Week 3 Practical

1 Question 1 - Basic Propositions

Which of the following are propositions, and what are the truth values of these propositions:

1. Just put the gun down - Not a proposition
2. Is it raining? - Not a proposition
3. $x^2 = 9$ - Not a proposition
4. The square root of banana is apple - Is a proposition - False
5. 5+6=10 - Is a proposition - False

2 Question 2 - Translating simple formulae

1. $q \Rightarrow p$ - If I am ill then I want to lie down
2. $\neg s$ - I can hide
3. $r \wedge \neg s$ - I can run and I can hide
4. $q \Rightarrow \neg r$ - If I am ill then I can't run
5. $p \vee (q \wedge \neg r)$ - I want to lie down or I am ill and can't run
6. $\neg s \Rightarrow (\neg q \wedge \neg p)$ - If I can hide then I don't want to lie down and I am not ill
7. $\neg q \wedge r \wedge \neg s \wedge \neg p$ - I am not ill and I can run and I can hide and I don't want to lie down
8. $(p \Leftrightarrow (q \wedge \neg r)) \vee (\neg s \wedge \neg p)$ - I want to lie down if and only if I am ill and I can't run. Or I can hide and I don't want to lie down

3 Question 3 - Translating Simple Formulae

1. $X \Rightarrow Y$
2. $\neg X \wedge Z$
3. $\neg Z \Rightarrow \neg X$
4. $Z \Rightarrow Y$
5. $X \Rightarrow Z$
6. $\neg X \Rightarrow Y$
7. $(Z \wedge \neg X) \Leftrightarrow \neg Y$
8. $Y \Leftrightarrow \neg X$

Q4

1.

p	x	(p ⇒ x)	∧	(¬p ⇒ x)
F	F	T	F	T F F
F	T	T	T	T T T
T	F	F	F	F T F
T	T	T	T	F T T

✱2.

p	q	r	((¬p ∨ q)	∧	(r ∨ ¬q))	⇒	(p ⇒ r)
F	F	F	T T F	T	F T T	T	F T F
F	F	T	T T F	T	T T T	T	F T T
F	T	F	T T T	F	F F T	T	F T F
F	T	T	T T T	T	T T F	T	F T T
T	F	F	F F F	F	F T T	T	T F F
T	F	T	F F F	F	T T T	T	T T F
T	T	F	F T T	F	F F T	T	T F F
T	T	T	F T T	T	T T F	T	T T T

3.

X	Y	Z	((X	∨	Y)	∨	Z)	⇔	(¬Z	∧	X)
F	F	F		F		F		T	T	F	F
F	F	T		F		T		F	F	F	F
F	T	F		T		T		F	T	F	F
F	T	T		T		T		F	F	F	F
T	F	F		T		T		T	T	T	T
T	F	T		T		T		F	F	F	T
T	T	F		T		T		T	T	T	T
T	T	T		T		T		F	F	F	T

Q5

1.

X	Y	Z	¬X	⇒	(Y⇒Z)	Y	⇒	(X ∨ Z)
F	F	F	T	T	T	F	T	F
F	F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	F	F
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T
T	F	T	F	T	T	F	T	T
T	T	F	F	T	F	T	T	T
T	T	T	F	T	T	T	T	T

Formulas equivalent as they have the same truth table

2.

p	q	r	s	(p⇒q)	⇒	(r⇒s)	(p⇒r)	⇒	(q⇒s)
F	F	F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	T	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	T	T
T	F	T	F	F	T	F	T	T	T
T	F	T	T	F	T	T	T	T	T
T	T	F	F	T	T	T	F	T	F
T	T	F	T	T	T	T	F	T	T
T	T	T	F	T	F	F	T	F	F
T	T	T	T	T	T	T	T	T	F

Remark: line, correct apart from that

Formulas not equivalent as they do not have the same truth table

3.

a	b	c	$(a \wedge b)$	$\Rightarrow c$	$(a \Rightarrow c)$	$\wedge$	$(b \Rightarrow c)$
F	F	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	T	T	F	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	F
T	T	T	T	T	T	T	T

Formulae not equivalent as truth tables not the same

Q 6

1.

p	q	$(\neg p$	$\wedge$	$(p \vee q))$	$\Rightarrow$	q
F	F	T	F	F	T	
F	T	T	T	T	T	
T	F	F	F	T	T	
T	T	F	F	T	T	

Always evaluates to true, is a tautology

2.

p	q	$(p$	$\wedge$	$(p \Rightarrow q))$	$\Rightarrow$	q
F	F	F	T		T	
F	T	F	T		T	
T	F	F	F		T	
T	T	T	T		T	

Always evaluates to true, is a tautology

3.

p	q	r	$((p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)) \Rightarrow r$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Always evaluates to true, is a tautology

Q7

X	Y	Z	Result
T	F	T	T
F		F	

$((X \wedge Z) \wedge (\neg Y)) \wedge (\neg X \vee \neg Z)$

2 truth tables as there is only 1 row in the truth table that is uncertain, TTT, so this can give rise to 2 different premises

Q6

Q8

	p	q	r	$p \vee \neg q$	$q \vee \neg r$	$\neg p \vee q$	$q \vee r$	$\neg q \vee \neg r$
4	F	F	F	T	T	T	F	T
4	F	F	T	T	F	T	T	T
3	F	T	F	F	T	T	T	T
3	F	T	T	F	T	T	T	F
4	T	F	F	T	T	F	T	T
3	T	F	T	T	F	F	T	T
5	T	T	F	T	T	T	T	T
4	T	T	T	T	T	T	T	F

$p = T \quad q = T \quad r = F$

Q9

If p is wholly equivalent to ψ and ψ is wholly equivalent to χ then p must be wholly equivalent to χ

If inputs to make it true, then input to ψ makes it true.

As ψ and χ are logically equivalent, it makes χ true

Q10

$(\neg p \wedge (p \vee q)) \Rightarrow q$

$q = F$

$\neg((p \vee \neg(p \vee q))$

Q10

1.

For $(\neg p \wedge (p \vee q)) \Rightarrow q$ to be false:

- $\neg p \wedge (p \vee q)$ must be true when Q is false
- As Q is false p must have to be true due to $p \vee q$ being on one side of the and, however this expresses to $\neg p \wedge p$, which is a contradiction, so it must be a tautology.

2. For $(p \wedge (p \Rightarrow q)) \Rightarrow q$, to be false

- $p \wedge (p \Rightarrow q)$ must be true when q is false
- When q is false for $p \Rightarrow q$ to be true p must be false, however $p \wedge (p \Rightarrow q)$ requires p to be true to be true, this is a contradiction, so it must be a tautology

3. For $((p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)) \Rightarrow r$ to be false

- $(p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow r)$ must be true when r is false
- For $p \Rightarrow r$ and $q \Rightarrow r$ to be true when r is false p and q must be false
- If both p and q are false $p \vee q$ cannot be true, so the full LHS of the statement cannot be true

Not LHS $\Rightarrow$ RHS $\equiv \neg$LHS $\vee$ RHS

LHS	⇒	RHS
F	F	T
F	T	T
T	F	F
T	T	T

F	F	T
F	T	T
T	F	F
T	T	T