SELECTIVE LASER SINTERING 3D PRINTER

ELEC 341 PROJECT

LUFEI LIU – 14090154

MODEL OVERVIEW

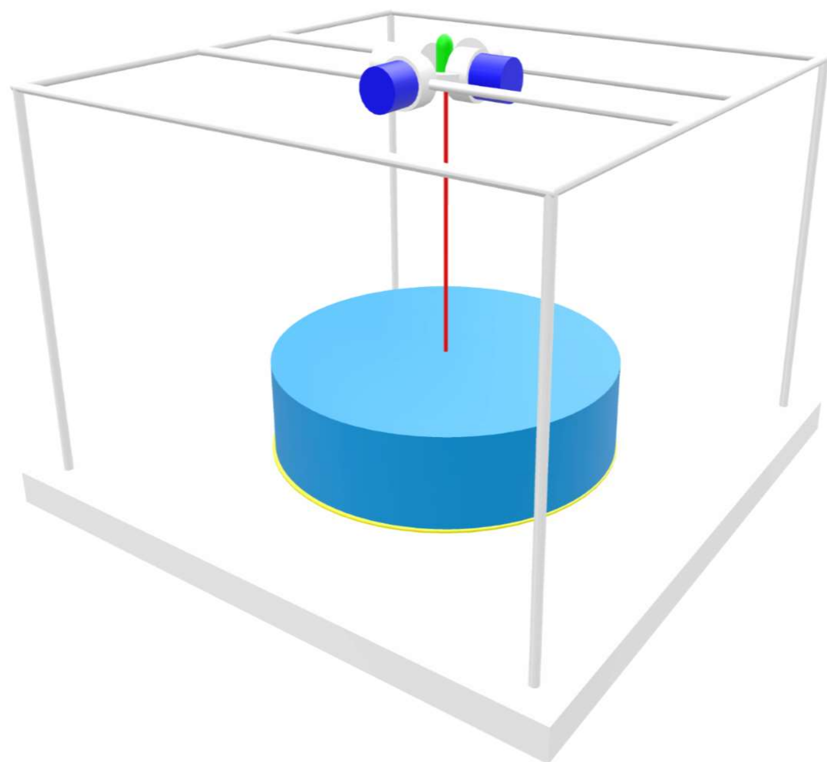
1. Inverse Kinematics — Convert X,Y coordinates to angles

2. Amplifier — Convert angles to voltages and amplify

3. Motor — Model motor behavior to convert voltages into resulting angles

4. Sensor — Use sensor to track progress and adjust as necessary

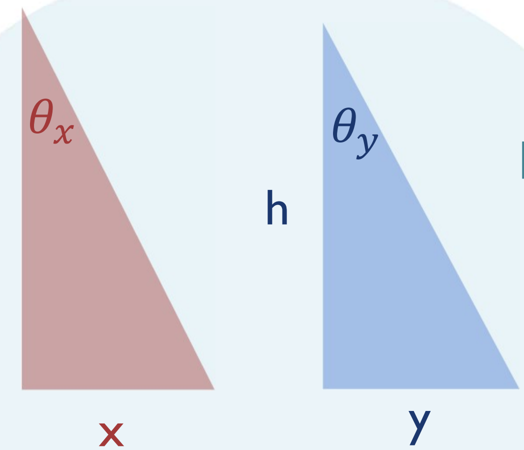
3D PRINTER



Input: Desired X,Y coordinates of object
Output: 3D printed object

DIRECT AND INVERSE KINEMATICS

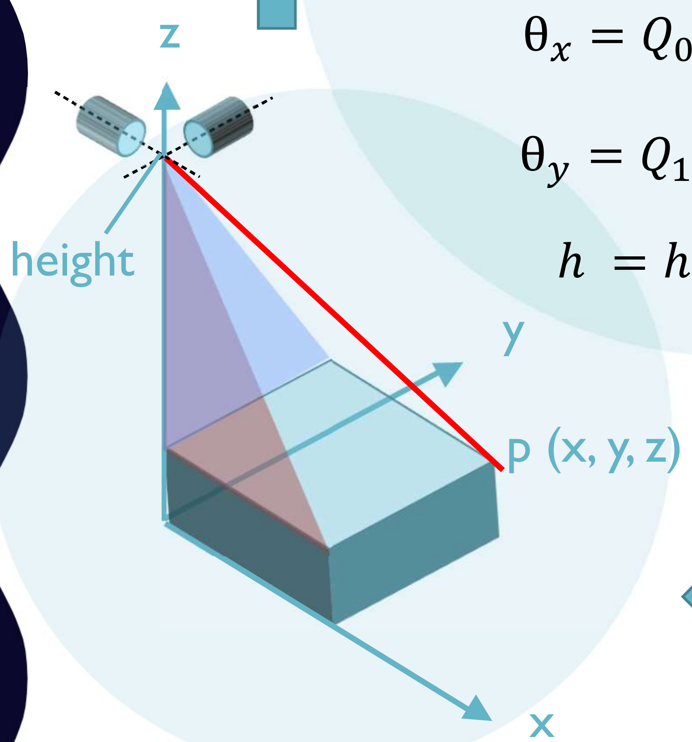
X,Y Coordinates

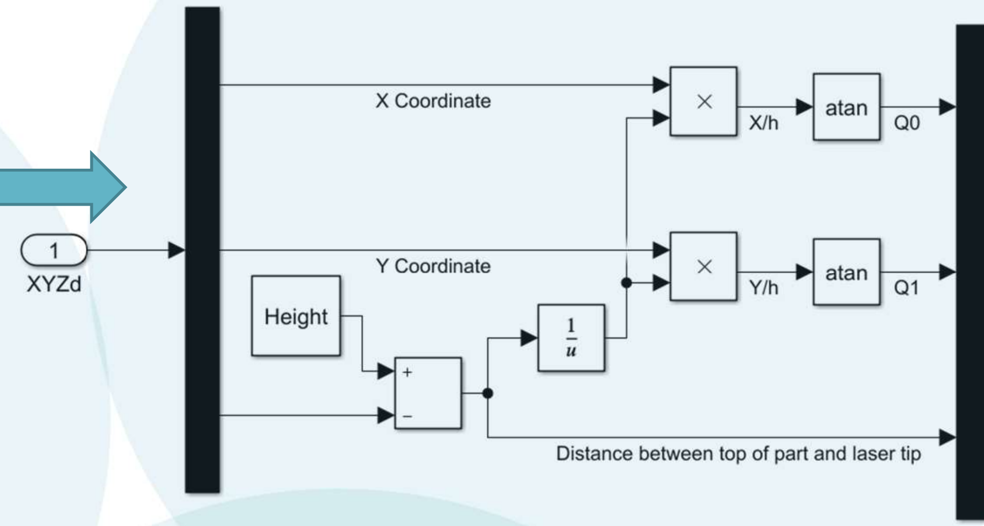


$$\theta_x = Q_0 = tan^{-1}\left(\frac{x}{h}\right)$$

$$\theta_y = Q_1 = tan^{-1}\left(\frac{y}{h}\right)$$

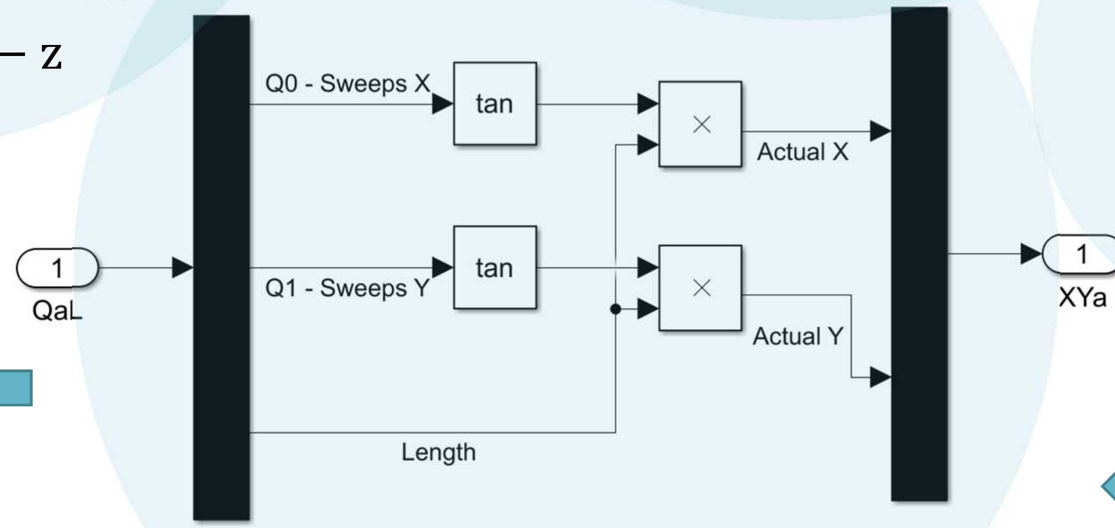
$$h = height - z$$





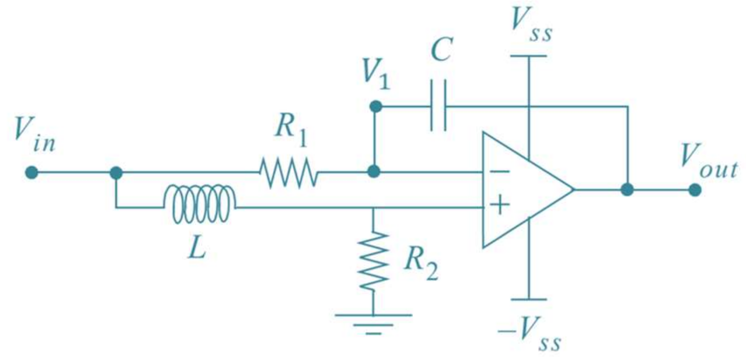
Q_0 Q_1 Angles

$$y = h \times tan(\theta_y)$$
$$= h \times tan(Q_1)$$
$$x = h \times tan(\theta_x)$$
$$= h \times tan(Q_0)$$



MODELING COMPONENTS

POWER AMPLIFIER



Using nodal analysis with $V_P = V_N = V_1$ due to negative feedback:

$$\frac{V_{in} - V_1}{R_1} = (V_1 - V_{out}) \times sC$$

$$\frac{V_{in} - V_1}{sL} = \frac{V_1}{R_2}$$

Substituting for the transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{CR_1R_2 - L}{CLR1s + CR_1R_2}$$

Set V_{ss} to motor nominal voltage to limit voltage into motor

STATIC FRICTION

Static friction for an object is described by:

$$F_{SF} = \mu_{SF} \times F_N = \mu_{SF} \times mg$$

Rotor mass is negligible in a DC motor.

