Weekly Practice Question (A)

1. Suppose A and B are events where $P(A^c) = 0.6$, $P(B) = 0.5$, and $P(A \text{ and } B)^c = 0.9$. Then $P(A \text{ or } B) =$ ___________. Answer:0.8

2. The probability that house sales will increase in the next 6 months is estimated to be 0.25. The probability that the interest rates on housing loans will go up in the same period is estimated to be 0.74. The probability that house sales or interest rates will go up during the next 6 months is estimated to be 0.89. What is the probability that both house sales and interest rates will increase during the next 6 months? answer 0.1

3. The probability that house sales will increase in the next 6 months is estimated to be 0.25. The probability that the interest rates on housing loans will go up in the same period is estimated to be 0.74. The probability that house sales or interest rates will go up during the next 6 months is estimated to be 0.89. What is the probability that house sales will increase but interest rates will not during the next 6 months? answer 0.15

4. The employees of York company were surveyed on questions regarding their educational background (university degree or no college degree) and marital status (single or married). Of the 600 employees, 450 had university degrees, 100 were single, and 60 were single college graduates. The probability that an employee of the company does not have a university degree is: answer 0.25

5. A survey is taken among customers of a fast-food restaurant to determine preference for hamburger or fried chicken. Of 200 respondents selected, 75 were children and 125 were adults. 120 preferred hamburger and 80 preferred fried chicken. 55 of the children preferred hamburger.

a. What is the probability that a randomly selected individual is an adult? 0.625

b. What is the probability that a randomly selected individual is an adult, or a child? 1

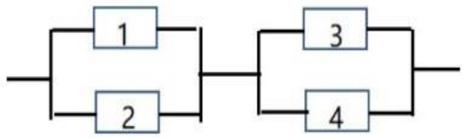
c. What is the probability that a randomly selected individual is a child and prefers fried chicken? 0.1

d. What is the probability that a randomly selected individual is a child or prefers hamburger? 0.7

6. Given that $P(C^c) = 0.82$, $P(B \cup C) = 0.41$, $P(A \cup C) = 0.52$, $P(A \cup B \cup C) = 0.69$. The event C is disjoint from events A and B

Compute a. $P(A^c \cup B^c \cup C)$ b. $P(A^c \cap B \cap C^c)$ **Answer a. 0.94 b. 0.17**

7. Some components are connected in parallel, then the subsystem works if at least one component work. Some components are connected in series, then the subsystem works if all components work. Assume all components are of the same type and are mutually independent, and P(component i work) = 0.88, i = 1, 2, ..., n. For the following system, calculate P(system works).



0.9714

8. There are 18 students in MATH2030 class, what is the probability that at least two students have the same birthday. (Assume 365 days in a year and each day is equally like). 0.3467

TRUE/FALSE

1. If event *A* and event *B* cannot occur at the same time, then events *A* and *B* are said to be independent.

2. If either event *A* or event *B* must occur, then events *A* and *B* are said to be mutually exclusive

3. The collection of all possible events is called a population space.

4. All the events in the sample space that are not part of the specified event are called independent events

5. Suppose *A* and *B* are independent events where $P(A) = 0.3$ and $P(B) = 0.4$, then $P\ (A\ or\ B) = 0.58$.

Weekly Practice Questions (B)

1. In how many ways can six students line up to get a bus to York University? In how many ways can they line up, if two of the students refuse to follow each other? 720, 480
2. How many distinct permutations are there of the letters in the word “statistics”? How many of these begin and end with the letter s? 50400, 3360
3. A shipment of ten tv sets includes three that are defective. In how many ways can a hotel purchase four of these sets and receive at least two of the defective sets? 70
4. Elizabeth has four skirts, seven blouses, and three sweaters. In how many ways can she choose two of the skirts, three of the blouses and one of the sweaters to take along on a trip? 630
5. If eight persons are having dinner together, in how many different ways can three order chicken, four order steak and one order lobster? 280
6. If nine students from York University having dinner together, in how many different ways can three order organic chicken, five order steak and one order lobster? 504
7. How many different bridge hands are possible containing five spades, three diamonds, three clubs and two hearts? 8211173256
8. In how many different ways can be marked for a true-false test that consists of 20 questions? 1048576
9. A true-false test consists of 20 questions. In how many ways can the questions be marked true or false so that
   a. 7 are right and 13 are wrong;
   b. 10 are right and 10 are wrong;
   c. At least 17 are right?

