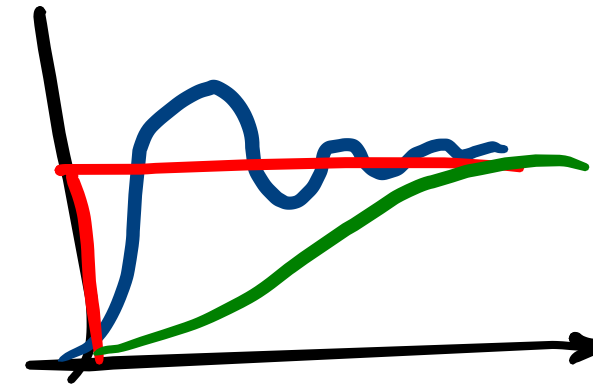


Minicurso

Sistemas

Lineares

Aula 4



Lucas Zischler

Exercício

① $\mathcal{L}\{t^2 G_2(t)\}$

$(-1)^2 \frac{d}{ds^2} \left(\frac{e^s - e^{-s}}{s} \right)$

$$\frac{d}{ds} \frac{e^s}{s} = \frac{e^s}{s} + \frac{-1e^s}{s^2}$$

$$\frac{e^s - e^{-s}}{s}$$

$$\frac{C}{s} + \frac{e^s}{s} + \frac{-1e^s}{s^2}$$

$$\frac{e^s}{s} - \frac{e^s}{s^2} \left[\frac{e^s}{s^2} - \frac{2e^s}{s^3} \right]$$

$$\frac{e^s}{s} - \frac{2e^s}{s^2} + \frac{2e^s}{s^3}$$

$$\frac{d}{ds} \frac{e^{-s}}{s}$$

$$\frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \frac{2e^{-s}}{s^3}$$

$$-\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2}$$



$$-\left[\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} \right] - \left[\frac{e^{-s}}{s^2} - \frac{2e^{-s}}{s^3} \right]$$

