathbf{h}^{(i)}\}$ are fixed over consecutive OFDM blocks and modeled as multipath Rician fading channel.

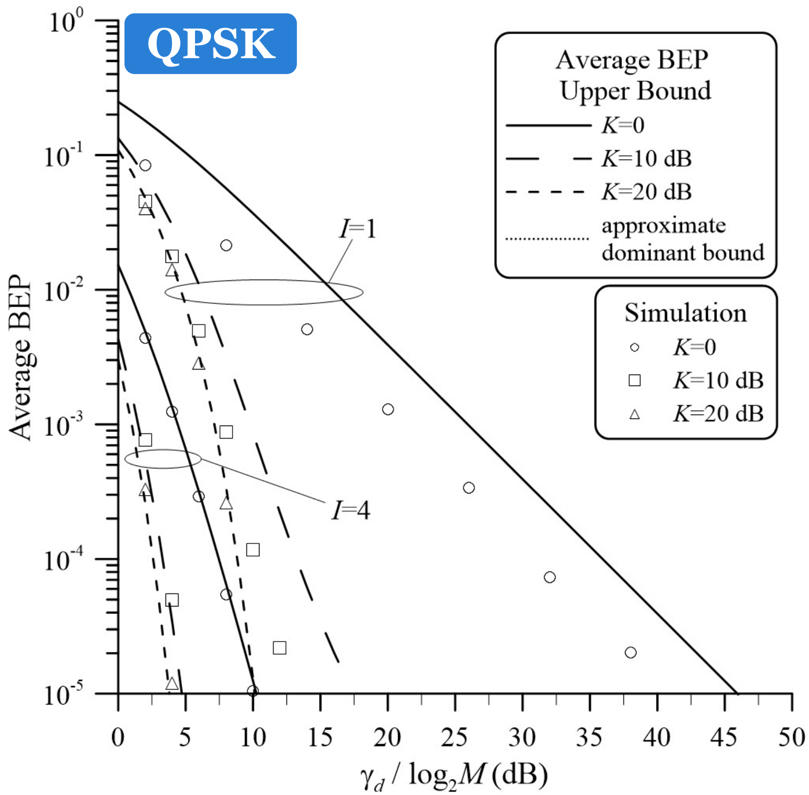


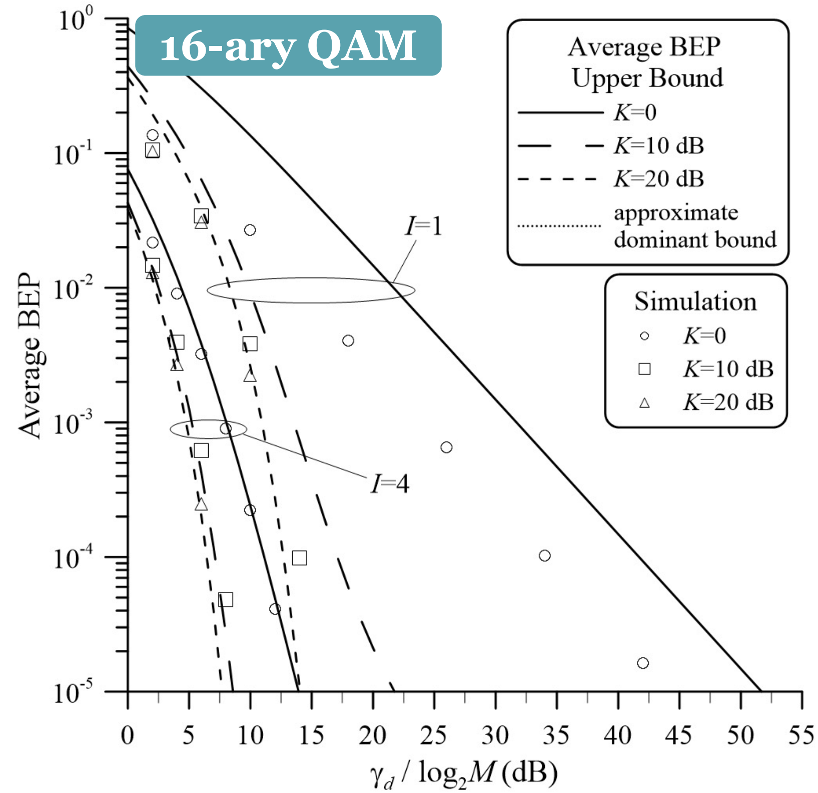
Parameters	Values
N	64
ϕ	4
L	3
D	2

Note

Normalize the channel power $E\left\{\left\|h^{(i)}\right\|^2\right\} = 1$, so that we can characterized the channel effect through the **factor *K*** by $\sigma^2_{\text{direct}} = \frac{K}{K+1}$ and $\sum_{l=0}^{L-1} \sigma^2_{\text{diffuse}}[l] = \frac{1}{(K+1)}$.

