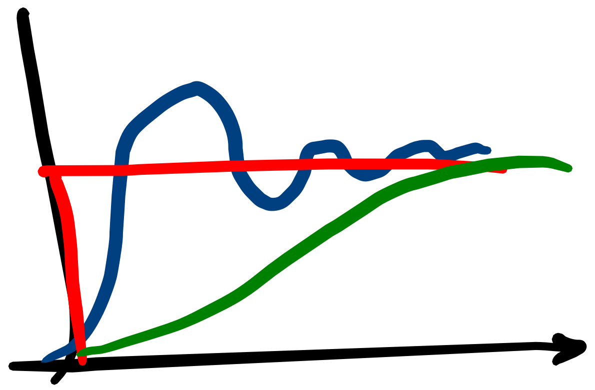
Minicurso Sistemas Lineares



Aula 3

CAEEL

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Exercício

$\mathcal{L}\{G_2(t)\}$ $\quad G_2 = u(t+1) - u(t-1)$

$$\mathcal{L}\{e^{2(t-b)}u(t-b)\} = \frac{e^{-sb}}{s-2}$$

$$\frac{e^{sb} - e^{-sb}}{s}$$

Exercício

$\mathcal{L}\{\cos(\theta t)\,u(t)\}$

$a = j\theta \quad a = -j\theta$

$$\cos(\theta t) = \frac{e^{j\theta t} + e^{-j\theta t}}{2}$$

$$\mathcal{L}\{e^{a(t-b)}\,u(t-b)\} = \frac{e^{-sb}}{s-a}$$

$$\frac{1}{2}\left[\frac{1}{s-j\theta} + \frac{1}{s+j\theta}\right]$$

$$\frac{1}{2}\left[\frac{1}{s-j\theta}+\frac{1}{s+j\theta}\right]=\frac{1}{2}\,\frac{(s+j\theta)+(s-j\theta)}{s^2+\theta^2}$$

$$\frac{s}{s^2+\theta^2}$$

Convolução em Laplace

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(t) * h(t) = y(t)$$

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$X(s) \cdot H(s) = Y(s)$$

Propriedade da Derivada em t

$$\mathcal{L}\left\{\frac{d}{dt^n} f(t)\right\} = s^n F(s)$$

$$\int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = F(s)$$

$$u = e^{-st}$$

$$dv = \dot{f}(t)$$

$$du = -s e^{-st}\, dt$$

$$v = f(t)$$

$$\cancel{\overset{0}{f(t)e^{-st}}}\Big|_0^\infty - \int_0^\infty -s e^{-st} f(t)\, dt$$

$$\int_{-\infty}^{\infty} \frac{d}{dt} f(t) e^{-st} dt = F(s)$$

$$u = e^{-st}$$

$$du = -s e^{-st} dt$$

$$dv = \frac{d}{dt} f(t)$$

$$v = f(t)$$

$$f(t) e^{-st} \Big|_0^{\infty} - \int_0^{\infty} -s e^{-st} f(t) dt$$

$$0$$

$$-\int_0^\infty -se^{-st} f(t)\,dt$$

$$s\underbrace{\int_0^\infty e^{-st} f(t)\,dt}_{F(s)}$$

$$-\int_0^\infty -s\,e^{-st} f(t)\,dt$$

$$s\int_0^\infty e^{-st} f(t)\,dt$$

$$s \cdot F(s)$$

Propriedade da Derivada em s

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\int_0^\infty (t)^n f(t) e^{-st} dt = (-1)^n \frac{d^n}{ds^n} F(s)$$

Exemplo

$$\mathcal{L}\{t^2 u(t)\}$$

$$\frac{e^{-sb}}{s-a} \quad b=0 \quad a=0$$

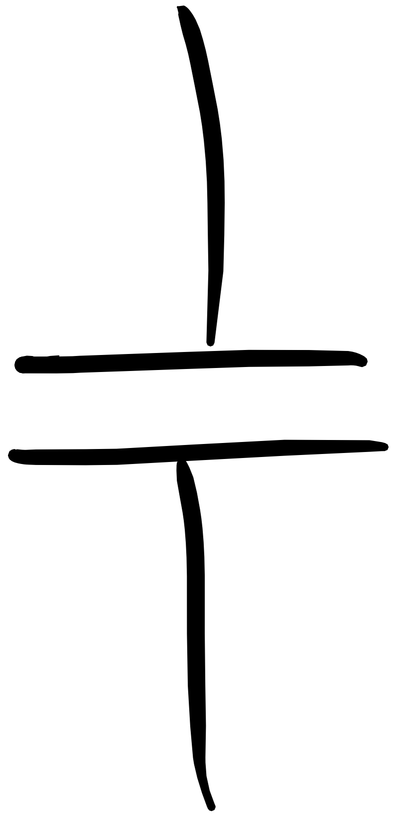
$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$(-1)^2 \frac{d^2}{ds^2}\frac{1}{s} = \frac{d}{ds}\frac{-1}{s^2} = \frac{2}{s^3}$$

