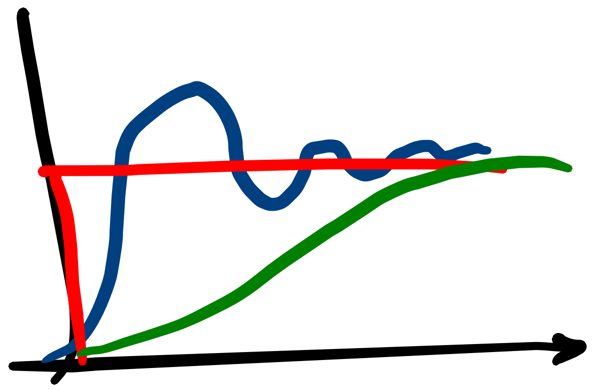
Minicurso Sistemas Lineares



Aula 4

CAEEL

Lucas Zischler

Exercício

1. $\mathcal{L}\{t^2 G_2(t)\}$

$$\cancel{(-1)^2}\,\frac{d^2}{ds^2}\left(\frac{e^{s}-e^{-s}}{s}\right)$$

$$\frac{e^{s}-e^{-s}}{s}$$

$$\frac{d}{ds}\frac{e^{s}}{s} = \frac{e^{s}}{s} + \frac{-1e^{s}}{s^2}$$

$$\frac{d}{s}\,\frac{e^{s}}{s} + \frac{-1e^{s}}{s^{2}}$$

$$\frac{e^{s}}{s} - \frac{e^{s}}{s^{2}} - \left[\frac{e^{s}}{s^{2}} - \frac{2e^{s}}{s^{3}}\right]$$

$$\frac{e^{s}}{s} - \frac{2e^{s}}{s^{2}} + \frac{2e^{s}}{s^{3}}$$

$$\frac{d}{ds}\frac{e^{-s}}{s}$$

$$-\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2}$$

$$-\left[-\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2}\right] - \left[-\frac{e^{-s}}{s^2} - \frac{2e^{-s}}{s^3}\right]$$

$$\frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \frac{2e^{-s}}{s^3}$$

?

