Questions likely to be on the exam

1. What are the two main components of an integrated circuit? (Both of these components usually appear in their millions.) [1]

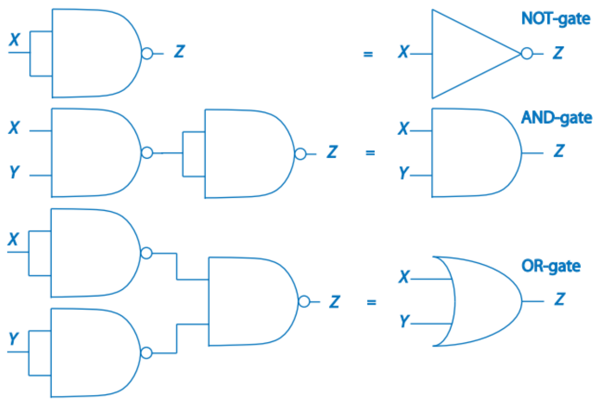
Solution:
Transistors [$\frac{1}{2}$] interconnected by microscopic wires [$\frac{1}{2}$].

2. What is the current trend in microprocessor design so as to overcome difficulties with power dissipation as we build faster and faster single processors? [2]

Solution:
Multi-core processors [1] where one CPU with a high clock-speed is replaced with a number of CPUs with lower clock-speeds but which, when working together, can give better computational power [1].

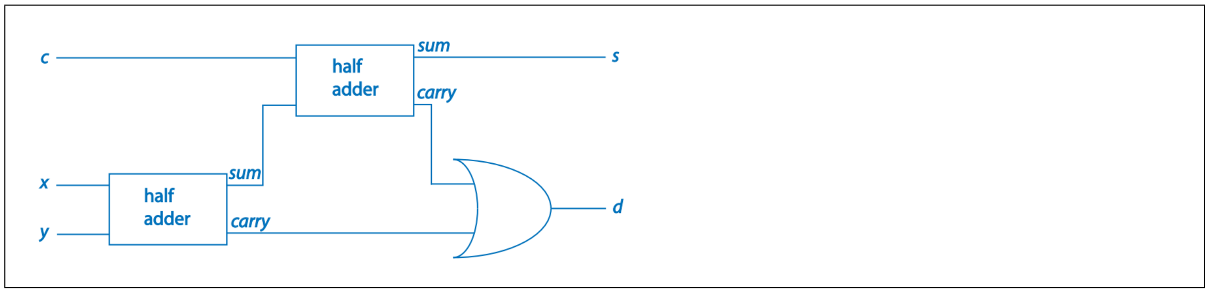
3. What is a NAND-gate? Show how a NOT-gate, an AND-gate and an OR-gate can be constructed using just NAND-gates [10]

Solution:



4. What are a half-adder and a full-adder? Show how a full-adder can be built using 2 half-adders [7]

Solution:
A half-adder takes x and y as inputs and computes the Boolean sum of x and y, where the Boolean sum of a collection of inputs is 1 if, and only if, an odd number of the inputs are 1, and also the resulting carry bit, which is 1 if, and only if, both x and y are 1 [2]. A full-adder takes x, y, and z as inputs and computes the Boolean sum of x, y, and z, resulting in the sum-bit and the carry-bit [2].



Solution:

The von Neumann bottleneck is a limitation of the rate of data transfer between the CPU and memory (data and instructions have to be fetched in sequential order and idle time is wasted whilst waiting for data items or instructions to be fetched from memory) [2]. The von Neumann bottleneck gave rise to the use of caches [1].

5. How does the Harvard architecture differ from the von Neumann architecture? [2]

Solution:

The Harvard architecture has memory that is partitioned into data memory and instruction memory with dedicated buses for each of them [2].

6. Name two types of bus within a CPU and explain the general purpose of each. What is the width of a bus? Explain how the width of a bus imposes memory or data limitations within a CPU. [6]

Solution:

Buses include a **data bus** [$\frac{1}{2}$], an **address bus** [$\frac{1}{2}$] and a **control bus** [$\frac{1}{2}$]. A data bus carries the contents of memory locations between the processor and main memory [$\frac{1}{2}$]. The address bus holds addresses of locations in main memory [$\frac{1}{2}$]. The control bus is used to transfer information between the CPU and various other devices within the processor [$\frac{1}{2}$]. The width of a bus is the number of parallel wires in a bus [1]. The width of the data bus determines the word-size of the computer [1]. The width of an address bus determines the size of addressable memory [1].