$$\frac{e^s(s^2-2s+2)}{s^3} - \frac{e^{-s}(s^2-2s+2)}{s^3}$$

$$\frac{e^s(s^2-2s+2) - e^{-s}(s^2-2s+2)}{s^3}$$

Exercício

$$\textcircled{2} \mathcal{L}^{-1} \left\{ \frac{s+1}{s^2+2s+1} \right\}$$

$$s' = -1$$

$$s'' = -1$$

$$\frac{\overset{''}{A}}{(s+1)^2} + \frac{B}{(s+1)}$$

$$A = F(s)(s+1)^2 \Big|_{s=-1} = s+1 \Big|_{s=-1} = 0$$

$$B = 1 \Big|_{s=-1} = 1$$

$$\frac{1}{s+1} \Rightarrow \frac{e^{-s\tau} \rightarrow 0}{s-2}$$

$$e^{-t} u(t)$$

Convolução distributiva

$$[u(t+1) - u(t-1)] * u(t)$$

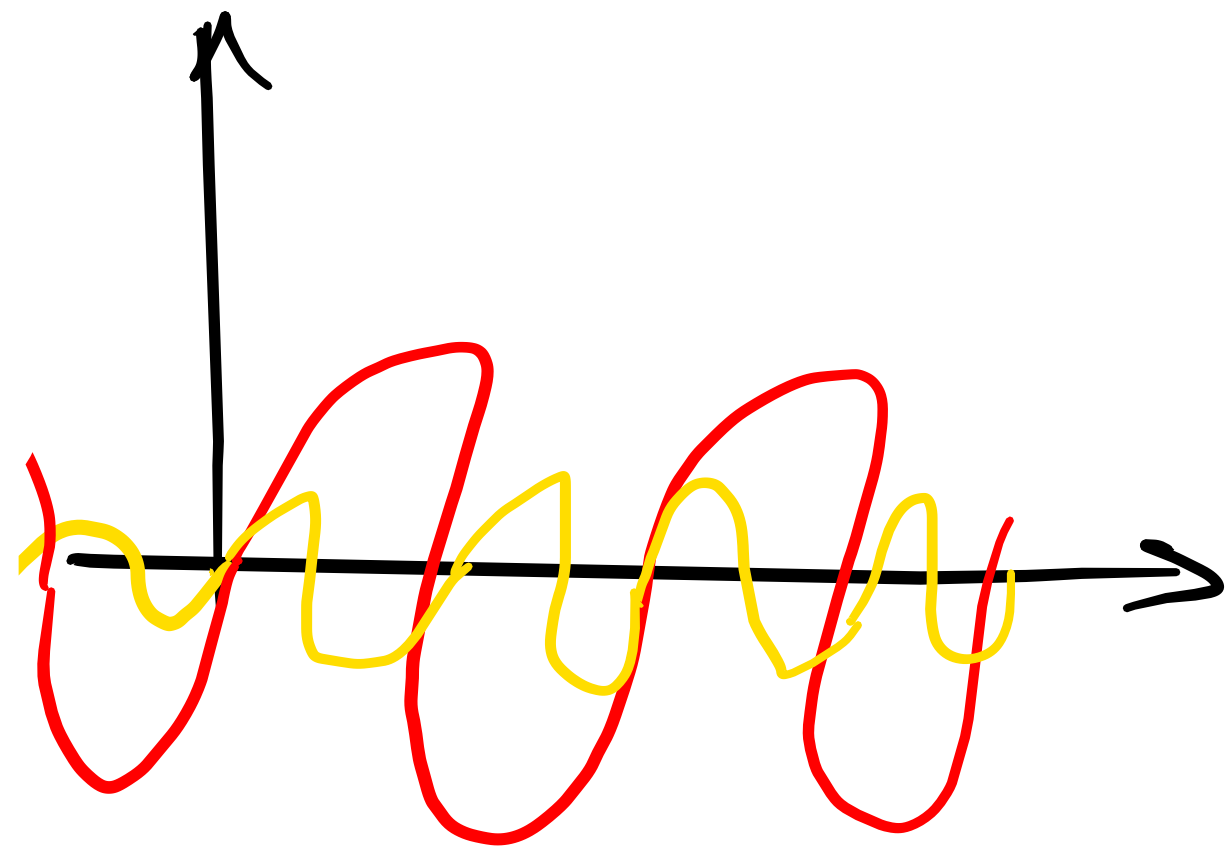
$$1(t)u(t+1) - 1(t-1)u(t-1)$$

Exemplo

$$[u(t) + u(t-1)] * [(t+1)u(t+1) + u(t)]$$

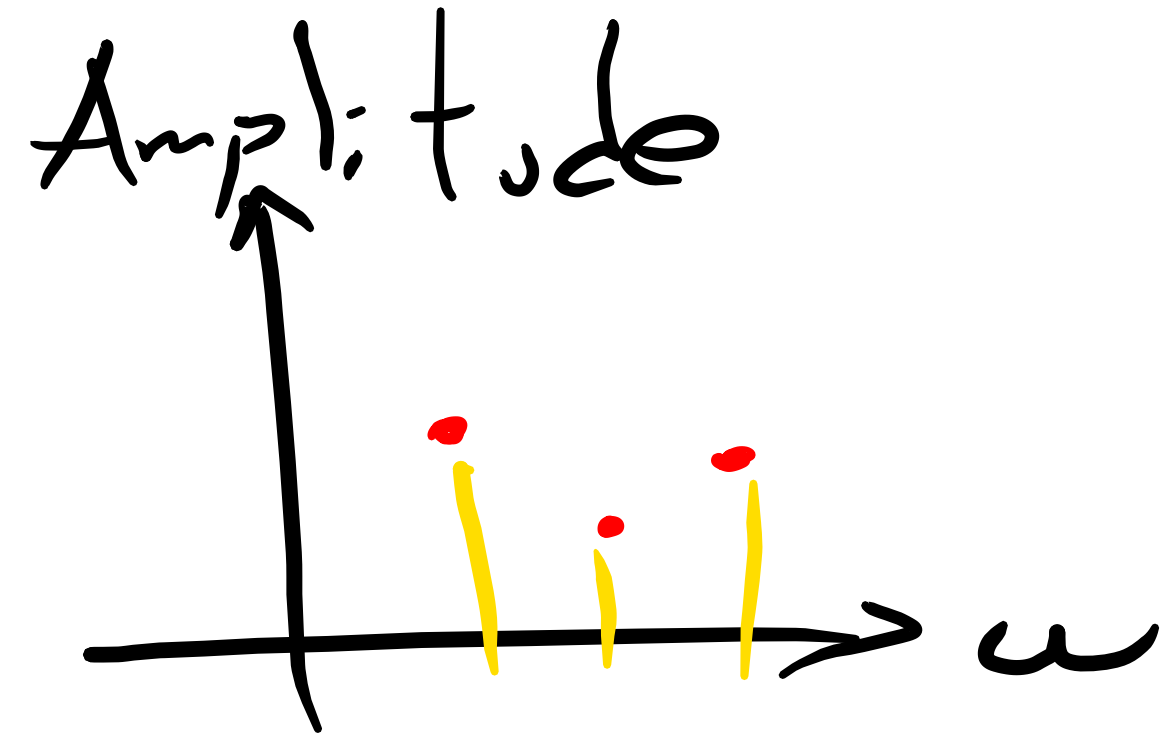
$$\frac{(t+1)^2}{2} u(t+1) + t u(t) + \frac{t^2}{2} u(t) + (t-1) u(t-1)$$