$$\frac{e^{s}(s^2-2s+2)}{s^3} - \frac{e^{-s}(s^2-2s+2)}{s^3}$$

$$\frac{e^{s}(s^2-2s+2)-e^{-s}(s^2-2s+2)}{s^3}$$

Exercício

② $$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+1}\right\}$$

$s' = -1$

$s'' = -1$

$$\frac{A''}{(s+1)^2} + \frac{B}{(s+1)}$$

$$A = F(s)(s+1)^2 \Big|_{s=-1} = s+1 \Big|_{s=-1} = 0$$

$$B = 1 \Big|_{s=-1} = 1$$

$$\frac{1}{s+1} \Rightarrow \frac{e^{-s3} \rightarrow 0}{s-2}$$

$$e^{-t}u(t)$$

Convolução distributiva

$$[u(t+1) - u(t-1)] * u(t)$$

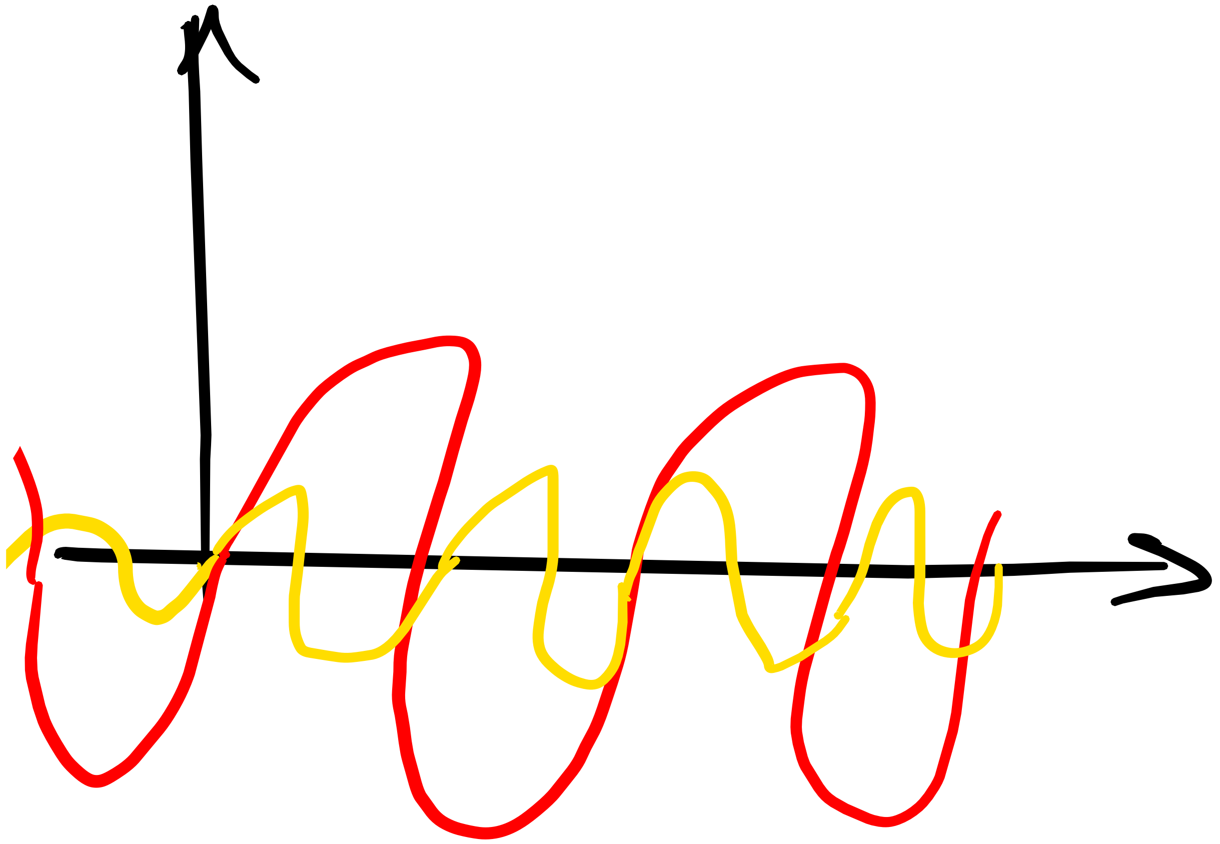
$$(t+1)\,u(t+1) - (t-1)\,u(t-1)$$

Exemplo

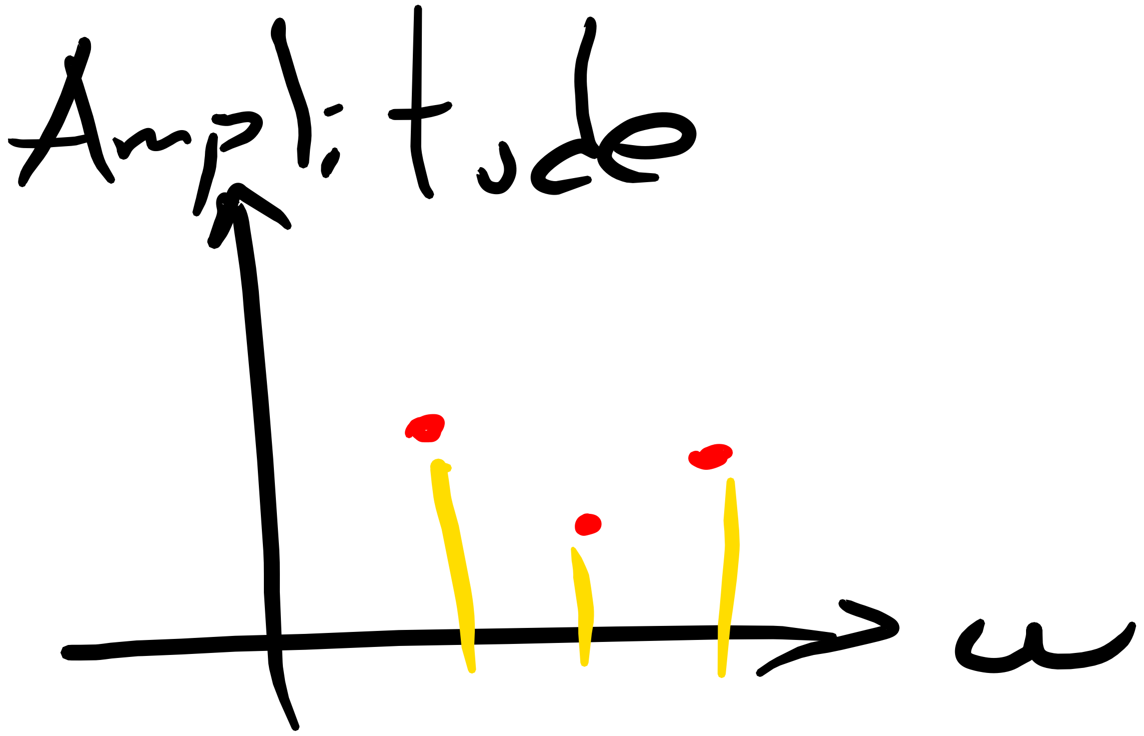
$$[u(t)+u(t-1)] * [(t+1)\,u(t+1)+u(t)]$$

