Graph Algorithms - Matchings in Graphs

1 Problem Definition

Let $G = (V, E)$ be an undirected graph

A set $M \subseteq E$ is a matching in G if no two edges in M have an end-vertex in common

A matching M is maximum if there is no matching of G with more edges than M

The matching number ν_G of G is the size of a maximum matching in G

Definition: Matching

Instance: a graph G
Task: determine ν_G

2 Alternating Paths and Cycles

Let $G = (V, E)$ be a graph and M be a matching inG

A path P is alternating with respect to M iff among every two consecutive edges of P, exactly one edge belongs to M

A cycle C is alternating with respect to M iff among every two consecutive edges of C, exactly one edge belongs to M.

A vertex u is matched by a matching M is it is the end-vertex of an edge of M; otherwise u is unmatched by M

For two sets A and B, the symmetric difference is the set

$$A \oslash B = (A \backslash B) \cup (B \backslash A)$$

Lemma
Let G be a graph with a matching M and an alternating path P with respect to M. If each end-point of P is either unmatched by M or matched by $M \cap P$, then $M \oslash P$ is another matching

3 Augmenting Paths