Transformada de Fourier



$$s = \sigma + j\omega$$

The equation $s = \sigma + j\omega$ is written in blue and red ink. A yellow arrow points from the σ term to a yellow 'D' above it, indicating the damping coefficient.

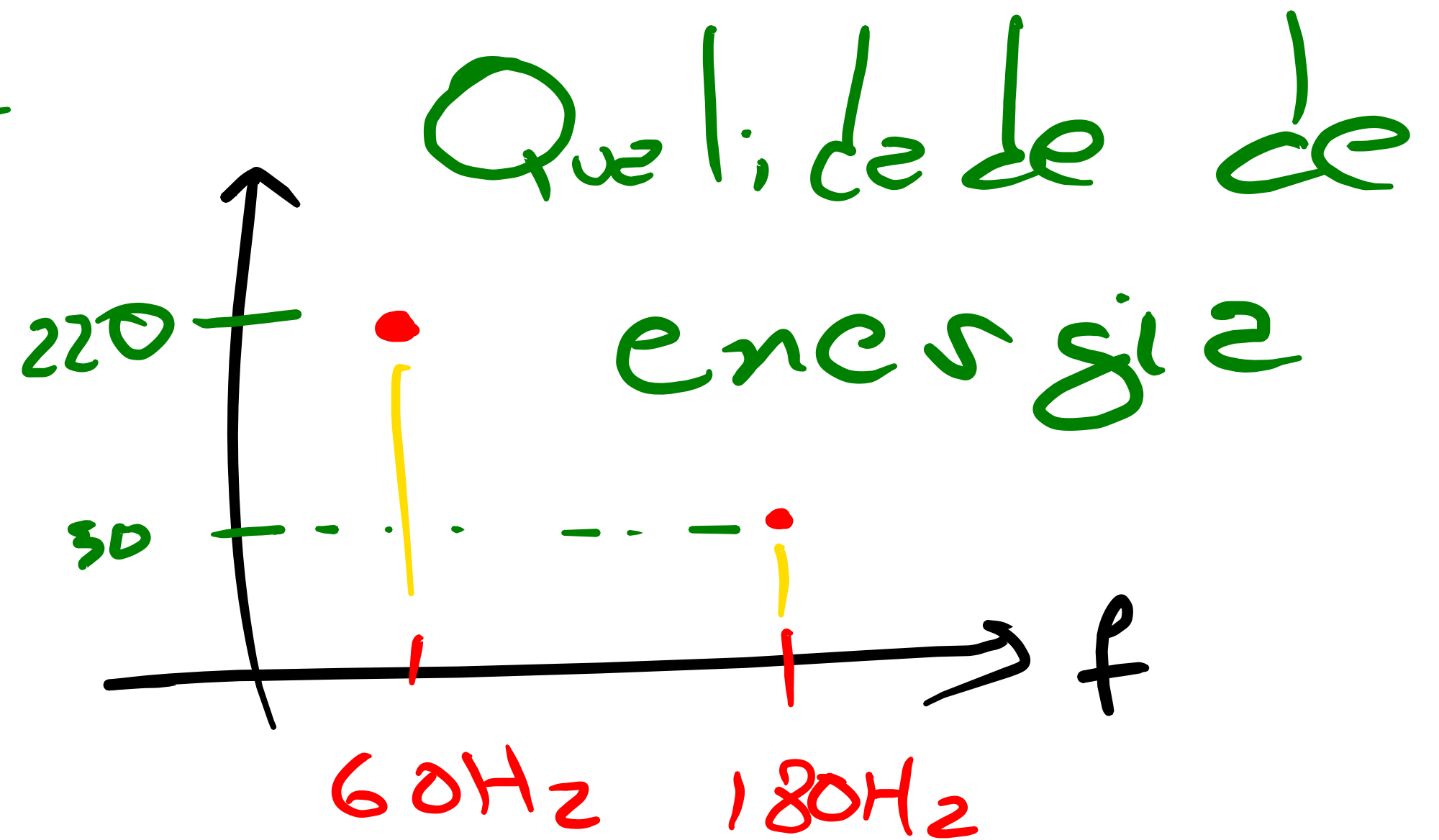
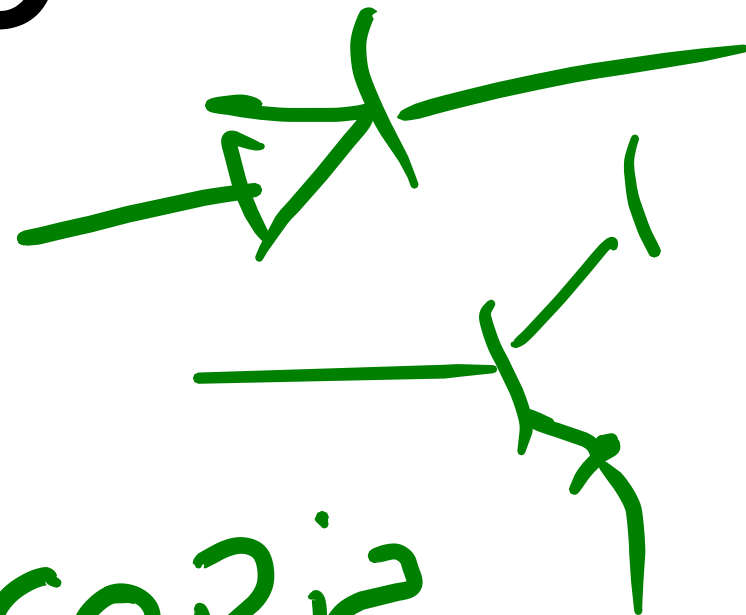
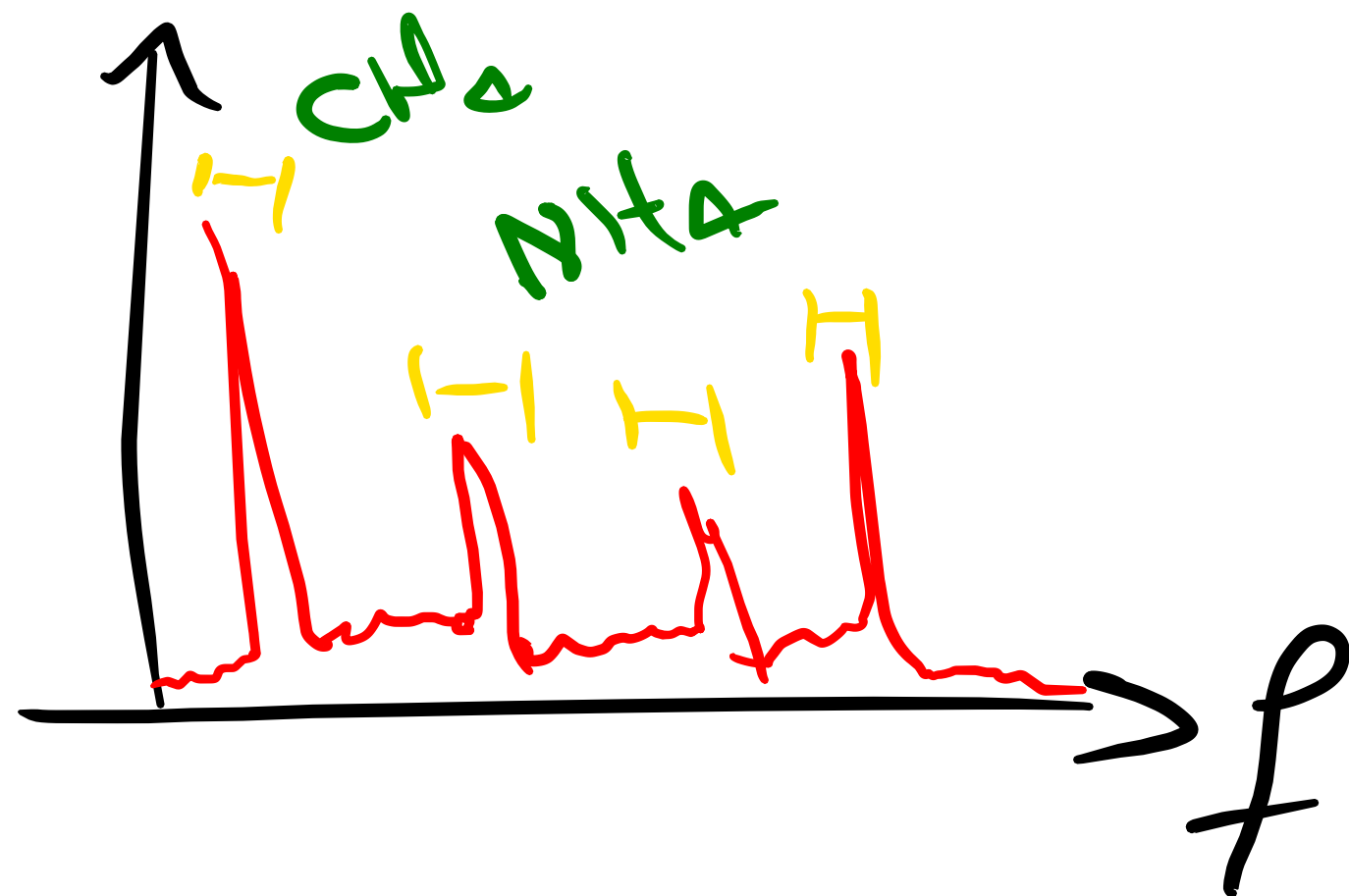