Motor 0

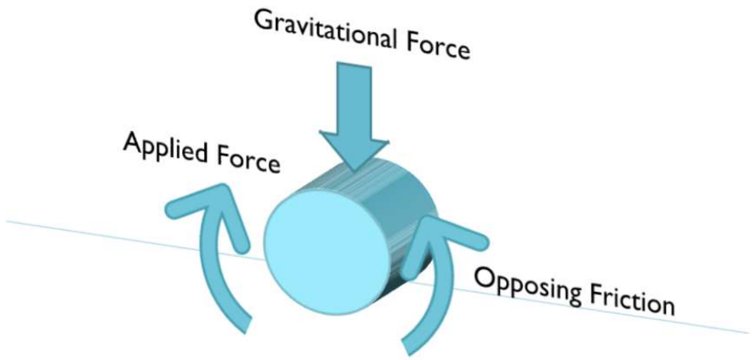
Weight attached includes motor, counterweight, and link.

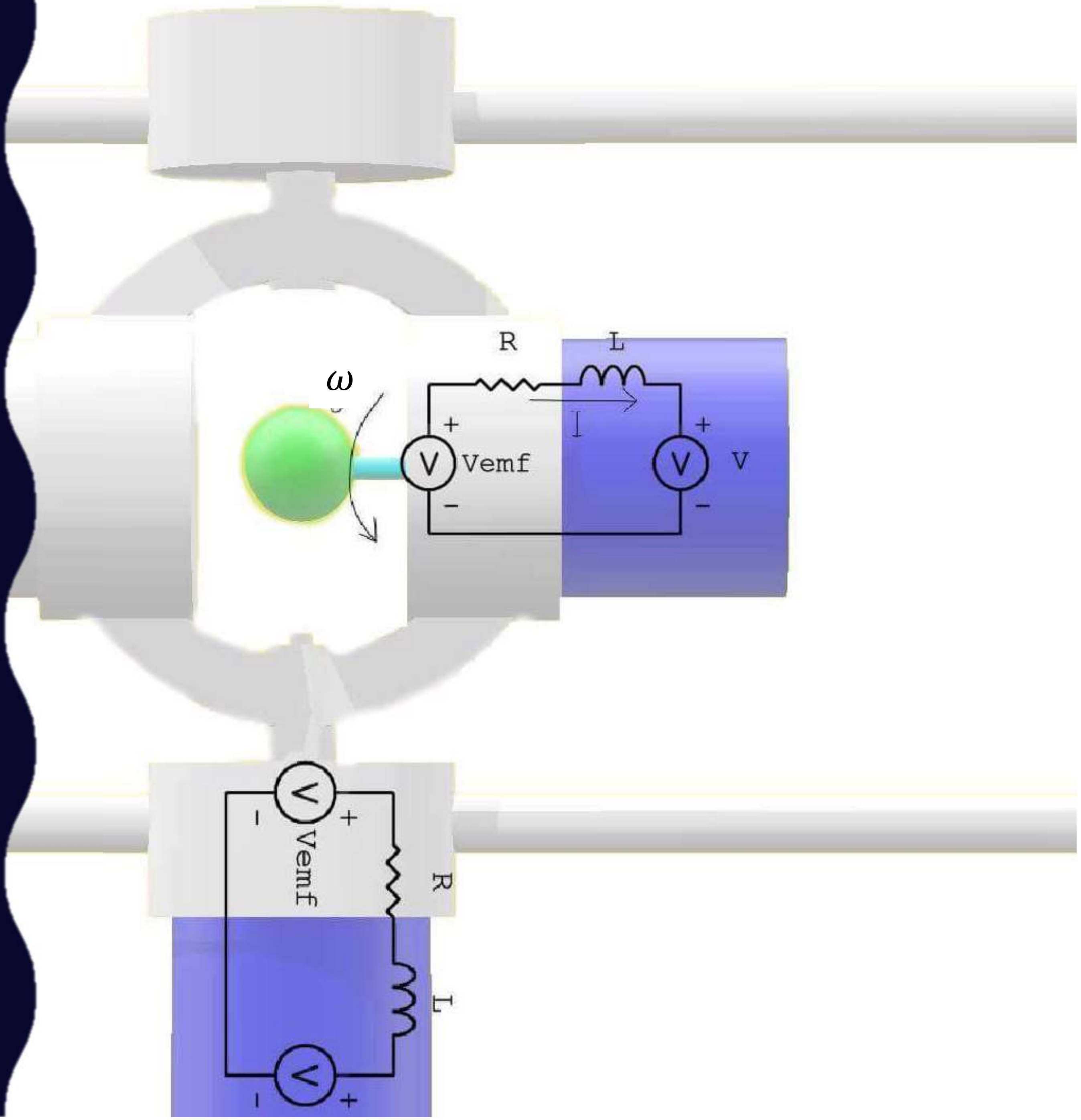
$$m = 2 \times m_{motor} + m_{link}$$

Motor 1

Weight attached is negligible.

$$m = 0$$





DC MOTOR CIRCUIT

A DC motor can be modelled as a simply circuit with input voltage v, back-emf resulting from the rotations v_{emf}, and an armature impedance of R and L.

Relationships from the circuit:

$$\frac{v - v_{emf}}{R + sL} = I$$

$$v_{emf} = k_b \times \omega$$

$$\tau = k_\tau \times I$$

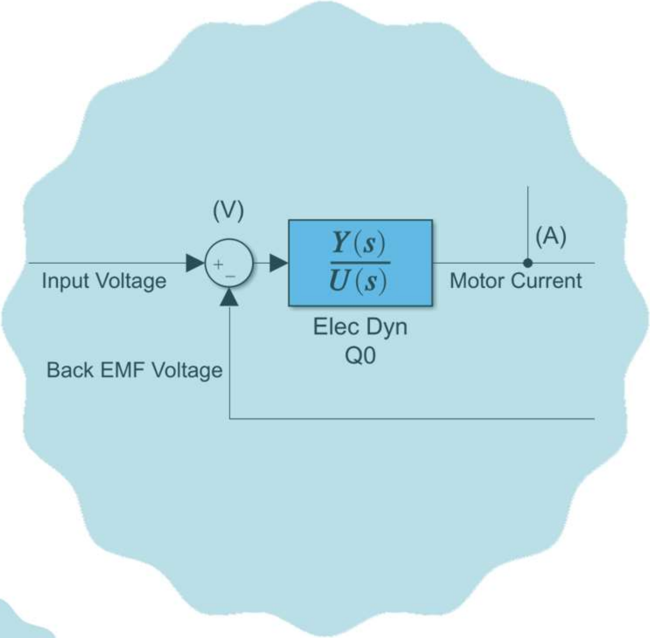
- The electrical transfer function relates current to the difference in voltage

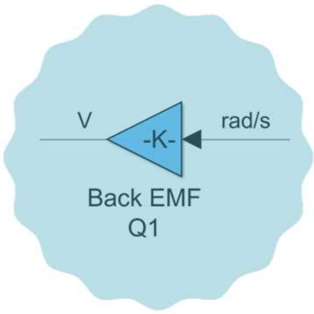
$$\frac{I}{v - v_{emf}} = \frac{1}{R + sL} = \frac{Y(s)}{U(s)}$$