   77520 ; 184756; 1351
10. A college team plays 10 football games during a season. In how many ways can it end the season with five wins, four losses and one tie? 1260
11. A fair coin is tossed 10 times. What is the probability that we get exactly 5 tails? 0.2461
12. We have a basket that contains 80 marbles, 50 of them blue and 30 green. Select 8 marbles at random. What is the probability that 5 are blue and 3 are green? 0.2968
13. A full deck of 52 cards contains 13 hearts. Pick 8 cards from the deck at random (a) without replacement and (b) with replacement. In each case compute the probability that you get no hearts. 0.08176 ; 0.1001

MATH2030-**Weekly Practice Questions**(C)

Question 1

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk? **0.243**

Question 2

Suppose we have additional information for the previous question. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk for hearth attacked. A single person is selected at random. What is the probability that it is a high-risk female? **0 .0392**

Question 3

From a previous question, we know that 49% of the population are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. A single person is selected at random and found to be high risk. What is the probability that it is a male? **0.6096**

Question 4

A pair of fair dice is rolled. What is the probability that the second die lands on higher value than does the first? **0.4167**

Question 5.

Warren, a financial analyst has determined that there is an 8% probability that a mutual fund will outperform the market over a year period provided that it outperformed the market the previous year. If only 3% of mutual funds outperform the market during any year, what is the probability that mutual fund will outperform the market **2 years** in a row? 0.0024

Question 6.

Suppose that customers of York Restaurant were asked whether they preferred water or whether they preferred bubble tea. 70% said that they preferred water. 60% of the customers were male. 80% of the males preferred water.

a. What is the probability that a randomly selected customer is a female who prefers bubble tea?

b. Suppose a randomly selected customer is a female, what is the probability that the customer prefers water is ___________. 0.55

0.18 , 0.55

Question 7.

Suppose a test using AI technology for diagnosing a certain serious disease is successful in detecting the serious disease in 99.7% of all person infected but that is incorrectly diagnoses 0.5% of all healthy people as having the serious disease. Suppose also that it incorrectly diagnoses 1.8% of all people having another minor disease as having the serious disease. It is known that 2% of the population have the serious disease, 93% of the population are healthy, and 5% have the minor disease. Given the test is positive, what is the probability that selected has the serious disease?

Use H to represent healthy, M to represent having minor disease, and D to represent having serious disease. 0.7823

Question 8.

A security system is manufactured with a "fail-safe" provision so that it functions properly if any two or more of its three main components, X, Y and Z, are functioning properly. The probabilities that components X, Y, and Z are functioning properly are 0.98, 0.88 and 0.78, respectively. What is the probability that the system functions properly? 0.9679

Question 9.

A recent report revealed that only 88% of active Gmail accounts use two-factor authentication(2FA). Suppose 3 active Gmail accounts are selected at random, compute the probability that at least 1 active Gmail account does not use 2FA.

0.3185

Question 10.

In an organic vegie packaging plant Machine A account for 60% of the plant's output, while Machine B accounts for 40% of the plant's output. In total, 4% of the packages are improperly sealed. Also, 3% of the packages are from Machine A and are improperly sealed.

a. If a package selected at random is improperly sealed, what is the probability that it came from machine A? 0.75
b. If a package selected at random came from Machine A, what is the probability that it is improperly sealed? 0.05
c. If a package selected at random came from Machine B, what is the probability that it is properly sealed? 0.975

Weekly Practice Questions (D)

1. How many distinct permutations are there of the letters in the word "MISSISSIPPI"? 34650

2. Jerry prepares for an exam by studying a list of 15 problems. He can solve 9 of them. For the exam, the instructor selects 7 questions at random from the list of 15. What is the probability that Jerry can solve all 7 problems on the exam? 0.00559

3. From a group of 6 women and 8 men, how many different committees consisting of 3 women and 4 men can be formed? What if 3 of the men are feuding and refuse to serve the committee together? 1400 , 1300