$$\frac{(t+1)^2}{2}u(t+1)+t\,u(t)+\frac{t^2}{2}u(t)+(t-1)\,u(t-1)$$

Transformada de Fourier

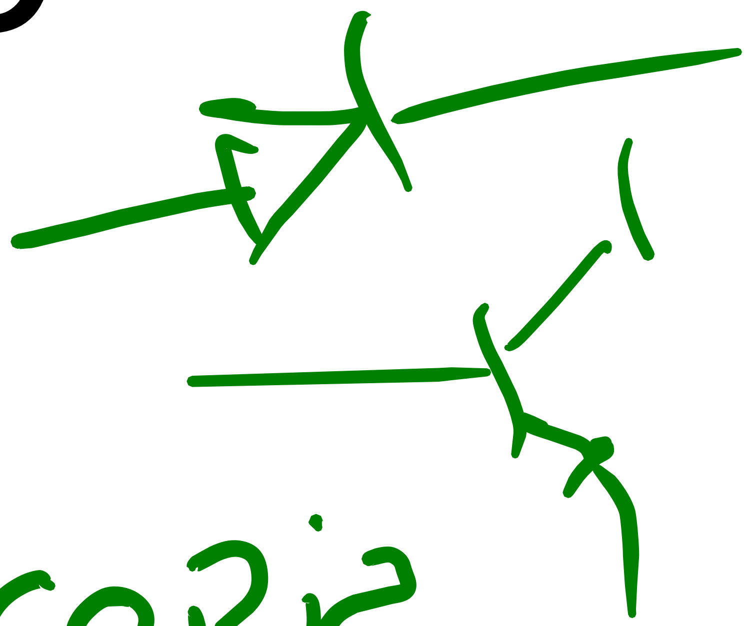


$$s = \cancelto{0}{\sigma} + j\omega$$

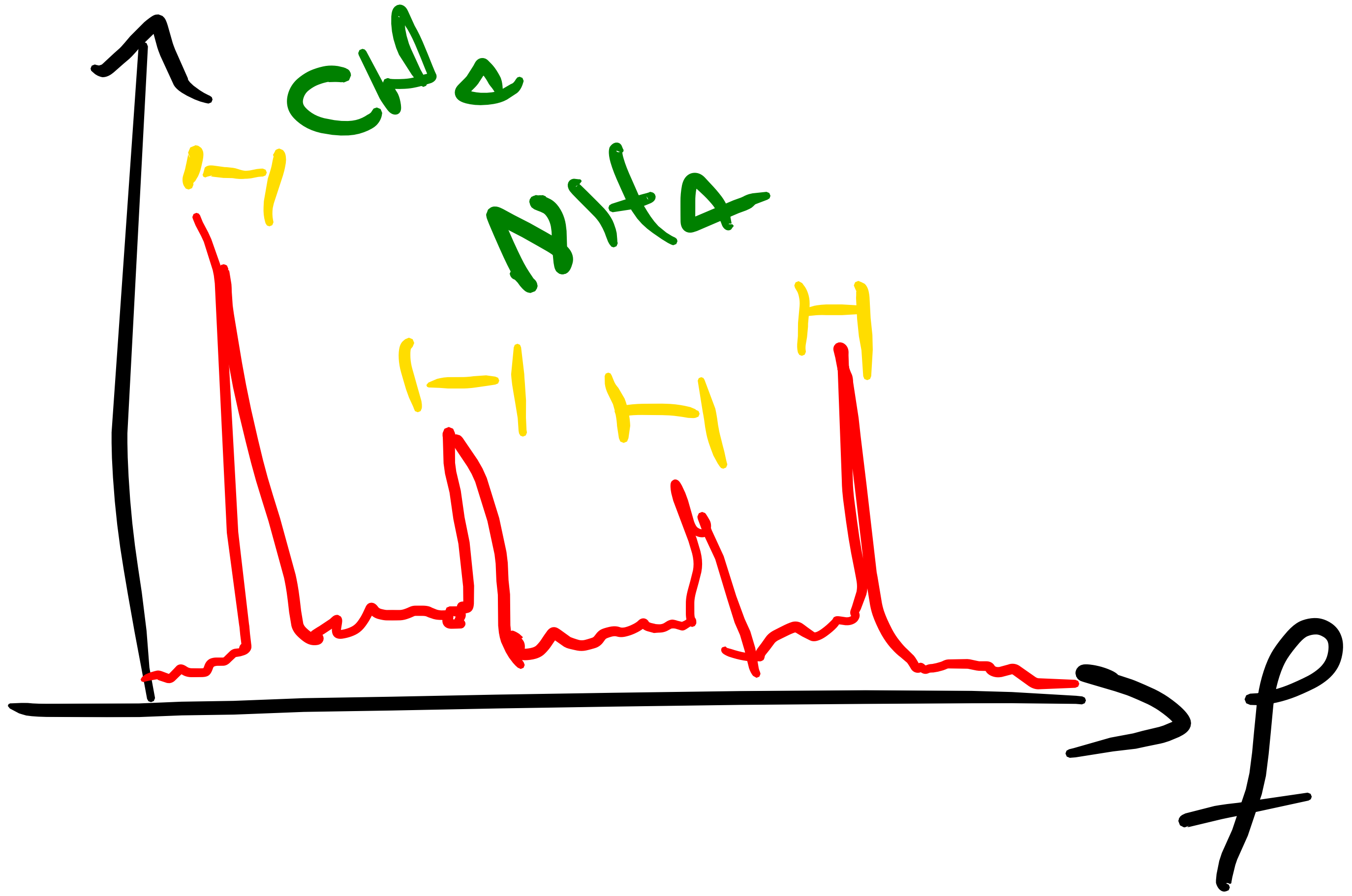


$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = F(\omega)$$

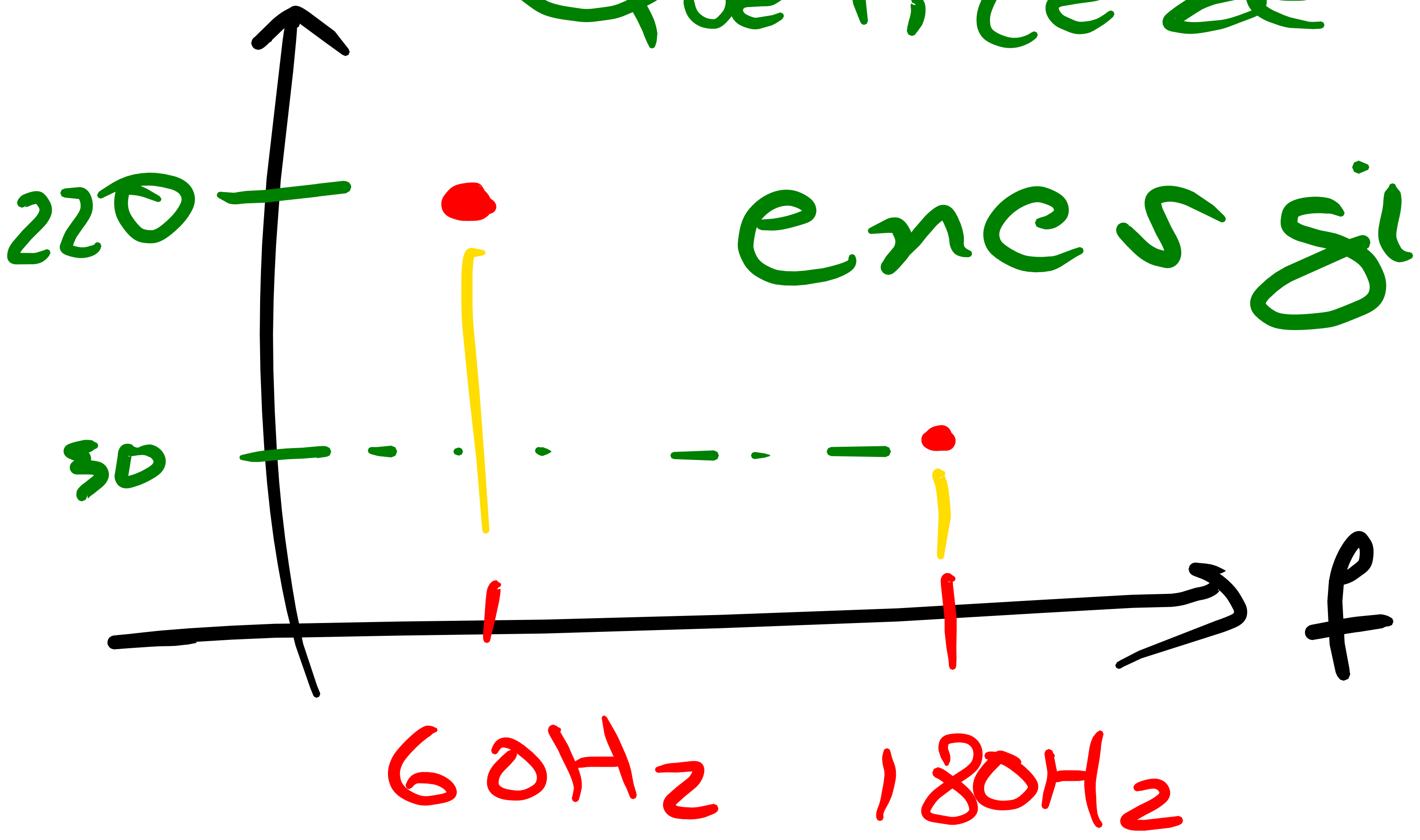
Aplicação



Espectroscopia



Qualidade de energia



$$\mathcal{F}\left\{e^{a(t-b)}u(t-b)\right\} = \frac{e^{-j\omega b}}{j\omega - a}$$

$$\mathcal{F}\left\{\cos(\theta t)\right\}$$

$$\int_{-\infty}^{\infty} \cos\theta t \, e^{j\omega t} \, dt$$

$$\int_{-\infty}^{\infty} \left[\frac{e^{j\theta t} + e^{-j\theta t}}{2} \right] e^{-j\omega t}$$

$$\int_{-\infty}^{\infty} \frac{e^{jt(\theta-\omega)}}{2} + \frac{e^{-jt(\theta-\omega)}}{2}$$

$$\int_{-\infty}^{\infty} \frac{e^{jtx}}{2}\,dt$$

$$x = \theta - \omega$$

$$\left.\frac{e^{jtx}}{2jx}\right|_{-\infty}^{\infty} = \lim_{t\to\infty} \frac{e^{jtx} - e^{-jtx}}{2jx}$$