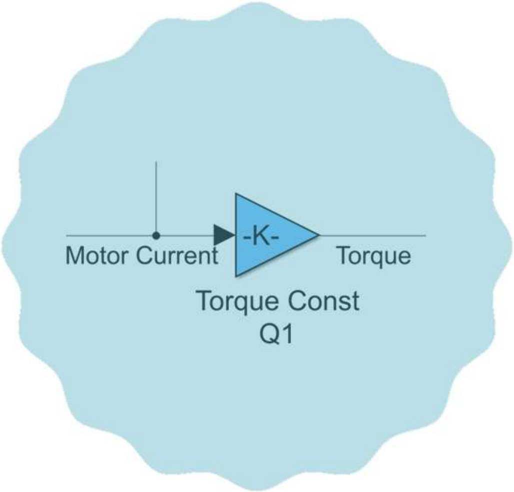
- k_b is represented by *speed constant* in the motor datasheet

$$k_b = \frac{1}{2\pi \times 60 \times speed\ constant}$$

- k_τ is represented by *torque constant* in the motor datasheet







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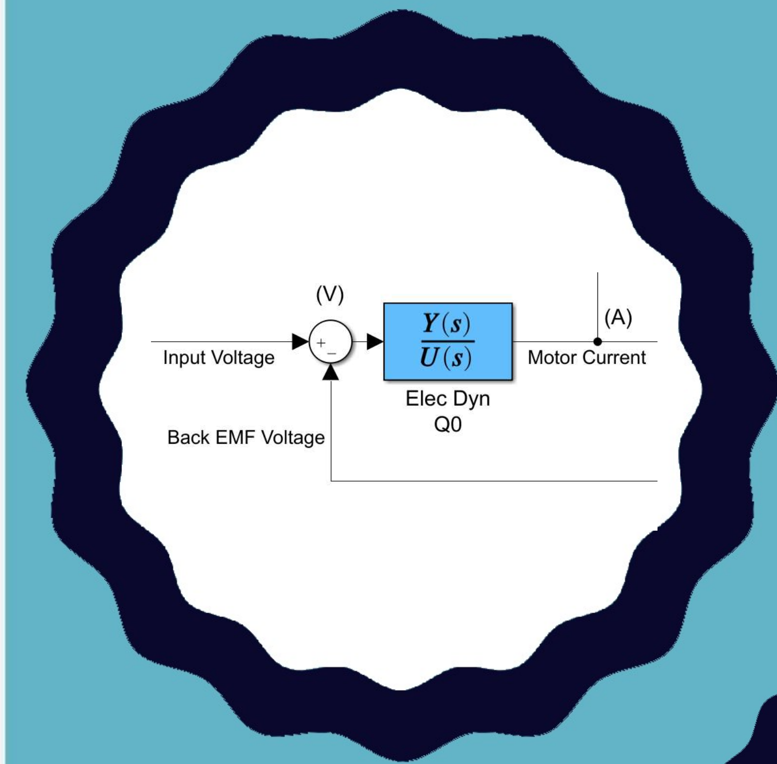
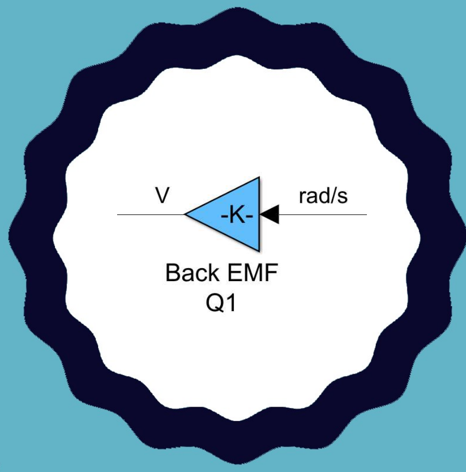
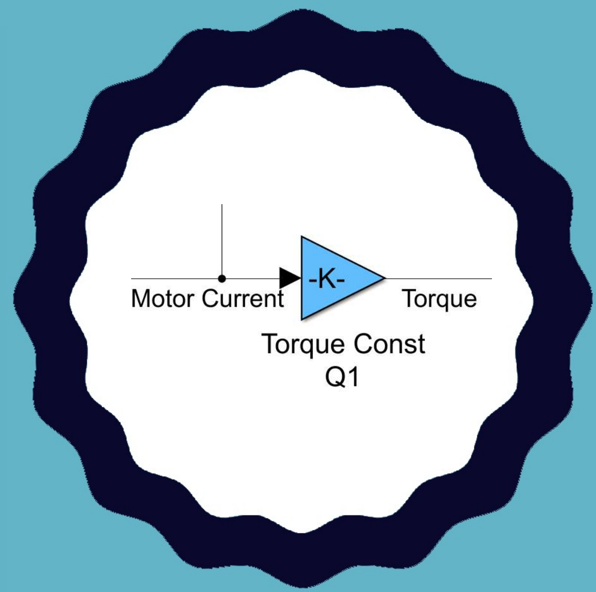
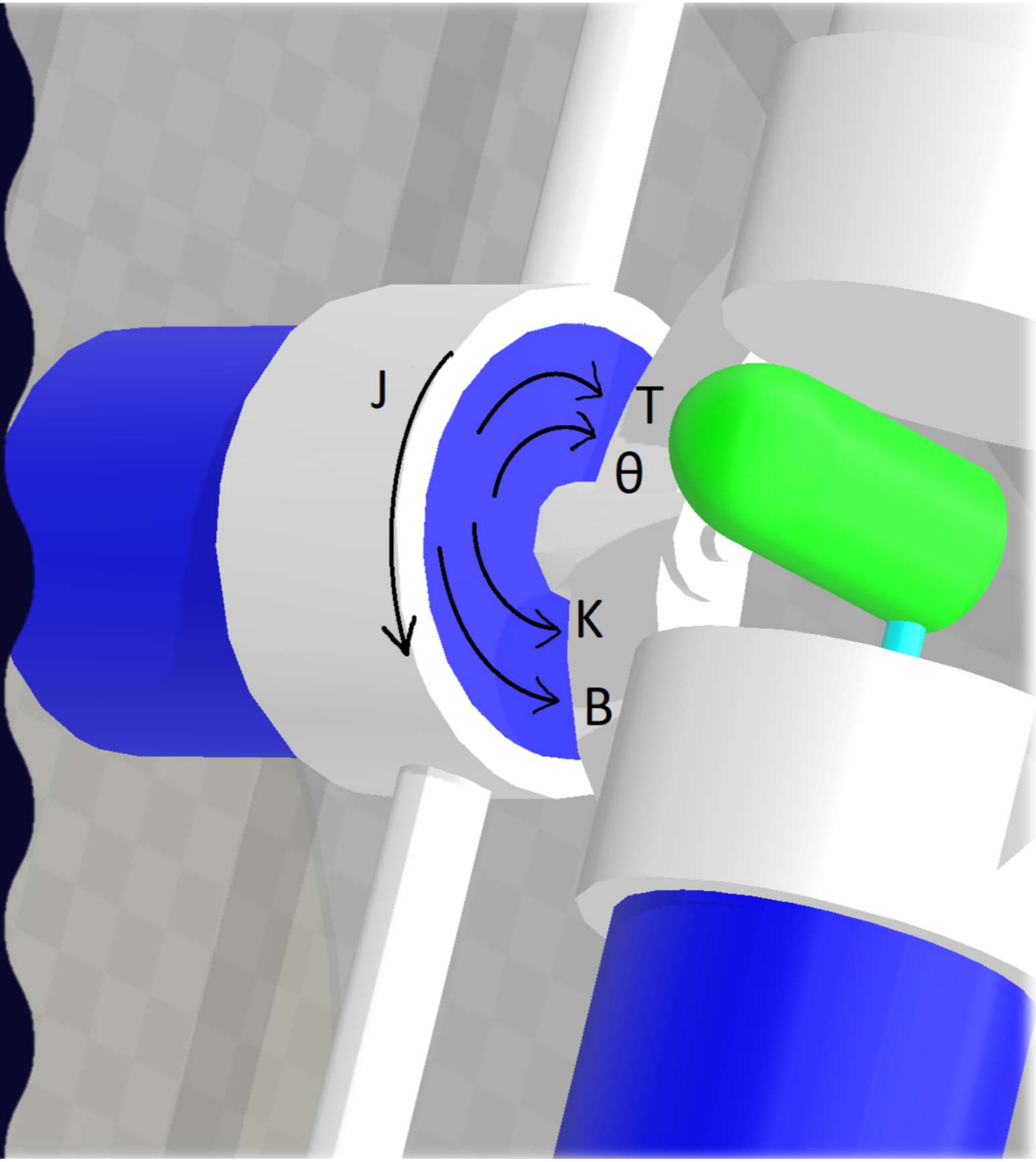








DC MOTOR MECHANICS

At the rotor, the forces can be modelled as a combination of inertia (J), spring (K), and friction (B). The effective torque is then:

$$\tau(t) = J\ddot{\theta} + B\dot{\theta} + K\theta$$

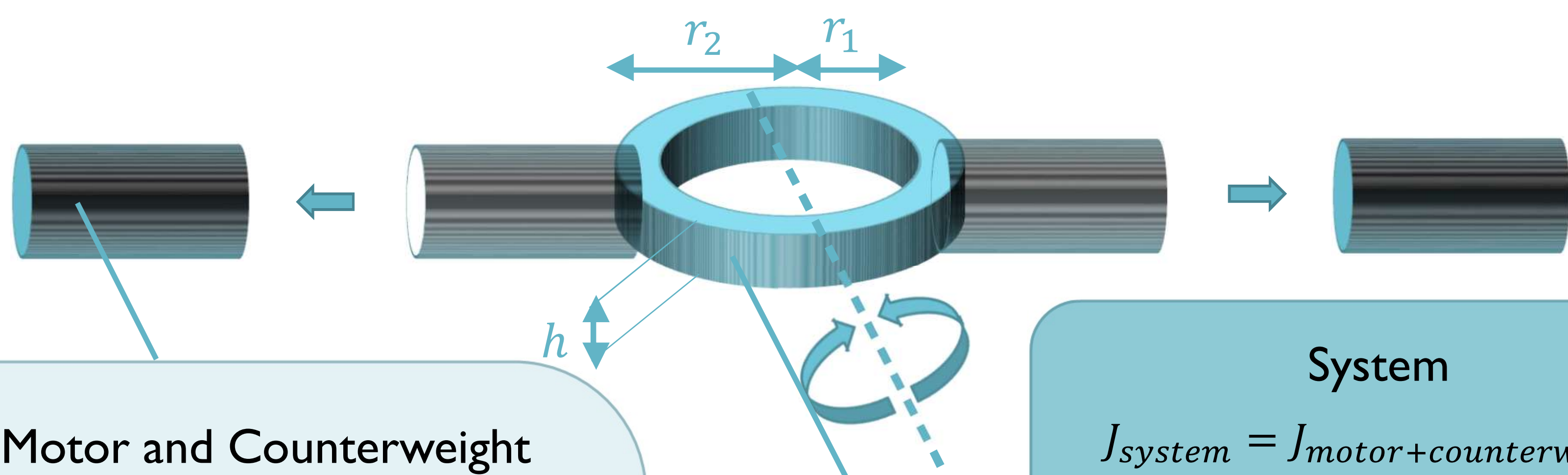
Or in terms of ω:

