Team notebook

CU BadToTheBone - University of Chittagong

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Contents

1 Data Structures

1.1 ordered$_s$*et*

```cpp
#include <bits/stdc++.h>

using namespace std;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template<class T> using ordered_set = tree<T, null_type,
     less<T>, rb_tree_tag, tree_order_statistics_node_update>;

int main() {
    ordered_set<int> ost; // ordered set

    ost.insert(2);
    ost.insert(3);
    ost.insert(3);
    ost.insert(4);
    ost.insert(4);

    cout << *ost.find_by_order(1) << endl; // 3
    cout << *ost.find_by_order(2) << endl; // 4
    cout << ost.order_of_key(5) << endl; // 3
    cout << ost.order_of_key(3) << endl; // 1

    ordered_set<array<int, 2>> omst; // ordered multiset

    /*omst.insert({2, 1});
    omst.insert({4, 2});
    omst.insert({3, 3});
    omst.insert({4, 4});
    omst.insert({3, 5});

    cout << omst.order_of_key({5, 0}) << endl; // 5
    cout << omst.order_of_key({3, 0}) << endl; // 1
    */

    omst.insert({1, 1});
    omst.insert({1, 2});
    omst.insert({2, 3});
    omst.insert({2, 4});
    cout << omst.size() - omst.order_of_key({1, 200}) << endl;
}
```

1.2 segment$_t$*ree*

```cpp
const int mx = 2e5+123;
ll t[mx*3], a[mx], prop[3*mx];
bool vis[3*mx];

void shift ( int id, int b, int e )
{
    int mid = ( b + e ) >> 1;
    t[id*2] += ( mid-b+1 * prop[id] );
    t[id*2+1] += ( e-(mid+1)+1 * prop[id] );

    prop[id*2] += prop[id];
    prop[id*2+1] += prop[id];

    vis[id*2] = vis[id*2+1] = 1;
    prop[id] = vis[id] = 0;
}

void init ( int id, int b, int e )
{
    if ( b == e ) {
        t[id] = a[b];
        return;
    }

    int mid = ( b + e ) >> 1;

    init ( id*2, b, mid );
```

```
    init ( id*2+1, mid+1, e );

    t[id] = t[id*2] + t[id*2+1];
}

void upd ( int id, int b, int e, int i, int j, ll val )
{
    if ( b > j || e < i ) return;
    if ( b >= i && e <= j ) {
        t[id] += ( val * e-b+1 );
        prop[id] += ( prop[id] * val );
        vis[id] = 1;
        return;
    }

    if ( vis[id] ) shift ( id, b, e );
    int mid = ( b + e ) >> 1;

    upd ( id*2, b, mid, i, j, val );
    upd ( id*2+1, mid+1, e, i, j, val );

    t[id] = t[id*2] + t[id*2+1];
}

ll ask ( int id, int b, int e, int i, int j )
{
    if ( b > j || e < i ) return 0;
    if ( b >= i && e <= j ) return t[id];

    if ( vis[id] ) shift ( id, b, e );
    int mid = ( b + e ) >> 1;

    ll ret1 = ask ( id*2, b, mid, i, j );
    ll ret2 = ask ( id*2+1, mid+1, e, i, j );

    return ret1 + ret2;
}
```

1.3 segment$_u$nion

```
int length_union(const vector<pair<int, int>> &a) {
    int n = a.size();
    vector<pair<int, bool>> x(n*2);
    for (int i = 0; i < n; i++) {
        x[i*2] = {a[i].first, false};
        x[i*2+1] = {a[i].second, true};
    }

    sort(x.begin(), x.end());

    int result = 0;
    int c = 0;
    for (int i = 0; i < n * 2; i++) {
        if (i > 0 && x[i].first > x[i-1].first && c > 0)
            result += x[i].first - x[i-1].first;
        if (x[i].second)
            c--;
        else
            c++;
    }
    return result;
}

int point_union ( vii v )
{
    int req_time = 0;
    sort ( all ( v ) );
    int lastr = 0;

        for (auto s : v){

                if (s.F <= lastr) {
                        req_time += max(0, s.S - lastr);
                        lastr = max(s.S, lastr);
                }

                else {
                        req_time += s.S - s.F + 1;
                        lastr = s.S;
                }
        }

        return req_time;
}
```

