Decision Problems

Definition: Time complexity

For any function f, we say that the time complexity of a decidable language $\mathscr{L}$ is $O(f)$, or $\mathscr{L}$ is decidable in $O(f)$ time, if there exists a TM T which decides $\mathscr{L}$, and constants n_0 and c such that for all inputs x with $|x| > n_0$

$$Time_T(x) \leqslant c \cdot f(|x|)$$

1 Complexity Classes

Definition: Time complexity class TIME[f]

The class of all problems for which there exists an algorithm with time complexity in $O(f)$

2 The complexity class P

Definition: P

$$\mathbf{P} = \bigcup_{k \geq 0} TIME\left[n^k\right]$$

The class P is a reasonable mathematical model of the class of problems which are tractable or solvable in practice

However, the correspondence is not exact:

- When the degree of the polynomial is high then the time grows so fast that in practice the problem is not solvable
- The constants may also be very large

3 Different models of computation

Lemma
We can simulate t steps of k-tape TM with an equivalent one tape TM in $O[t^2]$ steps

Lemma
We cans simulate t steps of a two-way infinite k-tape machine with an equivalent k-tape TM in $O[t]$ steps

Hence the class P is the same for all of these models of computation (and many others)

4 Different encodings

Lemma

For any number n, the length of the encoding of n in base b_1 and the length of the encoding of n in base b_2 are related by a constant factor (provided $b_1, b_2 \geqslant 2$)