7. What is the difference between static RAM and dynamic RAM? [3]

Solution:

Dynamic RAM (DRAM) is where a bit of data is stored using a transistor/capacitor combination [1]. Static RAM (SRAM) is where a bit of data is stored by a flip-flop, which incorporates 4-6 transistors [1]. Static RAM is stable and fast but takes up more memory [$\frac{1}{2}$] whereas dynamic RAM is cheap, slow and needs to be refreshed because of ‘leaky’ capacitors [$\frac{1}{2}$].

8. What is the purpose of cache memory in a CPU? [1]

Solution:

Caches are expensive memory that are used to store rapidly accessed items [1].

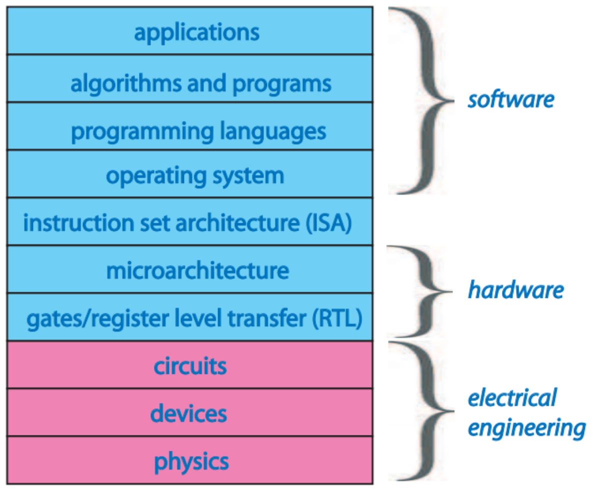
9. Explain carefully the four phases of the fetch-decode-fetch-execute processor cycle (be sure to explain the purpose of any CPU components you happen to mention). [5]

Solution:

The '**instruction fetch**' phase involves the supply of the instruction address (via the address bus) and the return from memory (via the data bus) of the instruction [1]. The '**instruction decode**' phase involves interpreting the stored instruction within the CPU [1]. The '**operand fetch**' phase involves the supply of the address of any required data (via the address bus) and the return from memory (via the data bus) of this data [1]. The '**execute instruction**' phase involves the CPU performing the necessary actions [1] (this phase is sometimes split into two with the 'execute instruction' phase followed by a 'write-back' phase where data is written back to memory, if needs be [1]).

10. Show different layers of abstraction in a modern computer system. [4]

Solution:



11. Briefly (and, necessarily, without being too precise) explain the difference between an algorithm and a program. Suppose we have an algorithm and wish to implement it in Python. How many different implementations are there of this algorithm? [5]

Solution:

An algorithm is a sequence of precise instructions that can be applied to specific data items. A program is the implementation of the algorithm in a form that can be executed by a computer, or at least compiled to a form that can be executed by a computer. There are many different implementations of an algorithm as a program.

12. Give 4 different programming paradigms and briefly explain the underlying principle for each paradigm. [6]

Solution:

Imperative: Statements change a programs state (closest to ”memory abstraction” of CPU)
Declarative: Programs say what to do, rather than how to do it
Data-Oriented: Programs work with data through manipulating and searching relations (tables). Tables have things in common that can be linked together to get more information
Scripting: Designed to automate frequently used tasks that involve calling or passing commands to external programs. These languages have lots of libraries to make things easier to do

13. Give 4 different drivers of the evolution of programming languages and briefly explain each of these driving motivations. [4]

Solution:

Productivity: Speed up software development process, reduce times and costs, support fast user interface development. This lead to the development of rapid application development (RAD) languages and scripting languages
Reliability: To try and reduce the number of errors caused during the execution of the program, this includes things such as type checking and exception handling.
Security: Scripting languages used for webpages can, when run on machines, enable malicious programmers to breach your security.
Execution: Different programming languages will work better for multi-threading and multi-core processing, allowing parallel computing

