Heuristic Search

1 General Heuristic Search

- We've seen that uniformed search can be extremely expensive

[Heuristic Search (Informed Search)] Additional information pertinent to the problem in hand is supplied (or constructed) and used We shall continue to base our strategies around the search tree [Best-First Search]

- Expand a fringe node z with a minimal value according to a given evaluation function $f(z)$
  - f is defined on the nodes of the search tree
- Put children on the fringe and repeat

Choice of evaluation function determined type of best-first search, e.g.

- f(z)="depth of a node" yields a breadth first search
- f(z)="1/(depth of a node)" yields a depth first search

In tandem with the choice of termination condition - up until now it has been "stop when a goal node appears on the fringe"

2 Best-First Search

```
newid = 1
register node:
S[1]=initial_state;
P[1]=-;
A[1]=-;
PC[1]=0;
D[1]=0;

initialise priority queue (of tuples):
fringe F = [(newid, S[1], P[1], A[1], PC[1], D[1])]
if node 1 is a goal-node then return 1
else
while F ≠ ∅ do:
  pop (id, state, parent_id, action, path_cost, depth) from F
  for each state child and action α for which we have that child ∈ ϕ (state, α) do:
    newid=newid+1
    register node:
    S[newid]=child;
    P[newid]=id;
    A[newid]=α;
    PC[newid]=PC[id]+ρ(state,α,child);
    D[newid]=D[id]_1;
    if node newid  is a goal node then return newid
    else add (newid, S[newid], P[newid], A[newid], PC[newid], D[newid]) to F
return 0
```

3 Heuristic Functions

- The evaluation function f is usually dependent upon an additional heuristic function, denoted h, where
  - $h(z) \geqslant 0$ can be thought of as the estimated cost of the cheapest path in the search tree from node z to any goal-node (recall that all step costs are non-negative)
- Every heuristic function must satisfy two constraints

- if z is a goal-node then necessarily h(z)=0 (if you're at the goal node, the cost of getting there is 0)
- h(z) only depends upon the state associated with the node z so h(z) is considered as the estimated cost of the cheapest path in the search space from the state associated with z to any goal state

Important: Heuristics

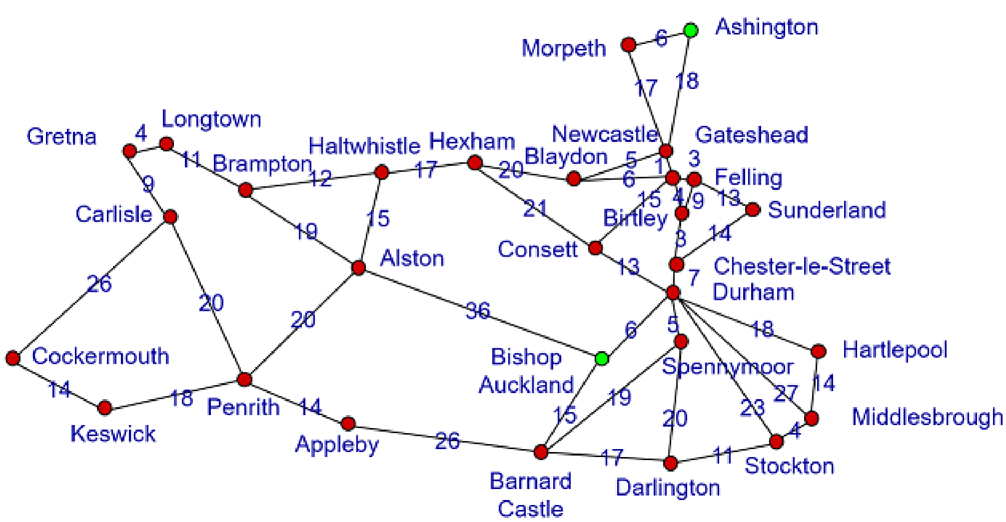
A heuristic is given as additional information (unless we build it ourselves)
The usefulness of a heuristic is almost always dependent upon how good an estimation it is

4 Greedy best-first search

- The evaluation function equals the heuristic function; that is, f(z)=h(z)
- Termination when a goal-node appears on the fringe

Consider the following problem:

- Initial state: Ashington
- Goal state
- One action and step costs-shown
- Heuristic h is a straight line distance to Bishop Aukland