1.4 sparse$_t$able

```
int st[MAXN][K];

void preprocess()
{
    for (int i = 1; i <= N; i++) st[i][0] = array[i];

    for (int j = 1; j <= K; j++) {
        for (int i = 1; i + (1 << j) <= N; i++) {
            st[i][j] = min(st[i][j-1], st[i + (1 << (j - 1))][j
                - 1]);
        }
    }
}

int getMinimum(int L, int R)
{
    int j = log2(R - L + 1);
    return min(st[L][j], st[R - (1 << j) + 1][j]);
}
```

2 Dynamic Programming

2.1 1D1D$_o$ptimization

```
/// Here l, r is range and p is optimal solution
struct node {
    int l, r, p;
    node(){}
    node ( int _l, int _r, int _p ) : l (_l), r (_r), p (_p) {

    }
};

node que[mx];

int dp[mx], n, c[mx][mx], a[mx];


/// This function calculates the cost of (i, j).
int calc ( int j, int i )
{
    return c[j][i];
}


/// This function compares if i is better ans than j for k
bool cmp ( int i, int j, int k )
{
    int v1 = dp[i] + calc ( i, k ), v2 = dp[j] + calc ( j, k );
    return ( v1 <= v2 );
}


/// This function finds the lowest position where i is optimal
     solution in node cur
int find ( node cur, int i )
{
    int l = cur.l, r = cur.r+1;
    while ( l < r ) {
        int mid = ( l + r ) >> 1;
        if ( cmp ( i, cur.p, mid ) ) r = mid;
        else l = mid+1;
    }

    return r;
}


void solve ()
{
    int s = 1, t = 1;
    dp[0] = 0;
    que[1] = node ( 1, n, 0 ); /// Initializing optimal value
         of all index as 0.

    for ( int i = 1; i <= n; i++ ) {
        while ( s < t && que[s].r < i ) s++; /// Deleting
             ranges from front until we get the range where i
             index lies
        dp[i] = dp[que[s].p] + calc(que[s].p, i ); ///
             calculation dp[i]
```

```
        if ( cmp ( i, que[t].p, n ) ) { /// Checking if i is
             better than the current optimal value of last range
            while ( s <= t && cmp ( i, que[t].p, que[t].l ) )
                t--; /// Deleting all range from back of deque
                where i is better.
            if ( s > t ) que[++t] = node ( i+1, n, i ); ///
                Creating new range when deque is empty.
            else {
                int pos = find( que[t], i ); /// Finding lowest
                    position where i is optimal solution.
                que[t].r = pos-1;
                que[++t] = node ( pos, n, i ); /// Creating new
                    range.
            }
        }
    }
}


int main()
{

    cin >> n;
    for ( int i = 1; i <= n; i++ ) cin >> a[i];

    for ( int i = 1; i <= n; i++ ) {
        for ( int j = i+1; j <= n; j++ ) {
            cin >> c[i][j];
        }
    }

    solve();
    cout << dp[n] << endl;

    return 0;
}
```

2.2 divide$_a$nd$_c$onquer$_u$sing$_k$nuth

```
const int mx = 5e3+123;
ll dp[mx][mx], a[mx], cost[mx][mx], opt[mx][mx];

int main()
{
    optimize();

    int t;
    cin >> t;
    for ( int tc = 1; tc <= t; tc++ ) {
        int n, k;
        cin >> n >> k;
        for ( int i = 1; i <= n; i++ ) cin >> a[i];

        for ( int i = 1; i <= n; i++ ) {
            cost[i][i] = a[i];
            for ( int j = i+1; j <= n; j++ ) {
                cost[i][j] = cost[i][j-1] | a[j];
            }
        }

        for ( int i = 1; i <= n; i++ ) {
            dp[1][i] = cost[1][i];
            opt[1][i] = 1;
        }

        for ( int i = 1; i <= k; i++ ) opt[i][n+1] = n;

        int pre = -1;
        for ( int i = 2; i <= k; i++ ) {
            for ( int j = n; j >= 1; j-- ) {

                int ml = opt[i-1][j];
                int mr = opt[i][j+1];

                if ( pre > mr ) return 0;
                pre = ml;

                dp[i][j] = 0;
                for ( int k = ml; k <= mr; k++ ) {
                    ll d = dp[i-1][k] + cost[k+1][j];

                    if ( d > dp[i][j] ) {
                        dp[i][j] = d;
                        opt[i][j] = k;
                    }
                }
            }
        }

        cout << dp[k][n] << endl;
    }

    return 0;
}
```