Average BEP Characteristic vs. $\gamma_d / \log_2 M$

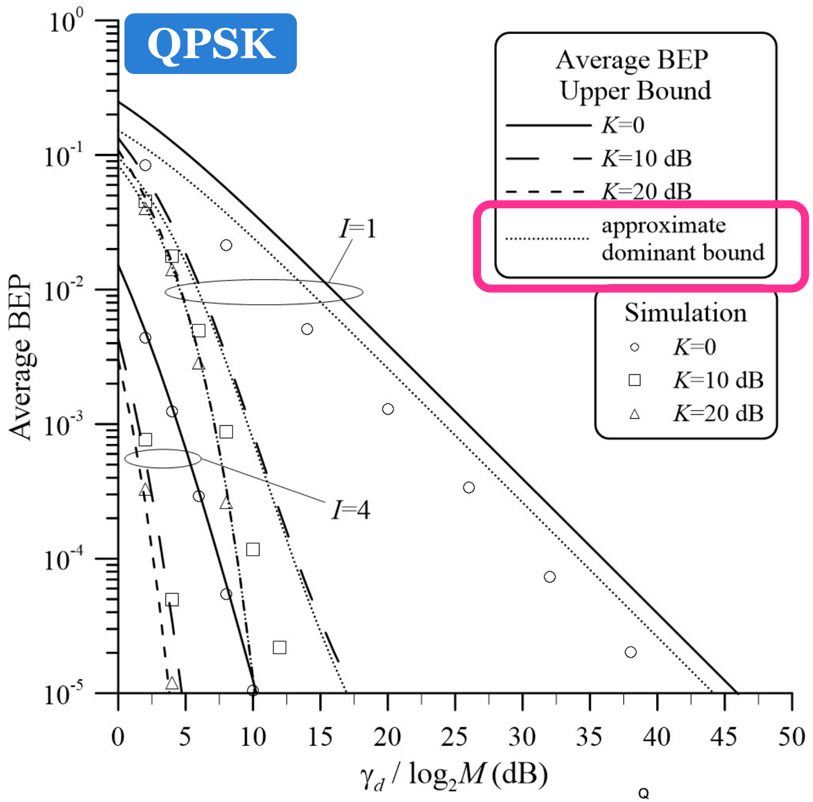


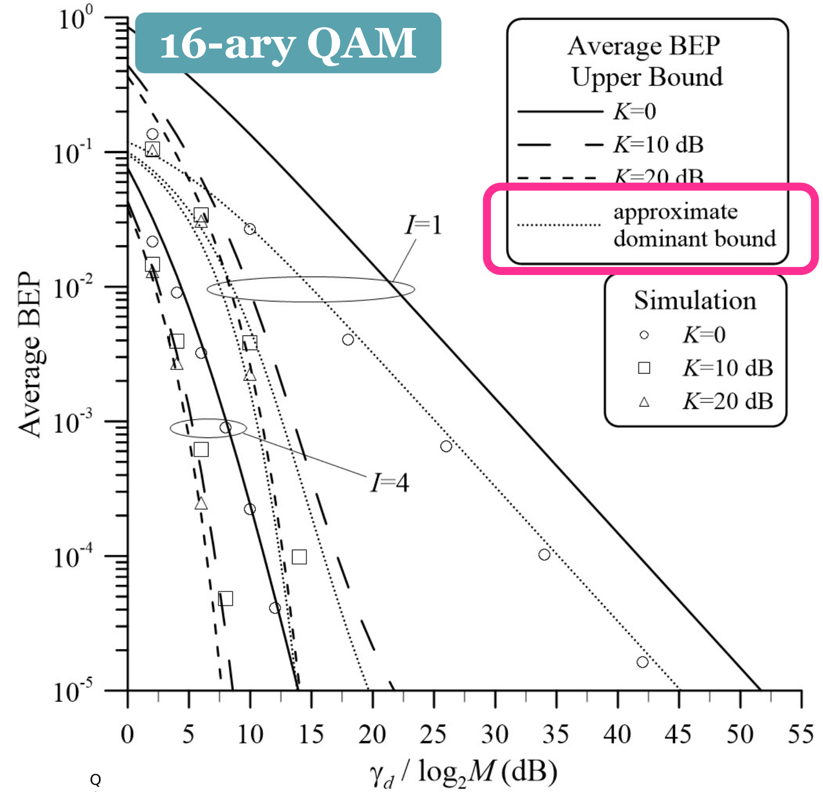


- The average BEP upper bound agrees very well with simulation when the BEP is below 10^{-2} for a larger $I \geq 4$ or $K \geq 20$ dB.
- The average BEP performance improves when more antenna paths are adopted and when the channel is less fluctuated.

