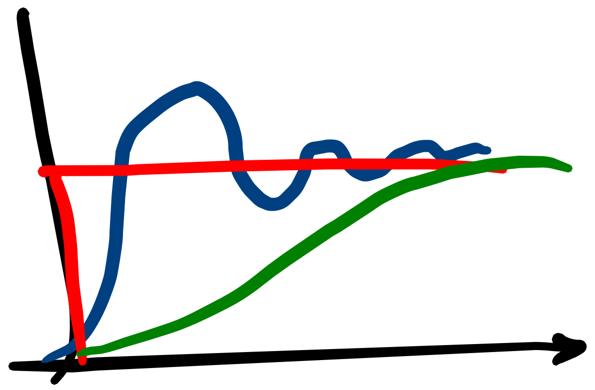
Minicurso Sistemas Lineares



Aula 6

CAEEL

Lucas Zischler

Exercício

1) $\mathcal{F}^{-1}\{\omega\, u(\omega)\}$

$\mathcal{F}\{t\,u(t)\} = \cancel{j}\,\dfrac{d}{d\omega}\,\dfrac{1}{\cancel{j}\omega} = \dfrac{-1}{\omega^2}$

$f(t) = t\,u(t)$

$F(\omega) = \dfrac{-1}{\omega^2}$

$$\mathcal{F}^{-1}\{f(\omega)\} = \frac{1}{2\pi} \cdot F(-t)$$

$$\frac{1}{2\pi} \frac{-1}{t^2}$$

$$\frac{-1}{2\pi t^2}$$

Exercício

2 $\mathcal{Z}\{G_2[n]\}$ $u[n+1]-u[n-1]$

$a=1$

$b=-1$ $\quad$ $b=1$

$$\frac{z}{1-z^{-1}}-\frac{z^{-1}}{1-z^{-1}}$$

$$\frac{z-z^{-1}}{1-z^{-1}}\cdot\frac{z}{z}=\left[\frac{z^2-1}{z-1}\right]$$

Propriedades

~ Algumas de Laplace são válidas ~

Deslocamento

$$\mathcal{Z}\{f[n-b]\} = z^{-b}F(z)$$

Derivada

$$\mathcal{Z}\{n^m f[n]\} = (-z)^m \frac{d}{dz^m}F(z)$$

$$\sum \frac{d}{dz} f[n] z^{-n} = \frac{d}{dz} F(z)$$

$$\sum -n f[n] z^{-n-1} = -z^{-1} \sum n f[n] z^{-n}$$

$$\sum n f[n] z^{-n} = -z \frac{d}{dz} F(z)$$

Frações Parciais

$$\mathcal{Z}^{-1}\{F(z)\}$$

$$\frac{z^{-b}}{1-(az)^{-1}} \cdot \frac{z}{z} = \frac{\boxed{z} \cdot z^{-b}}{z - a^{-1}}$$

Exemplo

$$\frac{z^2 - z}{z^2 + 3z + 2} = \frac{zA}{z+1} + \frac{zB}{z+2}$$

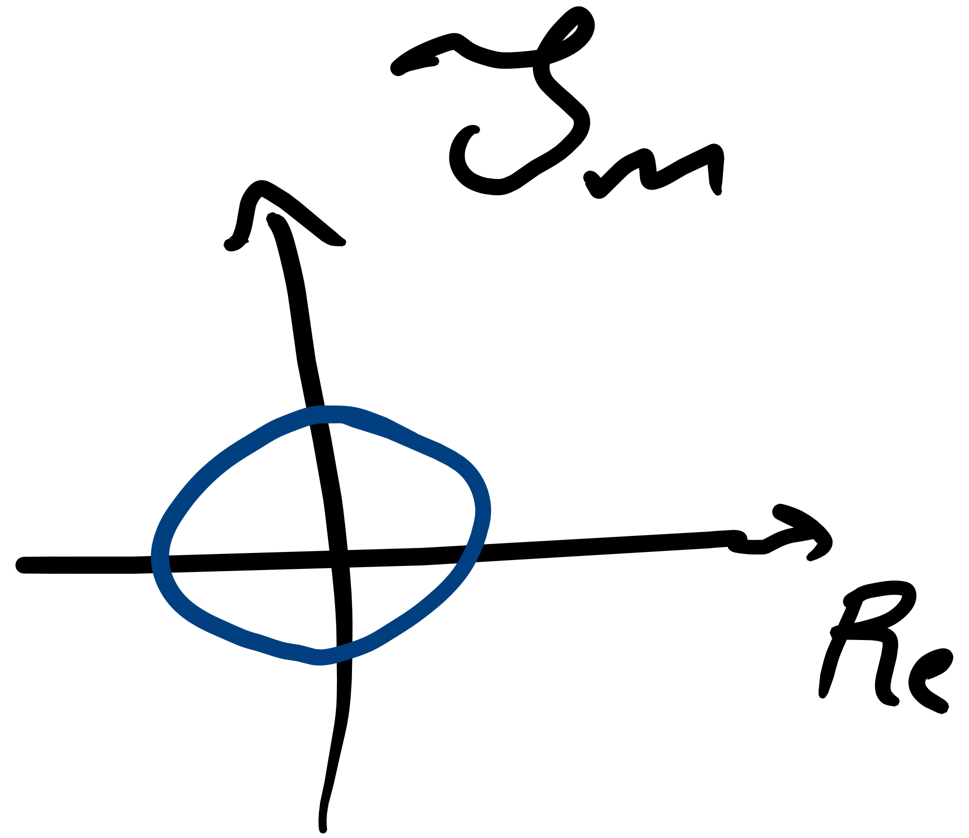
$$A = \left.\frac{z-1}{z+2}\right|_{z=-1} = -2$$