$$\tau(t) = J\dot{\omega} + B\omega + K\frac{1}{\omega}$$

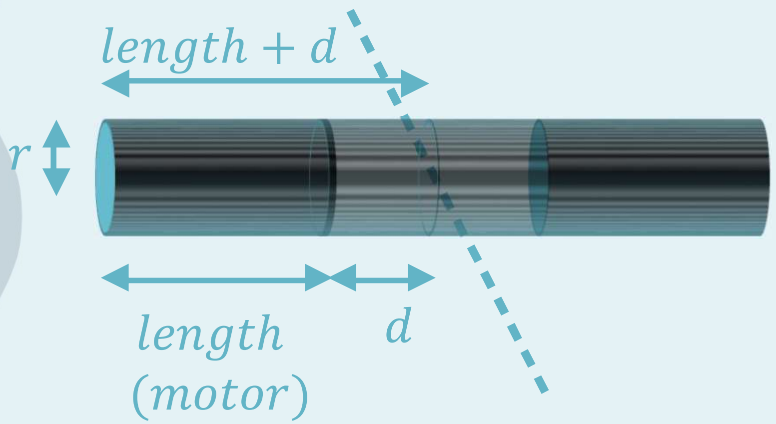
with s representing the derivative:

$$\tau = \frac{1}{s}\left(Js^2 + Bs + K\right)$$

INERTIA



Motor and Counterweight



$$Inertia = J_{2(length+d)} - J_{2d}$$

$$m_x = \frac{m_{motor}}{volume_{motor}} \times volume_x$$

$$J = \frac{1}{12} m_{object} \times \left(3r^2 + (2\,length + 2d)^2\right)$$
$$-\frac{1}{12} m_{imaginary} \times \left(3r^2 + (2d)^2\right)$$

Link

$$J_2 = \frac{\pi\rho h}{12}\left(3\left(r_2^2 + r_1^2\right) + h^2\right)$$

System

$$J_{system} = J_{motor+counterweight}$$
$$+ J_{link} + J_{rotor}$$

With no load, the generated torque equates to the damping torque (or kinetic friction)

$$\tau_{generated} = \tau_{friction}$$

$$k_\tau \times I_{no\ load} = B \times \omega_{no\ load}$$

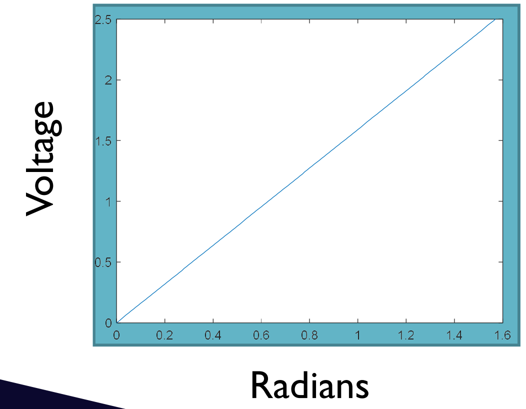
$$B = \frac{k_\tau I_{no\ load}}{\omega_{no\ load}}$$

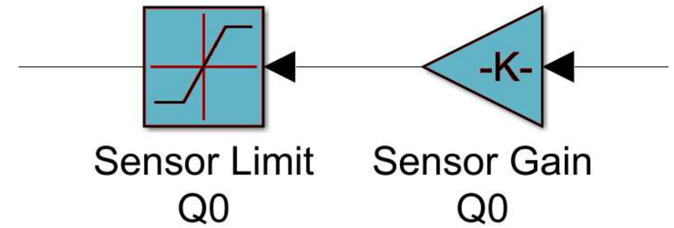
KINETIC FRICTION

TESTING THE MODEL

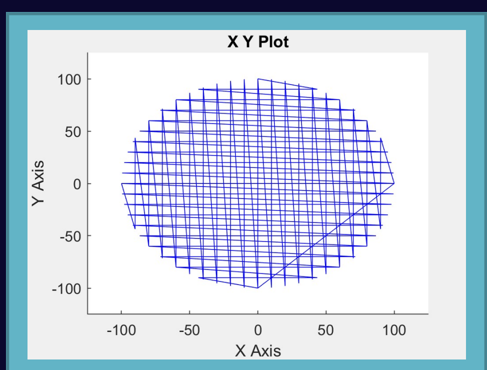
Sensor Feedback Loop

Sensor converts angles of motors proportionally to a voltage using ***sensor gain***





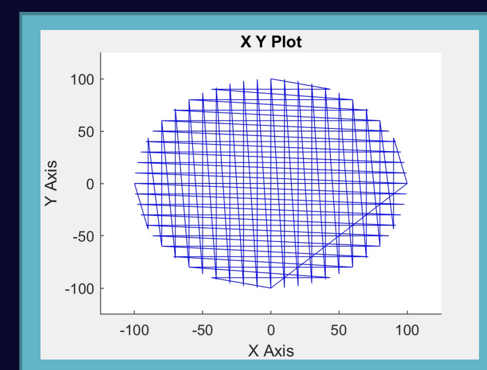
Confirming Direct and Inverse Kinematics



Desired

Inverse Kinematics → Direct Kinematics

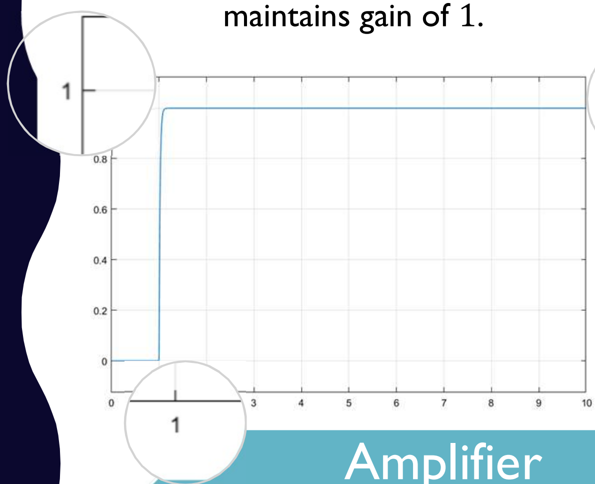
kinematic blocks should be an exact inverse of one another, effectively cancelling to generate identical actual plot



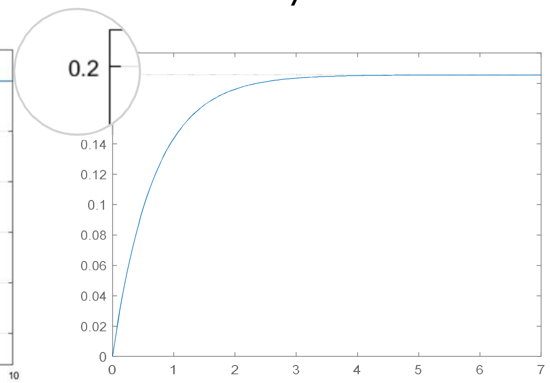
Actual

Step Response

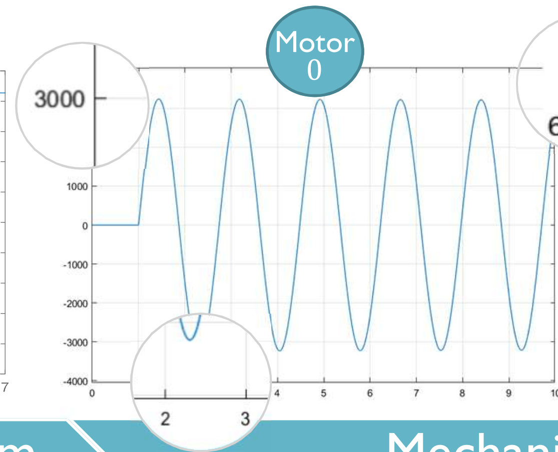
Voltage follower with constant gain of 1.
Step response maintains gain of 1.



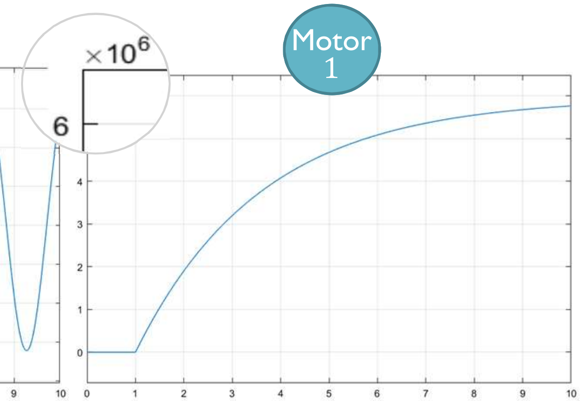
Electrical system outputs constant current for constant voltage. Initial slow slope caused by inductor



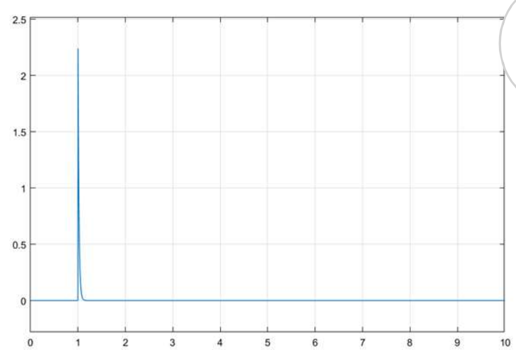
Oscillates due to continuous rotations from constant voltage. Tends towards 0 as spring force increases.

