Practical 1

1 Question 1

Prove that, for every positive integer n,

$$1 \cdot 2 + 2 \cdot 3 + ... + n(n+1) = n(n+1)(n+2)/3$$

Basis Step: The statement is valid for n=1 as $1 \cdot 2 = 1(1+1)(1+2)/3$

Assume the statement holds true for $n = k$:

$$1 \cdot 2 + 2 \cdot 3 + ... + k(k+1) = k(k+1)(k+2)/3$$

Is it true for $n = k + 1$

$$1 \cdot 2 + 2 \cdot 3 + ... + k(k+1) + (k+1)(k+2) =?$$

$$k(k+1)(k+2)/3 + (k+1)(k+2) =?$$

$$[k(k+1)(k+2) + 3(k+1)(k+2)]/3 =?$$

$$(k^3 + 6k^2 + 11k + 6)/3 =?$$

$$[(k+1)(k^2 + 5k + 6)]/3 =?$$

$$(k+1)(k+2)(k+3)/3$$

Since the statement is **valid for n=1**, and that is **valid for n=k+1 if it is valid for n=k**, we conclude that it is **valid for all positive integers n**

2 Question 2

Prove that 3 divides $n^3 + 2n$ for every positive integer n

Basis Step: The statement is valid for n=1 as $1^3 + 2 \times 1 = 3$

Assume that the statement holds true for n=k

3 divides $k^3 + 2k$ for every positive integer k

Is it true for $n = k + 1$?

$$(k+1)^3 + 2(k+1)$$

$$k^3 + 3k^2 + 3k + 1 + (2k + 2)$$

$$k^3 + 3k^2 + 5k + 3$$

$$(k^3 + 2k) + 3k^2 + 3k + 3$$

$$(k^3 + 2k) + 3(k^2 + k + 1)$$

As already assumed that $k^3 + 2k$ is divisible by 3, and $3(k^2 + k + 1)$ is divisible by 3, it must be true for $n = k + 1$

Since the statement is **valid for n=1**, and that is **valid for n=k+1 if it is valid for n=k**, we conclude that it is **valid for all positive integers n**