$$B = \left.\frac{z-1}{z+1}\right|_{z=-2} = 3$$

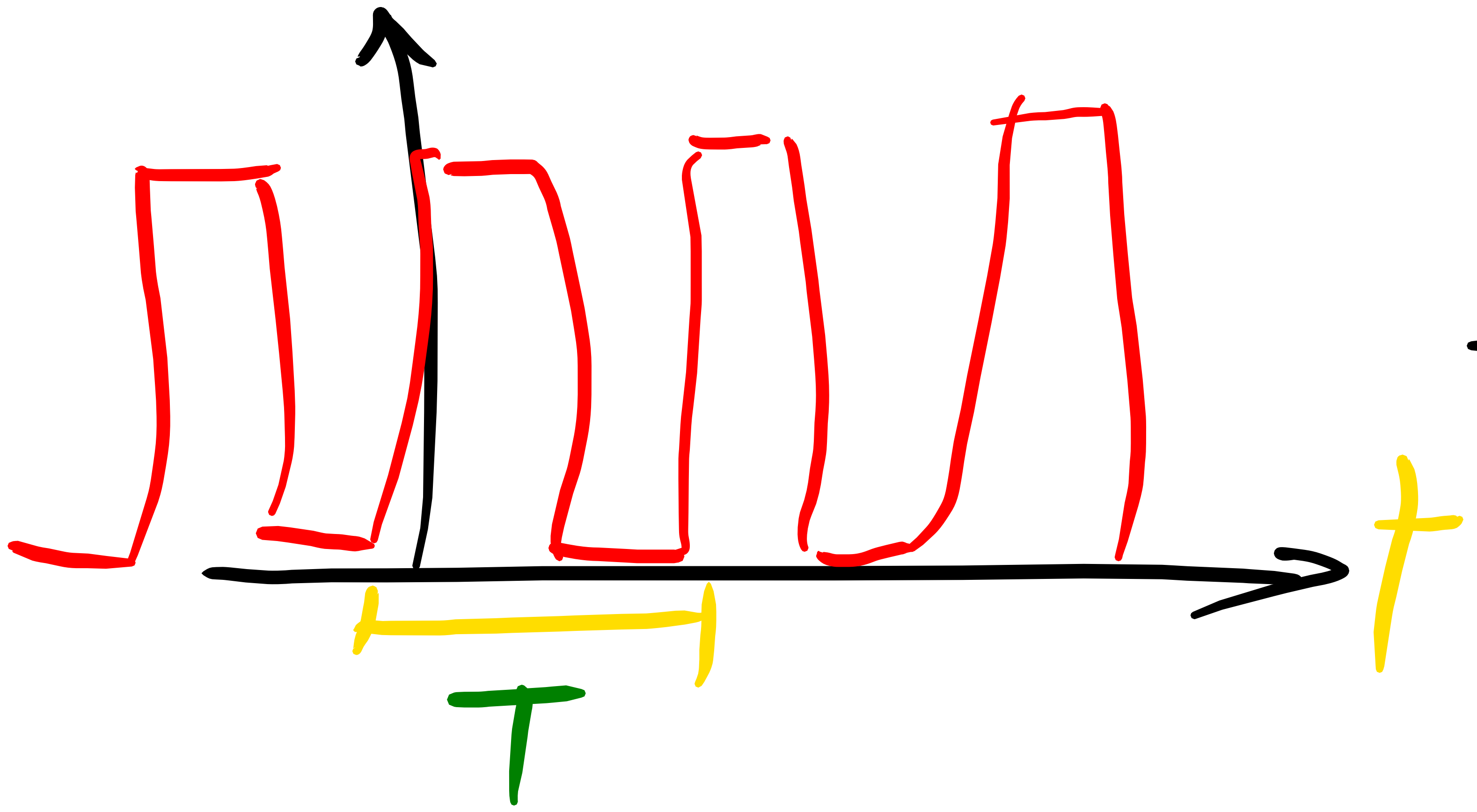
$$\frac{-2z}{z+1} + \frac{3z}{z+2} \qquad z^{-1} = -2 \qquad z = -0.5$$