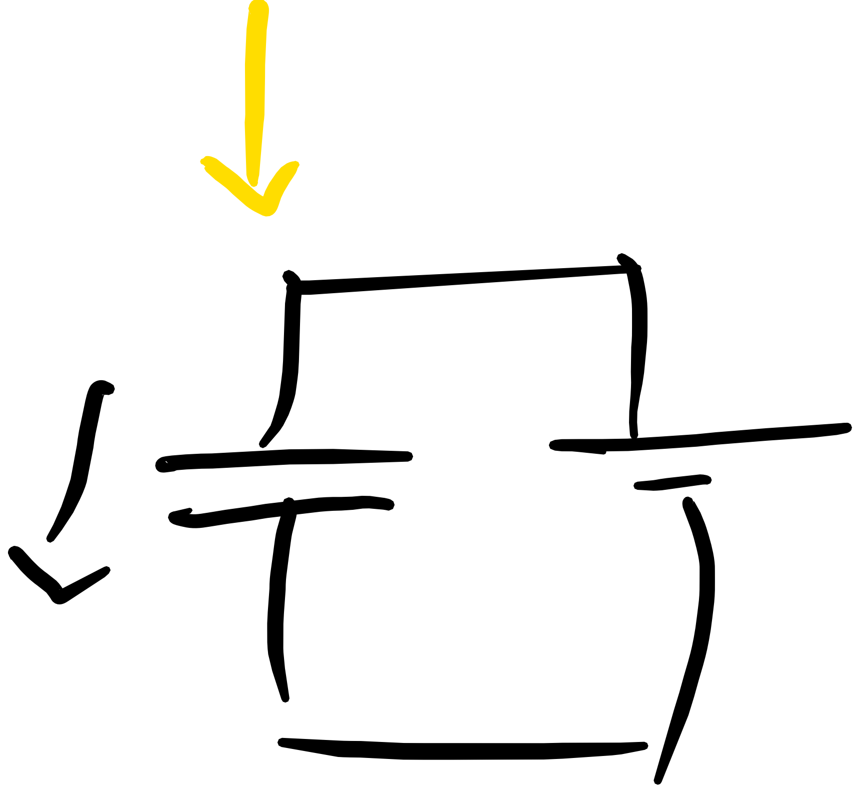
ELEMENTOS DE CIRCUITO

Capacitor

$$i(t) = C\frac{dv(t)}{dt}$$



$$\frac{V}{I} = \frac{1}{sC}$$



$$I = VsC$$

$$Z = \frac{1}{sC}$$

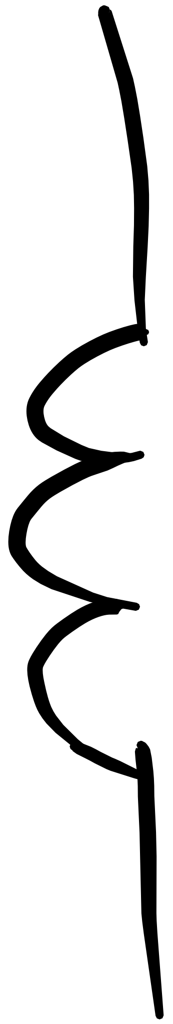
ELEMENTOS DE CIRCUITO

Indutor

$$v(t) = L\,\frac{d\,i(t)}{dt}$$

$$\downarrow$$

$$V(s) = L\,s\,I(s)$$



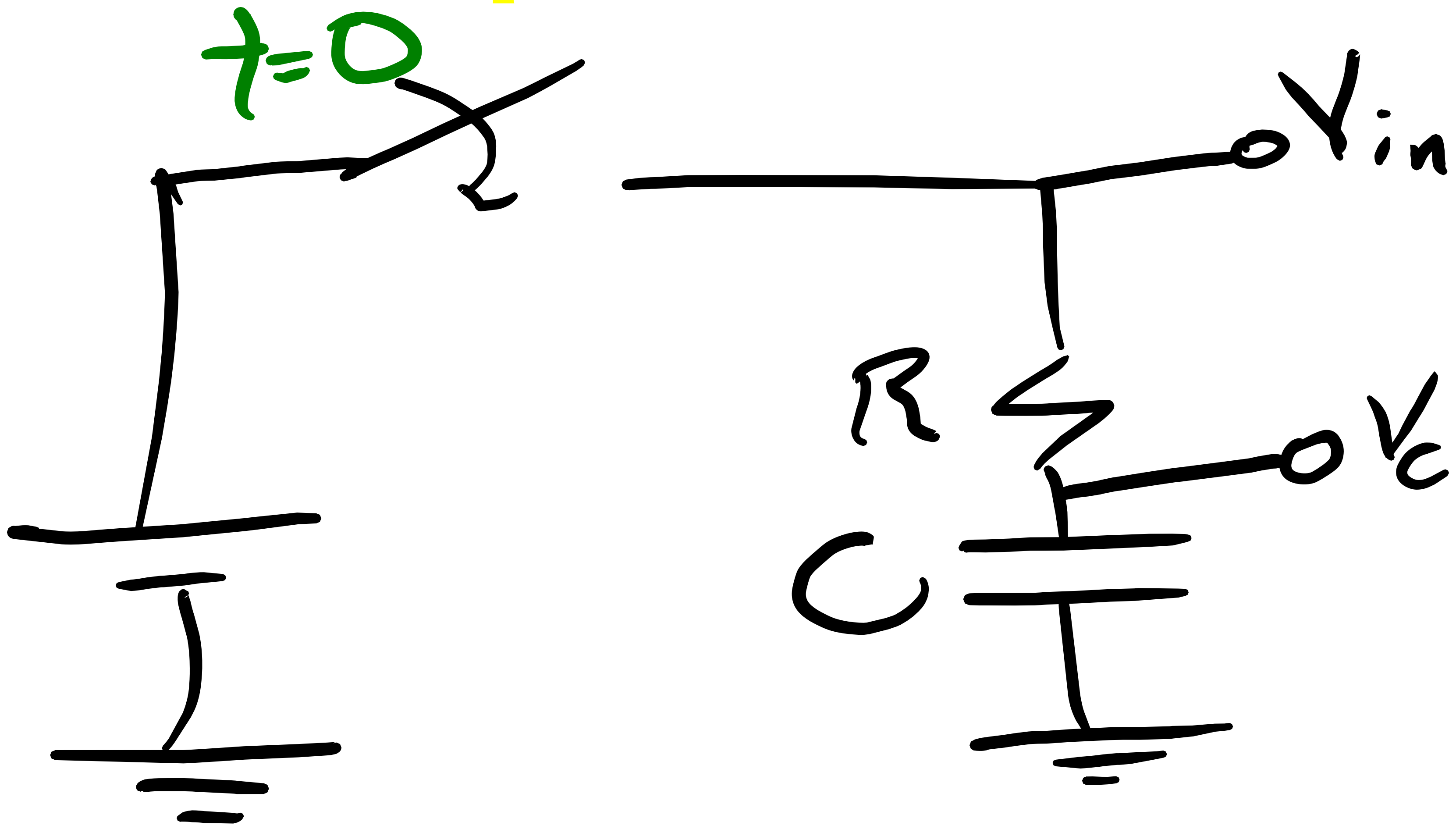
$$Z = sL$$

Valor inicial e final

Inicial $\rightarrow$ $t = 0$ $\quad s = \infty$

Final $\rightarrow$ $t = \infty$ $\quad s = 0$

Exemplo



$$V_{in} \longrightarrow \boxed{H(s)} \longrightarrow V_c$$

$$V_c = V_{in} \cdot \frac{\cancel{1/sC}}{\cancel{R} + \cancel{1/sC}}$$

∞ ∞

$$V_c(0) = 0$$

$$V_c(\infty) = V_{in} \frac{\cancel{1/sC}}{\cancel{1/sC}} = V_{in}$$

Frações Parciais

$$F(s) = \frac{2s-3}{s^2-3s+2} = \frac{A}{s-1} + \frac{B}{s-2}$$

