# Econometric Homework 4

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c. All values of $y$ and $x$ were divided by 10 before estimation.

**4.3** We have five observations on $x$ and $y$. They are $x_i = 3, 2, 1, -1, 0$ with corresponding $y$ values $y_i = 4, 2, 3, 1, 0$. The fitted least squares line is $\hat{y}_i = 1.2 + 0.8x_i$, the sum of squared least squares residuals is $\sum_{i=1}^{5} \hat{e}_i^2 = 3.6$, $\sum_{i=1}^{5}(x_i - \bar{x})^2 = 10$, and $\sum_{i=1}^{5}(y_i - \bar{y})^2 = 10$. Carry out this exercise with a hand calculator. Compute

a. the predicted value of $y$ for $x_0 = 4$.
b. the se($f$) corresponding to part (a).
c. a 95% prediction interval for $y$ given $x_0 = 4$.
d. a 99% prediction interval for $y$ given $x_0 = 4$.
e. a 95% prediction interval for $y$ given $x = \bar{x}$. Compare the width of this interval to the one computed in part (c).

(a.) $x_0 = 4$

$$\hat{y}_0 = 1.2 + 0.8 \times 4 = 1.2 + 3.2 = 4.4$$

(b.) $$se(f) = \sqrt{\hat{\sigma}^2\left[1 + \frac{1}{N} + \frac{(x_0 - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right]} = \sqrt{1.2\left[1 + \frac{1}{5} + \frac{(4-1)^2}{10}\right]} = 1.5875$$

$$\hat{\sigma}^2 = \frac{\Sigma \hat{e}_i^2}{N-2} = \frac{3.6}{5-2} = \frac{3.6}{3} = 1.2$$

$$\bar{x} = \frac{3+2+1-1+0}{5} = 1$$

(c.) $\alpha = 0.05$

$1 - \alpha = 0.95$

$$\hat{y}_0 \pm t_{(0.975,\,3)}\, se(f) = 4.4 \pm (3.1824 \times 1.5875) = 4.4 \pm 5.05206$$
$$= [-0.6520,\ 9.4520]$$

(d.) $\alpha = 0.01$

$1 - \alpha = 0.99$

$$\hat{y}_0 \pm t_{(0.995,\,3)}\, se(f) = 4.4 \pm (5.8409 \times 1.5875) = 4.4 \pm 9.2724$$
$$= [-4.8722,\ 13.6722]$$

(e.) $\alpha = 0.05$

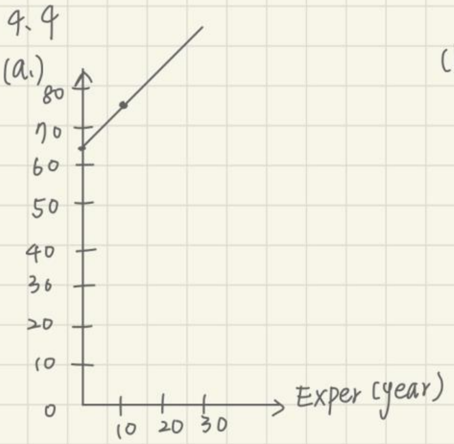
$1 - \alpha = 0.95 \qquad \hat{y} = 1.2 + 0.8 \times 1 = 2$

$$se(f') = \sqrt{1.2\left[1 + \frac{1}{5}\right]} = 1.2$$

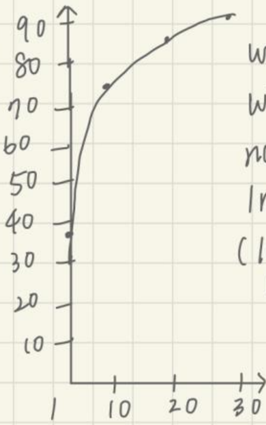
$$\hat{y}_0 \pm t_{(0.975,\,3)}\, se(f') = 2 \pm (3.1824 \times 1.2) = 2 \pm 3.8189$$
$$= [-1.8189,\ 5.8189]$$

4.9

(a.)




(b.)




why the four artists with no experience are not in model 2 is because ln's domain is >0 (ln(0) will cause -inf or math error)

(c.) marginal effect

$$m = \frac{d\,\widehat{RATING}}{d\,EXPER} = 0.990$$

(i) Marginal effect for an artist with 10 years of experience = 0.990

(ii) Marginal effect for an artist with 20 years of experience = 0.990

(d.) marginal effect

$$m = \frac{d\,\widehat{RATING}}{d\,EXPER} = \frac{0.990}{EXPER}$$

(i) Marginal effect for an artist with 10 years of experience = $\frac{0.990}{10} = 0.099$

(ii) Marginal effect for an artist with 20 years of experience = $\frac{0.990}{20} = 0.0495$

(e.) the model fits the data better. Because model 2 has higher $R^2$ (The $R^2$ is defined as the proportion of the variation is y about its mean that is explained by the regression model) Even if model 1 just using the technical artists with some experience, its $R^2$ is still lower than model 2, showing that after elimating the difference from data, model 2 still fit the data better.

(f.) Model 2 is more reasonable based on economic reasoning called "Diminishing marginal utility". As the explaintory variable increasing, the marginal utility (curve slope) decrease. The curve just like ln curve in our model 2.