$$-2(-1)^{-n}u[n] + 3(-0.5)^{-n}u[n]$$

$$F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$



$$x[n] = \frac{1}{2\pi} \int_{2\pi} F(e^{j\omega})\, e^{j\omega n}\, d\omega$$

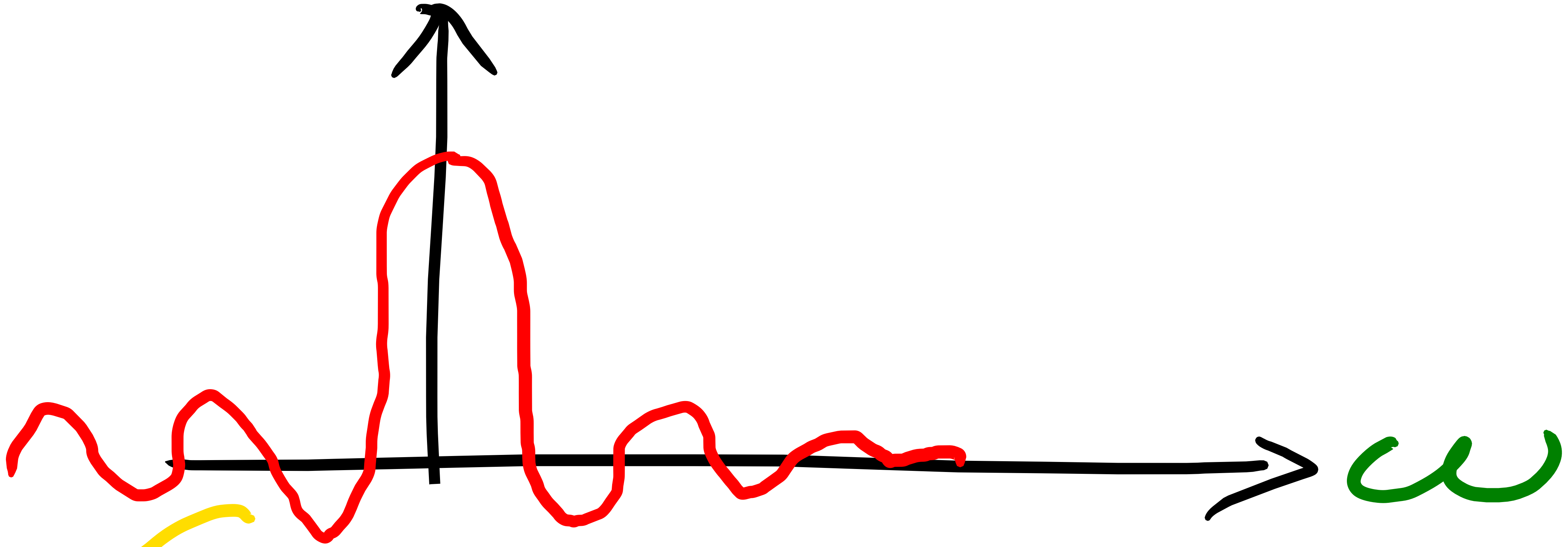


$$\frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

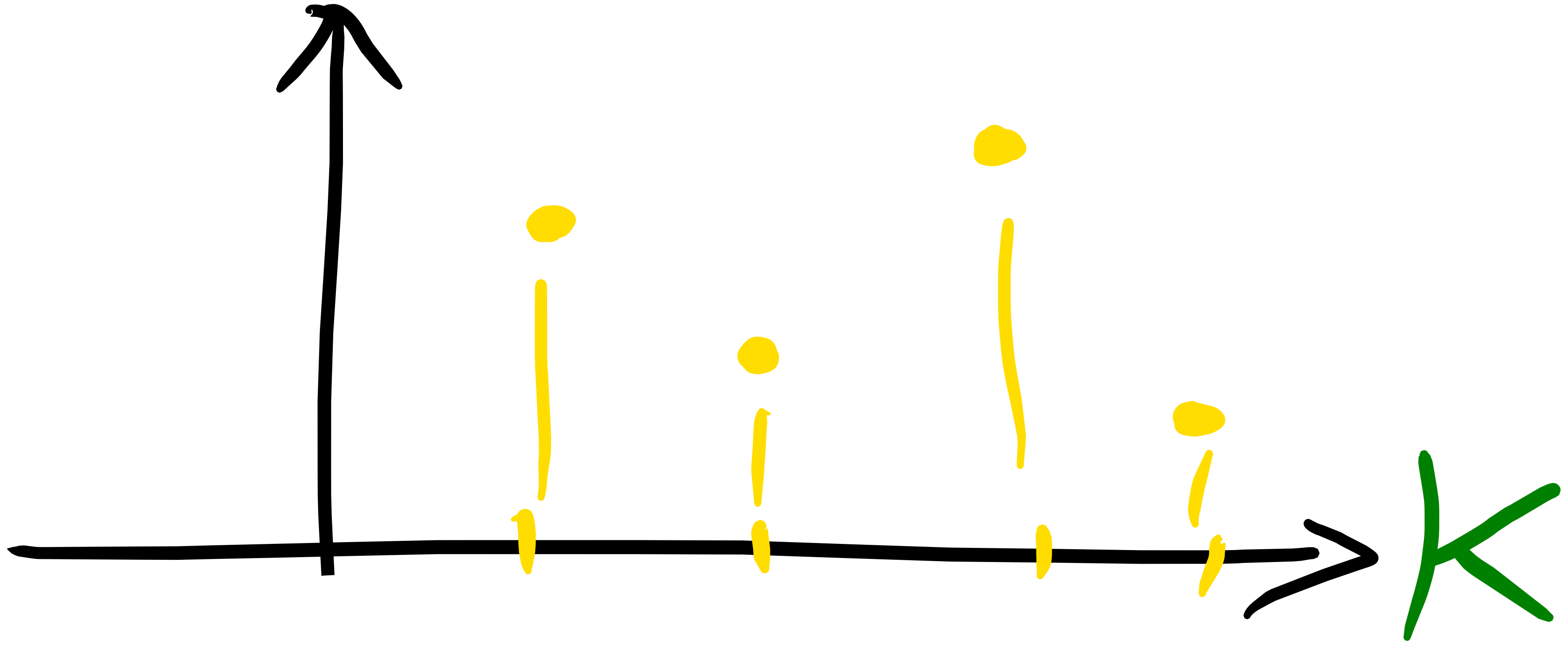
$$\omega = k\omega_0 = k\frac{2\pi}{T}$$

