

Q4

1.	p	x	$(p \Rightarrow x) \wedge (\neg p \Rightarrow x)$
	F	F	T F T F F
	F	T	T T T T T
	T	F	F F T F F
	T	T	T F T T T

* 2. $\boxed{p \quad q \quad r \quad ((\neg p \vee q) \wedge (r \vee \neg q)) \Rightarrow (p \Rightarrow r)}$

F	F	F	T T F	T F +	T	T	F T F
F	F	T	T T F	T T +	T	T	F T T
F	T	F	T T T	F F F	F	T	F T F
F	T	T	T T T	T T F	T	T	F T T
T	F	F	F F F	F T T	T	T	T F F
T	F	T	F F F	F T T	T	T	T T F
T	T	F	F T T	F F F	T	T	T F F
T	T	T	F T T	T T F	T	T	T T T

3. $\boxed{x \quad y \quad z \quad ((x \vee y) \vee z) \Leftrightarrow (\neg z \wedge x)}$

F	F	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	T	F	T	T	F	F	F
T	F	F	F	T	T	F	T
T	F	T	F	T	F	F	T
T	T	F	F	T	T	F	T
F	T	T	F	T	F	F	T

Q5

x	y	z	$\neg x \Rightarrow (y \Rightarrow z)$	$y \Rightarrow (x \vee z)$
F	F	F	T	F
F	F	T	T	F
F	T	F	T	F
F	T	T	T	T
T	F	F	F	F
T	F	T	T	T
T	T	F	F	T
T	T	T	F	T

Formulas equivalent as they have the same truth table

p	q	r	s	$(p \Rightarrow q) \Leftrightarrow (r \Rightarrow s)$	$(p \Rightarrow r) \Rightarrow (q \Rightarrow s)$
F	F	F	F	T	T
F	F	F	T	T	T
F	F	T	F	F	T
F	F	T	T	T	T
F	T	F	F	T	F
F	T	F	T	T	T
F	T	T	F	F	T
F	T	T	T	T	T
T	F	F	F	F	T
T	F	F	T	T	T
T	F	T	F	F	F
T	F	T	T	T	T
T	T	F	F	T	F
T	T	F	T	T	T
T	T	T	F	F	F
T	T	T	T	T	T

Remove line, correct apart from that

Formulas ^{not} equivalent as they do not have the same truth table

a	b	c	$(a \wedge b) \Rightarrow c$		$(a \Rightarrow c) \wedge (b \Rightarrow c)$	
F	F	F	F	T	T	T
F	F	T	F	T	T	T
F	T	F	F	T	T	F
F	T	T	F	T	T	T
T	F	F	F	T	F	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T

Formulas not equivalent as truth values not the same

Q6

1.

p	q	$(\neg p \wedge (p \vee q)) \Rightarrow q$
F	F	T F F T
F	T	T T F T
T	F	F F T T
T	T	F F T T

Always evaluates to true, is a tautology

2.

p	q	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
F	F	F T T T
F	T	F T T T
T	F	F F T T
T	T	T T T T

Always evaluates to true, is a tautology

3.

p	q	r	$((p \vee q) \wedge (p \Rightarrow r)) \wedge (q \Rightarrow r) \Rightarrow r$							
F	F	F	F	F	T	F	T			T
F	F	T	F	F	T	F	T			T
F	T	F	T	T	T	F	F			T
F	T	T	T	T	T	T	T			T
T	F	F	T	F	F	F	T			T
T	F	T	T	T	T	T	T			T
T	T	F	T	F	F	F	F			T
T	T	T	T	T	T	T	T			T

Always evaluate to true, is a tautology

Q7

x	y	z	Result
T	F	T	T
F		F	

$$((x \wedge z) \wedge (\neg y)) \wedge (\neg x \vee \neg z)$$

2 truth tables as there is only 1 row in the truth table that is correct, TTT, so this can give rise to 2 different formulas

Q6

Q8

	P	q	r	$p \vee q$	$q \vee r$	$\neg p \vee q$	$q \vee r$	$\neg q \vee \neg r$
4	F	F	F	T	T	T	F	T
4	F	F	T	T	F	T	T	T
3	F	T	F	F	T	T	F	T
3	F	T	T	F	T	T	T	F
4	T	F	F	T	T	F	T	T
3	T	F	T	T	F	F	T	T
5	T	T	F	T	T	T	T	F
4	T	T	T	T	T	T	T	F

$$p = T \quad q = T \quad r = F$$

Q9

If ψ is wholly equivalent to χ and ψ is wholly equivalent to χ then ψ must be wholly equivalent to χ .

If inputs two ψ make it true - then implications ψ makes it true.

As ψ and χ are logically equivalent, ψ makes χ true

Q10

$$(\neg p \wedge (p \vee q)) \Rightarrow q$$

$$\neg p \vee q$$

$$q = F$$

$$\neg(p \wedge \neg(p \vee q))$$

$$\neg p$$

ANS

Q10

1.

For $(\neg p \wedge (p \vee q)) \Rightarrow q$ to be false:

- $\neg p \wedge (p \vee q)$ must be true when Q is false
- As Q is false p must have to be true due to $p \vee q$ being on one side of the and, however this expresses to $\neg p \wedge p$, which is a contradiction, so it must be a tautology.

2. For $(p \wedge (p \Rightarrow q)) \Rightarrow q$ to be false

- $p \wedge (p \Rightarrow q)$ must be true when q is false
- When q is false for $p \Rightarrow q$ to be true p must be false, however $p \wedge (p \Rightarrow q)$ requires p to be true to be true, this is a contradiction, so it must be a tautology.

3. For $((p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)) \Rightarrow r$ to be false

- $(p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)$ must be true when r is false
- For $p \Rightarrow r$ and $q \Rightarrow r$ to be true when r is false p and q must be false
- If both p and q are false $p \vee q$ cannot be true, so the LHS of the statement cannot be true

Note LHS \Rightarrow RHS $\equiv \neg \text{LHS} \vee \text{RHS}$

LHS \Rightarrow RHS

FF	T
FT	T
TF	F
TT	T

✓

FF	T
FT	F
TF	F
TT	T