More on resolution for Propositional Logic

1 Example 1

- Let φ be the formula $\neg((p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q))$
- Is φ a theorem?
- In order to prove this using resolution we negate φ and put it in conjunctive normal form if necessary
- So $\neg\varphi$ is the formula $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ which is in conjunctive normal form already
- We will now try to apply resolution on $\neg\varphi$ until
  - Either we infer the empty clause, which means that $\neg\varphi$ is a contradiction, and hence φ is a theorem or,
  - We do not infer the empty clause but at some point we do not find any new clauses either; in that case we can find a truth assignment that makes $\neg\varphi$ true, and hence, φ false, which means that φ is not a theorem

Applying resolution:
$p \vee q$
$\neg p \vee q$
$p \vee \neg q$
$\neg p \vee \neg q$

$q \vee q$ (1 and 2)
$q \vee \neg q$ (2 and 3)
$\neg q \vee \neg q$ (3 and 4)
$\varnothing$
This implies that φ is a theorem

2 Example 2

- Use resolution to prove that if
  - "It is not raining or I have my umbrella" $\neg r \vee u$
  - "I do not have my umbrella or I do not get wet" $\neg u \vee \neg w$
  - "It is raining or I do not get wet" $r \vee \neg w$

  then
  - I do not get wet
- A formula $\varphi \Rightarrow \psi$ is logically equivalent to $\neg\varphi \vee \psi$
  - So that the negation of our formula is $\varphi \wedge \neg\psi$
    - That is
$$(\neg R \vee U) \wedge (\neg U \vee \neg W) \wedge (R \vee \neg W) \wedge W$$

So we must apply resolution on clauses - it is already in cnf so is easy to do

$$\neg R \vee U, \neg U \vee \neg W, R \vee \neg W, W$$

R - It is raining
U - I have my umbrella
W - I get Wet

$\varphi \Rightarrow \psi$
$\neg\varphi \vee \psi$
$\neg(\neg\varphi \vee \psi)$
$\neg\neg\varphi \wedge \neg\psi$

$\varphi \wedge \neg\psi$

$W, R \vee \neg W \Rightarrow R$
$W, \neg U \vee \neg W \Rightarrow \neg U$
R is true, so not R is false, not U is true, so U is false. Is a theorem

$u \vee \neg W$ (1 and 3)
$\neg W \vee \neg W \Rightarrow \neg W$ (new 1 and 2)

3 Example 3

Applying resolution to the following set of clauses

$$a \vee b \vee c \qquad a \vee \neg c \vee d \qquad \neg a \vee e \vee f \qquad c \vee \neg e \vee f \qquad c \vee d \vee \neg f$$

1. (1,3) $b \vee c \vee e \vee f$
2. (2,3) $\neg c \vee d \vee e \vee f$
3. (4,n2) $\neg d \vee f \vee \neg f$