Homework Set 4
"Galactic Dynamics" course at SJTU

Due Date: June 16, 2020

Warning: please start working on these problems as early as possible.

1. (10 points) The goal of this problem is to explore the behavior of the velocity dispersion near the center of a spherical galaxy. At radii $r < r_0$ assume that the density has the power-law form

$$\rho(r) = \rho_0 \frac{r_0^\gamma}{r^\gamma}, \quad 0 \le \gamma < 3. \tag{1}$$

Assume that the velocity dispersion is isotropic at all radii and equal to σ_0^2 at r_0.

(a) Why is the constraint $\gamma < 3$ necessary?

(b) What is the dispersion $\sigma^2(r)$ for $r < r_0$?

(c) For what value(s) of γ is $\sigma^2(r)$ independent of r as $r \to 0$? For what range of γ does $\sigma^2(r) \to 0$ as $r \to 0$? For what range of γ does $\sigma^2(r)$ diverge as $r \to 0$? What is the case for the NFW profile?

2. (10 points) (a). Consider a cloud of particles with total mass M, initial radius R and initial velocity $V = 0$, which collapse under its self-gravity. Without radiating energy lost, what is the temperature if the cloud is in virial equilibrium? Should the temperature be higher or lower if the radiation is considered?

(b). Suppose that our Galaxy formed by aggregating from an initial radius that was much larger than its present size. Assume that most of the galactic material is now moving at about $\sim 200\,\mathrm{km\ s^{-1}}$, whether on circular orbits in the disk or on eccentric and highly inclined halo orbits. Estimate how much energy, in terms of the Galaxy's rest-mass energy (Mc^2), must have been radiated away when the Galaxy formed.

3. (20 points) [BT08, Problem 4.8] Consider a spherical system with DF $f(\mathcal{E}, L)$. Let $N(\mathcal{E}, L)\mathrm{d}\mathcal{E}\mathrm{d}L$ be the fraction of stars with $\mathcal{E}$ and L in the

ranges $(\mathcal{E}, \mathcal{E}+d\mathcal{E})$ and $(L, L+\mathrm{d}L)$.
(a) Show that

$$N(\mathcal{E}, L) = 8\pi^2 L f(\mathcal{E}, L) T_r(\mathcal{E}, L). \tag{2}$$

where T_r is the radial period defined by equation (3.17).
(b) A spherical system of test particles with ergodic DF surrounds a point mass. Show that the fraction of particles with eccentricities in the range $(e, e+\mathrm{d}e)$ is $2e\,\mathrm{d}e$.

4. (10 points) [BT08, Problem 4.13] Show that when the DF of a spherical system depends only on the function $Q(x, v)$ defined by equation (4.73), the ratio of the mean-square tangential and radial speeds is

$$\overline{v_{\mathrm{t}}^2}/\overline{v_r^2} = \frac{2}{1+(r/r_{\mathrm{a}})^2}. \tag{3}$$

5. (10 points) [BT08, Problem 4.27] Show that a self-gravitating isothermal stellar system with velocity dispersion σ, cylindrical symmetry, and non-singular, non-zero density ρ_0 at $R=0$ has the density distribution

$$\rho(R, \phi, z) = \rho_0 \left(1 + \frac{\pi G \rho_0 R^2}{2\sigma^2}\right)^{-2}. \tag{4}$$

6. (15 points) [BT08, Problem 4.21] We may study the vertical structure of a thin axisymmetric disk by neglecting all radial derivatives and assuming that all quantities vary only in the coordinate z normal to the disk. Thus we adopt the form $f = f(E_z)$ for the DF, where $E_z \equiv \frac{1}{2}v_z^2 + \Phi(z)$. Show that for an isothermal disk in which $f = \rho_0 \left(2\pi\sigma_z^2\right)^{-1/2} \exp\left(-E_z/\sigma_z^2\right)$, the approximate form (2.74) of Poisson's equation may be written

$$2\frac{\mathrm{d}^2\phi}{\mathrm{d}\zeta^2} = \mathrm{e}^{-\phi}, \quad \text{where} \quad \phi \equiv \frac{\Phi}{\sigma_z^2}, \quad \zeta \equiv \frac{z}{z_0}, \quad \text{and} \quad z_0 \equiv \frac{\sigma_z}{\sqrt{8\pi G\rho_0}}$$