4. An Italian restaurant in Québec City offers a special summer menu in which, for a fixed dinner cost, you can choose from one of three salads, one of four entrees, and one of five desserts. How many different dinners are available? 60

5. A businesswoman in Toronto is preparing an itinerary for a visit to five major cities in Ontario. Each city will be visited once and only once. The distance travelled, and hence the cost of the trip, will depend on the order in which she plans her route. How many different itineraries (and trip costs) are possible? 120

6. York Car Rental Agency has 19 compact cars and 12 intermediate-size cars. If four of the cars are randomly selected for a safety check, what is the probability of getting two of each kind? 0.3587

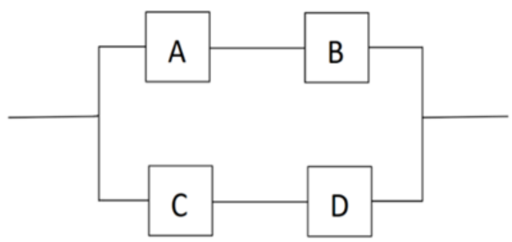
7. The Chartered Financial Analyst (CFA) is a designation earned after taking three annual exams (CFA I, II, and III). The exams are taken in early June. Candidates who pass an exam are eligible to take the exam for the next level in the following year. The pass rates for level I, II, and III are 0.23, 0.59, and 0.85, respectively. Suppose that 3100 candidates take the level I exam, 1400 take the level II exam, and 500 take the level III exam. A randomly selected candidate who took a CFA exam tells you that he has passed the exam, what is the probability that he took the CFA III exam? 0.2164

8. Given that $P(C^c) = 0.91$, $P(B \cup C) = 0.45$, $P(A \cup C) = 0.53$, $P(A \cup B \cup C) = 0.79$. The event C is disjoint from events A and B. Compute a. $P(A \cap B^c \cap C^c)$ b. $P(A \cup B^c \cup C)$ 0.34 , 0.74

9. The Chartered Financial Analyst (CFA) is a designation earned after taking three annual exams (CFA I, II, and III). The exams are taken in early June. Candidates who pass an exam are eligible to take the exam for the next level in the following year. The pass rates for level I, II, and III are 0.58, 0.74, and 0.80, respectively. Suppose that 5500 candidates take the level I exam, 3000 take the level II exam, and 1500 take the level III exam. A randomly selected candidate who took a CFA exam tells you that he has passed the exam. What is the probability that he took the CFA III exam? 0.1815

10. The probability that an individual randomly selected from a particular population has a certain disease is 0.038. A diagnostic test correctly detects the presence of the disease 98.5% of the time and correctly detects the absence of the disease 99.5% of the time. If the test is applied twice, the two test results are independent, and both are positive, what is the probability that the selected individual has disease? 0.9993

11.



In order for the circuit to work, current must be able to pass from left to right. Each component (A, B, C, D) has an independent probability of failure. Component A has a probability of failure of 0.07. Component B has a probability of failure of 0.06. Component C has a probability of failure of 0.05. Component D has a probability of failure of 0.04. What is the probability that the circuit does not work? 0.01107

12. Suppose *A* and *B* are independent events where *P*(*A*) = 0.18 and *P*(*B*) = 0.3.

Then a. P(A and B) = b. *P (A or B)* = . 0.054 , 0.426

MATH2030(Weekly Practice Questions(E)

Question 1.

Suppose X is a binomial random variable with n=18 and p=0.15. Find $P(X \le \mu - \sigma)$

0.2241

Question 2.

At a fast Food restaurant, the expected number of customers who will spend at least \$10 during the lunchtime period from 12:00 noon to 2:00 P.M is 221 and the variance is 33.15. Assume this is a binomial model. Determine the number of customers will be in this fast food restaurant on this Friday during the lunch time period from 12:00 noon to 2:00P.M.

[260]

Question 3.

Suppose X is a continuous random variable with cumulative distribution function given by

$$F(x) = \begin{cases} 1 - e^{-x^2/8} & if \quad x \ge 0 \\ 0 & otherwise \end{cases}$$

Determine

a. $P(X > 2)$ b. $P(1 \le X \le 3)$ c. $P(X \le 2 | X \le 4)$

Question 4.

