

1.

Code in Java

```
public boolean decideSuper(String w) {  
    String y;  
    for (int i = 0; i < w.length+1; i++) {  
        for (int j = 0; j < w.length+1; j++) {  
            y = w.substring( i , j ); // iterate over all possible substrings  
            if ( DecideL(y) == true ) { // if DecideL returns true for a substring, the  
                return true;           // string w is a superstring  
            }  
        }  
    }  
    return false;  
}
```

This algorithm works by iterating over all possible substrings  $y$  of the input string  $w$ . For each substring, it calls the `DecideL` subroutine to check if  $y$  is in  $L$ . If `DecideL(y)` returns `True` for any substring  $y$ , then  $w$  is a superstring of  $L$ , and the algorithm returns `True`. If no such  $y$  is found after checking all substrings, the algorithm returns `False`, indicating that  $w$  is not a superstring of  $L$ .

It is correct because it iterates over all possible substrings and therefore exhausts all possibilities.

2.

Code in Java

```
public boolean recognizeSUB(String w) {  
    for (String x :  $\Sigma^*$ ) { // iterate over all x in  $\Sigma^*$   
        for (String z :  $\Sigma^*$ ) { // iterate over all z in  $\Sigma^*$   
            if (recognizeL(x + w + z)) {  
                return true;  
            }  
        }  
    }  
    return false;  
}
```

This algorithm works by iterating over all possible strings  $x$  and  $z$  in  $\Sigma^*$ . For each pair of strings, it concatenates  $x$ ,  $w$ , and  $z$  and calls the `RecognizeL` subroutine. If `RecognizeL` returns True for any such concatenation, then the algorithm returns True, indicating that  $w$  is a substring of a string in  $L$ . If no such concatenation of  $w$  with  $x$  and  $z$  belongs to  $L$ , the algorithm returns False.

This algorithm is correct because it checks every possible way that  $w$  could be a substring of a string in  $L$  and correctly identifies whether such a string exists.