Finite Automata

1 Nondeterministic Finite Automata

A nondeterministic finite automaton (NFA) consists of

- A finite set S of states
- The input alphabet Σ (the set of input symbols)
- A start state $s_0 \in S$ (or initial state)
- A set F of final states (or accepting states)

Often we represent an NFA by a transition graph

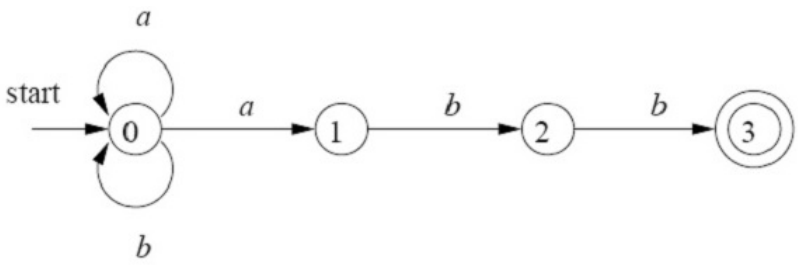
- Nodes are possible states
- Edges are directed and labelled
- The same symbol can label edges from a state to many different other states

1.1 Representation

$$\Sigma = \{a, b\}$$

$$s_0 = 0$$

$$F = \{3\}$$



Alternative representation is a transition table

- Rows → states
- Columns → symbols in $\Sigma \cup \{\epsilon\}$
- Entries → Transitions between states

STATE	a	b	ϵ
0	$\{0, 1\}$	$\{0\}$	$\emptyset$
1	$\emptyset$	$\{2\}$	$\emptyset$
2	$\emptyset$	$\{3\}$	$\emptyset$
3	$\emptyset$	$\emptyset$	$\emptyset$

Advantage of transition table: more visible transitions
Disadvantage of transition table: needs more space than the transition graph

1.2 Acceptance of NFA

An NFA accepts an input string x if there exists a path that:

- Starts at the start state s_0
- Ends at one of the accepting states in F
- Concatenation of the symbols on its edges gives exactly x

A language accepted (or defined) by an NFA:

- The set of strings that this NFA accepts

2 Deterministic Finite Automata

A deterministic finite automaton (DFA) is a special case of a NFA, where:

- No edge is labelled by the empty string ϵ
- For each state s and each input symbol a, there is exactly one edge out of s labelled with a

A direct algorithm to decide whether a given string x is accepted by a DFA:

- Start at the start state s_0
- Iteratively follow the edges labelled by the characters of x
- Check whether you reach a final state when x ends:
  - If yes, then the DFA accepts x
  - Otherwise not

3 NFA vs DFA

Theorem 1 *NFAs accept exactly the regular languages (i.e. the regular expressions)*

Therefore, simulation of an NFA can be used in the lexical analyser to recognise strings, identifiers etc

However the simulation of NFAs is not straightforward

- Many alternative outgoing edges from a state
- Transitions labelled with ϵ are possible

Theorem 2 *NFAs accept exactly the same languages as DFAs*

i.e. for every NFA, we can construct an equivalent DFA