Suppose that 89% of the account receivables of a company are free of errors. A random sample of eleven such accounts is drawn.

a. Determine the probability that exactly **ten** account receivables are free of errors.
b. Determine the probability that more than **ten** account receivables are free of errors.
c. Determine the probability that at least **ten** account receivables are free of errors.

a. 0.3773 b. 0.2775 c. 0.6548

Question 5.

Suppose that 18% of the account payables of a company have errors. A random sample of ten such accounts is drawn. Given that fewer than 3 such accounts have errors, what is the probability that at most one account has errors.

0.5956

Question 6.

The New York Times reported (Laurie J. Flynn, "Tax Surfing") that the mean time to download the homepage from the Internal Revenue Service Web site www.irs.gov is 0.9 second. Suppose that the download time is normally distributed with a standard deviation of 0.2 second.

a. What is the probability that a download time is above 0.9 second?
b. What is the probability that a download time is more than 1 second?
c. What is the probability that a download time is less than 1 second?

d. What is the probability that a download time is more than 0.8 seconds?
e. What is the probability that a download time is between 0.8 and 1.2 seconds?

a. 0.5 b. 0.3085 c. 0.6915 d. 0.6915 e. 0.6247

Question 7.

Police estimates that 8% of drivers are driving their cars above the posted speed limits of 100 km/h on the highway 401 in Toronto on Saturday. If a driver is driving above the posted limit on the highway 401, and is caught by a policeman, then s/he will receive a $50 ticket. If a policeman is planning to randomly test 150 cars moving on the highway 401 using a radar detector on next Saturday,

a. How many tickets on average would he expect to write?

b. What is the standard deviation of the number of tickets the policeman would write?

a. 12 b.3.323

Question 8.

York paper reported that 38% of York students spend less than 90 minutes per day on Internet and 27% of York students spend at least 2 hours each day on Internet.

a. Among 10 randomly selected York students, what is the probability that less than 9 of them will spend at least 90 minutes per day on Internet?

b. Among 10 randomly selected York students, what is the probability that more than 9 of them will spend less than 2 hours per day on Internet?

c. Among 10 randomly selected York students, what is the probability that at most one of them spend at least 90 minutes but less than 2 hours on Internet?

a. 0.9402 b. 0.04298 c.0.0860

Weekly Practice Questions (F)

Question #1

Jerry is out trick-or-treating and he goes to his friend Kyle's house. Kyle's mom has a bag of candy containing 78 Whoppers, 102 Kit-Kats and 20 Skittles. Jerry randomly draws 9 pieces of candy from the bag

a. What is the probability of getting at least 8 Whoppers?
b. What is the probability of getting at most 8 Kit-Kats?
c. What is the probability of getting more than 1 Skittle?
d. What is the probability that X is within 1 standard deviation of its mean value? X is defined as the number of Kit-Kats

a. 0.0031 b.0.9977 c.0.2252 d. 0.6653

Question #2

The amount spent by students from York U on Halloween costume is known to be normally distributed with a mean of $88.00 and a standard deviation of $28.88. A student is randomly selected, Compute the probability that he will spend

a. More than $50.00 b. between $60.00 and $100.00

(a) 0.9066, (b) 0.4968

Question #3

Adam is out trick-or-treating and he goes to his neighbor Brad's house. Brad has a bag of candy containing 320 Whoppers and 80 Skittles. Brad randomly draws eighty-eight pieces of candy from the bag to give to Adam.

What is the probability of getting at least 58 Whoppers?

0.9997

Question #4

One thousand independent rolls of a fair die will be made.

a. Find the probability that the number 6 will appear between 150 and 200 times inclusively.
b. Given that the number 6 appears exactly 200 times, find the probability that the number 5 will appear less than 150 times

Answer: a. 0.9258 b. 0.1762

Question 5

The annual commissions earned by sales representative of York Machine Products Inc. of light machinery, follow the normal distribution. The mean yearly amount earned is $40,000 and the standard deviation is $5000. The sales manager wants to award the sales representatives who earn the largest commissions a bonus of $8888. He can award a bonus to 20 percent of the sales representatives. What is the cutoff point between those who earn a bonus and those who do not?