$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = F(\omega)$$

The Fourier transform equation is written in green and red ink. The function $f(t)$ is in blue, and the exponential term $e^{-j\omega t}$ is in red.

Aplicação

Espectroscopia



$$\mathcal{F}\{e^{a(t-b)}u(t-b)\} = \frac{e^{-j\omega b}}{j\omega - a}$$

$$\mathcal{F}\{\cos(\theta t)\}$$

$$\int_{-\infty}^{\infty} \cos \theta + e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} \left[\frac{e^{j\theta t} + e^{-j\theta t}}{2} \right] e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} \frac{e^{j t (\theta - \omega)}}{2} + \frac{e^{-j t (\theta - \omega)}}{2} dt$$

$$\int_{-\infty}^{\infty} \frac{e^{j\omega t x}}{2} dt$$

$$x = \theta - u$$

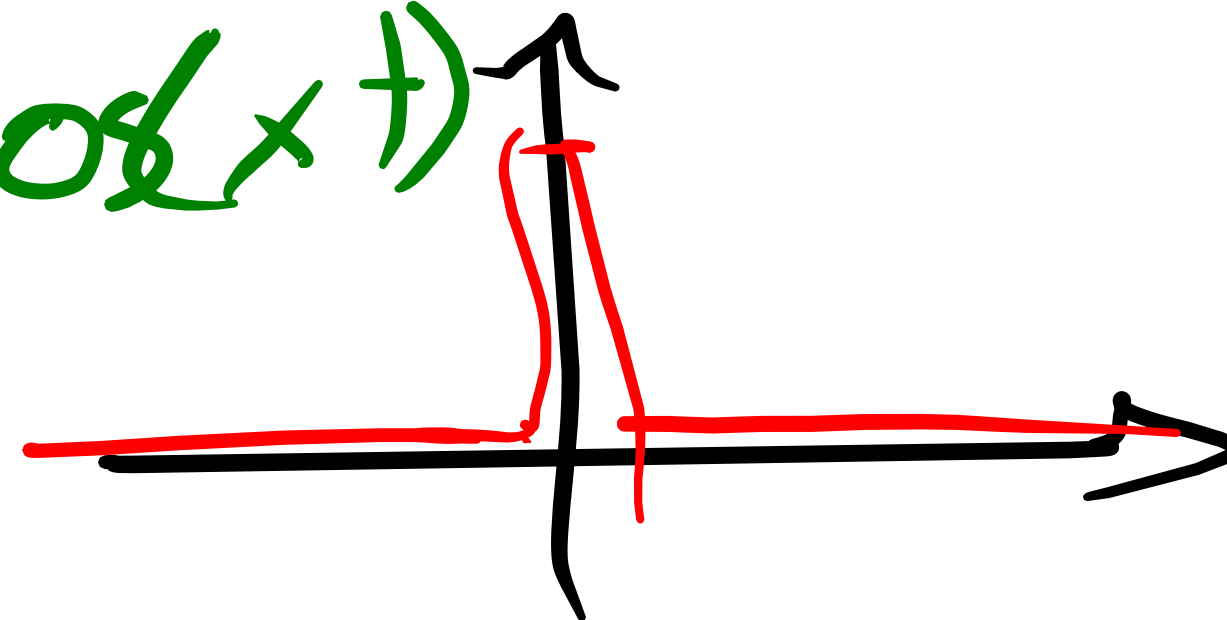
$$\left. \frac{e^{j\omega t x}}{2j\omega x} \right|_{-\infty}^{\infty}$$

$$= \lim_{t \rightarrow \infty}$$

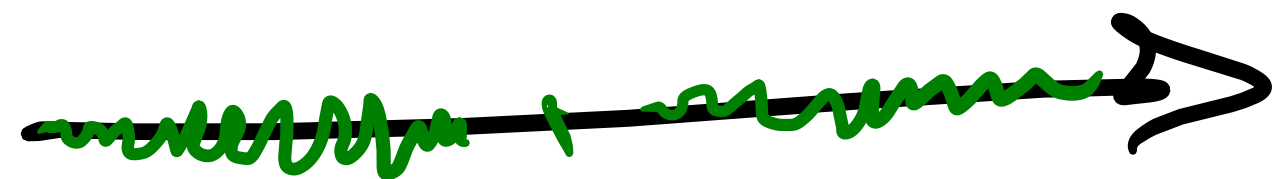
$$\frac{e^{j\omega t x} - e^{-j\omega t x}}{2j\omega x}$$

$$\frac{\sin(xt)}{x}$$

$$\lim_{t \rightarrow \infty} \frac{\sin(xt) \rightarrow \cos(xt)}{x \rightarrow 0}$$



$$\lim_{x \rightarrow 0}$$



$$\int_{-\infty}^{\infty} \cos \theta \left\{ \pi (\delta(\theta - \omega) + \delta(\theta + \omega)) \right\}$$

$$\int_{-\infty}^{\infty} \cos(\theta t) e^{-i\omega t} dt \quad \cos \theta t = \frac{e^{i\theta t} + e^{-i\theta t}}{2}$$

$$\int_{-\infty}^{\infty} e^{i t(\theta - \omega)} + e^{-i t(\theta + \omega)} dt$$

$$\frac{1}{2i(\theta - \omega)} \int_{-\infty}^{\infty} e^{i t(\theta - \omega)} dt$$