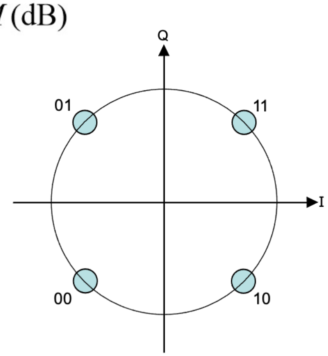
Parameters	Values
φ	1
I	1 and 4
K	0, 10 dB and 20 dB

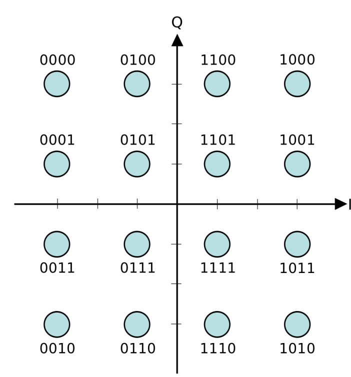
Average BEP Characteristic vs. $\gamma_d / \log_2 M$





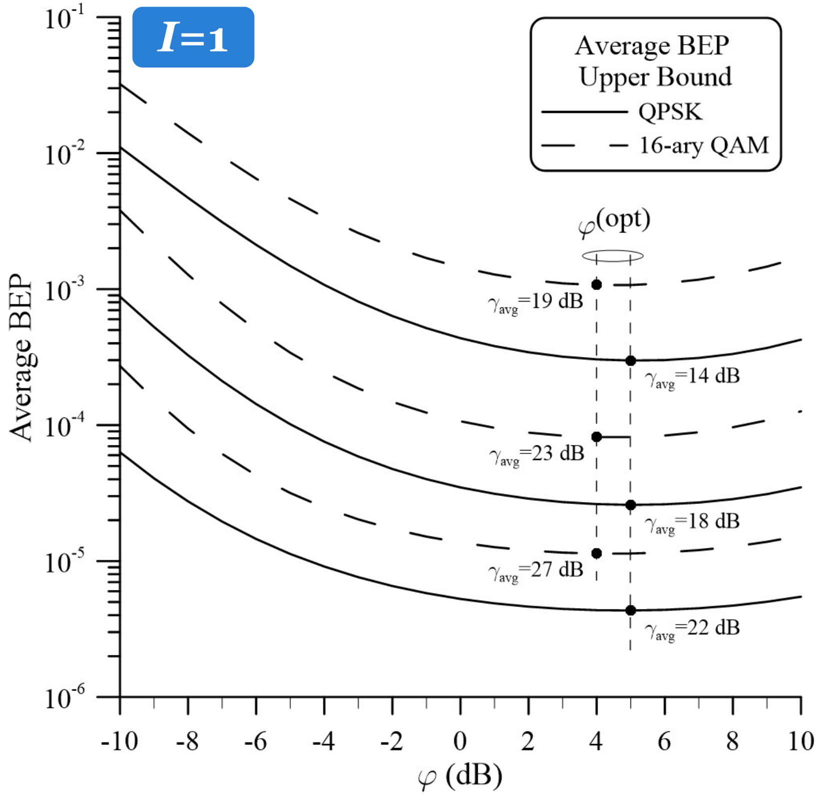
- **Dominant Terms**

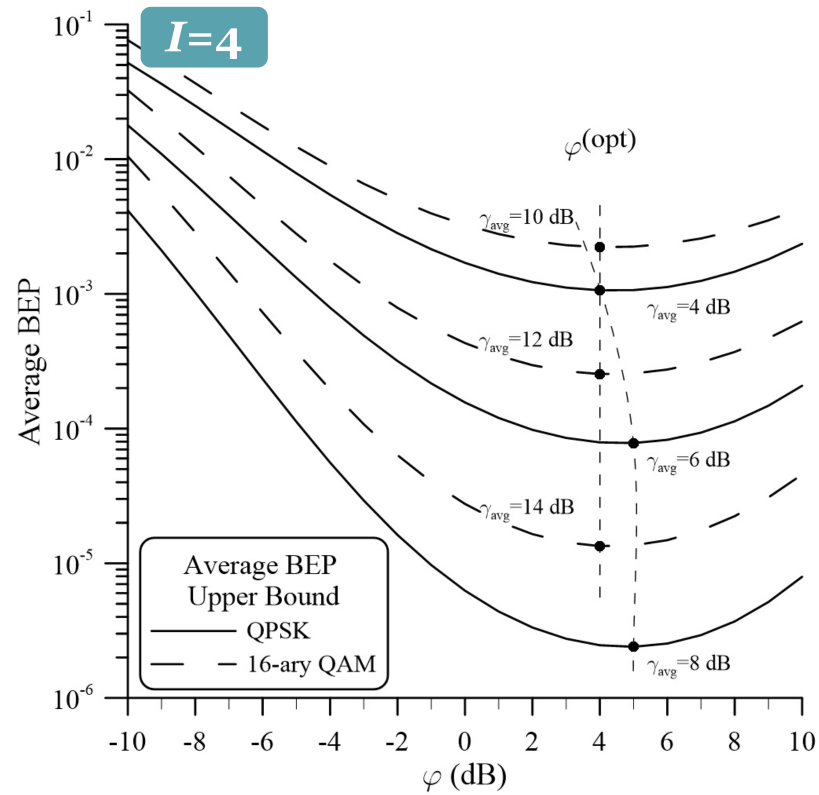




Parameters	Values
φ	1
I	1 and 4
K	0, 10 dB and 20 dB

Average BEP Characteristic vs. φ



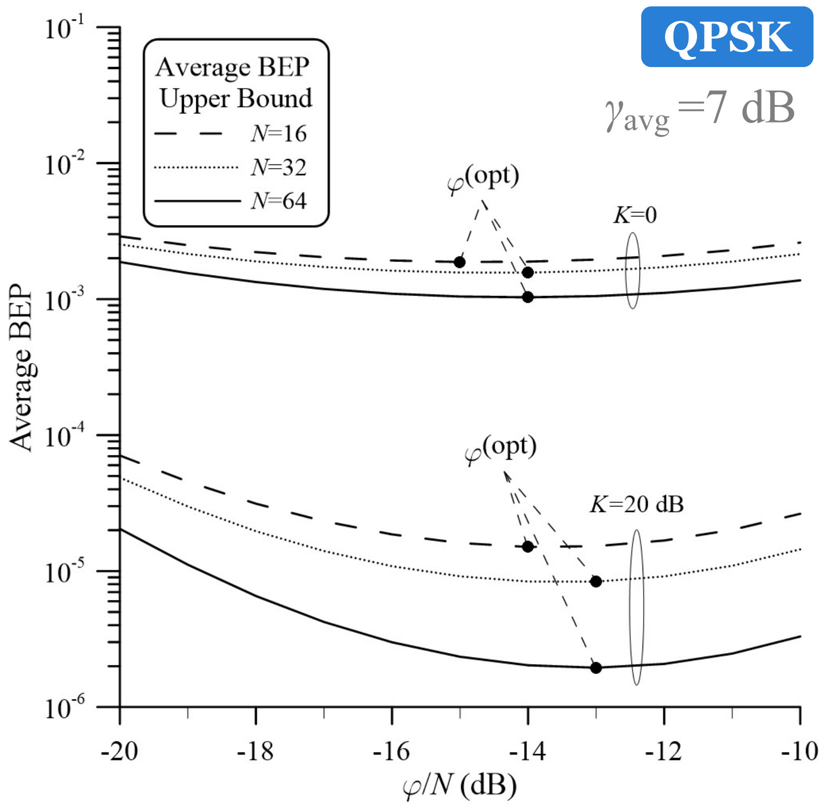


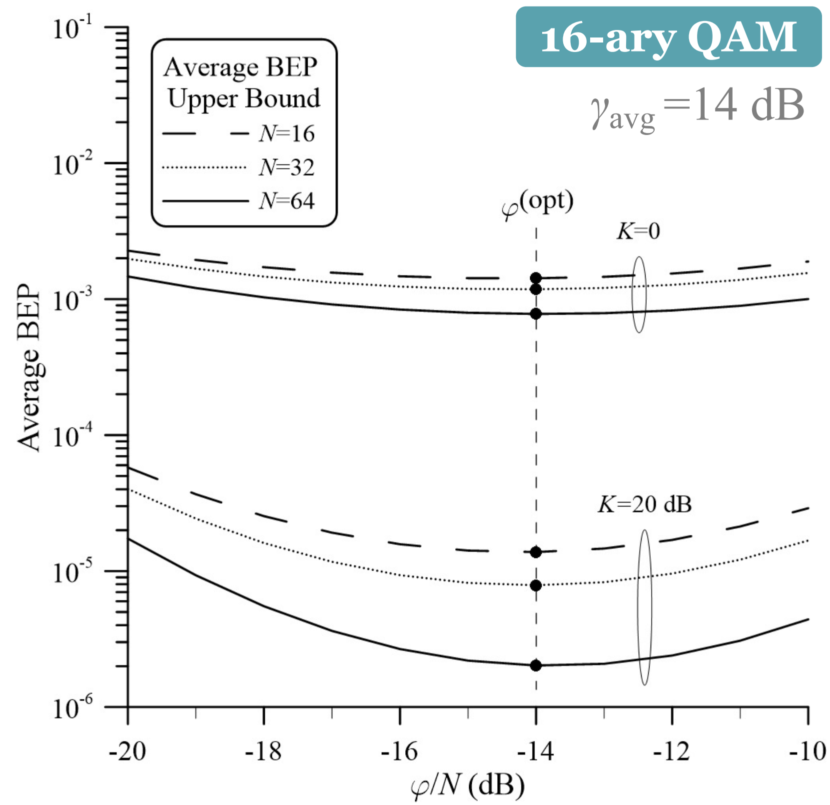
- Curve Fitting Table

component modulation	I	curve-fitted polynomial function in dB
$M=4$ (QPSK)	1	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 5$ dB for $\gamma_{\mathrm{avg}} \in [4, 26]$ dB
	2	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = -0.0001037\gamma_{\mathrm{avg}}^5 + 0.003447\gamma_{\mathrm{avg}}^4 - 0.03502\gamma_{\mathrm{avg}}^3 + 0.1007\gamma_{\mathrm{avg}}^2 + 0.3315\gamma_{\mathrm{avg}} + 3.5$ dB for $\gamma_{\mathrm{avg}} \in [1, 13]$ dB
