Conditional Probability and Bernoulli Trials

1 Conditional Probability

Let E and F be events and let p(F) > 0. The conditional probability of E given F, denoted p(E—F) is

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

Intuition: if we assume F, we make F the new sample space, and then apply the usual definition of probability.

1.1 Examples

1.1.1 Example 1

What is the probability that a randomly generated 4-bit string has two consecutive 0s if we know that the first bit is 0?

- Sample space S: all 4-bit strings, S = 2^4 = 16
- Event E: 4-bit string has two consecutive 0s
- Event F: first bit is 0
- We know that $p(E|F) = p(E \cap F)/p(F)$
- $E \cap F$ consists of strings 0000, 0001, 0010, 0011, 0100. So $p(E \cap F) = 5/16$
- $p(F) = 8/16 = 1/2$, so $p(E|F) = p(E \cap F)/p(F) = \frac{5/16}{1/2} = 5/8$

1.2 Example 2

What is the probability that a family with two children has two boys, given that they have at least one boy? Assume that all possibilities BB, GG, BG, and GB are equally likely

- Sample space S: the 4 combinations abov
- Event E: BB
- Event F: BB, GB, BG
- We know that $p(E|F) = p(E \cap F)/p(F)$
- Event $E \cap F : BB$. So $p(E \cap F) = 1/4$
- $p(F) = 3/4$, so $p(E|F) = p(E \cap F)/p(F) = \frac{1/4}{3/4} = 1/3$

2 Independence

- One can say that event E is independent from F if p (E) = p (E — F)
- Since $p(E|F) = p(E \cap F)/p(F)$, we have

$$p(E) = p(E|F) \text{ iff } p(E \cap F) = p(E)p(F)$$