14. Give 3 general properties any programming language should have. [3]

Solution:

- Be easy to use, with its programs easy to read, write and understand
- Support abstraction so that adding new features and concepts should be possible
- Support testing, debugging and program verification
- Be inexpensive to use, in terms of execution time, memory usage and maintenance costs

15. In order to execute a high-level program we must first convert it into machine code so that the CPU can 'understand' it. Briefly explain the difference between compilation and interpretation. How are Java programs executed? [6]

Solution:

Compilation is where the entire program is converted into a form that can be executed by the processor before the program is run, whilst interpretation is where the program is converted into an executable form while it is run.
Java, however, is compiled into bytecode before run time, this bytecode is then interpreted at runtime

16. Define carefully what a regular expression is and explain how a regular expression is used to denote a set of strings [14]

Solution:
A regular expression over some alphabet Σ is defined as follows:

- Any $a \in \Sigma$ is a regular expression
- $\emptyset$ and ϵ are special regular expressions
- If ω and ω' are regular expressions, then so are
  - $(\omega\omega')$
  - $(\omega|\omega')$
  - (ω^*)

Every regular expression denotes a set of strings over Σ

- $a, \emptyset$ and ϵ denote the sets of strings $\{a\},$ and $\{\epsilon\}$ respectively
- If ω and ω' denote the sets of strings R and S then:
  - $(\omega\omega')$ denotes $\{xy : x \in R, y \in S\}$
  - $(\omega|\omega')$ denotes $R \cup S$
  - (ω^*) denotes $\{x_1x_2 \ldots x_n : n \geq 0, x_1, x_2, \ldots, x_n \in R\}$

17. Define carefully a finite state machine and explain how one is used to accept a set of strings [8]

Solution:
A finite state machine is defined as:

$$M = (\Sigma, Q, \delta : Q \times \Sigma \to Q, q_0 \in Q, F)$$

where

- Σ is some finite alphabet
- Q is some finite set of states with initial state q_0 and set of final states $F \subseteq Q$
- $\delta : Q \times \Sigma \to Q$ is the transition function

On input any string $a_1a_2...a_n$ over Σ (where $n \geq 0$, and so we may input the empty string if we wish), M yields a sequence of states $q_0, q_1, q_2, ..., q_n$ via:

$$q_1 = \delta\left(q_0, a_1\right), q_2 = \delta\left(q_1, a_2\right), q_3 = \delta\left(q_2, a_3\right), \ldots, q_n = \delta\left(q_{n-1}, a_n\right)$$

The input string is accepted by our FSM M if the resulting sequence of states $q_0, q_1, q_2, ..., q_n$ ends in a final state; that is, is such that $q_n \in F$. This M accepts a set of strings over Σ

18. What is the Instruction Set Architecture (ISA) and what is the primary component of the ISA? [3]

Solution:
The ISA is the interface between the hardware and software. The primary component of the ISA is its assembly language

19. Give three instructions of the MIPS (Microprocessor without Interlocked Pipeline Stages) ISA and explain carefully what they do. [6]

Solution:

- `lw $s1, ($s2)` - Register \$s1 takes the value of the memory location held in register \$s2
- `addi $s1, $s2, x` - register \$s1 takes the value of register \$s2 plus the number x
- `beq $s1, $s2, ,` - if the value of register \$s1 is equal to the value of the register \$s2 then go to memory location m

20. Detail four different functions of the operating system. [4]

Solution:

- Virtualises a machine
  - Provides abstractions that present clean interfaces to make the computer easier to use.
- Starts and stops programs
  - Ensures that when a program is stopped, it's memory is freed up
- Manages memory
  - Ensures programs can still run, even if the memory they request is in use.
- Handles input/output
  - Handles interrupts from inputs and outputs so that they don't have to wait for the processor to finish what it is doing.