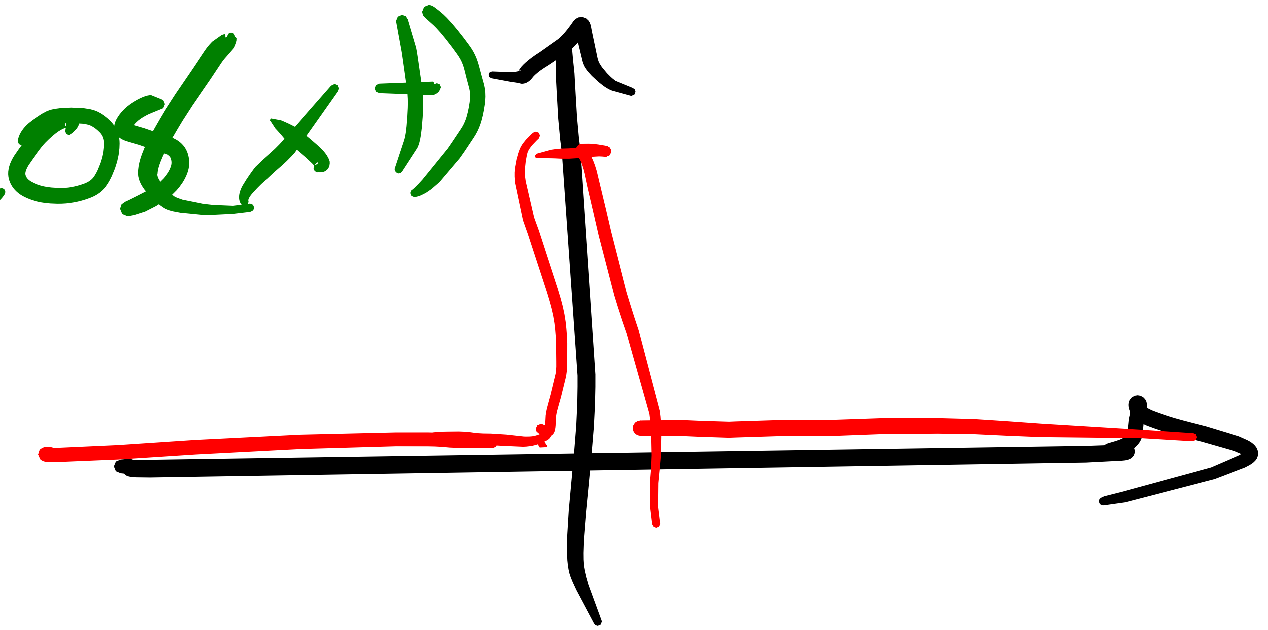
$$\frac{\operatorname{sen}(xt)}{x}$$

$$\lim_{t\to\infty}\frac{\operatorname{sen}(\cancel{xt})}{\cancel{x}}$$

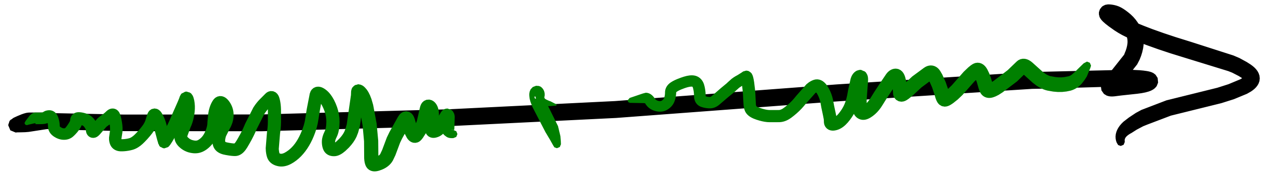
$xt \to 0$

$\to \cos(xt)$

$x \to 0$



$\lim_{x\to 0}.$



$$\tilde{\mathcal{F}}\left\{\cos\theta t\right\} \pi\left(\delta(\theta-\omega)+\delta(\theta+\omega)\right)$$

$$\int_{-\infty}^{\infty} \cos(\theta t)\, e^{-j\omega t}\, dt \qquad \cos\theta t = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\int_{-\infty}^{\infty} e^{jt(\theta-\omega)} + e^{-jt(\theta+\omega)}\, dt$$