2.3 knuth$_o$ptimization

```
const int mx = 1e3+123;
long long dp[mx][mx], c[mx];
int opt[mx][mx];

int main()
{
    optimize();

    ll m, n;
    while ( cin >> m >> n ) {
        mem ( dp, 0 );
        c[n+1] = m;

        for ( int i = 1; i <= n; i++ ) cin >> c[i];

        for ( int i = 0; i <= n+1; i++ ) {
            for ( int l = 0; l+i <= n+1; l++ ) {
                int r = l + i;

                if ( i < 2 ) {
                    dp[l][r] = 0;
                    opt[l][r] = l;
                    continue;
                }

                int ml = opt[l][r-1];
                int mr = opt[l+1][r];

                dp[l][r] = inf;
                for ( int k = ml; k <= mr; k++ ) {
                    int d = dp[l][k] + dp[k][r] + c[r] - c[l];
                    if ( dp[l][r] > d ) {
                        dp[l][r] = d;
                        opt[l][r] = k;
                    }
                }
            }
        }

        cout << dp[0][n+1] << endl;
    }

    return 0;
}
```

2.4 partition$_d$p$_t$rick

```
const int mx = 5e3+123;
int n, num[mx], dp[mx], c[mx][mx], a[mx];

/// Here l, r is range and p is optimal solution
struct node {
    int l, r, p;
    node(){}
    node ( int _l, int _r, int _p ) : l (_l), r (_r), p (_p) {

    }
};

node que[mx];

/// This function compares if i is better ans than j for k
bool cmp ( int i, int j, int k )
{
    int v1 = dp[i] + c[i+1][k], v2 = dp[j] + c[j+1][k];
    if ( v1 == v2 ) return num[i] <= num[j];
    return ( v1 < v2 );
}

/// This function finds the lowest position where i is optimal
     solution in node cur
int find ( node cur, int i )
{
    int l = cur.l, r = cur.r+1;
```

```
    while ( l < r ) {
        int mid = ( l + r ) >> 1;
        if ( cmp ( i, cur.p, mid ) ) r = mid;
        else l = mid+1;
    }

    return r;
}

int solve ( int mid )
{
    int s = 1, t = 1;
    dp[0] = num[0] = 0;
    que[1] = node ( 1, n, 0 ); /// Initilaising optimal value
         of all index as 0.

    for ( int i = 1; i <= n; i++ ) {
        while ( s < t && que[s].r < i ) s++; /// Deleting
             ranges from front until we get the range where i
             index lies
        dp[i] = dp[que[s].p] + c[que[s].p+1][i] + mid; ///
             calculating dp[i] with slop mid
        num[i] = num[que[s].p] + 1; /// calculating num[i].

        if ( cmp ( i, que[t].p, n ) ) { /// Checking if i is
             better than the current optimal value of last range
            while ( s <= t && cmp ( i, que[t].p, que[t].l ) )
                t--; /// Deleting all range from back of queue
                where i is better.
            if ( s > t ) que[++t] = node ( i+1, n, i ); ///
                Creating new range when deque is empty.
            else {
                int pos = find( que[t], i ); /// Finding lowest
                     position where i is optimal solution.
                que[t].r = pos-1;
                que[++t] = node ( pos, n, i ); /// Creating new
                     range.
            }
        }
    }

    return num[n];
}

int main()
{
    int k;
    cin >> n >> k;
    for ( int i = 1; i <= n; i++ ) cin >> a[i];

    for ( int i = 1; i <= n; i++ ) {
        for ( int j = i; j <= n; j++ ) cin >> c[i][j];
    }

    int l = 0, r = 3e7+123, ans = 0;

    /// Binary search on slop
    while ( l <= r ) {
        int mid = ( l + r ) >> 1;
        if ( solve ( mid ) <= k ) {
            ans = dp[n] - ( k * mid ); /// As mid is added in
                 dp[n], k times.
            r = mid-1;
        }

        else l = mid+1;
    }

    cout << ans << endl;


    return 0;
}
```