$$c_k = \mathcal{F}_S\{f(t)\}$$

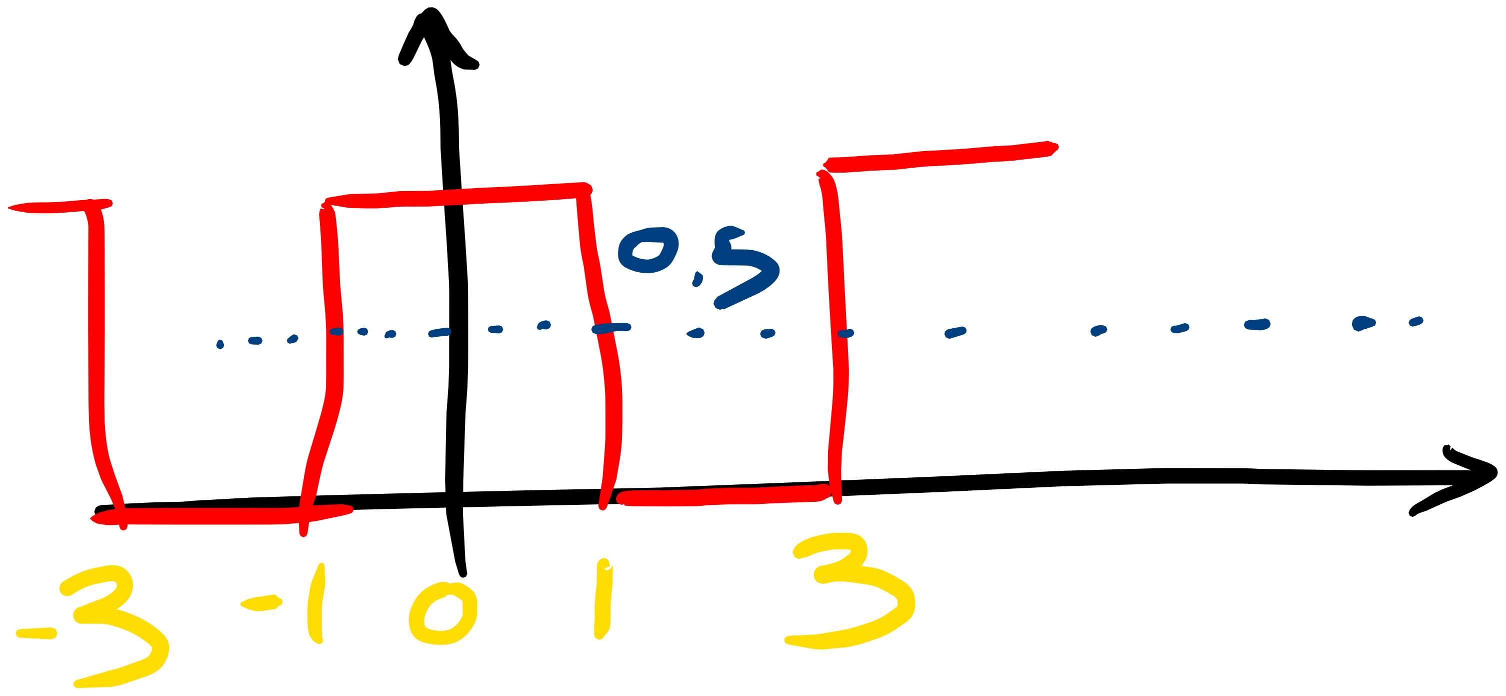
Transformada



Série



Exemplo



$T = 4$

$$\frac{1}{4}\int_{-1}^{1} e^{-jk\omega_0 t}\,dt =$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

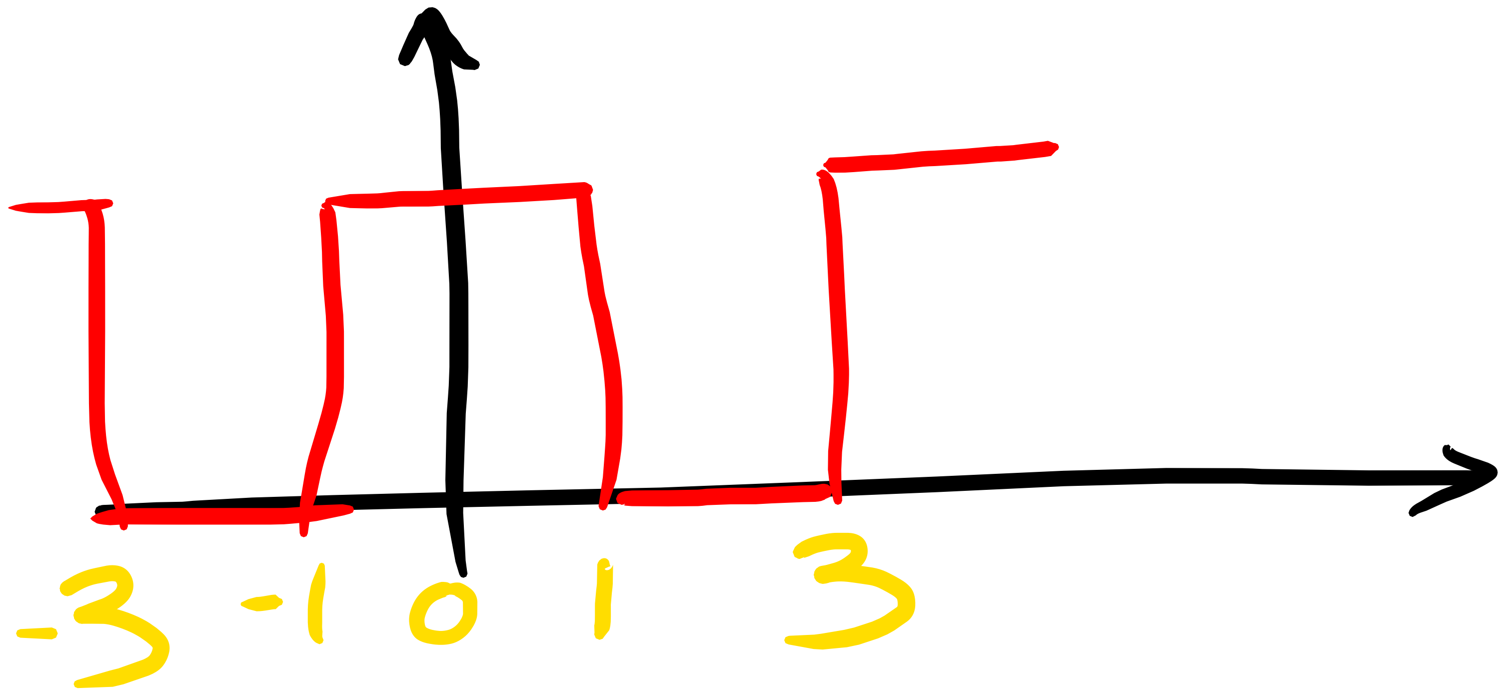
$$\frac{-e^{-jk\omega_0} + e^{jk\omega_0}}{+jk\omega_0 4} = \frac{sen(k\pi/2)}{2k\pi/2}$$

$C_0 = 1/2 = 0.5$

$C_1 = 1/\pi$

Exemplo

$T=4$



$$\frac{1}{4}\int_{-1}^{1} e^{-jK\omega_0 t}\,dt =$$

$$\frac{e^{-jK\omega_0} - e^{jK\omega_0}}{-jK\omega_0} = \frac{1}{2}\,\frac{\text{sen}\left(K\frac{\pi}{2}\right)}{K\pi/2}$$