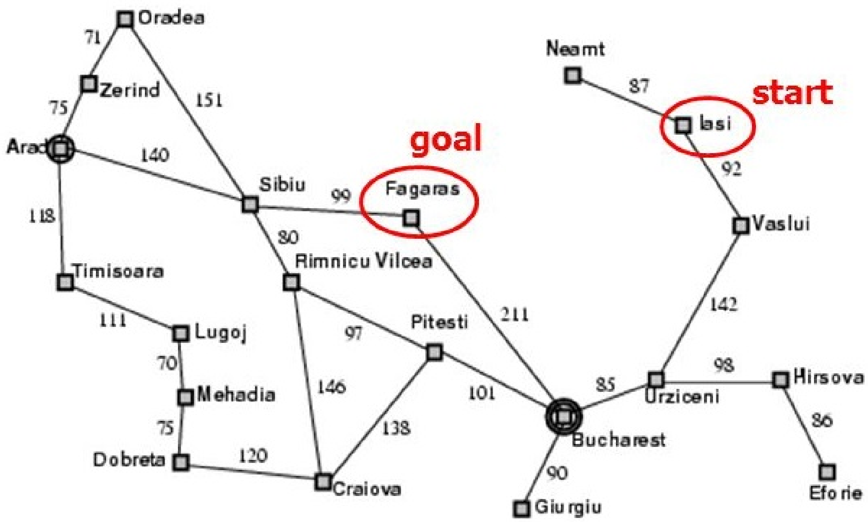


The greedy expansion of this doesn't lead to an optimal path

4.1 Performance of greedy best-first search

- Greedy best-first search need not return an optimal solution
  - In fact, greedy best first search need not terminate, i.e. it is incomplete
  - The diagram below does not complete as Neamt is closest in distance to Fagaras
  - It keeps looping between Iasi and Neamt
- If d is the depth of the shallowest goal-node in the search tree and b is the branching factor then
  - In the worst case, the time and space complexity is $\Omega(b^d)$, for the heuristic function could be useless
  - e.g., h(z)=0 for every z and ties could be broken so that the greedy best-first search proceeds exactly as does BFS

- The improvement one gains with greedy best first search is strongly dependent upon the quality of the heuristic function



5 Building a heuristic function

- We can sometimes manufacture our own heuristic function if additional information is not supplied
- Consider TSP formulated so that
  - States are partial tours $(c_1, c_2, ..., c_i)$ i.e., paths where $0 \leqslant i \leqslant n$ (where n is the number of cities)
  - Initial state is the empty partial tour ()
  - Goal states are the partial tours containing every city, e.g. $(c_1, c_2, ..., c_n)$
  - Transitions are obtained by "visiting an unvisited city" (there is one action)
    * $(c_1, c_2, ..., c_i) \rightarrow (c_1, c_2, ..., c_i, c_{i+1})$ where $c_{i+1} \neq c_j$ for all $j = 1, 2, .., i$
  - Step-cost $\sigma((c_1, c_2, .., c_i), (c_1, c_2, ..., c_i, c_{i+1})) = \delta(c_i, c_{i+1})$ unless
    * $i = n - 1$ when $= \delta(c_{n-1}, c_n) + \delta(c_n, c_1)$ (loop back to complete tour)
- Define the heuristic function $h(z)$ as follows
  - If the state associated with tree-node z is $(c_1, c_2, ..., c_i)$, where $i \neq n$, then

$$h(z) = \min\{\delta(c_i, c') : \text{c' is an unvisited city}\}$$

  - This is the shortest extension you can make to get to an unvisited city from any fringe node
  - if the state associated with tree-node z is $(c_1, c_2, ..., c_n)$ then $h(z) = 0$
- This heuristic function satisfies the two criteria required for any heuristic function
- Our greedy best-first search differs from the "visit-nearest-neighbour" algorithm
  - visit-nearest-neighbour goes down, extending from the last extended node

6 A* Search

- The most widely known form of best-first search is A* search
  - Evaluates nodes by combining $h(z)$
    * The heuristic cost to get from the node z to a goal node and g(z) (g is the sum of the step costs to get to z from the root)

        - The path-cost (available from the search tree data structure) to reach node z from the root
    - Evaluation function $f(z) = g(z) + h(z)$

- Another way of looking at $f(z)$
    - It is the estimated cost of the optimal solution through node z
- An A* Search terminates when
    - A goal node z is on the fringe and f(z) is minimal amongst fringe nodes
        - A* search outputs the path of action from the root to z (of cost f(z)=g(z)) (note that h(z)=0 for a goal node)
    - or, when there are no nodes to expand. So no goal-node exists, and this is signalled in output

Important: A* Search

A goal-node is never expanded under A* Search (at that point the heuristic would be 0)
A* Search can be both complete and optimal