3 FFT

3.1 FFT

```
typedef complex<dl> cd;
typedef vector<cd> vcd;

void fft ( vcd &a, bool invert )
{
    int n = sz ( a );

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for ( int len = 2; len <= n; len <<= 1 ) {
        dl ang = ( ( 2.0 * PI ) / (dl)len ) * ( invert ? -1 : 1
             );
        cd wlen ( cos ( ang ), sin ( ang ) );

        for ( int i = 0; i < n; i += len ) {
            cd  w(1);
            for ( int j = 0; j < ( len >> 1 ); j++ ) {
                cd u = a[i+j], v = w * a[i+j+(len>>1)];
                a[i+j] = u + v;
                a[i+j+(len>>1)] = u - v;
                w *= wlen;
            }
        }
    }

    if ( invert ) {
        for ( int i = 0; i < n; i++ ) {
            a[i] /= n;
        }
    }
}

vl mul ( vi a, vi b )
{
    vcd fa ( all ( a ) ), fb ( all ( b ) );
    int n = 1;
    while ( n < sz ( a ) + sz ( b ) ) {
        n <<= 1;
    }

    fa.resize ( n ), fb.resize ( n );

    fft ( fa, 0 );
    fft ( fb, 0 );

    for ( int i = 0; i < n; i++ ) {
        fa[i] *= fb[i];
    }

    fft ( fa, 1 );

    vl ret(n+1);
    for ( int i = 0; i < n; i++ ) {
        ret[i] = round ( fa[i].real() );
    }

    return ret;
}

int main()
{
    optimize();

    int t;
    cin >> t;

    while ( t-- ) {
        int n;
        cin >> n;
        vi a(n+1), b(n+1);
        for ( int i = 0; i < n+1; i++ ) cin >> a[i];
        for ( int i = 0; i < n+1; i++ ) cin >> b[i];

        vl ans = mul ( a, b );

        for ( int i = 0; i < (2*n)+1; i++ ) cout << ans[i] << "
             ";
        cout << endl;
    }
    return 0;
}
```

3.2 $\mathbf{fft}_{s}tring_{m}atching$

```
using cd = complex<double>;

int reverse(int num, int lg_n) {
```

```
    int res = 0;
    for (int i = 0; i < lg_n; i++) {
        if (num & (1 << i))
            res |= 1 << (lg_n - 1 - i);
    }
    return res;
}

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    int lg_n = 0;
    while ((1 << lg_n) < n)
        lg_n++;

    for (int i = 0; i < n; i++) {
        if (i < reverse(i, lg_n))
            swap(a[i], a[reverse(i, lg_n)]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd & x : a)
            x /= n;
    }
}


vector<int> multiply(vector<int> const& a, vector<int> const&
     b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);

    vector<int> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}
```

```
const int mx = 5e5+123;
int ans[mx], n, m;
string s, p;

void solve ( char ch )
{
    vector < int > a, b, c;
    for ( auto u : s ) {
        a.push_back ( ( u == ch ) );
    }

    for ( auto u : p ) {
        b.push_back ( ( u == ch ) );
    }

    c = multiply( a, b );

    for ( int i = m-1; i < n; i++ ) {
        ans[i-m+1] += c[i];
    }

}

int main()
{
    optimize();

    cin >> s >> p;

    reverse( p.begin(), p.end() );

    n = s.size();
    m = p.size();

    solve ( 'A' );
    solve ( 'T' );
    solve ( 'G' );
    solve ( 'C' );

    int sol = INT_MAX;
    for ( int i = 0; i <= n-m; i++ ) {
        sol = min ( sol, m - ans[i] );
    }

    cout << sol << endl;


    return 0;
}
```