$44200

Question 6

York Manufacturing Inc. offers dental insurance to its employees. A recent study by the human resource director shows the annual cost per employee per year followed the normal distribution, with a mean of 1280 dollars and a standard deviation of 420 dollars. What was the minimum cost for the 10 percent of employees that incurred the highest dental expense?

$1817.60

Question 7
The number of traffic accidents in a day on a certain section of highway 488 is known to be Poisson distributed with a mean of 2.16. Compute the probability of no accidents occurring on this section of highway during a six-hour period?
Ans. 0.5827

Question 8
If the cdf of a continuous random variable is given by

$$F(x) = \begin{cases} 1 - \dfrac{4}{x^2} & x > 2 \\ 0 & x \le 2 \end{cases}$$

Compute a. P(X<3) b. P(4 < X <5)

Ans. a.0.5556 b.0.09

Weekly Practice Questions(G)

Question #1
The probability density function of X, the lifetime of a certain type of electronic device (measured in hour), is given by

$$f(x) = \begin{cases} \dfrac{10}{x^2} & x > 10 \\ 0 & x \le 10 \end{cases}$$

What is the probability that of 6 such types of devices, at least 2 will functions for at least 15 hours? 0.98217

Question #2
Suppose that X is a normal random variable with mean 5. If P(X > 9)=0.2. What is Var(X)?
22.66

Question #3
Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that P(X > c)=0.10. 14.56

Question #4
A survey indicates that a customer spent an average of 18 minutes with a variance of 16 minutes ^{2} in the store. The length of time spent in the store is normally distributed.
a. What is the probability that the customer will be in the store between 20 minutes and half an hour?
b. If there are 9 customers shop in the store, what is the probability that more than 7 customers will be in the store for less than 20 minutes?
a. 0.3072 b.0,1813

Question #5
York Hospital finds that 28% of accounts are at least 8 months in arrears. A random sample of 288 accounts was taken.
What is the probability that

a. Less than 88 accounts in the sample were at least 8 months in arrears?
b. the number of accounts in the sample at least 8 months' arrears was between 70 and 100 (inclusive)

a.0.8165 b. 0.9044

Weekly Practice Questions(H)

Question# 1

Suppose X is a discrete random variable with the probability distribution given in the following table

x	-20	-10	0	10
P(x)	0.3	0.4	0.2	0.1

Compute E(5X+3) and Var(3X+7)

42, 801

Question# 2

Suppose E[X]=5 , E[X(X-1)]=27.5

Determine Var[X] 7.5

Question#3

The following table shows the probability that a certain laptop will malfunction 0, 1, 2, 3, 4 and 5 times on any given day:

Number of malfunction , x	0	1	2	3	4	5
Probability , f(x)	0.44	2a	b	1.5b	3a	2b

Given that b=2a

a. Find the value of a

b. Determine the probability that a certain laptop will malfunction more than 3 times on next Wednesday. a.0.04 b. 0.28

Question #4

The density function of X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

If E[X]= $\frac{3}{5}$, find a and b answer a = 3/5 b= 6/5

Question 1 (3 marks)

Prove that $P(AB^C) = P(A) - P(AB)$

Question 2(3 marks)

Show that the probability that exactly one of the events A or B occurs equals $P(A) + P(B) - 2P(AB)$

Question 3 (9 marks)

For celebrating family's day, Yes Frills at the Centre Point Mall has a special turkey sale for the first 15 customers, \$0.88 for a turkey weighing less than 10 pounds, \$1.88 for a turkey weighing between 10 and 20 pounds, and \$2.88 for a turkey weighing more than 20 pounds. There will be 8 turkeys weighing less than 10 pounds, 18 turkeys that weighing between 10 pounds and 20 pounds, and 19 turkeys weighing at least 20 pounds available during the sales. What is the probability that

a. At least 13 turkeys weighing at least 20 pounds will be sold in this special turkey's sales
b. At least 6 turkeys weighing between 10 and 20 pounds, and at least 8 turkeys weighing at least 20 pounds will be sold in this special turkey's sales
c. At least 4 turkeys weighing less than 10 pounds, at least 5 turkeys weighing between 10 and 20 pounds, and at least 5 turkeys weighing at least 20 pounds will be sold in this special turkey's sales

