

Introduction to Logic

1 What is logic?

There are two parts of logic

- Formal Language - Making statements about certain objects
- Formal System - Reasoning about the properties of objects

Objective of logic:

- To carry out **precise** and **rigorous** arguments about **assertions** and **proofs** and to **implement** these arguments and **proofs**
- We need a **language** whose structure (**syntax**) can be **precisely** described and whose meaning (**semantics**) can be **unambiguously** defined

Proof System - A system of **deduction** by which **proofs** can be constructed

Semantics - A notion of **meaning** by which the **truth** of some property of some object can be determined

Propositional logic- Joining propositions to create more complicated propositions

First order logic - Statement is broken down into a subject and a **predicate**, the predicate **modifies** the subject

Logic comprises of 3 components:

- **Syntax** - The definition of the formulae of the logic
- **Semantics** - The association of meaning and truth to the formulae of the logic
- **Proof System** - The manipulation of formulae according to a system of rules

What is desired from these components is:

- **Completeness** - All the "true" (semantics) formulae should be "provable" (syntax, proof system)
- **Soundness** - A formula that is "provable" (syntax, proof system) should be "true" (semantics)

2 Logic in action

2.1 Programming languages

Syntax - Exactly which combinations of symbols constitute a legible program

2.2 Circuits

Logic Gate - Performs a Boolean operation on digital inputs and provides the result of this operation as an output

Logic Circuit - A collection of logic gates connected together

Truth Table - A model of the intended behaviour of a logic gate

Propositional logic can be used to create a truth table

A specification of a logic circuit as a truth table may be incomplete due to a large circuit with multiple combinations that could be given, and not all need to be tested

2.3 Databases

Database - A structured collection of logical records

Database Query Language - A language for asking and answering questions of the structured data

The expressive power of SQL is very closely related to that of predicate logic

2.4 Formal Methods

Formal Methods - The use of mathematically based techniques for the specification and verification of computer systems - these prove that programs have certain property and don't just rely on testing

Model Checking - A branch of formal methods where a computer system is modelled as a mathematical structure then a specific property that this system might have is expressed by a formula of some logic

Model checking used for rapid prototyping systems

Examples of use of formal methods:

- Microprocessor design
- Design of data-communications protocol software
- Critical Software
- Operating Systems

Note: Non examinable from here on

3 A history lesson

Gottlob Frege attempted to show that all of mathematics grew out of logic, in doing so inventing first order predicate logic. His intention was to show that there was:

- A set of axioms (basic and obvious facts)
- A set of logical rules (unambiguous)

This was so that all true mathematical statements could be expressed in this way

4 Hilbert's Programme

Hilbert believed that:

- All mathematical statements could be written in a formal language and manipulated according to formal rules
- All true mathematical statements could be proved in the formalism
- There would be an "Algorithm" to decide whether or not a mathematical statement is true or not

5 Logic and Computation

Part of Hilbert's Programme was to solve the Entscheidungsproblem which is an algorithm that takes the inputs:

- A description of a formal language
- A mathematical statement in the language

Then give the output true or false depending if the statement is true or false

Later Alonzo Church and Alan Turing independently proved that a general solution to the problem is impossible, both using different methods

6 Computational Complexity

Graph colouring is NP Complete as the correct colouring cannot be easily determined for a graph, however given a coloured graph it can be checked quickly.