21. What is process within an operating system and what are the two essential elements of a process? Is it the case that an operating system for a CPU with one processor can only have one process? [5]

Solution:

- Process - Each executing program
- A process consists of
  - **Thread** - A sequence of instructions in the context of a sequential execution
  - **Address space** - Consists of some memory locations that the corresponding process can read from and write to
- Multiple processes can be run on one processor, providing there is no mutual exclusion

22. Explain the principle of mutual exclusion and give an illustration to show that ensuring mutual exclusion is important. [6]

Solution:

- **Mutual Exclusion -** Ensuring that two threads are not in the critical selection at the same time
- **Critical Selection -** Exclusive access to some shared resource such as memory location
- Two threads have access to the same counter, and each want to increment the counter, depending on how the read, increment, store is handled, the result will be different

23. Describe the life-cycle of processes within the operating system (be sure to explain each component of the life-cycle). [10]

Solution:

- **new**: the process being created
- **ready:** the process not executing on the CPU but is ready to execute
- **running:** the process is executing on the CPU
- **blocked:** the process is waiting for an event (and so not executable)
- **exit:** the process has finished

There are the following state transitions

- **admit:** the control overheads as regards a process have been set up and the process is moved to the run queue
- **dispatch:** the scheduler allocates the CPU to an executable process
- **timeout/yield:** the executing process is forced to/volunteers to release access to cpu
- **event-wait:** a process is waiting for an input/output event, for example, and gives up access to the cpu
- **event:** an event occurs and wakes up a process
- **release:** a process terminates and releases access to the CPU and other resources

24. What is a process control block (PCB) and what is one used for by the operating system? Name 3 items in a PCB. [6]

Solution:

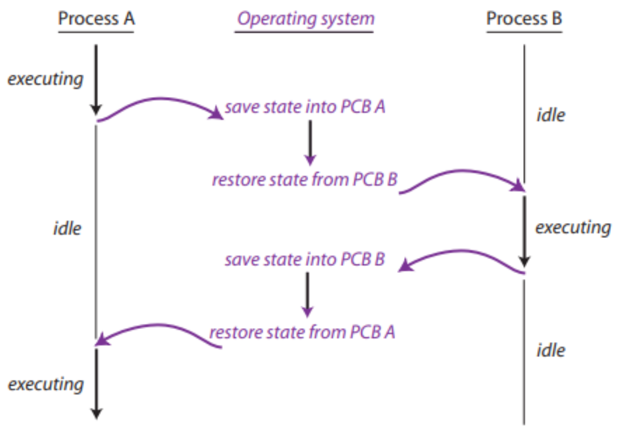
- **Process control block(PCB) -** a data structure that the kernel uses in order to manage a process

- It is used so that the current situation of any process is fully understood, this means that the CPU can better handle processes
- Components
  - the process's unique ID
  - The current state of the process
  - CPU scheduling information such as the process priority

25. Explain the principle of context switching with regard to process control blocks. [5]

Solution:

Context switching - Where the operating system pauses one process and resumes another process



26. What is the real-world map-colouring problem? Explain how we might abstract this problem as a graph colouring problem. What is a planar graph? Explain how planar graphs and maps are related. Is every graph a planar graph? [7]

Solution:

- Given a plane map of regions, each contained within a continuous border, can you colour the regions of the map with 3 colours so that if any two regions touch they must be coloured differently?
- This can be abstracted into a graph, as all the important information is just if two regions touch
- **Planar Graph -** A graph that can be drawn in a plane without any graph edges crossing
- Every map can be turned into a planar graph
- Not every graph is a planar graph, some will have edges that cross

27. Give a precise definition of a decision problem. [2]

Solution:

A decision problem D consists of:

- a set of instances I
- A subset $Y \subseteq I$ of yes instances

28. What is a graph? What is a proper colouring of a graph and an optimal colouring of a graph? Describe two different algorithms (in English) that enable you to colour graphs. Explain whether or not your algorithms yield an optimal colouring (you may use graphs to illustrate your answer if you wish). Show how your algorithms proceed when you wish to decide whether the graph in Fig. 1 can be coloured using 4 colours [20]

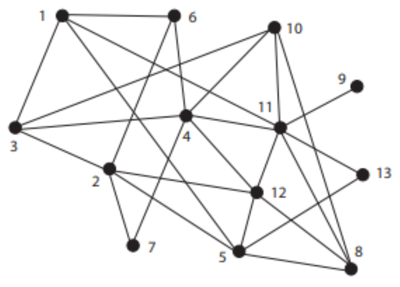


Fig. 1: a graph.