$$C_0 = 0.5$$

$$C_1 = 1/\pi$$

$$C_2 = 0$$

$$C_3 = -1/3\pi$$

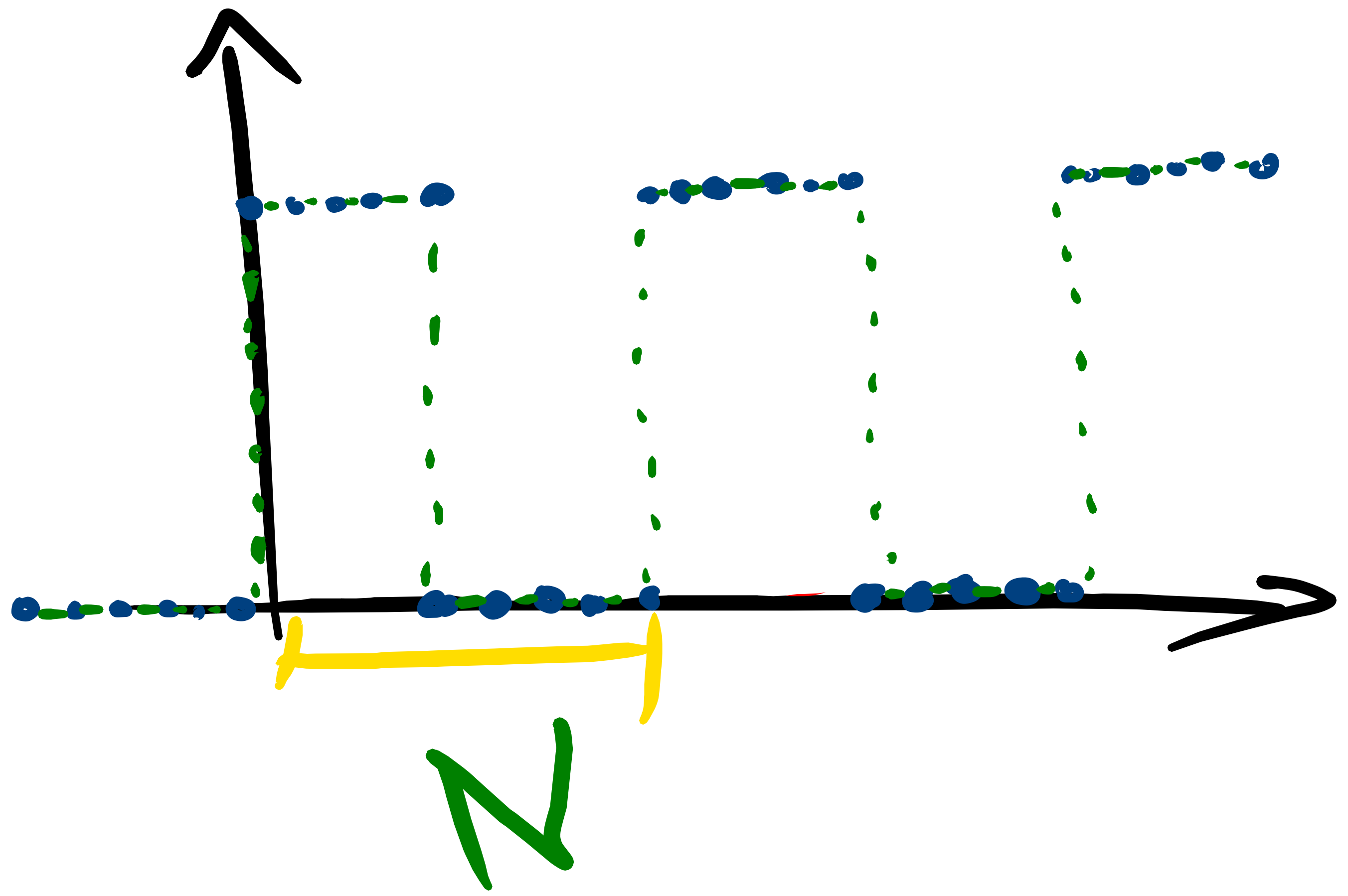
$$\vdots$$

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = a_k + j b_k$$

$$e^{j\theta} = \cos\theta + j\,\mathrm{sen}\,\theta$$

$$e^{-j\theta} = \cos(-\theta) + j\,\mathrm{sen}(-\theta)$$

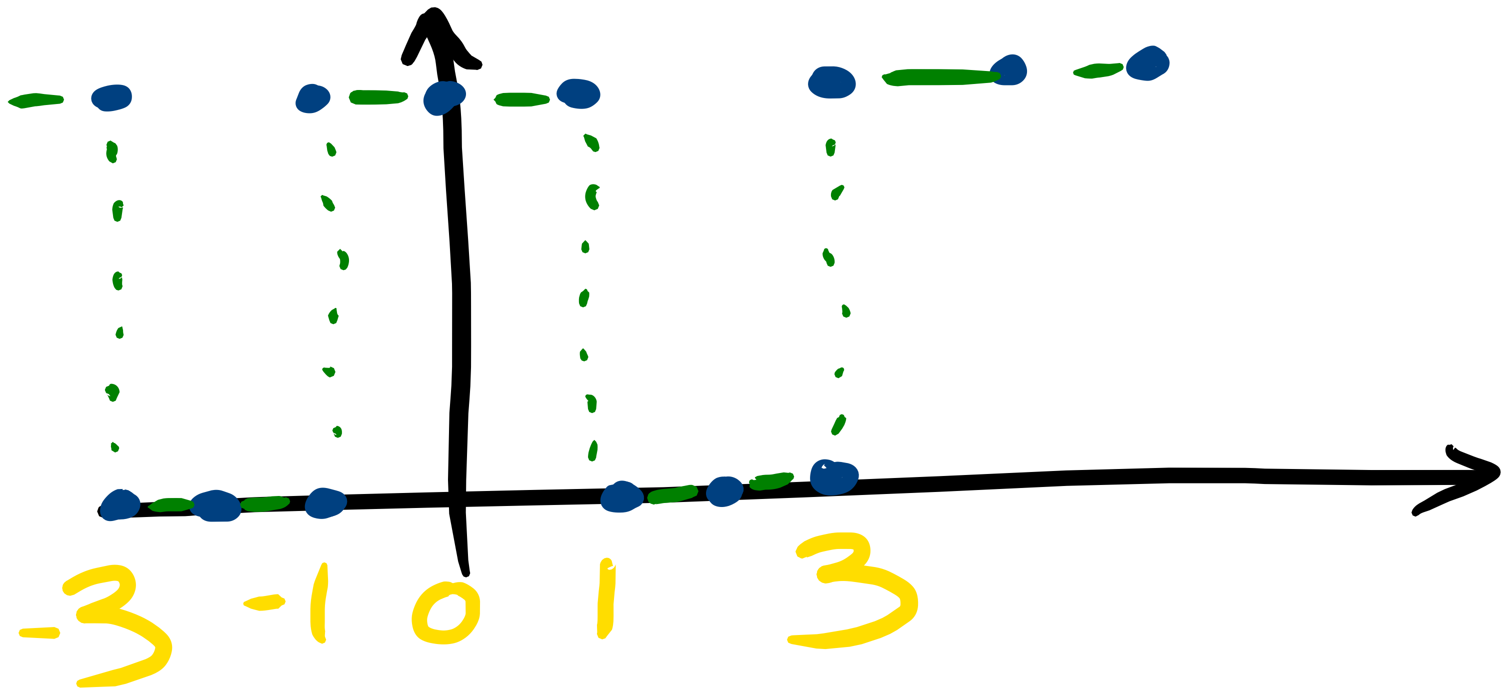


$$\frac{1}{N}\sum_{n}^{N} f[n]\, e^{-jk\frac{2\pi}{N}n}$$

$$\omega_0 = \frac{2\pi}{N}$$

$$C_K = \mathcal{F}_S\left\{f[n]\right\}$$

Exemplo



$N=4$

$$\frac{1}{4}\sum_{0}^{1} e^{-jk\frac{\pi}{2}\cdot n}$$

$$\frac{e^{-jk\frac{\pi}{2}}+1}{4} = \frac{\overset{1}{\cos\left(k\frac{\pi}{2}\right)} - j\,\overset{0}{\mathrm{sen}\left(k\frac{\pi}{2}\right)}+1}{4}$$