3.3 NTT

```
///  ***  ---  |||       In the name of ALLAH     |||  ---
     ***  ///
```

```
#include<bits/stdc++.h>
using namespace std;


typedef long long ll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<vi> vvi;
typedef vector<vl> vvl;
typedef pair<int,int> pii;
typedef pair<double, double> pdd;
typedef pair<ll, ll> pll;
typedef vector<pii> vii;
typedef vector<pll> vll;
typedef double dl;

#define endl '\n'
#define PB push_back
#define F first
#define S second
#define all(a) (a).begin(),(a).end()
#define rall(a) (a).rbegin(),(a).rend()
#define sz(x) (int)x.size()

const double PI = acos(-1);
const double eps = 1e-9;
const int inf = 2000000000;
const ll infLL = 9000000000000000000;
#define MOD 998244353

#define mem(a,b) memset(a, b, sizeof(a) )
#define sqr(a) ((a) * (a))

#define optimize()
     ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
#define fraction() cout.unsetf(ios::floatfield);
     cout.precision(10); cout.setf(ios::fixed,ios::floatfield);
#define file()
     freopen("input.txt","r",stdin);freopen("output.txt","w",stdout);

#define dbg(args...) do {cerr << #args << " : "; faltu(args); }
     while(0)
void faltu () {          cerr << endl;}
template < typename T, typename ... hello>void faltu( T arg,
     const hello &... rest) {cerr << arg << '
     ';faltu(rest...);}

ll gcd ( ll a, ll b ) { return __gcd ( a, b ); }
ll lcm ( ll a, ll b ) { return a * ( b / gcd ( a, b ) ); }

inline void normal(ll &a) { a %= MOD; (a < 0) && (a += MOD); }
inline ll modMul(ll a, ll b) { a %= MOD, b %= MOD; normal(a),
     normal(b); return (a*b)%MOD; }
inline ll modAdd(ll a, ll b) { a %= MOD, b %= MOD; normal(a),
     normal(b); return (a+b)%MOD; }
inline ll modSub(ll a, ll b) { a %= MOD, b %= MOD; normal(a),
     normal(b); a -= b; normal(a); return a; }
inline ll modPow(ll b, ll p) { ll r = 1; while(p) { if(p&1) r =
     modMul(r, b); b = modMul(b, b); p >>= 1; } return r; }
```

```
inline ll modInverse(ll a) { return modPow(a, MOD-2); }
inline ll modDiv(ll a, ll b) { return modMul(a, modInverse(b));
     }

int getK ( int m )
{
    for ( int i = 30; i >= 0; i-- ) {
        if ( (m-1) % ( 1 << i ) == 0 ) return i;
    }
}

int generator (int p) {
    vector<int> fact;
    int phi = p-1, n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back (i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1)
        fact.push_back (n);

    for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= (int)modPow( res, phi / fact[i]) != 1;
        if (ok) return res;
    }
    return -1;
}

const int mod = MOD;
const int K = getK ( mod );
const int root = modPow( generator( mod ), ( mod-1 ) / ( 1 << K
     ) );
const int root_1 = modInverse( root );
const int root_pw = 1 << K;


void fft(vector<int> & a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <<= 1)
            wlen = (int)(1LL * wlen * wlen % mod);

        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w
                     % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                w = (int)(1LL * w * wlen % mod);
            }
        }
    }

    if (invert) {
        int n_1 = modInverse(n);
        for (int & x : a)
            x = (int)(1LL * x * n_1 % mod);
    }
}

vector<int> multiply(vector<int> const& a, vector<int> const&
     b) {
    vector<int> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] = modMul( fa[i], fb[i] );
    fft(fa, true);

    vector<int> result(n);
    for (int i = 0; i < n; i++) {
        result[i] = fa[i];
    }
    return result;
}


int main()
{
    optimize();


    return 0;
}
```

4 Flow

4.1 ford$_f$ulkerson

```
/**
Ford-Fulkerson method
Complexity O ( V * E^2 )
**/

struct Ford {

    int n, s, t;
    const int inf = 2147483647;
    vector < vector < int > > capacity;
    vector < vector < int > > adj;
    int parent[mx];

    Ford ( int n, int s, int t ): n(n), s(s), t(t), adj(n+1),
         capacity( n+1, vector < int > (n+1, 0) ) {}

    void addEdge( int u, int v, int cap ) {
        adj[u].push_back ( v );
        adj[v].push_back ( u );
        capacity[u][v] = cap;
        /**
        For undirected graph :
        capacity[u][v] = cap;
        capacity[v][u] = cap;
        **/
    }

    int bfs() {
        mem ( parent, -1 );
        parent[s] = -2;
        queue<pair<int, int>> q;
        q.push({s, inf});

        while (!q.empty()) {
            int cur = q.front().first;
            int flow = q.front().second;
            q.pop();

            for (int next : adj[cur]) {
                if (parent[next] == -1 && capacity[cur][next] >
                     0 ) {
                    parent[next] = cur;
                    int new_flow = min(flow, capacity[cur][next]);
                    if (next == t)
                        return new_flow;
                    q.push({next, new_flow});
                }
            }
        }

        return 0;
    }

    int maxflow() {
        int flow = 0;
        int new_flow;

        while (new_flow = bfs()) {
            flow += new_flow;
            int cur = t;
            while (cur != s) {
                int prev = parent[cur];
                capacity[prev][cur] -= new_flow;
                capacity[cur][prev] += new_flow;
```

```
                cur = prev;
            }
        }

        return flow;
    }

};



int main()
{
    optimize();
    int n, m, s, t;
    cin >> n >> m >> s >> t;

    Ford ford ( n, s, t );

    for ( int i = 1; i <= m; i++ ) {
        int u, v, w;
        cin >> u >> v >> w;
        ford.addEdge( u, v, w );
    }

    cout << ford.maxflow();

    return 0;
}
```

