Image Noise and Spatial Filtering I

1 Noise in images

No digital image is a perfect representation of the original 2D signal
They are limited in resolution by sampling and contain noise

Noise removal is a major goal of image processing to limit the effects on image visualisation and analysis

2 Where does image noise come from?

Capture:

- Variations in sensor temperature
- Electrical sensor noise
- Sensor non-uniformity
- Dust in the environment
- Vibration
- Lens distortion
- Focus limitations
- Sensor saturation (too much light)
- Under exposure (too little light)

Sampling:

- Limitations in sampling and intensity quantization (aliasing)

Processing:

- Limitations in numerical precision
- Potential integer overflow
- Mathematical approximations

Image compression:

- Lossy image compression techniques remove information from image to save space
- JPEG/MPEG are examples of widely used lossy compression formats
- Remove non-perceivable detail aiming at ”no noticeable difference”
- Result: ”compression artefacts” in the image

3 Scene noise

Lighting:

- Sunlight changes
- Varying artificial light sources
- Interior light sources oscillate with power supply frequency
- Objects cast shadows causing false image features
- As objects move shadows change

Occlusion:

- Objects are frequently obscured by other objects: occlusion
- Big problem for recognition systems in image understanding tasks

4 Types of theoretical noise models

4.1 Salt and pepper noise (impulse noise)

- Random white or black value pixels into the image
- Follows a binary high-low bi-modal noise distribution

4.2 Gaussian noise (additive noise)

- Small random variation of the image signal around its true value following the Gaussian distribution
- This is the most common noise model in image processing

5 Image noise removal

5.1 Neighbours of a pixel

A pixel p at coordinates (x,y) has four horizontal and vertical neighbours whose coordinates are given by

$$(x+y,y),(x-1,y),(x,y+1),(x,y-1)$$

It also has four diagonal neighbours whose coordinates are given by:

$$(x+1,y+1),(x+1,y-1),(x-1,y+1),(x-1,y-1)$$

Together they form the 3×3 local pixel neighbourhood

Local image neighbourhoods define local areas of influence, relevance or interest

Image filtering and many other operations use $N\times M$ neighbourhoods

In most cases:

- N=M (we treat both directions equally)
- N is odd (simplifies implementation)

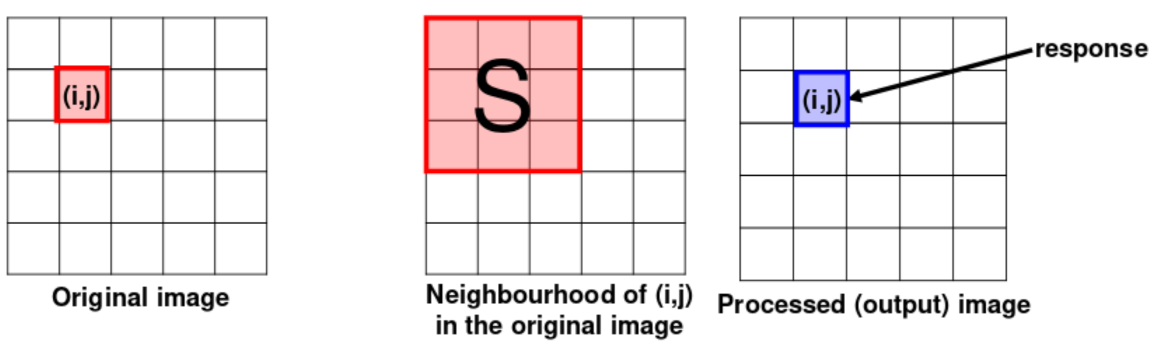
6 Spatial filtering

We go iteratively through the pixels we want to process (perhaps the whole image)

for each pixel (i,j):

- Consider a neighbourhood S of (i,j). Usually (i,j) will be at the centre of S
- Process the pixel values of S (i.e. apply to them a many to one function) to find a new value for pixel (i,j)

Replace the original pixel values with the new ones (called the filter's responses)



7 Filtering

Definition: Linear filtering

Output pixel is a linear combination of the corresponding input pixel's neighbourhood

Definition: Non-linear filtering

Output pixel is not linear function of the corresponding input pixel's neighbourhood. In practice, some decision based algorithm is employed

8 Mean filter

Operation: Replace a given pixel with the mean (unweighed average) of its $N \times N$ image neighbourhood

$$I_{output}(i, j) = \frac{1}{N^2} \sum_{(i,j) \in S} I_{input}(i, j)$$

S is the neighbourhood, a rectangular window centred at pixel (i,j) enclosing $N \times N$ neighbours

Effect: Eliminates sudden intensity jumps which could be caused by some noise processes, i.e. eliminates large deviations from the norm.