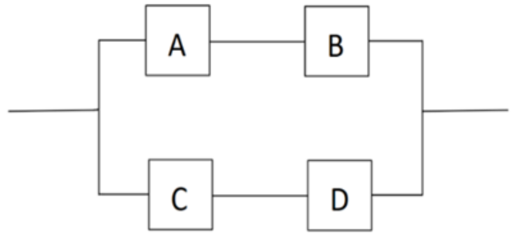
Question 4. (6 marks)

A bag contains 68 hearth-shaped Kit-Kat Chocolate numbered 1, 2, …, 68. Before the game starts, Nicholas selects 9 different numbers from 1, 2, …, 68 and writes them on a piece of paper. Then, he randomly selects 12 hearth-shaped Kit-Kat from the bag without replacement

What is the probability that

a. All the numbers are selected?
b. Exactly 8 of his numbers are selected?

Question 5. (9 marks)



In order for the circuit to work, current must be able to pass from left to right. Each component (A, B, C, D) has an independent probability of failure. Component A has a probability of failure of 0.03. Component B has a probability of failure of 0.04. Component C has a probability of failure of 0.05. Component D has a probability of failure of 0.06.

Given the circuit does not work, what is the probability that components C and D have failed?

Question 6. (9 marks)

The probability that a new Computer Science building in Markham campus, York University will get an award for its design is 0.75, the probability that it will get an award for the efficient use of materials is 0.48, and probability that it will get both awards is 0.39.

a. What is the probability that it will get at most one reward?

b. What is the probability that it will get at least one reward?

c. What is the probability that it will get less than one reward?

Question 7. [6 marks]
Two thousand randomly selected adults were asked if they are financially better off than their parents. The following table gives the two-way classification of the responses based on the education levels of the persons included in the survey and whether they are financially better off, the same, or worse off than their parents.

	Less than High School	High School	More than high School	
Better off	140	450	420	**1010**
Same	60	250	110	**420**
Worse off	200	300	70	**570**
	400	**1000**	**600**	**2000**

If one adult is selected at random from these 2000 adults, determine the probability that this adult is

a. financially better off than his/her parents or he/she has high school education

b. financially the same as his/her parents given he/she has more than high school education

Question 8 (3 marks)
Alice, Bob, Claire, Donald, Edward, Felix, Gary and Joe sit at random in a row. Determine the probability that Donald and Joe sit next to each other in a row?

Question 9. (3 marks)

Given that P(C) = 0.14, P (B ∪ C) = 0.46, P (A ∪ C) = 0.53, P (C ∪ D) = 0.32, P (A ∪ B ∪ C) = 0.72. The event C is disjoint from events A, B and D. The event D is disjoint from events A, B and C. Compute P ($A \cap B^c \cap C^c \cap D^c$).

A. O.04
B. 0.07
C. 0.12
D. 0.14
E. 0.26
F. 0.28
G. 0.31
H. 0.39
I. 0.42
J. 0.51
K. 0.53
L. 0.64
M. 0.65
N. 0.69
O. 0.72
P. 0.75
Q. 0.76
R. 0.78
S. 0.79
T. 0.81
U. 0.82
V. 0.83
W. 0.84
X. 0.85
Y. 0.88
Z. None of the above

Question 10. [6 marks]

In a binary transmission channel, a 1 is transmitted with probability 2/3 and a 0 with probability 1/3. The conditional probability of receiving a 1 when a 1 was sent is 0.94, the conditional probability of receiving a 0 when a 0 was sent is 0.87. Given that a 1 is received, what is the probability that a 1 was transmitted?

Question 1. [5 marks] 

Two candies are chosen randomly from a bag containing 9 crunches, 4 Kit-Kats, and 2 Snickers. Suppose that we win $8 for each Kit-Kat selected and we lose $5 for each crunch selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?

Present your answer in a probability distribution table form.

Question 2 [10 marks]

The cumulative distribution function for a discrete random variable X is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.54 & 1 \le x < 2 \\ 0.67 & 2 \le x < 3 \\ 0.83 & 3 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

a. P(X>2) b. P(X $\geq$ 2) c. P(X < 2) d. P(X $\leq$ 2) e. P (1 < X < 3)

f. P(1 < X $\leq$ 3) g. P(1 $\leq$ X $\leq$ 3) h. P(1 $\leq$ X < 3)

i. What is the pmf of X?