$$\frac{e^{jt(\theta-\omega)}}{2j(\theta-\omega)}\Bigg|_{-\infty}^{\infty}$$

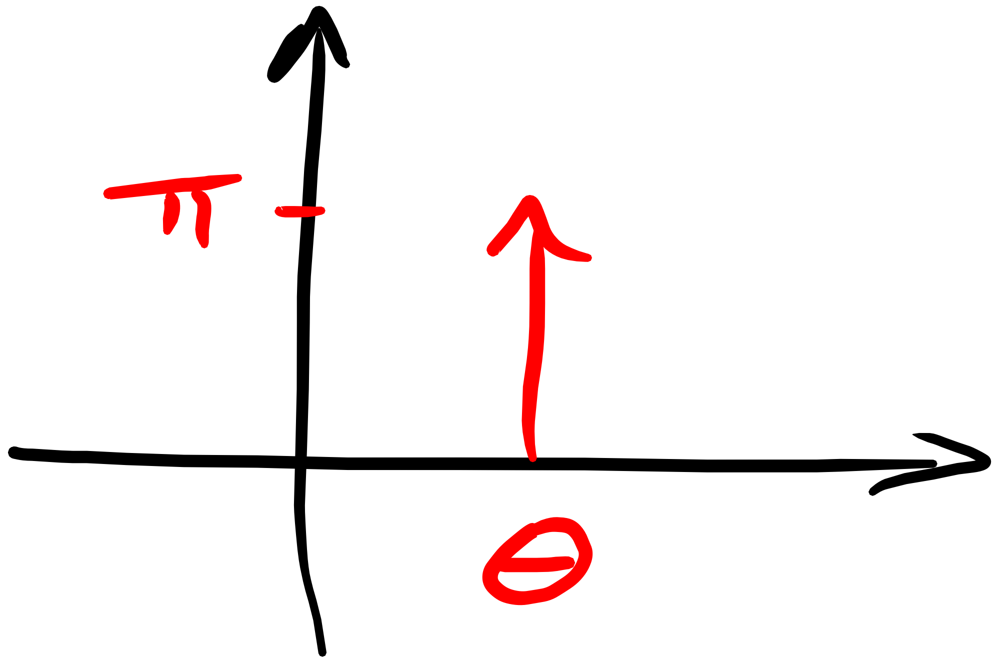
$$\frac{e^{jt(\theta-\omega)}}{2j(\theta-\omega)}\Bigg|_{-\infty}^{\infty}$$

$$\lim_{t\to\infty} \frac{e^{jt(\theta-\omega)} - e^{-jt(\theta-\omega)}}{2j(\theta-\omega)}$$

$$\theta-\omega=x$$

$$\frac{e^{jtx} - e^{-jtx}}{2jx} = \frac{\operatorname{sen}(tx)}{x}$$

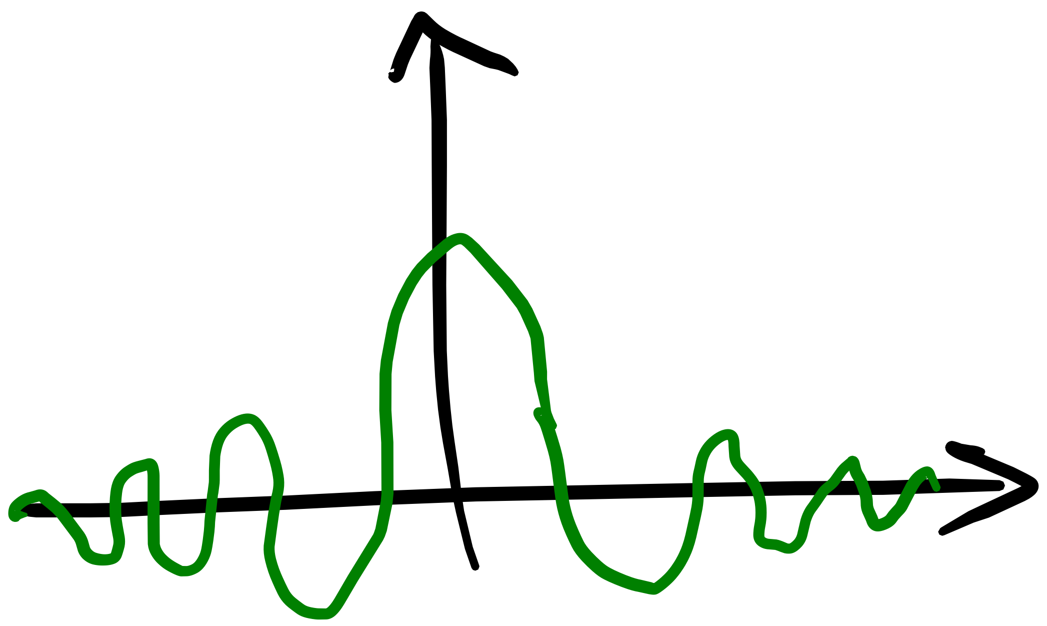
$$\lim_{t \to \infty} \frac{\operatorname{sen}(t(\theta-\omega))}{(\theta-\omega)} = \pi\,\delta(\theta-\omega)$$



