Odds and Logistic Regression

1 Odds

Definition: Odds

A numerical expression, expressed as a pair of numbers

Important: Odds vs Probabilities

Odds are not probabilities, odds is the number of successes vs number of failures, whereas probabilities is number of successes against total number of cases

$$\frac{\text{Probabilitiy(success)}}{\text{Probability(Failure)}} = \text{Odds}$$

1.1 Log of odds

Odds against success is between 0 and 1
Odds in favour of success is between 1 and $+\infty$

This makes the odds against success look way smaller, so take the logs to make everything symmetrical

1.2 Odds ratios

2 Logistic Regression

This is similar to linear regression except it predicts true or false

Logistic regression provides probabilities and classifies new samples using continuous and discrete measurements

$$\log(\frac{p}{1-p}) = \log(odds)$$

$$\frac{p}{1-p} = e^{\log(odds)}$$

$$p = (1-p)e^{\log(odds)} = e^{\log(odds)} - pe(\log(odds))$$

$$p + pe^{\log(odds)} = e^{\log(odds)}$$

$$p(1 + e^{\log(odds)}) = e^{\log(odds)}$$

$$p = \frac{e^{\log(odds)}}{1 + e^{\log(odds)}}$$