February 12, 2014
Quiz 3

Your name: Key

Quiz rules:

(a) This quiz is closed book, but you are allowed a two-sided sheet of paper of notes and a calculator.
(b) Each question is worth 6 points.
(c) A normal table is provided on the last page.
(d) You have 50 minutes to complete this quiz.
(e) If you fail to show your work and/or explain how you arrived at your answer then no points will be awarded.
(f) You do not need to solve all the problems to do well. Try your best.
(g) **Please write expressions for probabilities instead of numerical answers, unless otherwise specified.**

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Quiz 3

1. To play the Pick-3 lotto, you choose three digits between 0 and 9. Each day, three random draws are made (with replacement) from a jar containing balls marked 0 – 9. You win if the three digits drawn match yours (in any order).

(a) Suppose you bet on the digits 0-3-3. Show that your chance of winning is 0.6%. → 0.3%

Counting rule: 0-3-3, 3-0-3, 3-3-0,

total # of outcomes: $10^3 = 1000$

$3/1000$

Note: There was a mistake in this part, so this part will be worth up to 3 extra credit points.

(b) Suppose you bet on 0-3-3 every day. How many days would you have to play in order to guarantee winning at least once with a more than 50% probability?

This part was worth 6 points.

Correct answer:

$P(\text{don't win}) = 1 - .003 = .997$

$P(\text{win at least once in } n \text{ tries}) = 1 - (.997)^n > .50$

$n > \frac{\log .50}{\log .997} \approx 231$

Also accepted for full credit (if you used 0.6% from part a):

$P(\text{don't win}) = 1 - .006 = .994$

$P(\text{win at least once in } n \text{ tries}) = 1 - (.994)^n > .50$

$n > \frac{\log .50}{\log .994} \approx 116$

2. Sage Francis is a rapper from Rhode Island. He is well known for his intelligent, emotionally-honest lyrics. He's also known for his ability to improvise lyrics in battle-rap situations – for instance winning Scribble Jam in 2000.

   Suppose that Sage Francis has spent the afternoon talking trash about his improvisational skills with his two rapper-buddies – B. Dolan and Buck 65. Finally fed up, B. Dolan and Buck 65 challenge Sage to prove himself by playing the *[broken]mirror* game.

   Here's how the game works: Buck 65 mixes up six beats. For each beat Sage would have to improvise a rap. At the halfway mark of each beat, B. Dolan would flip a fair coin. If the coin comes up heads then B. Dolan would call out "mirror" and Sage would have to "mirror" the rap he had created for the first half of the beat (i.e., similar rhyme structure and topics in the same order as the first half). If the coin comes up tails then at the half way mark B. Dolan would call out "broken-mirror" and Sage would have to improvise a rap for the second half of the song that doesn't repeat any of the first half's rhymes or topics.

   a) Calculate the probability that Sage will have to create exactly 4 "mirror" raps.

Binomial $\binom{6}{4}(0.5)^6$

   b) Calculate the probability that Sage will have to create at least 1 "broken-mirror" rap.

Complement rule: $1 - (0.5)^6$

3. **Ten percent of Stats 60 students do all the reading in the textbook. Among students who do all the reading, 90% earn an A in the class. Among those who do not, only 30% earn an A. What percentage of students who earned an A would you expect to have done all the reading?**

$A = \{\text{earning an A}\}$

$B = \{\text{doing all the reading}\}$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

$$= \frac{90\% \cdot 10\%}{90\% \cdot 10\% + 30\% \cdot 90\%}$$

$$= 25\%$$

4. Walt and Jesse enjoy cooking up math. They make three kinds of questions: hard, medium and easy. It takes students 5 minutes to solve a hard question, 3 minutes for a medium and 2 minutes for an easy. (To keep things simple, you may assume all students take exactly 5, 3 and 2 minutes for these kinds of problems.)
Walt and Jesse have been asked to cook up a batch of 6 high-quality problems for this quiz. They decided to do something a little special for this batch: They would let chance dictate how challenging the quiz will be. For each of the six problems, they will roll a die. If the die comes up even, then they will write a hard question. If it comes up 1 or 3 then they'll write a medium difficulty question. And if they roll a 5 they'll write an easy question.

a) Create a box model that will be useful for estimating the duration of the quiz. Note: full marks will only be provided if all four parts of a box model are correctly identified.

2	x 1
3	x 2
5	x 3

$$\text{avg} = \frac{2}{6} + \frac{3 \times 2}{6} + \frac{5 \times 3}{6} = \frac{23}{6}$$

$$SD(\text{box}) = \sqrt{\frac{2^2 + 3^2 \times 2 + 5^2 \times 3}{6} - \left(\frac{23}{6}\right)^2} = 1.2$$

b) **What is the expected duration of a quiz that is generated this way? If they were to create many different quizzes this way, how much difference in duration would you anticipate seeing between the quizzes?**

$EV = n \times \text{avg}(\text{box}) = 23$

$SE = \sqrt{n} \times SD(\text{box}) = 2.94$

$$\boxed{23 \pm 2.94}$$

c) Calculate exactly (i.e., do not approximate) the chance that the quiz will take at least 27 minutes.

27 minutes = $5 \times 5 + 2$ (5 hard pbs + 1 easy)

$$\binom{6}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{6}\right) = 3.125\%$$

$$6\left(\frac{1}{2}\right)^6$$

$$7\left(\frac{1}{2}\right)^6$$

28 minutes = $5 \times 5 + 3$.

$$\binom{6}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{3}\right) = 6.25\%$$

30 minutes = 5×6

$$\left(\frac{1}{2}\right)^6$$

Addition rule: $7\left(\frac{1}{2}\right)^6 \approx \boxed{11\%}$

5. Four hundred draws will be made at random with replacement from a box with two 3's and two 2's.

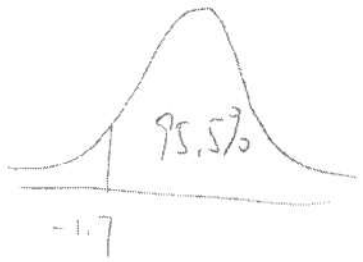
a) Approximate the chance that the sum of the draws will be more than 983.