$$\mathcal{F}\{\cos(\theta t)\} = \pi\left(\delta(\theta-\omega)+\delta(\theta+\omega)\right)$$

$$\mathcal{F}\{\operatorname{sen}(\theta t)\} = \frac{\pi}{j}\left(\delta(\theta-\omega)-\delta(\theta+\omega)\right)$$

$$\mathcal{F}\{e^{j\theta t}\} = 2\pi\,\delta(\theta-\omega)$$



$$\frac{sen(t)}{t} \quad \text{ou} \quad \frac{sen(\pi t)}{\pi t}$$

Sa(t)

$$\mathcal{F}\{G_T(t)\} = \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\omega T/2} \cdot \frac{T/2}{T/2}$$

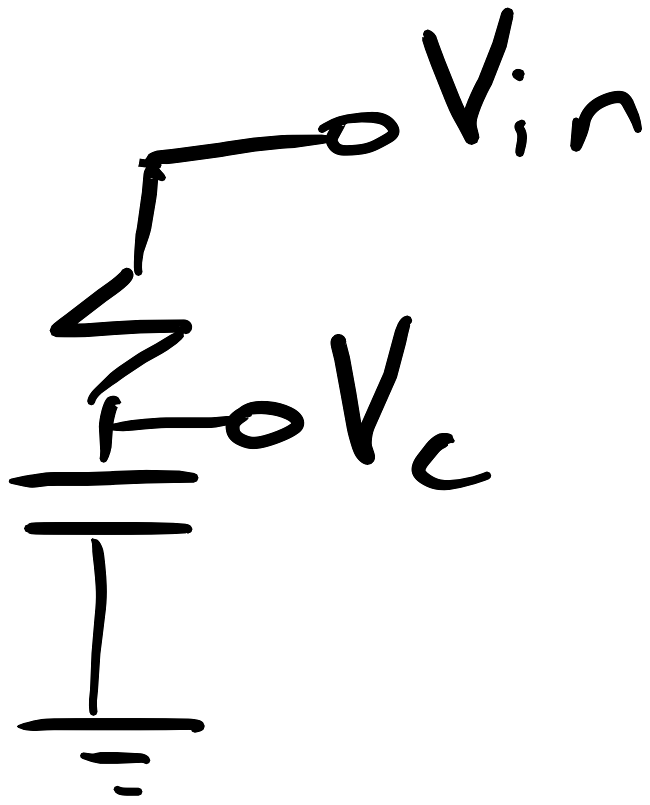
$$= T\,Sa(\omega T/2)$$

$$T\,\frac{\operatorname{sen}(\omega T/2)}{\omega T/2}$$

$$T \, \frac{\operatorname{sen}(\omega T/2)}{\omega T/2} = T \, S_a(\omega T/2)$$

Exercício

Ache $H(\omega)$ $\quad R = 1\Omega \quad C = 100\mu F$



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Obrigado

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