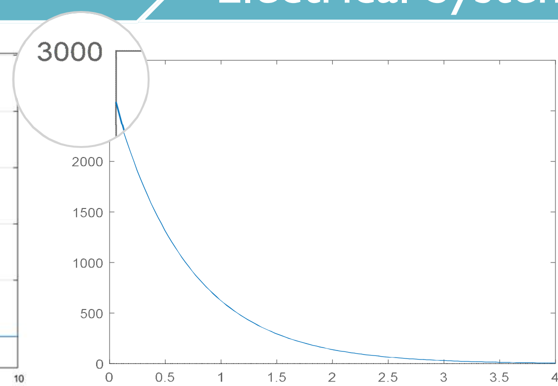


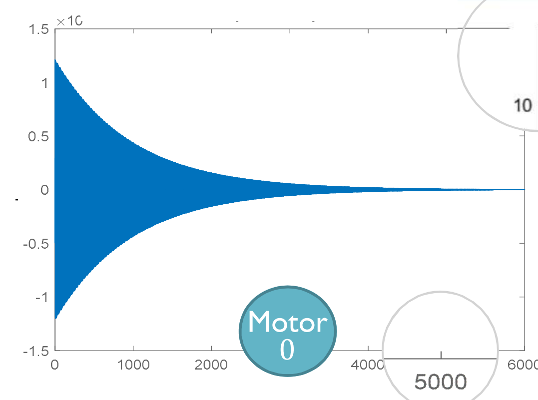
Slowly overcomes friction and inertia, eventually reaching maximum angle.

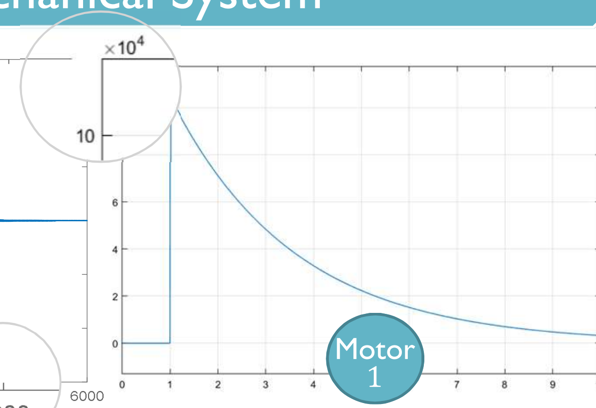


Amplifier | Electrical System | Mechanical System









Voltage follower with constant gain of 1.
Impulse response almost immediately drops back to 0.

Electrical system outputs constant current for constant voltage. Initial slow slope caused by inductor

For an impulse response, the motor oscillates due to continuous rotation until all energy from impulse has dissipated.

Energy slowly dissipates after initial impulse input due to inertia and friction

Impulse Response

EVALUATING THE SYSTEM

Transfer Functions

$$\frac{49.138}{(s+49.17)} \qquad \frac{2762.4}{(s+1.489e04)} \qquad \frac{12161\ s}{(s^2 + 0.002045s + 13.55)} \qquad \frac{2.2936e06\ s}{s\ (s+0.3856)}$$

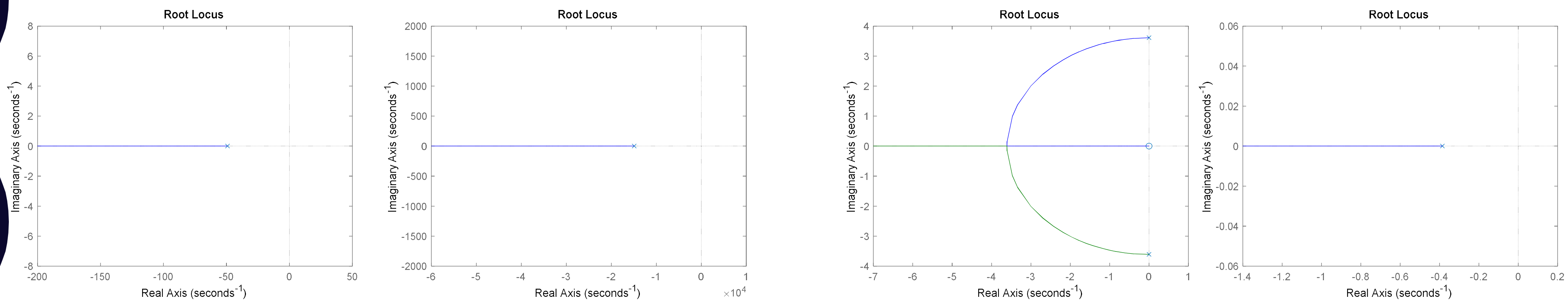
Amplifier

Electrical System

Mechanical System

Motor 0

Motor 1



Root Locus

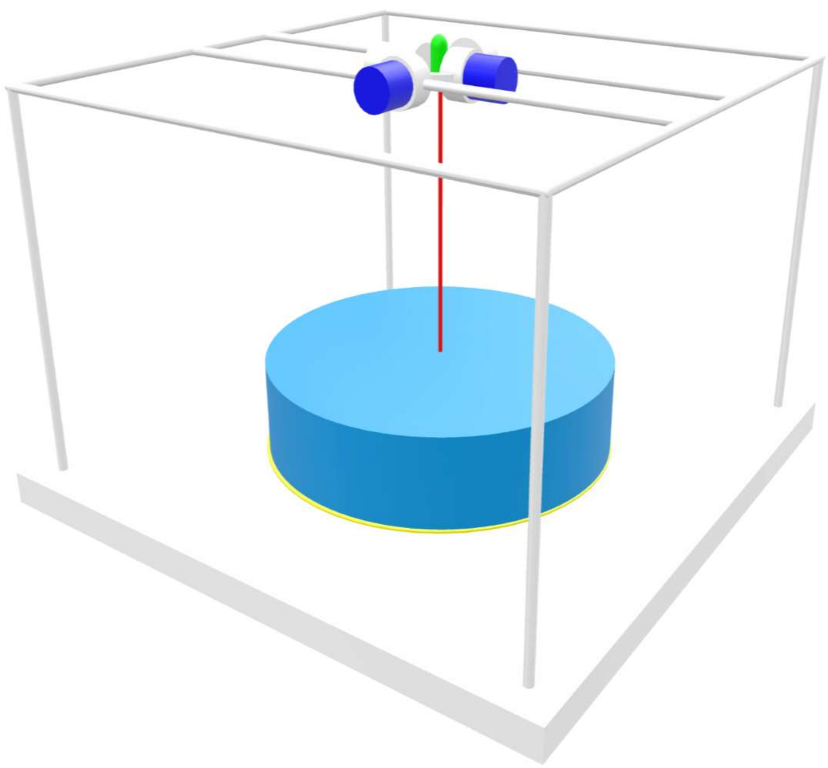
PID Controller

Sensor

CONTROL OVERVIEW

1. Motor — Choose appropriate motor to optimize the system
2. Root Locus — Plot root locus to determine characteristics of the system
3. PID — Explore and select values for a PID controller
4. Tune — Tune PID controller to minimize errors
5. Compare — Tune PID controller using PID controller box and compare

3D PRINTER



Input: Desired X,Y coordinates of object
Output: 3D printed object

CHOOSING MOTORS

MOTOR 0

Motor 0 holds a large weight, so choose most powerful motor

Choose largest stall torque

Motor	AMAX12 p75W SB	AMAX16 2W SB	AMAX19 2p5W SB	AMAX22 5W SB	AMAX22 6W SB
Stall Torque	1.52	4.84	9.47	24.3	23.7

MOTOR 1

Evaluate step responses with set motor 0 to determine most fitting motor 1

Choose shortest rise time, least overshoot and fastest peak time

Motor	AMAX12 p75W SB	AMAX16 2W SB	AMAX19 2p5W SB	AMAX22 5W SB	AMAX22 6W SB
Rise Time	0.12	0.16	0.23	0.29	0.29
Overshoot	71.34	79.04	85.21	88.25	88.25
Peak Time	0.35	0.46	0.66	0.84	0.84

AMAX22 6W

Motor 0 → AMAX22 5W M0

AMAX19 2p5W

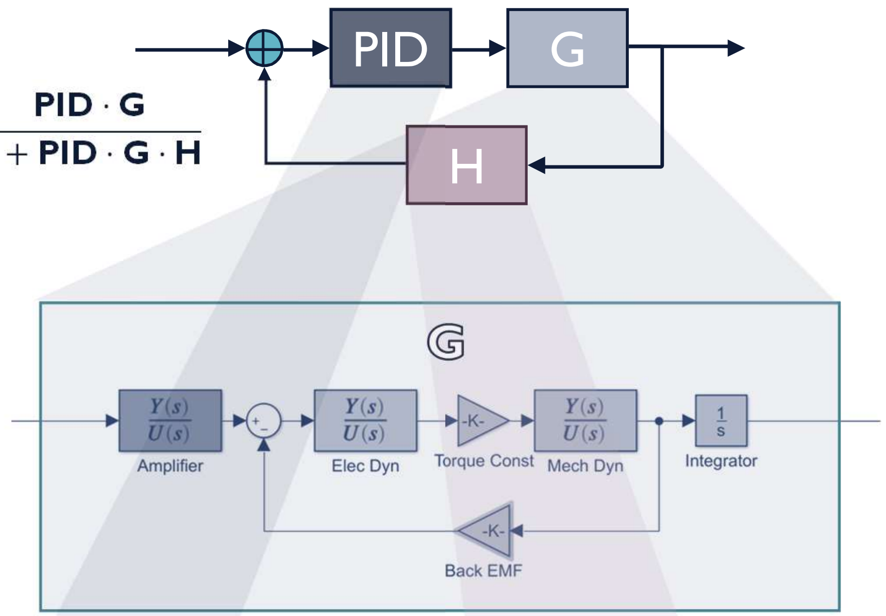
AMAX16 2W

Motor 1 → AMAX12 p75W M1

THE SYSTEM

Linearized for Evaluation

$$TF = \frac{PID \cdot G}{1 + PID \cdot G \cdot H}$$

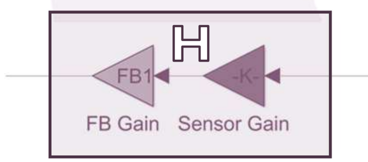


$$TF = TF_{amp} \cdot \frac{TF_{elec} \cdot K_{\tau} \cdot TF_{mech}}{1 + TF_{elec} \cdot K_{\tau} \cdot TF_{mech} \cdot K_B} \cdot \frac{1}{s}$$