Solution:

A graph, G=(V,E), has a finite set V of vertices and a set E of pairs of distinct vertices where no edge is repeated and (in an undirected graph) (u,v)=(v,u). A proper colouring is a colouring in which any two adjacent vertices are coloured differently, the optimal colouring is a proper colouring that uses the smallest number of colours.

Algorithm one: work through the nodes in increasing order and colour using the smallest available colour. This will always yield a proper colouring, but may not yield an optimal colouring, however if it does yield an optimal colouring, that colouring is correct.

Algorithm two: chose a node that can be coloured using the lowest colour possible, if there are multiple nodes that can be coloured with that colour, then chose the node with the lowest vertex order. Once no other nodes can be coloured using that colour, move onto the next colour.

Both algorithms produce an identical 3-colouring for the graph.

1:1, 2:1,3:2,4:1,5:2,6:2,7:2,8:1,9:1,10:3,11:2,12:3,13:1

29. Give a (not necessarily precise) definition of what it means for an algorithm to be greedy. [2]

Solution:

An algorithm that works through a list "greedily" choosing the best thing to do

30. Give a precise definition of a search problem and of a solution of a search problem. [5]

Solution:
A search problem S consists of:

- A set of instances I
- A set of solutions J
- A binary search relation $R \subseteq I \times J$

In order to solve a search problem, for any instance $x \in I$ we need to:

- find a solution $y \in J$ such that $(x, y) \in R$ if there is one, or
- Answer no if there is no solution y for which $(x, y) \in R$

31. Give a precise definition of an optimisation problem and of a solution of an instance of an optimisation problem. [5]

Solution:
An optimisation problem O is defined as follows:

- It consists of a set of instances I
- for every instance $x \in I$ there is a set of feasible solutions f(x)
- For every instance $x \in I$ and for every feasible solution $y \in f(x)$ there is a value $v(x, y) \in \mathbb{N} = \{0, 1, 2, ...\}$ giving the measure of the feasible solution y for the instance x
- There is a goal which is either min or max

To solve an optimisation problem, given $x \in I$, we need to find a feasible solution of maximum or minimum measure. It may be the case that:

- There are no feasible solutions for an instance
- For a maximisation problem, there are feasible solutions of increasingly large measure

32. What is an independent set in a graph? What is the Independent Set (optimisation) problem? [3]

Solution:

- An independent set is a set of vertices in a graph, no two of which are adjacent
- The independent set problem:
  - An independent set U in a graph $G = (V, E)$ is a subset $I \subseteq V$ of verticies so that no vertex in U is joined by an edge to any other vertex of U
  - A maximum independent set in G is an independent set of the greatest size possible

33. Detail, using pseudo-code, two different versions of Euclid's algorithm: a recursive one; and an iterative one [6]

Solution:

Recursive:

```
IF n==0:
output m
ELSE:
set m=n and n=m mod n
Euclid(m,n)
```

Iterative:

```
WHILE n ≠ 0:
m=m-n
IF n>m
swap m and n
output m
```

34. First, give a natural language description of the algorithm Bubble-sort, and, next, give pseudo-code for your algorithm. Describe how your algorithm works on the input list of numbers 4,3,6,6,2,1 [10]

Solution:

- The algorithm Bubble-sort repeatedly 'passes' through the input list of numbers, comparing and swapping adjacent numbers in the list.
- In a pass through the input list, consecutive pairs of numbers are compared in turn, and these numbers are swapped (in their locations) if the first number is greater than the second.
- If a swap has been made in a pass through the list then another pass is undertaken, otherwise the algorithm halts.

```
change = true
WHILE change == true:
change = false
i = 0
WHILE i < n - 1:
IF A[i] > A[i + 1]:
swap A[i] and A[i + 1]
change = true
i = i + 1
output A
```

35. Here is the pseudo-code for the algorithm selection-sort

```
pass=0
WHILE pass < n-1
```

```
x=A[pass]
i=pass+1
WHILE $i \leqslant n-1$
IF A[i]<x
swap x and A[i]
i=i+1
A[pass]=x
pass=pass+1
output A
```