$EV = 2.5 \times 400 = 1000$

$SE = (3-2)\sqrt{0.5 \cdot 0.5} \times \sqrt{400} = 10$

$z = \frac{983 - 1000}{10} = -1.7$

95.5%



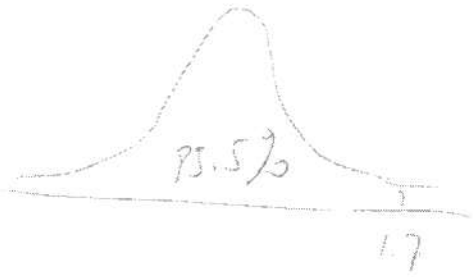
b) What are the chances that 2's will be drawn less than 217?

$EV = 200$

$SE = \sqrt{0.5 \cdot 0.5} \times \sqrt{400} = 10$

$z = \frac{217 - 200}{10} = 1.7$

95.5%



6. Twenty mice (10 male and 10 female) live in a house. The resident cat, Annie, is bored and decides to capture 7 of the mice. You may assume that Annie is equally likely to encounter any of the remaining mice at any point.

a) What is the chance that the first 3 mice she captures are female and the last 4 are male?

$\frac{10}{20} \quad \frac{9}{19} \quad \frac{8}{18} \quad \frac{10}{17} \quad \frac{9}{16} \quad \frac{8}{15} \quad \frac{7}{14}$

$= 0.93\%$

b) What is the chance the first 2 and last 2 mice she captures are male, with 3 female mice in between?

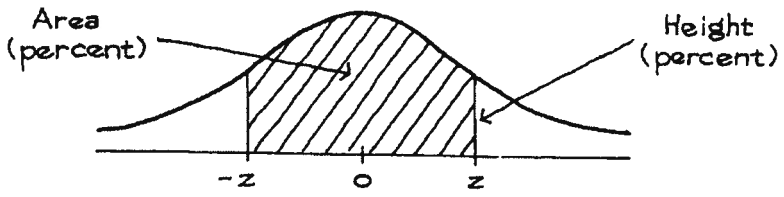
$$\frac{10}{20} \quad \frac{9}{19} \quad \frac{10}{18} \quad \frac{9}{17} \quad \frac{8}{16} \quad \frac{8}{15} \quad \frac{7}{14}$$

$$= 0.93\%$$

c) What is the chance she captures 4 male and 3 female mice in total?

addition rule: $\binom{7}{4} \times$ answer (a)

$$= 32.5\%$$



A NORMAL TABLE

z	*Height*	*Area*	*z*	*Height*	*Area*	*z*	*Height*	*Area*
0.00	39.89	0	1.50	12.95	86.64	3.00	0.443	99.730
0.05	39.84	3.99	1.55	12.00	87.89	3.05	0.381	99.771
0.10	39.69	7.97	1.60	11.09	89.04	3.10	0.327	99.806
0.15	39.45	11.92	1.65	10.23	90.11	3.15	0.279	99.837
0.20	39.10	15.85	1.70	9.40	91.09	3.20	0.238	99.863
0.25	38.67	19.74	1.75	8.63	91.99	3.25	0.203	99.885
0.30	38.14	23.58	1.80	7.90	92.81	3.30	0.172	99.903
0.35	37.52	27.37	1.85	7.21	93.57	3.35	0.146	99.919
0.40	36.83	31.08	1.90	6.56	94.26	3.40	0.123	99.933
0.45	36.05	34.73	1.95	5.96	94.88	3.45	0.104	99.944
0.50	35.21	38.29	2.00	5.40	95.45	3.50	0.087	99.953
0.55	34.29	41.77	2.05	4.88	95.96	3.55	0.073	99.961
0.60	33.32	45.15	2.10	4.40	96.43	3.60	0.061	99.968
0.65	32.30	48.43	2.15	3.96	96.84	3.65	0.051	99.974
0.70	31.23	51.61	2.20	3.55	97.22	3.70	0.042	99.978
0.75	30.11	54.67	2.25	3.17	97.56	3.75	0.035	99.982
0.80	28.97	57.63	2.30	2.83	97.86	3.80	0.029	99.986
0.85	27.80	60.47	2.35	2.52	98.12	3.85	0.024	99.988
0.90	26.61	63.19	2.40	2.24	98.36	3.90	0.020	99.990
0.95	25.41	65.79	2.45	1.98	98.57	3.95	0.016	99.992
1.00	24.20	68.27	2.50	1.75	98.76	4.00	0.013	99.9937
1.05	22.99	70.63	2.55	1.54	98.92	4.05	0.011	99.9949
1.10	21.79	72.87	2.60	1.36	99.07	4.10	0.009	99.9959
1.15	20.59	74.99	2.65	1.19	99.20	4.15	0.007	99.9967
1.20	19.42	76.99	2.70	1.04	99.31	4.20	0.006	99.9973
1.25	18.26	78.87	2.75	0.91	99.40	4.25	0.005	99.9979
1.30	17.14	80.64	2.80	0.79	99.49	4.30	0.004	99.9983
1.35	16.04	82.30	2.85	9.69	99.56	4.35	0.003	99.9986
1.40	14.97	83.85	2.90	0.60	99.63	4.40	0.002	99.9989
1.45	13.94	85.29	2.95	0.51	99.68	4.45	0.002	99.9991