PID
To Be Calculated

$TF = 1$

NOTE: Until values are determined



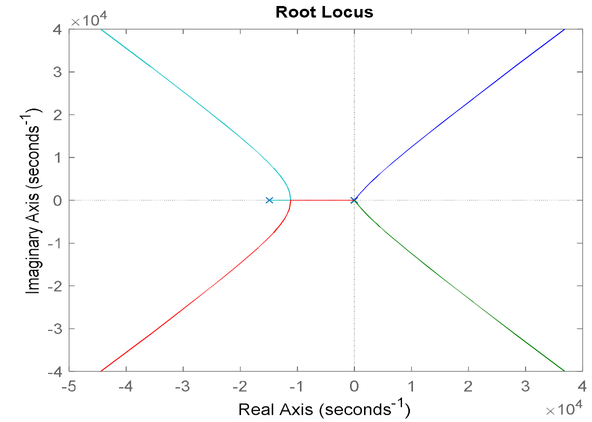
$TF = K_{gain} \cdot K_{FB\ gain} = 1$

NOTE: Set FB Gain to achieve this

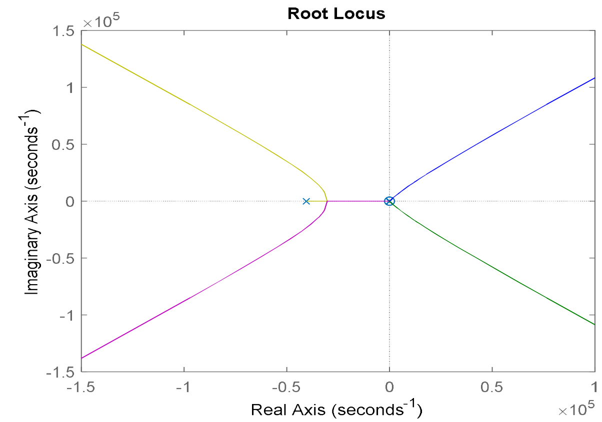
OPEN LOOP (KGH)

```
                 1.2843e+08
---------------------------------------------
(s+1.489e04) (s+49.17) (s^2 + 1.945s + 96.72)
```

MOTOR 0



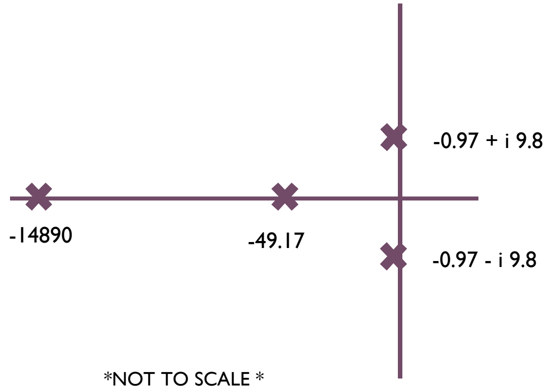
MOTOR 1




```
          1.4146e10 (s^2 + 0.02919s + 96.7)
-----------------------------------------------------
s (s+4.049e04) (s+49.17) (s+1.347) (s^2 + 0.1308s + 96.7)
```

SELECT INITIAL POSITION

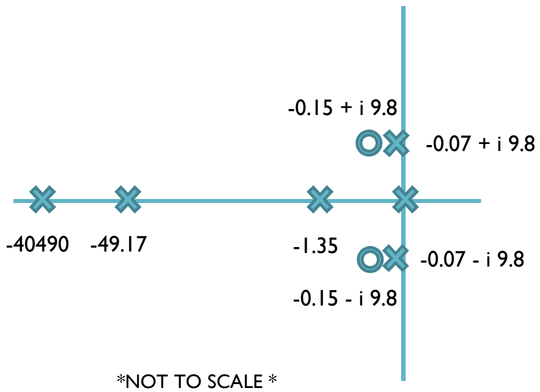
MOTOR 0



M0

Choose to eliminate complex poles closest to positive plane

MOTOR 1



M1

Must eliminate pole at 0 to remain stable
Choose to eliminate next closest pole (-1.35)

PID CONTROLLER

k_p Proportional Gain
k_i Integral Gain
k_d Derivative Gain

$$\mathrm{TF_{PID}} = k_d \frac{s^2 + \frac{k_P}{k_d}s + \frac{k_i}{k_d}}{s}$$

Set zeros at:

$s^2 + 1.945s + 96.72$

k_p	$1.945\ k_d$
k_i	$96.72\ k_d$
k_d	K_0

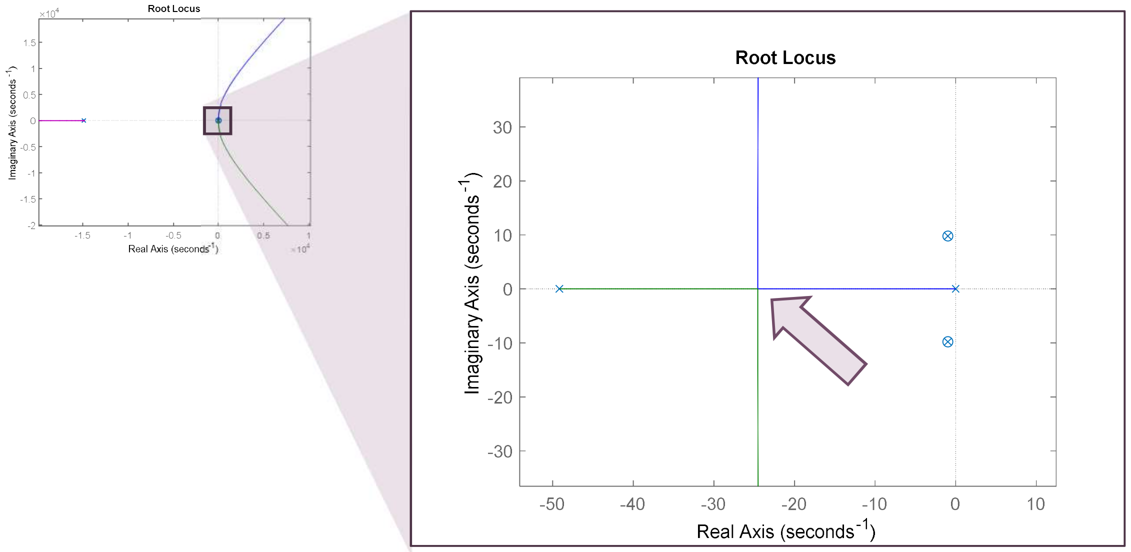
Set zeros at:

$s(s + 1.35) = s^2 + 1.35s$

k_p	$1.35\ k_d$
k_i	0
k_d	K_1

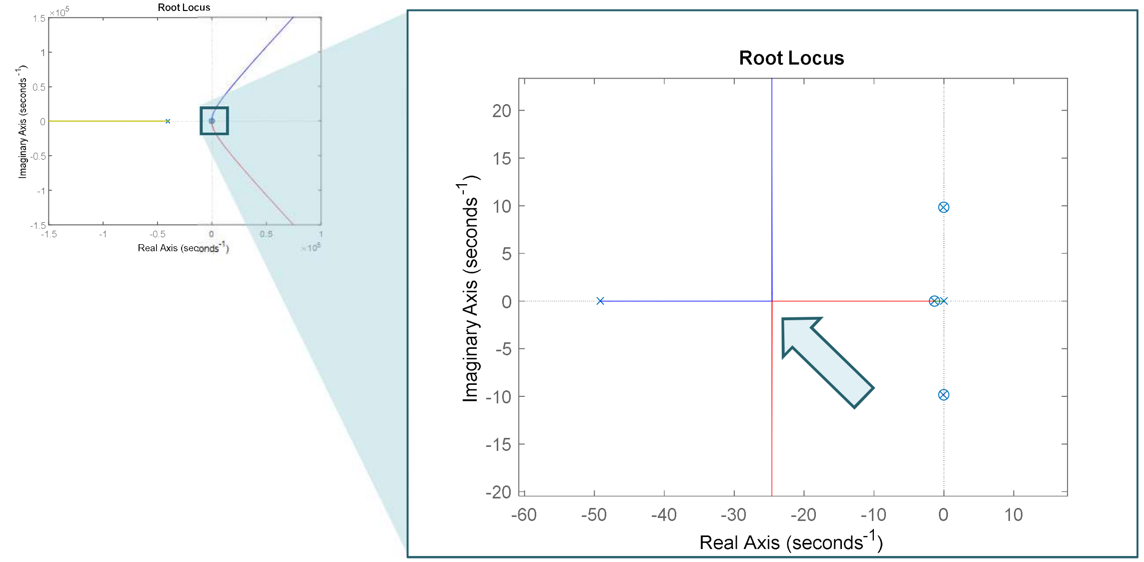
MOTOR 0

M0



MOTOR 1

M1



Choose K such that poles are critically damped

→ measure K at breakpoint and apply to previous equations

→ minimizes rise time, minimizes settle time, avoids oscillations

STARTING PID VALUES

K_0	k_p	k_i	k_d
0.07	0.135	6.77	0.07

K_1	k_p	k_i	k_d
0.0017	0.0023	0	0.0017

TUNING PROCESS

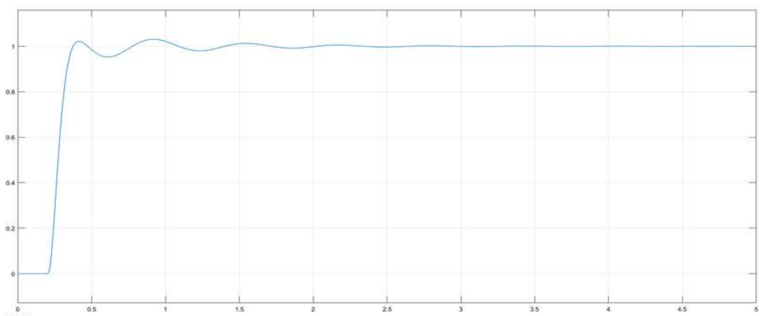
Slowly iterate through PID values to minimize build error