(a) Explain how you might amend the algorithm Selection-sort so that you no longer need the variable x [4]

Solution:

```
pass=0
WHILE pass < n-1
i=pass+1
WHILE $i \leqslant n-1$
IF A[i]<A[pass]
swap A[i] and A[pass]
i=i+1
pass=pass+1
output A
```

(b) Explain how you might amend the algorithm Selection-sort so that you lessen the number of data swaps made [4]

Solution:

```
pass = 0
WHILE pass < n-1
x=pass
i=pass+1
WHILE i⩽ n-1:
IF A[i]<A[x]:
x=i
i=i+1
swap A[pass] and A[x]
pass=pass+1
output A
```

Total for Question 35: 8

36. First, give a natural language description of the algorithm Merge-Sort, and, next,give pseudo-code for your algorithm (you need not provide pseudo-code to describe how two ordered lists are merged into an ordered list). Describe how your algorithm works on the input list of numbers 4,3,6,6,2,1 [14]

Solution:

- Chop the input list into roughly two 'halves' so that both 'halves' either have the same length or their lengths differ by 1

- recursively sort each half' and
- merge the two sorted lists together

```
Merge-sort(l,r)
if l<r
m=$\lceil (r-l+1)/2 \rceil - 1$
Merge-sort(l,m)
Merge-sort(m+1,r)
Merge(l,m,r)
Mergesort(0,n-1)
output A

Merge(l, m, r)
leftptr = l and rightptr = m + 1
counter = l
WHILE leftptr $\leqslant$ m and rightptr $\leqslant$ r:
IF A[leftptr] $\leqslant$ A[rightptr]:
B[counter] = A[leftptr]
leftptr = leftptr + 1
ELSE:
B[counter] = A[rightptr]
rightptr = rightptr + 1
counter = counter + 1
IF leftptr $\leqslant$ m:
B[counter..r] = A[leftptr..m]
ELSE:
B[counter..r] = A[rightptr..r]
A[l..r] := B[l..r]
```

On the input list given, it will be split into two sublists (4,3,6) and (6,2,1)
These will then be recursively split until the digits are individual.
Then the digits are merged back together, sorting them through the comparisons between the two sublists.
The merging goes all the way up the recursion stack until the input list is sorted

37. Give two different ways in which a software bug can arise and give two different effects of a software bug [2]

Solution:

- Programming Error
- Secondary Program Error like a compiler

38. Give the different aspects of the software engineering software design process known as the Waterfall model and very briefly explain what they are [5]

Solution:

- Requirements Phase - Produce a specification of what a program is supposed to do
- Design Phase - Compose a plan for the solution involving algorithms, data structures etc
- Implementation phase - Write the code
- Verification Phase - Test and Debug
- Maintenance - Continual modification/updating

39. When it comes to establishing program correctness, briefly compare and contrast the different approaches taken in software testing and in formal methods [2]

Solution:
Formal Methods - Mathematically based techniques for the formal specification and verification of software and hardware systems
Software Testing - Evaluating an attribute or capability of a program or system and determine that it meets its required results

40. What is the difference between an algorithm being totally correct and being partially correct? [2]

Solution:
Totally Correct - Correct with respect to some specification and terminates on every input
Partially Correct - Only correct with respect to the specification

41. Outline how the mathematical principle of induction and the algorithmic construction of recursion are often related in the realm of program correctness [4]

Solution:
Induction has 3 steps:

- Formulate a property P(n)
- Decide upon a base case
- The inductive step of proving for k+1

With recursion we have a base case and a recursive call
It is often easier to inductively prove a recursive algorithm as the base case and recursive call mirrors the inductive step

42. Consider the following algorithm where the input is a positive integer supplied by the variable n: [7]

```
algorithm: f(n):
(x,y)=(1,n)
```

```
while y!= 0:
x=x*(n-y+1)
y=y-1
output x
```

What does this algorithm do? Prove that it is totally correct

Solution:
This algorithm iteratively computes factorials.
By observation, the variable y decreases by 1 at the end of every iteration of the while loop. So there will be exactly n iterations of the while loop; consequently, our algorithm always terminates.

