Mathematics For Computer Science
Logic and Discrete Structures

Practical Questions Week 5

Question 1: Propositional reasoning

On an island there are knights who always tell the truth and knaves who always lie. You meet three people: A, B, and C.

- A says: "I am a knave and B is a knight."
- B says: "Exactly one of the three of us is a knight."

What are A, B, and C?

Question 2: Logical equivalence

The logical connective NAND is such that:

$$p \text{ NAND } q \text{ is } false \text{ if, and only if, } p \text{ and } q \text{ are both } true.$$

Prove that p NAND (q NAND r) and (p NAND q) NAND r are not equivalent.

Question 3: De Morgan Laws and distribution laws

Show that the following propositional formulae are tautologies without using truth tables:

1. $(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg q$
2. $((p \vee q) \wedge \neg p) \Rightarrow q$

Question 4: De Morgan Laws and distribution laws

Show that $(a \vee c) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a)$ and $(b \Rightarrow c) \wedge a$ are logically equivalent without using truth tables.

Question 5: Functional completeness

The logical connective NAND is such that:

$$p \text{ NAND } q \text{ is } false \text{ if, and only if, } p \text{ and } q \text{ are both } true,$$

and the logical connective NOR is such that:

$$p \text{ NOR } q \text{ is } true \text{ if, and only if, } p \text{ and } q \text{ are both } false.$$

1. Write p NAND q and p NOR q using only $\vee$, $\wedge$ and $\neg$.
2. Write $p \vee q$, $p \wedge q$, and $\neg p$ using only NAND and NOR.

(So, the set of connectives $\{\text{NAND}, \text{NOR}\}$ is functionally complete.)

Question 6: Functional completeness

Find a propositional formula involving only the logical connective NOR that is logically equivalent to $X \Rightarrow Y$.

Question 7: Conjunctive normal form

Which of the following propositional formulae are satisfiable (that is, are such that there is a satisfying truth assignment)?

1. $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
2. $(\neg p \vee \neg q \vee s) \wedge (\neg p \vee q \vee s) \wedge (p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee s) \wedge (p \vee r \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee r)$
3. $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$.

Question 8: Conjunctive normal form

Reduce the following formulae φ to conjunctive normal form by first reducing $\neg\varphi$ to disjunctive normal form using truth tables.

1. $((p \wedge \neg q) \vee r) \Rightarrow (\neg p \wedge \neg r)$
2. $((p \wedge (q \Rightarrow r)) \Rightarrow s)$
3. $(p_1 \wedge p_2) \vee (p_3 \wedge (p_4 \vee p_5))$ (* not so much hard as very messy!)

Question 9: Conjunctive normal form

Reduce the following formulae φ to conjunctive normal form without using truth tables.

1. $((p \wedge \neg q) \vee r) \Rightarrow (\neg p \wedge \neg r)$
2. $(p \wedge (q \Rightarrow r)) \Rightarrow s$
3. $(p_1 \wedge p_2) \vee (p_3 \wedge (p_4 \vee p_5))$