Gain Increased	Rise Time	Overshoot	Settle Time	SS Error
K_p	Decrease	Increase	Increase	Decrease
K_i	Decrease	Increase	Increase	0
K_d	Increase	Decrease	Decrease	N/A

STEP RESPONSES

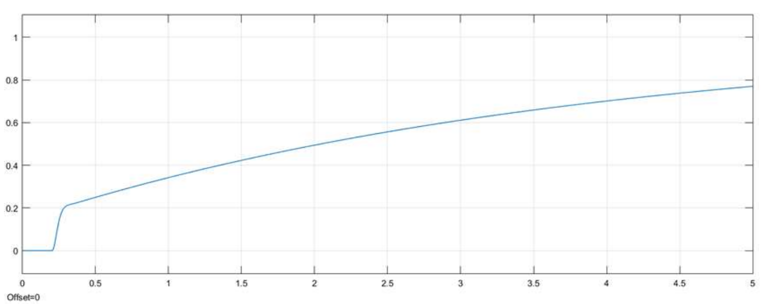
Initial PID Values

M0



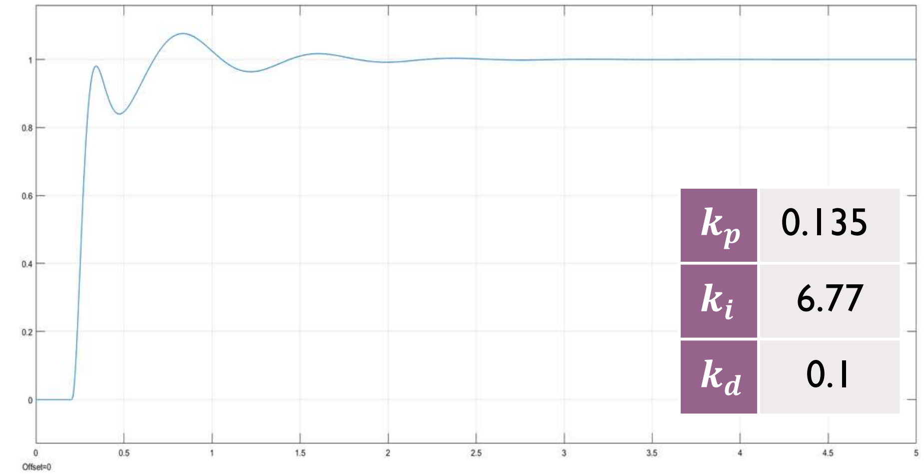
Initial PID Values

M1



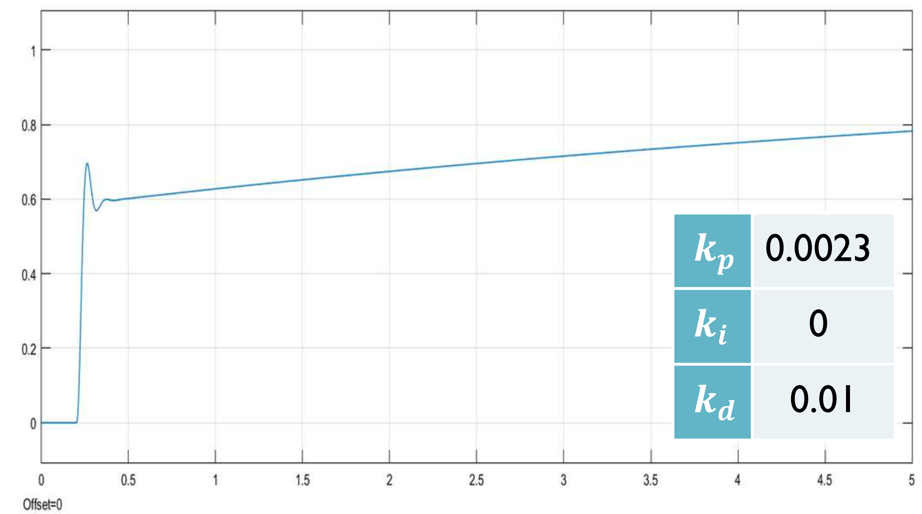
- M0 rise time can be improved, increase K_d
- M1 initial peak very low, increase K_d