For correctness. We can also see that at the end of each iteration of the while loop, the value of n-y+1 increases by 1. Prior to the first execution of the while-loop, the value of n-y+1 is 1. Thus, by observation, during the ith iteration of the while loop, x is multiplied by i, having been initialized at 1. Consequently, after the nth iteration of the while-loop, x has the value n!.

43. Give two different resources used by an algorithm that we might measure? [2]

Solution:

- Time
- Memory

44. Why do we usually measure the time taken by an algorithm rather than the time taken by an implementation of an algorithm? [2]

Solution:

- Not affected by language
- Not affected by hardware

45. In pseudo-code we assume that any 'basic' instruction takes c units of time to execute. What is this constant c supposed to reflect? [2]

Solution:
The time to do one operation on a given processor

46. What is the worst-case time complexity of an algorithm? Why is it expressed as a function on the natural numbers? [4]

Solution:

- Can't give a fixed number as a bigger input will always take more time
- The greatest amount of time that an algorithm will take for an input
- How the function reacts as the input increases

47. Define precisely what we mean when we say that two functions $f(n) : \mathbb{N} \to \mathbb{N}$ and $g(n) : \mathbb{N} \to \mathbb{N}$ are such that $f = \mathcal{O}(g)$ [2]

Solution:
There exists $n_0 \in \mathbb{N}$ and $k \in Q$ such that $f(n) \leqslant k \cdot g(n)$ wherever $n \geqslant n_0$

Define precisely what we mean when we say that two functions $f(n) : \mathbb{N} \to \mathbb{N}$ and $g(n) : \mathbb{N} \to \mathbb{N}$ are such that $f = \Omega(g)$ What does it mean in practice if we say that any algorithm solving the sorting problem has time complexity $\Omega(n \log n)$?

Solution:
If there exists $n_0 \in \mathbb{N}$ and $k \in Q$ such that $f(n) \geqslant k \cdot g(n)$ wherever $n \geqslant n_0$. The algorithm takes at least $n \log n$ steps

48. The algorithm Bubble-sort is as follows: [5]

```
change = true
WHILE change == true:
change = false
i = 0
WHILE i < n - 1:
IF A[i] > A[i + 1]:
swap the numbers in A[i] and A[i + 1]
change = true
i = i + 1
output A
```

What is the time complexity of bubble-sort? (you should explain your answer)

Solution:

- The inner while loop will loop n-1 times
- The outer loop will loop n-1 times before no swaps happen
- $\mathcal{O}((n-1)n + n) = \mathcal{O}(n^2)$
- Best case $\Omega(n)$

49. What do we mean when we say that a decision problem is tractable? What is the complexity class **P** [4]

Solution:
Tractable - It can be solved by an algorithm of time complexity $\mathcal{O}(n^k)$; that is, polynomial time
P - The complexity class of efficiently solvable problems

50. Give two objections to the definition of a tractable problem as a problem solvable in $\mathcal{O}(n^k)$ time [4]

Solution:

- What about the hidden constant
- What if the k is massive

51. What do we mean when we say that a decision problem is efficiently checkable? [3]

Solution:
Given a potential witness that some instance is a yes-instance, we can check in polynomial time whether the witness is indeed a witness

52. What is the complexity class **NP** [2]

Solution:
The complexity class of decision problems that are efficiently checkable

53. Define precisely the notion of a polynomial-time transformation [4]

Solution:
A polynomial time transformation from X to Y is an algorithm α that:

- Takes an instance I of X as input and provides an instance $\alpha(I)$ of Y as output
- Is such that an instance I of X is a yes-instance iff the instance $\alpha(I)$ of Y is a yes-instance

54. Define what it means for a problem to be NP-complete. Prove that if Y is an NP-complete problem then **P=NP** if, and only if, $Y \in P$ [6]

Solution:
Suppose there is a problem $Y \in NP$ such that for any problem $X \in NP$, we have $X \rightarrow_{poly} Y$
Such problems are NP complete.
From the previous theorem

- If $Y \in NP$ then every problem $X \in NP$ is also in P; that is P=NP
- Conversely, suppose that P=NP, so, as $Y \in NP$, we must have that $Y \in P$

55. What is Cook's theorem? [2]

Solution:
P=NP iff $SAT \in P$