Counting

1 Question 1

How many licence plates can be made using either three letters followed by three digits or four letters followed by two digits?

$$26^3 \times 10^3 + 26^4 \times 10^2 = 62373600$$

2 Question 2

A palindrome is a string whose reversal is identical to the string.

2.1 Part 1

How many bit strings of length 12 are palindromes?

$$2^6 = 64$$

How many bit strings of length 12 are palindromes?

$$2^7 = 128$$

3 Question 3

Let S be the set of numbers 1,2,3,4 and 5

3.1 Part 1

List all the 2 permutations of S

$$[1,2], [1,3], [1,4], [1,5]$$
$$[2,1], [2,3], [2,4], [2,5]$$
$$[3,1], [3,2], [3,4], [3,5]$$
$$[4,1], [4,2], [4,3], [4,5]$$
$$[5,1], [5,2], [5,3], [5,4]$$

List all the 3 combinations of S

$$[1,2,3], [1,2,4], [1,2,5]$$
$$[1,3,4], [1,3,5]$$
$$[1,4,5]$$
$$[2,3,4], [2,3,5]$$
$$[3,4,5]$$

4 Question 4

Find the value of each of these quantities

- $P(6,3) = 120$
- $P(8,5) = 6720$
- $P(8,8) = 40320$

5 Question 5

Find the value of each of these quantities

- $\binom{5}{3} = 10$
- $\binom{8}{4} = 70$
- $\binom{8}{8} = 1$

6 Question 6

Expand the binomials:

6.1 Part 1

$(1+z)^4$

$$1+4z+6z^2+4z^3=z^4$$

6.2 Part 2

$(x+y)^5$

$$x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$

7 Question 7

Find the number of 5 permutations of a set with 8 elements

$\frac{8!}{3!}=6720$

8 Question 8

- BCD $5!=120$
- CFGA $4!=24$
- BA and GF $5!=120$
- ABC and DE $4!=24$
- ABC and CDE $3!=6$
- CBA and BED 0

9 Question 9

How many bit strings of length 10 contain

9.1 Part 1

Exactly four 1s

$$\frac{10!}{4!6!}=420$$

9.2 Part 2

At most four 1s

$$\binom{10}{1}+\binom{10}{2}+\binom{10}{3}+\binom{10}{4}$$

9.3 Part 3

At least 4 1s

$$\binom{10}{4}+\binom{10}{5}+\binom{10}{6}+\binom{10}{7}+\binom{10}{8}+\binom{10}{9}+\binom{10}{10}$$

9.4 Part 4

An equal number of 0s and 1s

$$\binom{10}{5}=252$$

10 Question 10

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and the women alternate?

$$2(n!)^2$$

11 Question 11

Find the number of anagrams of the word ABRACADABRA

$$83160$$

12 Question 12

Find the number of solutions (without listing them) in non-negative integers

To the equation $x_1 + x_2 + x_3 = 17$

$$78$$

To the inequality $x_1 + x_2 + x_3 \leqslant 11$ This is solved by having an imaginary x_4, which covers the remainder of the inequality so that it can be imagined to be equal to 11, rather than in an inequality.