MOTOR 0



k_p	0.135
k_i	6.77
k_d	0.1

MOTOR 1

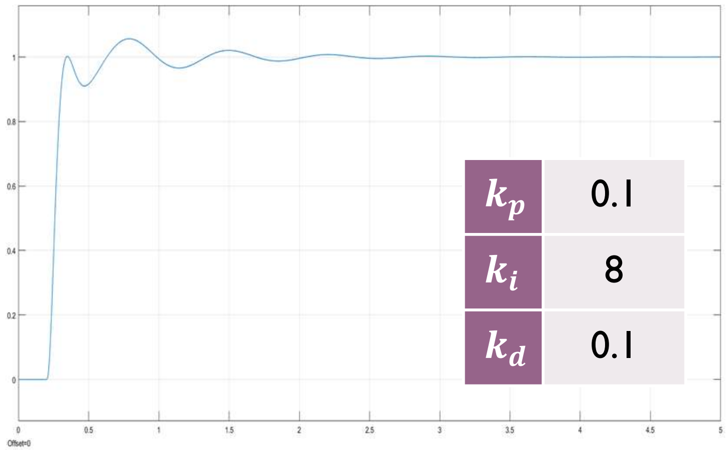


k_p	0.0023
k_i	0
k_d	0.01

Improve M0 settle time, decrease K_p, increase K_i

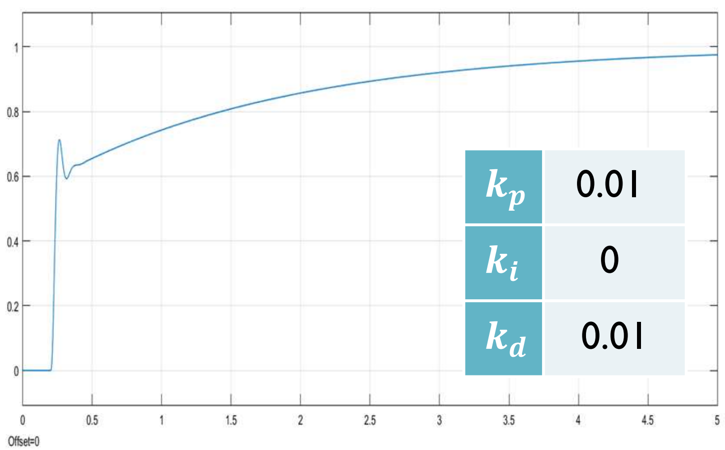
Improve M1 rise time, increase K_p

MOTOR 0



k_p	0.1
k_i	8
k_d	0.1

MOTOR 1

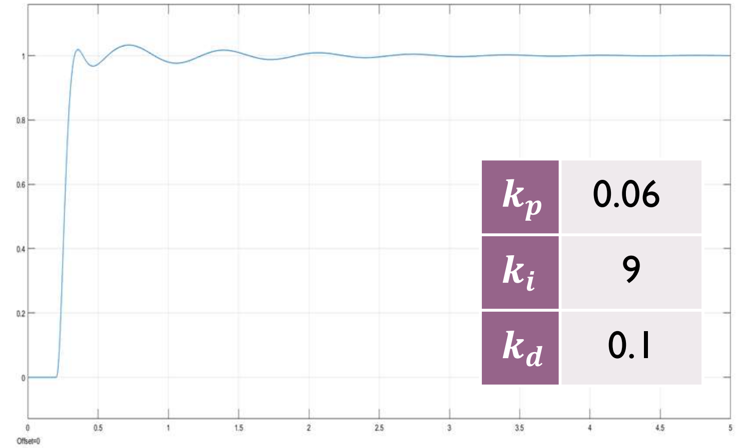


k_p	0.01
k_i	0
k_d	0.01

Continue to decrease K_p, increase K_i

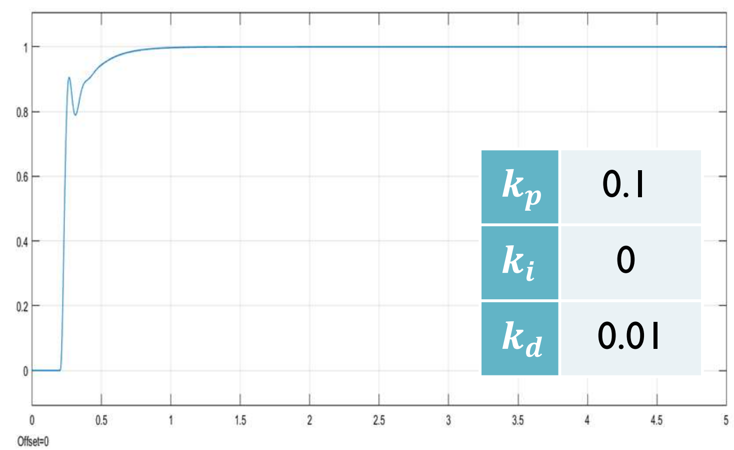
Continue to increase K_p

MOTOR 0



k_p	0.06
k_i	9
k_d	0.1

MOTOR 1



k_p	0.1
k_i	0
k_d	0.01

Continue to decrease K_p, increase K_i

Continue to increase K_p

continue iterating

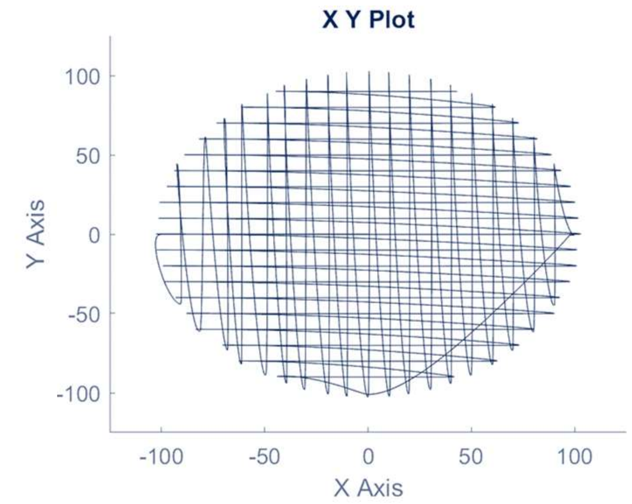
keep adjusting

RESULT

Final PID values

→ Optimized for 125ms step interval

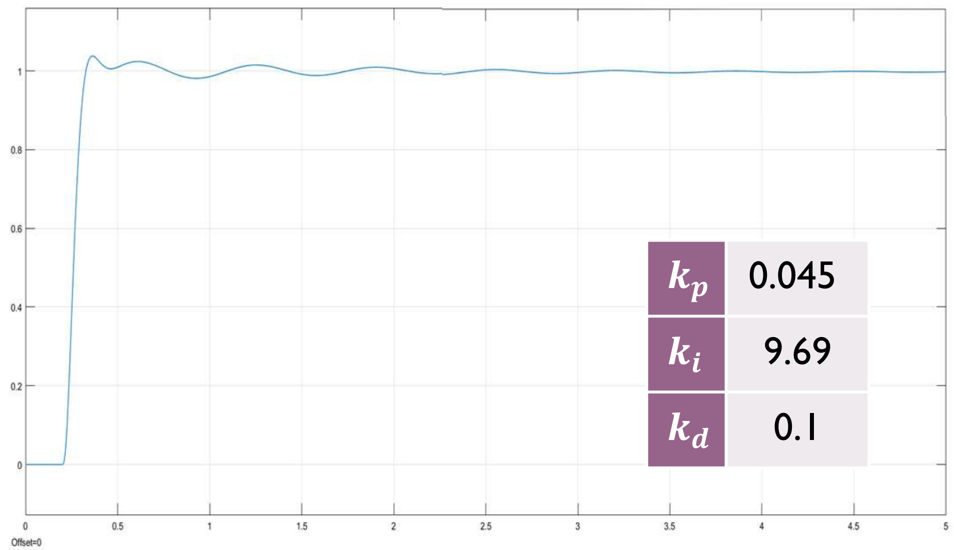
→ Minimized position error



Max Position Error
3.253

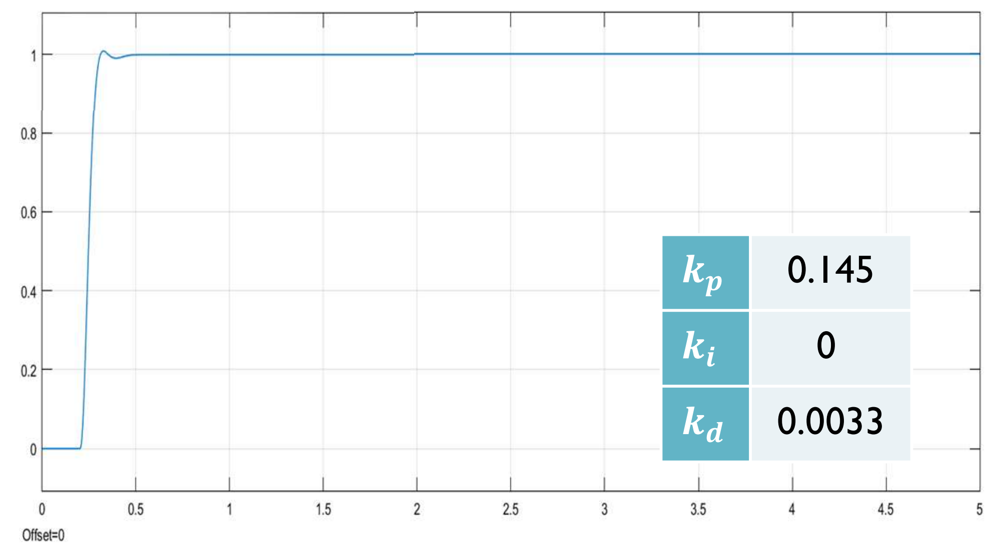
NOTE: Runtime of 20s

MOTOR 0



k_p	0.045
k_i	9.69
k_d	0.1

MOTOR 1

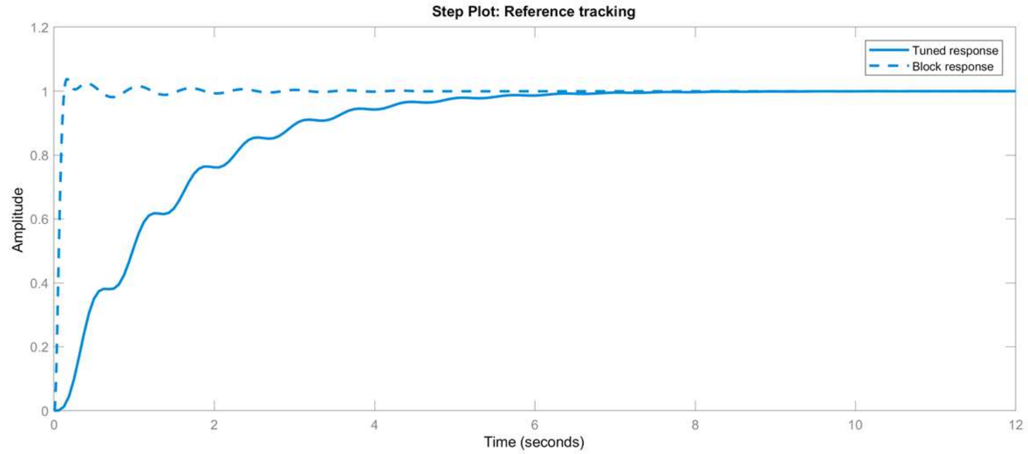


k_p	0.145
k_i	0
k_d	0.0033

COMPARISON

Hand Tuned PID Values VS Auto Tuned PID Values

MOTOR 0

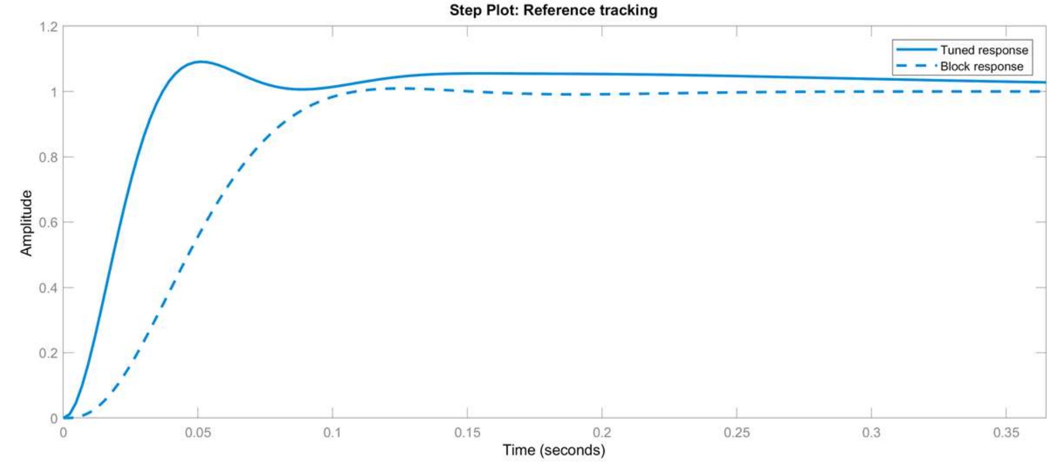


	k_p	k_i	k_d
Hand Tuned	0.045	9.69	0.1
Auto Tuned	0	0.398	0

	Rise Time	Settle Time	Overshoot
Hand Tuned	0.078	0.47	3.83%
Auto Tuned	2.78	5.45	0

Less overshoot, slower rise time, not ideal

MOTOR 1



	k_p	k_i	k_d
Hand Tuned	0.145	0	0.0033
Auto Tuned	0.35	1.43	0.015

	Rise Time	Settle Time	Overshoot
Hand Tuned	0.061	0.099	0.91%
Auto Tuned	0.025	0.42	9.1%

Fast rise time, larger overshoot, not ideal

THANK YOU