	4	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 0.000641\gamma_{\mathrm{avg}}^5 - 0.1515\gamma_{\mathrm{avg}}^4 + 0.1157\gamma_{\mathrm{avg}}^3 - 0.2914\gamma_{\mathrm{avg}}^2 + 0.1902\gamma_{\mathrm{avg}} + 4.007$ dB for $\gamma_{\mathrm{avg}} \in [-4, 9]$ dB
$M=16$ (16-ary QAM)	1	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 0.0007896\gamma_{\mathrm{avg}}^8 - 0.06672\gamma_{\mathrm{avg}}^7 + 3.273\gamma_{\mathrm{avg}}^6 - 102.7\gamma_{\mathrm{avg}}^5 + 2137\gamma_{\mathrm{avg}}^4$ dB for $\gamma_{\mathrm{avg}} \in [15, 27]$ dB
	2	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 4$ dB for $\gamma_{\mathrm{avg}} \in [8, 20]$ dB
	4	$\varphi^{(\mathrm{opt})}(\gamma_{\mathrm{avg}}) = 4$ dB for $\gamma_{\mathrm{avg}} \in [7, 17]$ dB

Parameters	Values
φ	1
I	1 and 4
K	10 dB

Average BEP Characteristic vs. φ/N





- The sensitivity of optimum $\varphi^{(\mathrm{opt})}/N$ is relatively higher to channel fluctuation than than the sequence length, but still whin a small variation.

Parameters	Values
φ	1
I	4
K	0 and 20 dB

Conclusion

- ✓ The GLRT-based SIMO-OFDM demodulation system is analyzed over both deterministic and random dispersive channels in terms of the BEP union bounds.
- ✓ The derived upper bounds are applicable to all two-dimensional component modulations.
- ✓ The random dispersive channel is generally modeled to have independent but not necessarily identical distributed random paths over various antenna elements.
- ✓ Within the channel model that all channel paths in the SIMO channel are independent, the presented analysis is useful as long as the marginal statistic of squared CFR magnitudes is available.
- ✓ Performance results are illustrated for both QPSK and 16-ary QAM component modulations in terms of the error performance characteristics with respect to data-to-noise power ratio, data-to-pilot power ratio, channel conditions, antenna diversity, and pilot length.

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[7] 3GPP TS 36.211 V12.3.0, *LTE; Evolved universal terrestrial radio access (E-UTRA): Physical channels and modulation*, Oct. 2014.

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Thank you!

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