Question 3 [6 marks]

A recent report revealed that 92% of Gmail accounts do not use two-factor authentication(2FA). 10 Gmail accounts are selected at random. Given that less than two Gmail accounts use 2FA, what is the probability that none of the Gmail account uses 2FA?

Question 4. [4 marks]

One of the leading dot-com companies has found that 23% of customers who come into its web site actually make a purchase of less than $250 and 16% of customers make apurchase of at least $500. In a sample of 10 customers who come into its web site, what is the probability that at most 90% of them make a purchase of less than $500?

Question 5. [6 points]

a. Check whether the following can be defined as probability mass function, and explain why?

   i. $f(x) = \frac{x}{12}$ for $x = 0,1,2,3,4$

   ii. $f(x) = \frac{3x+1}{50}$ for $x = 0,1,2,3,4,5$

b. Given that $f(x) = \frac{k}{2^x}$ is a probability mass function for a random variable that can take on the values $x = 0,1,2,3,$ and 4. Determine the value of k

Question 6. [3 marks]
If the probability density of a random variable is given by

$$g(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & elsewhere \end{cases}$$

Compute P(X< 2)

Question 7. [8 marks]

If the probability density of X is given by

$$\mathbf{f}(\mathbf{x}) = \begin{cases} \mathrm{kx}(\mathbf{1}-\mathbf{x}) & \mathbf{0} < \mathbf{x} < \mathbf{1} \\ \mathbf{0} & \mathbf{elsewhere} \end{cases}$$

Compute

a. the value of k
b. P(X > 0.5)
c. the cumulative distribution function of this random variable

Question 8 [8 marks]

If bolt thread length is normally distributed, what is the probability that the thread length of a randomly selected bolt is

a. Within 1.5 standard deviations of its mean value?
b. Farther than 2.5 standard deviations from its mean value?
c. Between 1 and 2 standard deviations from its mean value?

Question 9 [7 marks]

There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation 0.1 cm. The second machine produces corks with diameters that are normally distributed with mean 3.04 cm and standard deviation 0.02 cm. Acceptance corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

Question 1. [3 marks]

Show that Cov (X; Y) = E[XY] – E[X]E[Y]

Question 2. [5 marks]

Show that for all positive integer n and k with $n \geq k$

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

Question 3. [6 marks]
Let X be a binomial random variable with parameter n and p. Show that

$$E\left[\frac{1}{1+X}\right]=\frac{1-(1-p)^{n+1}}{(n+1)p}$$

Question 4. [6 marks]

Suppose X and Y are independent, X has mean 5 and variance 9, Y has mean -3 and variance 6.

Compute

a. $E[(3X + 2Y)(X + 4Y + 8)]$. b. $\mathrm{Var}[8XY]$

Question 5 [5 marks]

York Corporation manufactures an AI car that requires three AA batteries. The mean life of these batteries in this product is 35.0 hours. The battery life is known to be normally distributed with a standard deviation of 5.5 hours. As part of its testing program, Adam tests a sample of 25 batteries.

a. What is the standard error of the distribution of the sample mean?
b. What is the probability that the sample mean life is between 34.5 and 36.0 hours?

Question 6 [4 marks]

The number of hits per day on the web site of York AI Inc. is normally distributed with a mean of 700 and a standard deviation of 120. Determine the number of hits such that only 5% of the days will have the number of hits below this number.

Question 7 [4 marks]

Scores for MATH2030 final exam are normally distributed with a mean of 72.8% and a standard deviation of 11.8%. What is the minimum exam score needed in order to be the top 10% of all students taking MATH2030 final exam?