4.2 highest$_{l}abel_{p}reflow_{p}ush$

```
/*
 * Highest Label Preflow Push
 * Complexity : O(V^2 * sqrt(E))
 *
 * Fastest max flow implementation
 * 1. Works on directed graph
 * 2. Works on undirected graph
 * 3. Works on multi-edge(directed/undirected) graph
 * 4. Works on self-loop(directed/undirected) graph
 *
 * Can't find the actual flow.
 *
 * Status: Tested and OK
 */

template <class flow_t> ///int/long long;
struct HighestLabelPreflowPush {
    struct Edge {
        int v, rev;
        flow_t cap, tot;
        Edge(int a, flow_t b, int c) : v(a), rev(c), cap(b),
             tot(b) {}
    };

    const flow_t maxf = numeric_limits<flow_t>::max();

    int ht, S, T, N, H, labelcnt;

    vector<flow_t> exflow;
    vector< vector<Edge> > G;
    vector< vector<int> > hq, gap;
    vector<int> h, cnt;

    HighestLabelPreflowPush(int NN) : exflow(NN), G(NN),
         hq(NN), gap(NN) {}

    void addEdge(int u, int v, flow_t cap) {
        G[u].emplace_back(v, cap, G[v].size());
        G[v].emplace_back(u, 0, G[u].size() - 1);
    }

    void update(int u, int newh) {
        ++labelcnt;
        if (h[u] != H)
            --cnt[h[u]];
        h[u] = newh;
        if (newh == H)
            return;
        ++cnt[ht = newh];
        gap[newh].push_back(u);
        if (exflow[u] > 0)
            hq[newh].push_back(u);
    }

    void globalRelabel() {
        queue<int> q;
        for (int i = 0; i <= H; i++) hq[i].clear(),
             gap[i].clear();
        h.assign(H, H);
        cnt.assign(H, 0);
        q.push(T);
        labelcnt = ht = h[T] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (Edge& e : G[u]) {
                if (h[e.v] == H && G[e.v][e.rev].cap) {
                    update(e.v, h[u] + 1);
                    q.push(e.v);
                }
            }
            ht = h[u];
        }
    }

    void push(int u, Edge& e) {
        if (exflow[e.v] == 0)
            hq[h[e.v]].push_back(e.v);
        flow_t df = min(exflow[u], e.cap);
        e.cap -= df;
        G[e.v][e.rev].cap += df;
        exflow[u] -= df;
        exflow[e.v] += df;
    }

    void discharge(int u) {
        int nxth = H;
        if (h[u] == H)
            return;
        for (Edge& e : G[u])
            if (e.cap) {
                if (h[u] == h[e.v] + 1) {
                    push(u, e);
                    if (exflow[u] <= 0)
                        return;
                } else if (nxth > h[e.v] + 1)
                    nxth = h[e.v] + 1;
            }
        if (cnt[h[u]] > 1)
            update(u, nxth);
        else
            for (; ht >= h[u]; gap[ht--].clear()) {
                for (int& j : gap[ht]) update(j, H);
            }
    }

    flow_t maxFlow(int s, int t, int n) {
        S = s, T = t, N = n, H = N + 1;
        fill( exflow.begin(), exflow.end(), 0 );
        exflow[S] = maxf;
        exflow[T] = -maxf;
        globalRelabel();
        for (Edge& e : G[S]) push(S, e);
        for (; ~ht; --ht) {
            while (!hq[ht].empty()) {
                int u = hq[ht].back();
                hq[ht].pop_back();
                discharge(u);
                if (labelcnt > (N << 2))
                    globalRelabel();
            }
        }
        return exflow[T] + maxf;
    }
};


int main() {
    optimize();
    int T;
    cin >> T;
    for( int test = 1; test <= T; ++test ) {
            int N, M, s, t; ///no. of nodes; no. of edges;
                 source; sink;
        cin >> N >> M >> s >> t;
            HighestLabelPreflowPush<int> hlpp(N+2); ///int
                 to long long for flow of long long; total
                 no. of nodes+2(nodes+1 does not work);
            for( int i = 1; i <= M; ++i ) {
                int u, v, w;
                cin >> u >> v >> w;
                hlpp.addEdge(u, v, w); ///For directed
                     graph

                /**
                      For undirected graph:
```

```
                        hlpp.addEdge(u, v, w);
                        hlpp.addEdge(v, u, w);
                    **/
                }
                cout << hlpp.maxFlow(s, t, N) << endl;
                     ///source; sink; number of nodes;
    }
    return 0;
}
```

