Finite-state Automata over infinite words

A Finite-sate Automaton (FA) consists of

- A finite input alphabet Σ
- A finite set of states Q
- A transition relation $\Delta \subseteq Q \times \Sigma \times Q$
- A start state $q_0 \in Q$
- A set of accepting states $F \subseteq Q$

Then

1. If the input is finite, i.e. in Σ^*, we have a non-deterministic FA (with no ϵ transitions)
2. If the input is infinite, i.e. in Σ^ω, we have A Buchi Automaton
3. If Δ is a partial function $Q \times \Sigma \rightarrow Q$, we have a Deterministic Automaton

1 Acceptance conditions

NFA or DFA accepts a finite word $w_1 w_2 ... w_n \in \Sigma^*$ if there is a sequence of states $r_0, r_1, r_2, ..., r_n$ satisfying the following conditions

1. $r_0 = q_0$
2. $(r_i, w_{i+1}, r_{i+1}) \in \Delta$ for every $i, - \leqslant i \leqslant n-1$
3. $r_n \in F$

Buchi Automaton accepts $w_1 w_2 ... \in \Sigma^\omega$ if there is a sequence of states $r_0, r_1, r_2, ... \in Q^\omega$ satisfying the following conditions

1. $r_0 = q_0$
2. $(r_i, w_{i+1}, r_{i+1}) \in \Delta$ for every $i, i > 0$
3. There are infinitely many r_i's in F

2 Regular Languages

Regular language - Some DFA/NFA recognises it

Theorem - A language is regular iff it could be described by a regular expression

A regular language/expression is built upon the basic ones, which are any $s \in \Sigma$, the regular symbol ϵ or the empty language $\emptyset$, using the following operations (where A and B are regular)

1. $A \cup B$, which is the set-theoretic union
2. $A \circ B$ (or simply AB) which is $\{ab | a \in A, b \in B\}$
3. A*, which is $\{a_1 ... a_n | a_i \in A, n \geqslant 0\}$

3 ω-regular languages

Definition: ω regular language

An ω-regular language/expression is built upon regular languages, using the following expressions

1. $A \cup B$, where both A and B are ω-regular
2. AB, where A is regular and B is ω-regular
3. A^ω, which is $\{a_1 ... | a_i \in A\}$, i.e. an infinite sequence of words from A, where A is regular and doesn't contain the empty word

Theorem - An ω-language is ω-regular iff some non-deterministic Buchi Automaton recognises it

4 Limits of Regular Languages

Definition: Limit of a regular language

Let A be a regular language. The limit of A limA is the language $\{a \in \Sigma^{\omega}|$a has infinitely many prefixes in A$\}$

Theorem: An ω-language is a limit of a regular language iff some deterministic Buchi Automaton recognises it