$C_0 = 0.5 \quad C_1 = \frac{1}{4} - j\frac{1}{4}$

$C_0 = 0.5 \quad C_1 = \frac{1}{4} - j\frac{1}{4}$

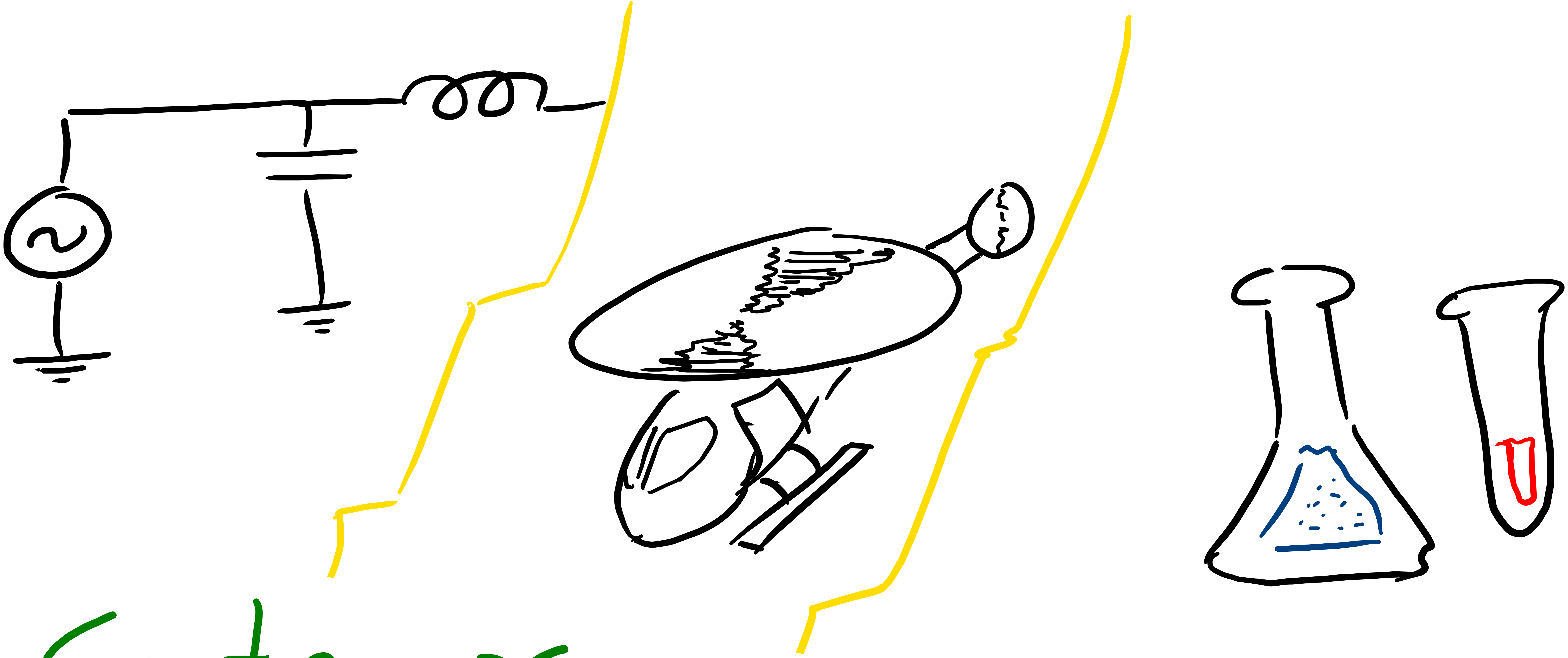
$C_2 = 0 \quad C_3 = \frac{1}{4} + j\frac{1}{4}$