By solving this equation subject to the boundary conditions $\phi(0) = \phi'(0) = 0$, show that the density in the disk is given by (Spitzer 1942)

$$\rho(z) = \rho_0 \operatorname{sech}^2\left(\frac{1}{2}z/z_0\right). \tag{5}$$

Show further that the surface density of the disk is

$$\Sigma = \frac{\sigma_z^2}{2\pi G z_0} = 4\rho_0 z_0. \tag{6}$$

7. (20 points) Figure 1 shows the surface-brightness profiles of three galaxies. The units for μ_V are magnitudes/arcsec^2 in the V band. The central line-of-sight velocity dispersions in these galaxies are: σ(N1400)=265 km s^{-1}, σ(N2832)=330 km s^{-1}, σ(N3608)=195 km s^{-1}. Assume that the galaxies are spherical and the velocity dispersion is isotropic, find the core mass-to-light ratio of each galaxy in solar units. The V-band absolute magnitude of the Sun is $M_V(\odot) = 4.83$. Hint: You may first prove that the unit conversion from magnitudes/arcsec^2 to $L_\odot/\,\mathrm{pc}^3$ is $\mu_V = 26.4 - 2.5 \log I_V$, similar to Equation (2.60) in the Binney & Merrifield (1998) book (i.e. Problem 2 in Homework Set 1).

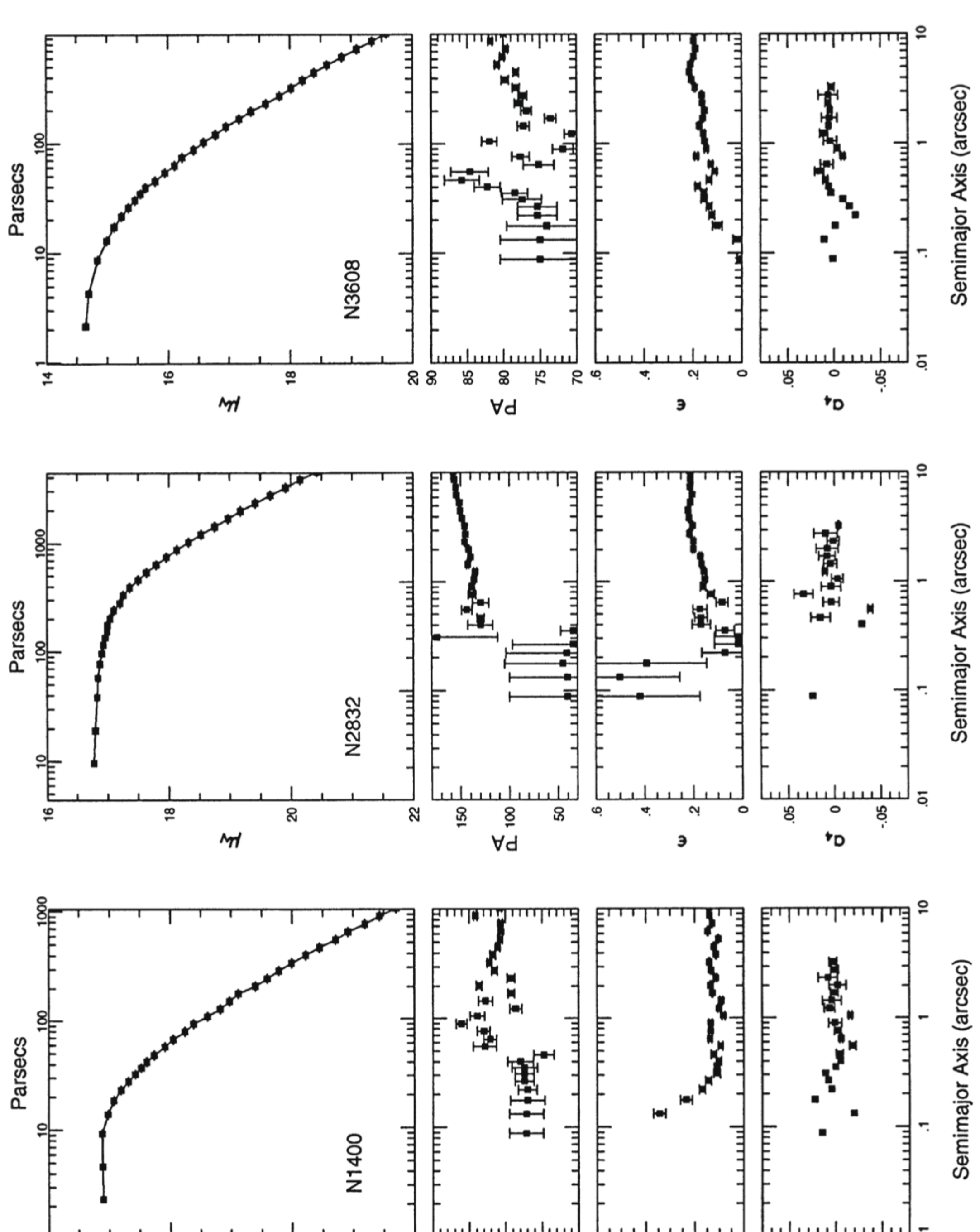

Figure 1: Figure of Problem 7