4.3 hungarian

```
const int INF = inf;
int a[123][123];

/// for set s1 and s2 what is maximum matching with minimum cost


int main()
{
    optimize();

    int T;
    scanf ( "%d", &T );
    for ( int tc = 1; tc <= T; tc++ ) {
        int n;
        scanf ( "%d", &n );
        for ( int i = 1; i <= n; i++ ) {
            for ( int j = 1; j <= n; j++ ) {
                scanf( "%d", &a[i][j] );
                a[i][j] *= -1; /// for max cost.
            }
        }
        int m = n;

        vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
        for (int i=1; i<=n; ++i) {
            p[0] = i;
            int j0 = 0;
            vector<int> minv (m+1, INF);
            vector<char> used (m+1, false);
            do {
                used[j0] = true;
                int i0 = p[j0], delta = INF, j1;
                for (int j=1; j<=m; ++j)
                    if (!used[j]) {
                        int cur = a[i0][j]-u[i0]-v[j];
                        if (cur < minv[j])
                            minv[j] = cur, way[j] = j0;
                        if (minv[j] < delta)
                            delta = minv[j], j1 = j;
                    }
                for (int j=0; j<=m; ++j)
                    if (used[j])
                        u[p[j]] += delta, v[j] -= delta;
                    else
                        minv[j] -= delta;
                j0 = j1;
            } while (p[j0] != 0);
            do {
                int j1 = way[j0];
                p[j0] = p[j1];
                j0 = j1;
            } while (j0);
        }

        printf ( "Case %d: %d\n", tc, v[0] );

        /// v[0] is the cost.
        /// -v[0] for min cost
        /// v[0] for max cost

    }

    return 0;
}
```