8.1 Example

Suppose we have the 3×3 neighbourhood:

$$\begin{matrix} 2 & 2 & 3 \\ 3 & 30 & 2 \\ 1 & 3 & 2 \end{matrix}$$

Thee value 30 is relatively large - noise spike

Mean filtering will suppress it:

$$\begin{matrix} 2 & 2 & 3 \\ 3 & 5.3 & 2 \\ 1 & 3 & 2 \end{matrix}$$

8.2 Effect on types of noise

Gaussian noise: Distributed around the original value - the mean can easily handle this
Salt and pepper noise: Significant deviation from the local distribution

8.3 Drawbacks

Mean filter is not robust to large noise deviations (statistical outliers). For example, a single pixel with a very unrepresentative value.

Mean filter causes edge blurring, that is, removes the high frequency sharp detail.

9 Spatial non-linear filters

For an $N \times N$ image neighbourhood, N_{xy}, centred at pixel (x,y) and index by (s,t) the following simple statistical filters can be defined to replace each pixel with the min/max/median from the input neighbourhood

Min:

$$\min_{(s,t) \in N_{xy}} \left\{ I_{input}(s, t) \right\}$$

Max:

$$\max_{(s,t)\in N_{xy}} \left\{ I_{\text{input}}(s,t) \right\}$$

Median:

$$\text{median}_{(s,t)\in N_{xy}} \left\{ I_{input}(s,t) \right\}$$

9.1 Other spatial non-linear filters

Alpha tripped mean:

$$I_{output}(x,y) = \frac{1}{N^2 - 2d} \sum_{(s,t)\in N_{xy}} I_{\text{Input}_r}(s,t)$$

Here, the dimension of the neighbourhood N_{xy} is $N \times N$. The d lowest and the d highest intensity levels of the image in N_{xy} are deleted (set to 0) with $I_{Input_r}(s,t)$ denoting the remaining (reduced set of) $N^2 - d$ pixels in N_{xy}

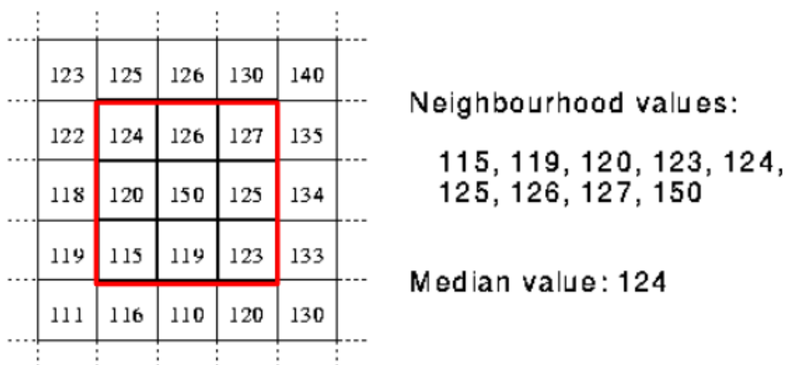
Harmonic mean:

$$I_{output}(x,y) = \frac{N^2}{\sum_{(s,t)\in N_{xy}} \frac{1}{I_{input}(s,t)}}$$

10 Median Filter

Operation: Replace a given pixel with the median of its $N \times N$ image neighbourhood

Example: for 3×3



Effect: Eliminates sudden intensity jumps which could be caused by some noise processes, i.e. large deviations from the norm.

But, it is robust to statistical outliers (unlike the mean filter)

Example: Suppose we have the 3×3 neighbourhood:

$$\begin{array}{ccc} 2 & 6 & 3 \\ 14 & 81 & 2 \\ 13 & 4 & 1 \end{array}$$

The value 81 is relatively large - noise spike (an outlier)

Median filtering will suppress it

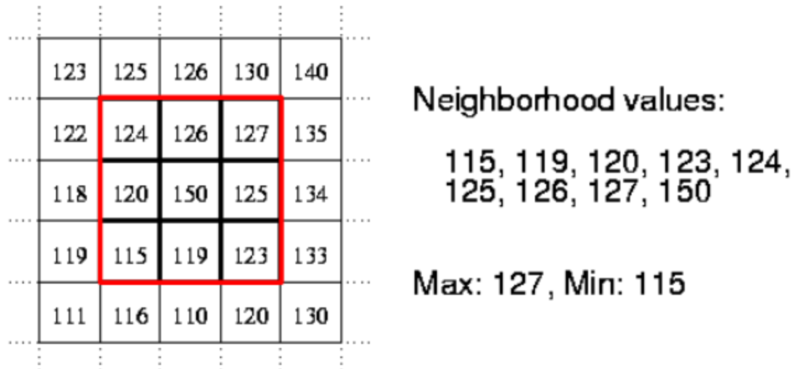
$$\begin{array}{ccc} 2 & 6 & 3 \\ 14 & 4 & 2 \\ 13 & 4 & 1 \end{array}$$

11 Conservative smoothing

Operation: Compare a pixel value to min and max of the other $(N \times N - 1)$ neighbourhood pixels:

- Replace by min if < min
- Replace by max if > max

Example: For $N = 3$



Effect: Eliminates sudden intensity jumps

Here, 150 is replaced by 127 (max of its 8 neighbours)

A conservative approach to smoothing:

- A pixel value will change only if it is outside the range of its neighbours
- By the minimum required amount to just bring it in to range