Histograms and Histogram based image enhancement

1 What is a histogram?

Definition: Histogram function

A function defined over all possible intensity levels. For each intensity level, its value is equal to the number of pixels with that intensity

2 Constructing a histogram

Simply count occurrences of each intensity value

Listing 1 Constructing a histogram

```
initialise all histogram array entries to 0
for each pixel I(i,j) within the image I
    histogram(I(i,j)) = hostogram(I(i,j)) +1
end
```

3 Contrast stretch/normalisation

Operation: Stretches the pixel range over a larger dynamic range

Approach: use four intensity values

a: upper pixel quantisation limit

b: lower pixel quantisation limit

c: maximum pixel value present

d: minimum pixel value present

$$I_{output}(i,j) = (I_{input}(i,j) - c)\left(\frac{a-b}{c-d}\right) + a$$

Potential problem - outliers in the image

- Possible that $c \sim a$ and $d \sim b$ (or even $c = a$ and $d = b$)
- Result: contrast stretch has no effect on the image

3.1 Solutions

Use a robust against outliers method to select c and d, instead of the min and max values in the image
Method 1

- Select c and d at fixed percentile points of the histogram distribution
- If any of new intensity values are below b or above a, map them to b and a respectively

Method 2

- Find the most frequent image value (histogram peak)
- Select a cut-off as a percentage of the peak
- Scan down from peak in either direction until last values above cut-off are reached and select these as c and d
- If any of new intensity values below b or above a, map them to b and a respectively

Method 2 is marginally weaker for complex, multi-peak histograms

4 Histogram modelling

Definition: Histogram modelling

Modify an image so that its histogram conforms to a given shape

Definition: Histogram equalisation

Histogram modelling via an intensity transformation function aiming at producing an output image with uniform histogram distribution

5 Cumulative histogram function

Let the dynamic range of a grayscale image be

$$i = 1, 2, ..., L$$

For a histogram function $h(i)$ we construct the cumulative histogram function $C(i)$

$$C(i) = \sum_{j=1}^{i} h(j)$$

That is, the values of $C(i)$ record the sum of the occurrence of each grey level up to and including i

$C(i)$ is a monotonically increasing function

6 Histogram equalisation

In an ideally equalised image, all intensity values would appear the same number of times, i.e. N/L each, where N is the number of pixels in the image

The cumulative function would then be

$$C_{ideallyEqualised}(i) = (N/L)i$$

Histogram equalisation corresponds to the intensity transform

$$t(i) = (L/N)C_{input}(i)$$

Which computes the cumulative histogram at intensity i, and maps i to the intensity of the ideally equalised image for that value of the cumulative histogram

A schematic example of how histogram equalisation works:

- Let L = 100 (dynamic range of 100 levels)
- Let $C(50) = 0.8 \cdot N$ (80% of the N pixels have value 50 or lower)
- $t(50) = (L/N) \cdot 0.8 \cdot N = 100 \cdot 0.8 = 80$
- 50 is mapped 80 and this, 80% of the pixel of the equalised image have value 80 or lower

7 Implementation

Technical issues requiring attention in practice:

- a dynamic range with L levels usually consists of the values $i = 0, 1, .., L - 1$
- L/N might not be an integer, thus some of the values of t() might not be integers

As a solution we can instead use the formula

$$t(i) = \lfloor ((L - 1)/N) \cdot C_{input}(i) \rfloor$$

8 Limitations

As a fully automated technique (no parameters) the effect of histogram equalisation is highly input dependant

In some image the global contrast can be over-exposed or under-exposed

8.1 Solution

Use the histogram from (a well balanced) sub-part of original image as the input histogram of the equalisation algorithm

9 Localised histogram equalisation

Split the image into a set of discrete, non-overlapping neighbourhoods of size $N \times N$

Histogram equalisation of each neighbourhood in isolation (tiling)

10 Adaptive histogram equalisation

Perform histogram equalisation at each pixel (rather than neighbourhood) using overlapping local $N \times N$ neighbourhoods

Adaptive histogram equalisation is slower than localised histogram equalisation

Tiling artefacts can be avoided but choice of neighbourhood size N crucial

11 Localised/adaptive equalisation

In terms of the aesthetics of the output, performance is poor because a global transform (of the entire dynamic range) is computed and applied locally