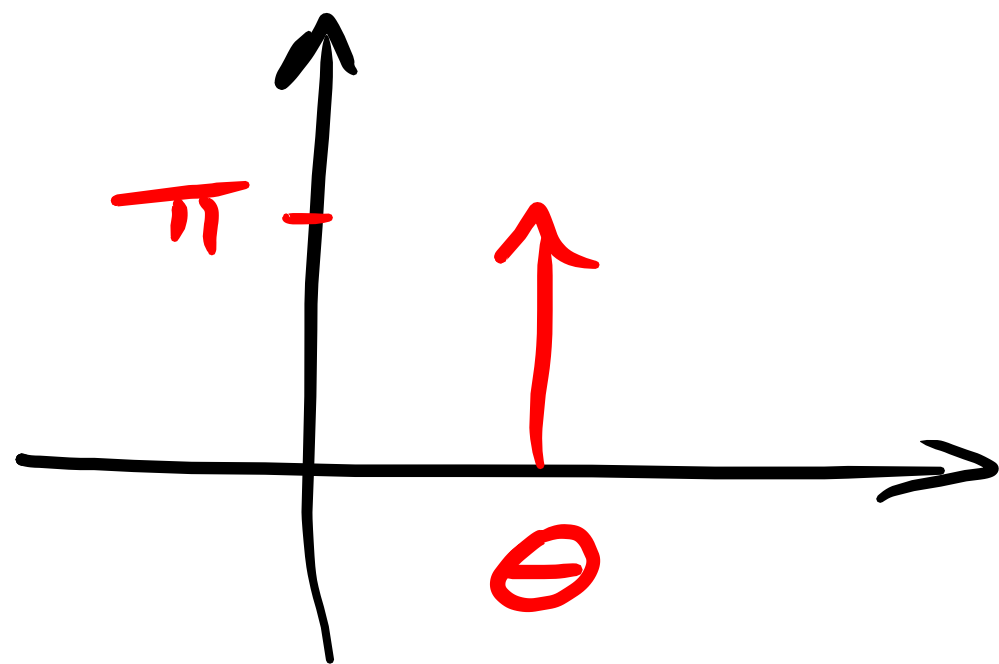
$$\frac{e^{z t(\theta-\omega)}}{2j(\theta-\omega)} \Big|_{-\infty}^{\infty}$$

$$\lim_{t \rightarrow \infty} \frac{e^{z t(\theta-\omega)} - e^{-z t(\theta-\omega)}}{2j(\theta-\omega)}$$

$$\theta - \omega = x$$

$$\frac{e^{jtx} - e^{-jtx}}{2jx} = \frac{\text{sen}(tx)}{x}$$

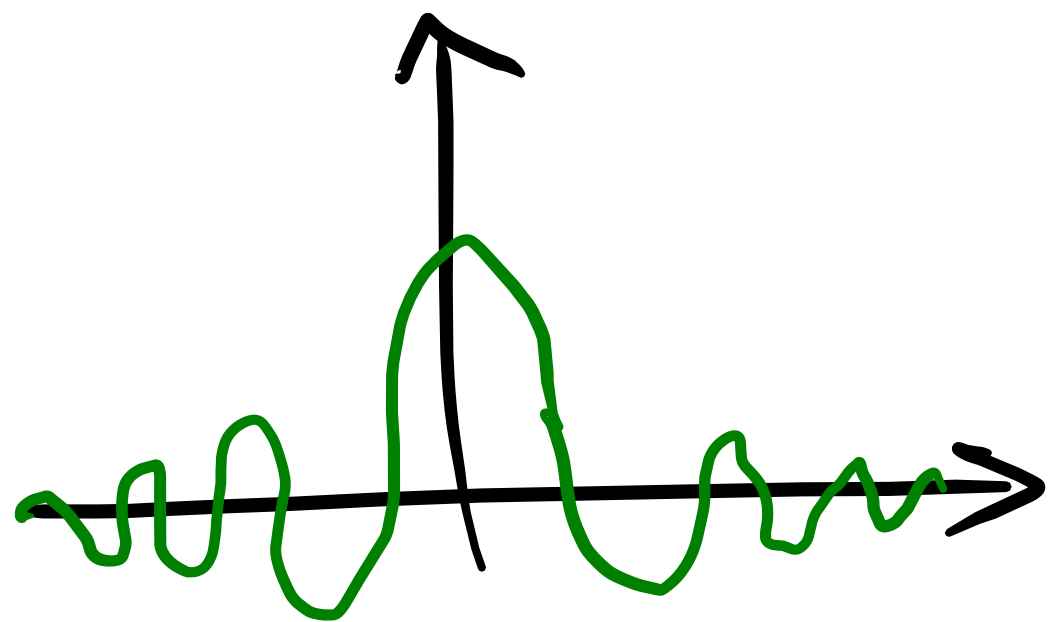
$$\lim_{t \rightarrow \infty} \frac{\text{sen}(t(\theta - \omega))}{(\theta - \omega)} = \pi \delta(\theta - \omega)$$