Question 8 [6 marks]
888 independent rolls of a fair die will be made. Given that the number 5 appears exactly 128 times and the number 3 appears exactly 160 times, find the probability that the number 1 will appear less than 123 times

Question 9 [5 marks]

The mean amount spent by each customer on non-medical mask at Chopper Drug Mart is 28 dollars with a standard deviation of 8 dollars. The population distribution for the amount spent on non-medical mask is positively skewed. For a sample of 36 customers, what is the probability that the sample mean amount spent on non-medical mask is greater than 22 dollars but less than 25 dollars

Question 10. [7 marks]
If the probability density of a random variable is given by

$$g(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & elsewhere \end{cases}$$

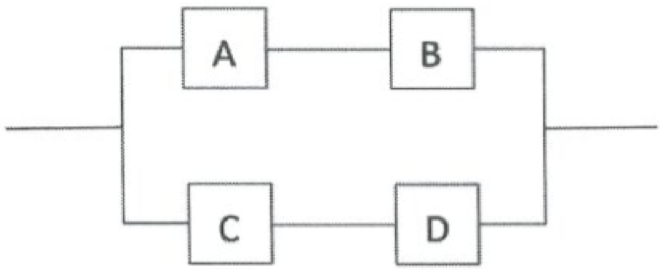
Compute μ and σ

2. A data engineer attempting to predict a tech company's earnings in the year of 2025 believes that the tech's business is quite sensitive to the level of interest rates. He believes that, if the average interest rates in the year of 2025 are more than 1% higher than this year, the probability of significant earnings growth is 0.1. If the average interest rates in the year 2025 are more than 1% lower than this year, the probability of significant earnings growth is estimated to be 0.8. If the interest rates for year 2025 are within 1% of this year's rates, the probability for significant earnings growth is at 0.5. The data analyst estimates that the probability is 0.03 that the average interest rates in the year 2025 will be more than 1% higher than this year and 0.85 that they will be more than 1% lower than this year.

a. What is the probability that this tech company will experience significant earnings growth?

b. If the company exhibits significant growth, what is the probability that the average interest rates will have been more than 1% lower than in the current year? **(10 pts)**

3.



In order for the circuit to work, current must be able to pass from left to right. Each component (A, B, C, D) has an independent probability of failure. The probability that component A works is 0.96. The probability that component B works is 0.97. The probability that component C works is 0.98. The probability that component D works is 0.99. Determine the probability that the circuit does not work. **(4 pts)**

4. Given that $P(C^c) = 0.87$, $P(B \cup C) = 0.49$, $P(A \cup C) = 0.53$, $P(C \cup D) = 0.35$, $P(A \cup B \cup C) = 0.68$. The event C is disjoint from events A, B and D. The event D is disjoint from events A, B and C. Compute $P(A^c \cap B \cap C^c \cap D^c)$ **(4 pts)**

5. 4 men and 5 women sit at random in a row. Determine the probability that either all the men or all the women end up sitting together? **(6 pts)**

6. Nuclear engineers in charge of maintaining our nuclear fleet must continually check for corrosion inside the pipes that are part of the cooling systems. The inside condition of the pipes cannot be observed directly but a nondestructive test can give an indication of possible corrosion. This test is not infallible. The test has probability 0.95 of detecting corrosion when it is present but it also has probability 0.08 of falsely indicating internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.03. Given the test is applied twice, the two test results are independent, and both of the test results indicate the presence of corrosion, what is the probability that a section of pipe has internal corrosion. **(9 pts)**

1. If the probability density function of the random variable X is given by

$$f(x)=\begin{cases} x & for \quad 0<x<1 \\ 2-x & for \quad 1\leq x<c \\ 0 & elsewhere \end{cases}$$

Determine the value of c and the cdf of X **(7 pts)**

2. Suppose X is a discrete random variable with the probability distribution

x	-20	-15	-10	-5
P (x)	a	2a	2b	0.5a

Given that E[2X+19] = -8. **(6 pts)**

Compute

a. the value of a and b
b. Var[5X+3]

3. The life times of interactive computer chips produced by York Semiconductor Manufacturer are normally distributed with a mean of 1.8×10^7 hours and a standard deviation of 3×10^6 hours. Compute the probability that a batch of 9 chips will contains at most 7 whose lifetimes are less than 2.1×10^7 hours. **(8 pts)**

4. It is known that 90% of all items produced by a particular manufacturing process are not defective. From the very large output of a single day, 400 items are selected at random, what is the probability that between 25 and 48 (inclusive) of the selected items are defective? **(6 pts)**

5. The random variable of X has the probability density function

$$f(x)=\begin{cases} ax+bx^2 & 0<x<1 \\ 0 & otherwise \end{cases}$$

Given that P (X < 0.5) = 0.35.
Find a and b. **(7 pts)**