$C_4 = C_0$

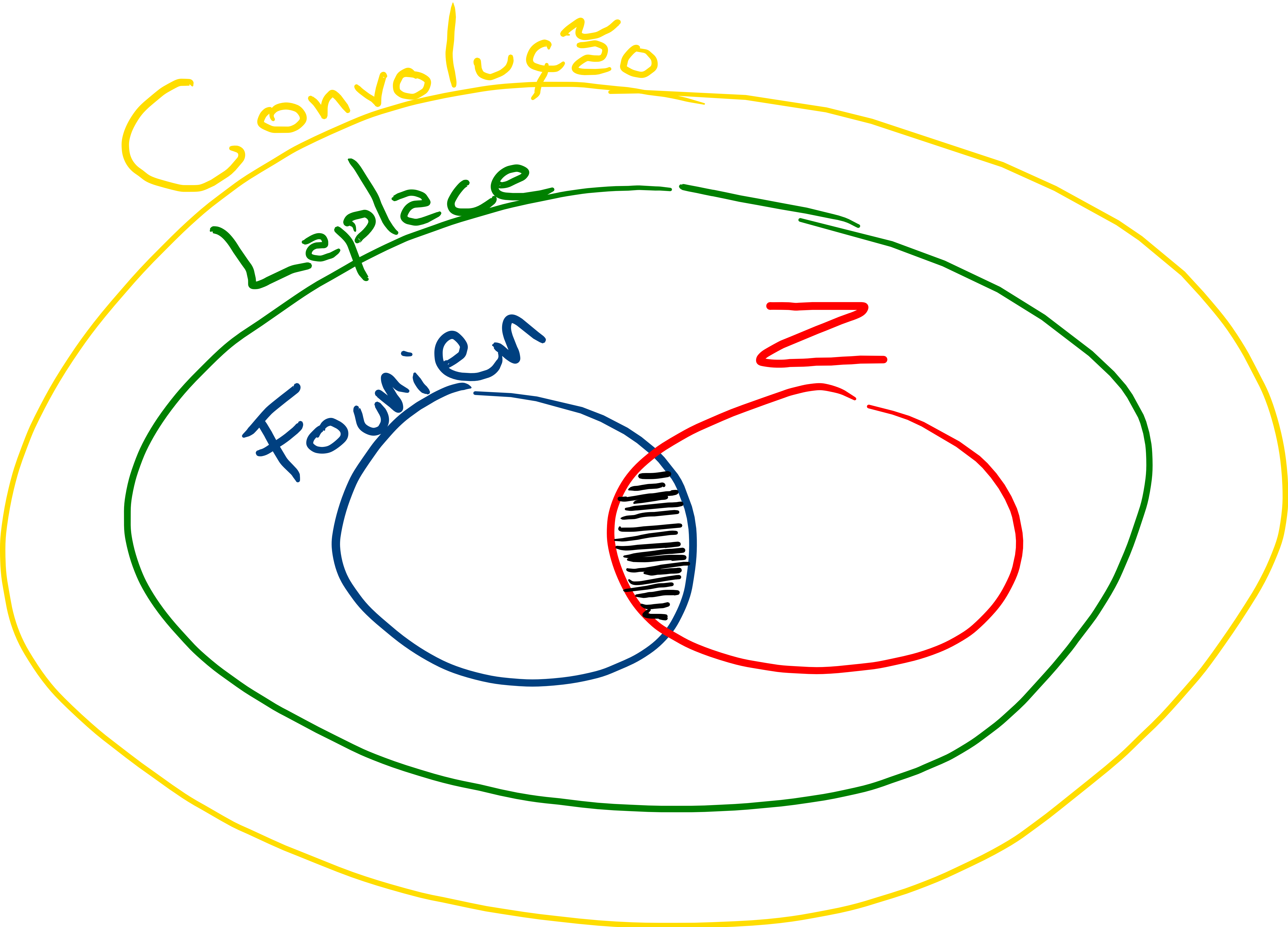
$N = K$

$$\mathcal{F}_s^{-1}\left\{c_k\right\} = \sum_{k}^{N} c_k e^{jk\frac{2\pi}{N}n}$$