$2-3=-1$

$$A = \frac{2s-3}{s^2-3s+2}\cdot(s-1)\Bigg|_{s=1} = \frac{2s-3}{s-2}\Bigg|_{s=1} = 1$$

$1-2$

$=-1$

$4-3$

$$B = \frac{2s-3}{s-1}\Bigg|_{s=2} = 1$$

$2-1$

Frações Parciais Pólos Iguais

$$F(s) = \frac{s}{s^2-2s+1} = \frac{A}{(s-1)^2} + \frac{B}{s-1}$$

$$A = \frac{s}{\cancel{s^2-2s+1}}\cancel{(s-1)^2}\Big|_{s=1} = 1$$

$$B = \frac{ds}{ds}\Big|_{s=1} = 1$$

$$\frac{1}{(s-1)^2} \quad ? \quad \frac{1}{s-1}$$

$$\longrightarrow e^{-t}u(t)$$

$$-\frac{d}{ds}\frac{1}{s-1} = \frac{1}{(s-1)^2}$$

$$t\,e^{-t}u(t)$$

Exercício

1) $\mathcal{L}\{t^2\, G_2(t)\}$

$$\frac{e^{s} - e^{-s}}{s}$$

2) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+1}\right\}$

Material e informações de contato:

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Obrigado

pela

atenção