$$\mathcal{F}\{\cos(\theta t)\} = \pi (\delta(\theta - \omega) + \delta(\theta + \omega))$$

$$\mathcal{F}\{\sin(\theta t)\} = \frac{\pi}{j} (\delta(\theta - \omega) - \delta(\theta + \omega))$$

$$\mathcal{F}\{e^{j\theta t}\} = 2\pi \delta(\theta - \omega)$$



$$\frac{\sin(t)}{t} \text{ ou } \frac{\sin(\pi t)}{\pi t}$$

$S_2(t)$



$$\begin{aligned}
 \mathcal{F}\{G_T(t)\} &= \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\omega T/2} \cdot \frac{T/2}{T/2} \\
 &= T \operatorname{sinc}(\omega T/2)
 \end{aligned}$$

The fraction $\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\omega T/2}$ is circled in red, and a red arrow points from it to the expression $T \operatorname{sinc}(\omega T/2)$, which is underlined in yellow.

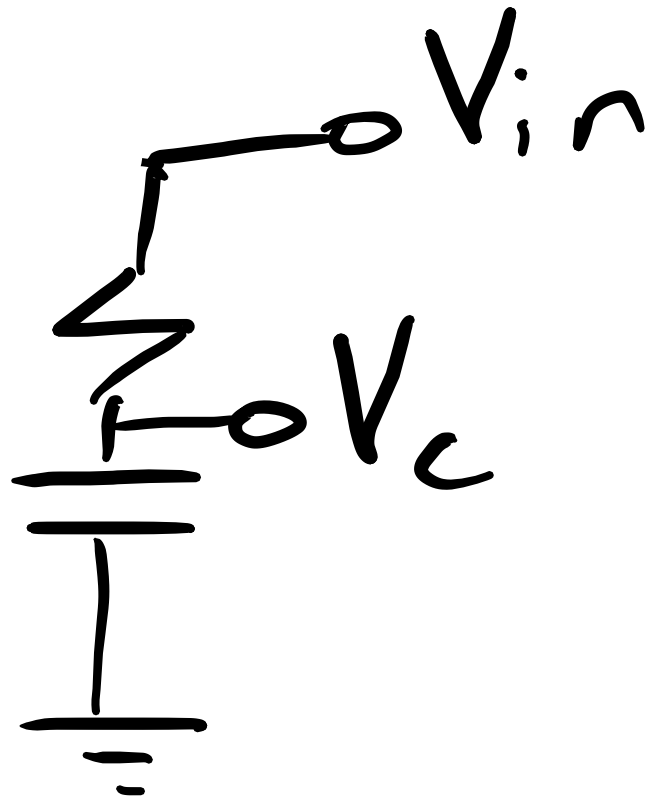
$$T \frac{\text{sen}(\omega T/2)}{\omega T/2} = T S_2(\omega T/2)$$

Exercício

Ache $H(\omega)$

$$R = 1\Omega$$

$$C = 100\mu F$$



Material e informações de contato:

www.lucas.zischler.nom.br

Obrigado
pela
atenção