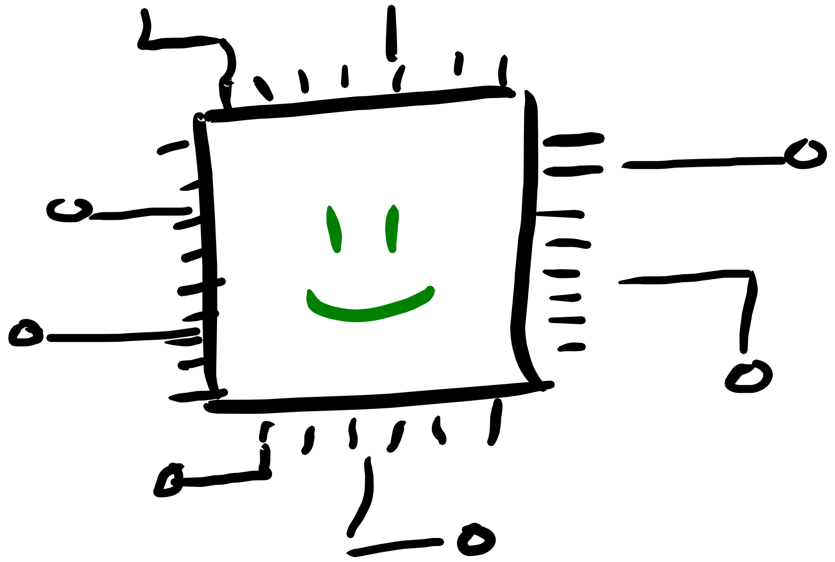
REVISÃO



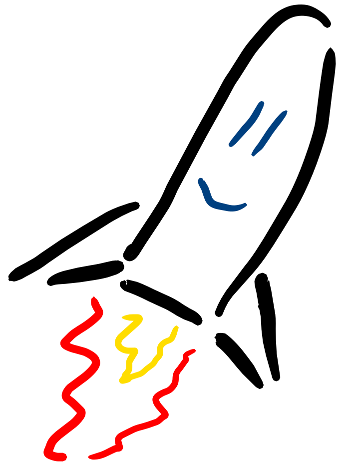
Sistemas Dinâmicos

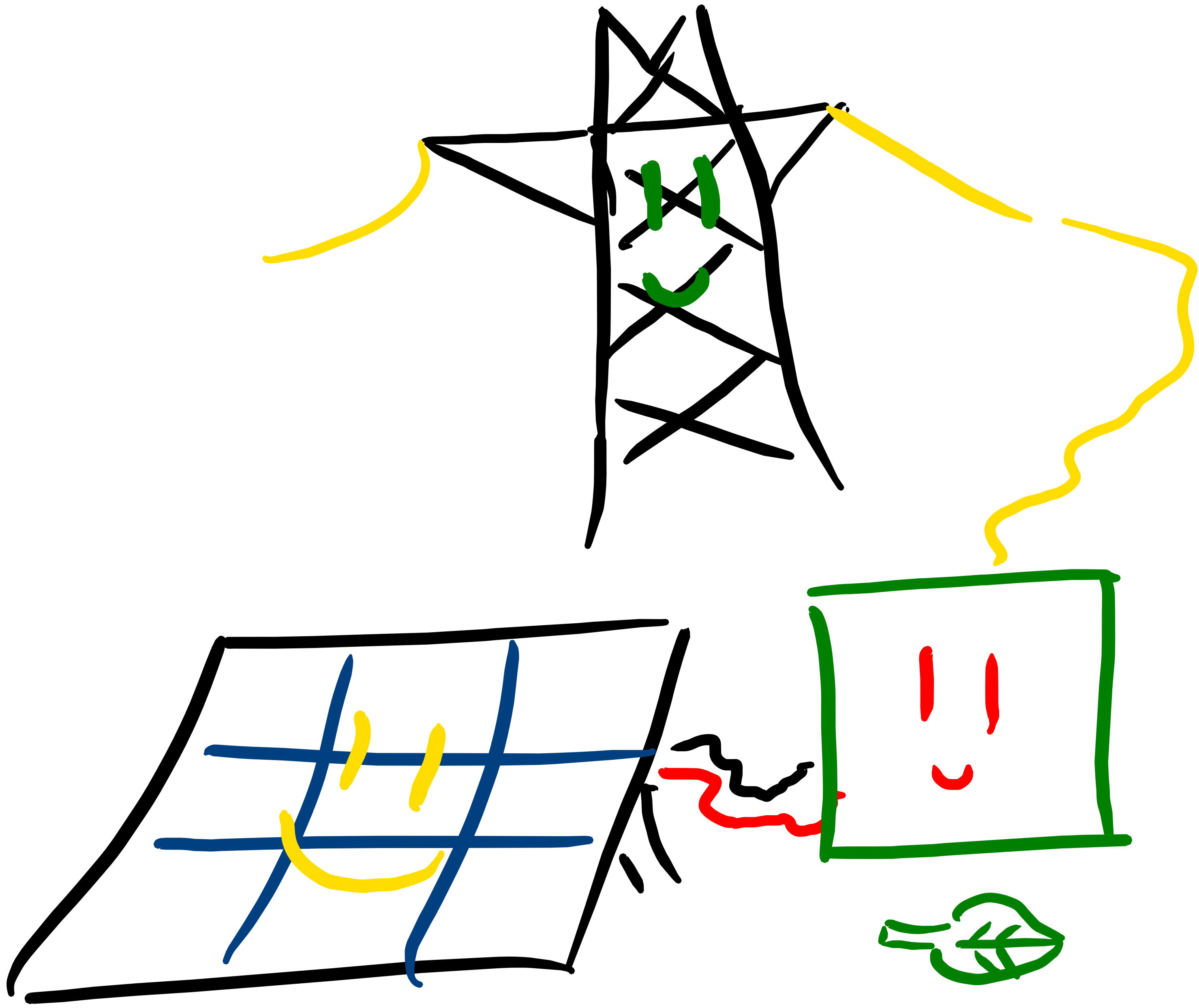

Convolução
Laplace
Fourier
Z

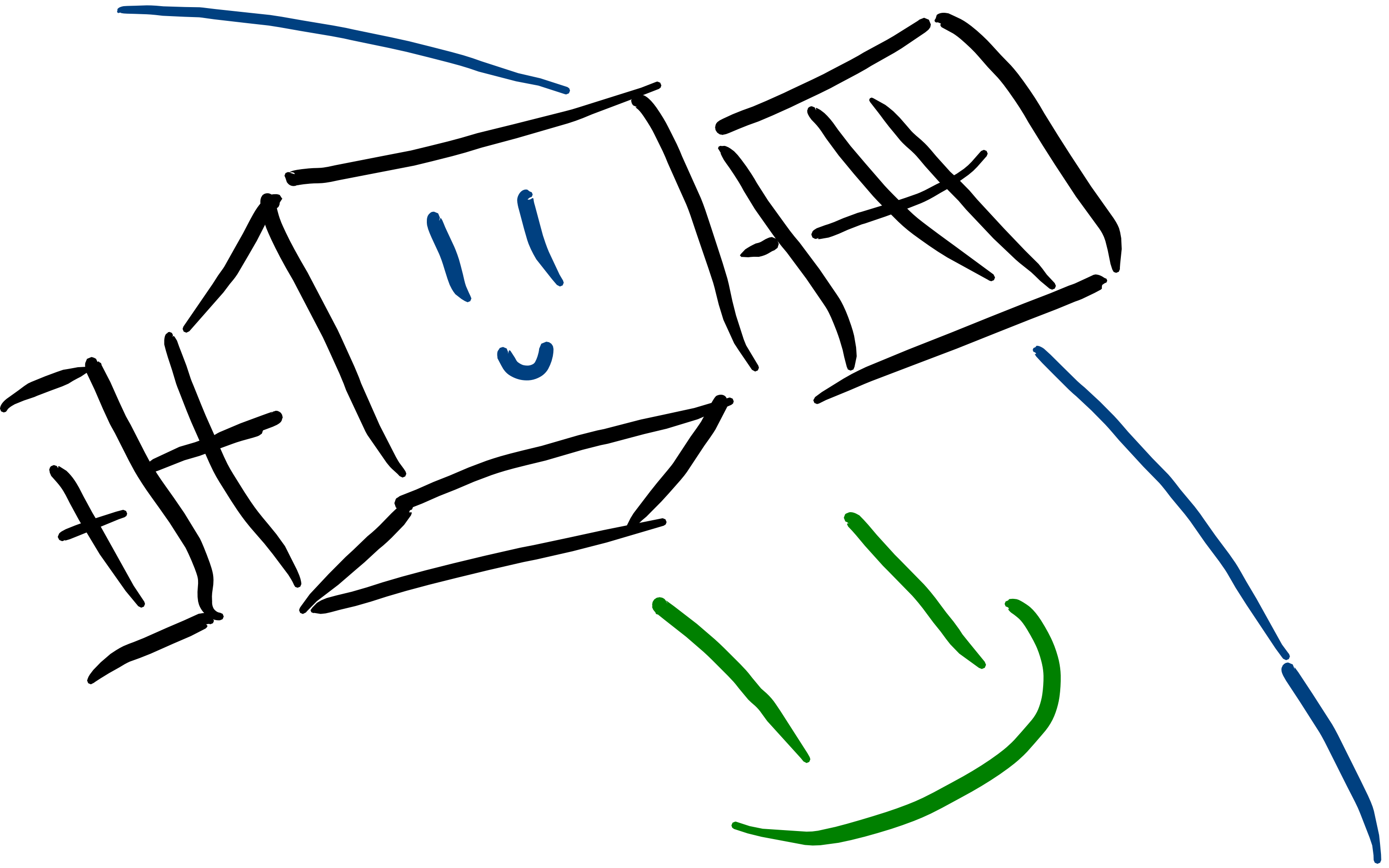
Transformada contínua	Transformada discreta
Série contínua	Série discreta



Fim da matéria







Material e informações de contato:

www.lucas.zischler.nom.br

Obrigado pela atenção