5 Geometry

5.1 geometry$_{t}$*emplate*

```
double INF = 1e100;
double EPS = 1e-12;

struct PT {
        double x, y;
        PT() {}
        PT(double x, double y) : x(x), y(y) {}
        PT(const PT &p) : x(p.x), y(p.y) {}
        PT operator + (const PT &p) const { return PT(x+p.x,
             y+p.y); }
        PT operator - (const PT &p) const { return PT(x-p.x,
             y-p.y); }
        PT operator * (double c)  const { return PT(x*c, y*c );
             }
        PT operator / (double c)  const { return PT(x/c, y/c );
             }
        bool operator <(const PT &p) const {
              return x < p.x || (x == p.x && y < p.y);
        }
};

double dot(PT p, PT q)   { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}

// checks if a-b-c is CW or not.
bool isPointsCW(PT a, PT b, PT c) {
    return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y)+EPS < 0;
}
// checks if a-b-c is CCW or not.
bool isPointsCCW(PT a, PT b, PT c) {
    return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y) > EPS;
}
// checks if a-b-c is collinear or not.
bool isPointsCollinear(PT a, PT b, PT c) {
    return abs(a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y)) <=
         EPS;
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)  { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) { // rotate a point CCW t degrees
     around the origin
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
       return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
       double r = dot(b-a,b-a);
       if (fabs(r) < EPS) return a;
       r = dot(c-a, b-a)/r;
       if (r < 0) return a;
       if (r > 1) return b;
       return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
       return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
       return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or
     collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
       return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
       return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
```

```
// intersection exists;
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
       b=b-a; d=c-d; c=c-a;
       assert(dot(b, b) > EPS && dot(d, d) > EPS);
       return a + b*cross(c, d)/cross(b, d);
}

// shift the straight line passing through points a and b
// by distance Dist.
// If Dist is negative the line is shifted rightwards or
     upwards.
// If Dist is positive the line is shifted leftwards or
     downwards.
// The new line passes through points c and d
//
     https://math.stackexchange.com/questions/2593627/i-have-a-line-i-want-to-move-the-line-a-certain-distance-away-parallelly/2594547
pair<PT,PT> ShiftLineByDist(PT a, PT b, double Dist) {
       double r = sqrt( dist2(a, b) );
       double delx = (Dist*(a.y-b.y))/r;
       double dely = (Dist*(b.x-a.x))/r;
       PT c = PT(a.x+delx, a.y+dely);
       PT d = PT(b.x+delx, b.y+dely);
       return MP(c, d);
}

// This code computes the area or centroid of a (possibly
     nonconvex)
// polygon, assuming that the coordinates are listed in a
     clockwise or
// counterclockwise fashion. Note that the centroid is often
     known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
       double area = 0;
       for(int i = 0; i < p.size(); i++) {
              int j = (i+1) % p.size();
              area += p[i].x*p[j].y - p[j].x*p[i].y;
       }
       return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
       return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
       PT c(0,0);
       double scale = 6.0 * ComputeSignedArea(p);
       for (int i = 0; i < p.size(); i++){
              int j = (i+1) % p.size();
              c = c + (p[i]+p[j])*(p[i].x*p[j].y -
                   p[j].x*p[i].y);
       }
       return c / scale;
}

// angle from p2->p1 to p2->p3, returns -PI to PI
double angle(PT p1, PT p2, PT p3)
{
    PT va = p1-p2,vb=p3-p2;
    double x,y;
    x=dot(va,vb);
    y=cross(va,vb);
    return(atan2(y,x));
}


int main()
{
       // expected: (-5,2)
       cerr << RotateCCW90(PT(2,5)) << endl;

       // expected: (5,-2)
       cerr << RotateCW90(PT(2,5)) << endl;

       // expected: (-5,2)
       cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

       // expected: (5,2)
       cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7))
            << endl;

       // expected: (5,2) (7.5,3) (2.5,1)
       cerr << ProjectPointSegment(PT(-5,-2), PT(10,4),
            PT(3,7)) << " "
              << ProjectPointSegment(PT(7.5,3), PT(10,4),
                   PT(3,7)) << " "
              << ProjectPointSegment(PT(-5,-2), PT(2.5,1),
                   PT(3,7)) << endl;

       // expected: 6.78903
       cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;


       // expected: 1 0 1
       cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1),
            PT(4,5)) << " "
              << LinesParallel(PT(1,1), PT(3,5), PT(2,0),
                   PT(4,5)) << " "
              << LinesParallel(PT(1,1), PT(3,5), PT(5,9),
                   PT(7,13)) << endl;

       // expected: 0 0 1
       cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1),
            PT(4,5)) << " "
              << LinesCollinear(PT(1,1), PT(3,5), PT(2,0),
                   PT(4,5)) << " "
              << LinesCollinear(PT(1,1), PT(3,5), PT(5,9),
                   PT(7,13)) << endl;

       // expected: (1,2)
       cerr << ComputeLineIntersection(PT(0,0), PT(2,4),
            PT(3,1), PT(-1,3)) << endl;

       // area should be 5.0
       // centroid should be (1.1666666, 1.166666)
       PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
       vector<PT> p(pa, pa+4);
       PT c = ComputeCentroid(p);
       cerr << "Area: " << ComputeArea(p) << endl;
       cerr << "Centroid: " << c << endl;

       // expected: 0
       cerr << isPointsCCW( PT(5, 6), PT(10, 10), PT(11, 5) )
            << endl;
       // expected: 1
       cerr << isPointsCCW( PT(5, 6), PT(10, 2), PT(11, 5) )
            << endl;
       // expected: 1
       cerr << isPointsCW( PT(5, 6), PT(10, 10), PT(11, 5) )
            << endl;
       // expected: 0
       cerr << isPointsCW( PT(5, 6), PT(10, 2), PT(11, 5) ) <<
            endl;
       // expected: 0
       cerr << isPointsCollinear( PT(5, 6), PT(10, 2), PT(11,
            5) ) << endl;
       // expected: 1
       cerr << isPointsCollinear( PT(5, 6), PT(10, 6), PT(11,
            6) ) << endl;

       // expected: (-0.437602,12.6564) (2.5624,14.6564)
       cerr << ShiftLineByDist( PT(4, 6), PT(7, 8), 8 ).F << "
            " << ShiftLineByDist( PT(4, 6), PT(7, 8), 8 ).S <<
            endl;
       // expected: (8.4376,-0.656402) (11.4376,1.3436)
       cerr << ShiftLineByDist( PT(4, 6), PT(7, 8), -8 ).F <<
            " " << ShiftLineByDist( PT(4, 6), PT(7, 8), -8 ).S
            << endl;


}
```

5.2 half$_{planner}$

```
// OFFLINE
// Complexity: O(NlgN)
// very easy concept and implementation
// https://codeforces.com/blog/entry/61710

double INF = 1e100;
double EPS = 1e-12;

struct PT {
       double x, y;
       PT() {}
       PT(double x, double y) : x(x), y(y) {}
       PT(const PT &p) : x(p.x), y(p.y) {}
       PT operator + (const PT &p) const { return PT(x+p.x,
            y+p.y); }
       PT operator - (const PT &p) const { return PT(x-p.x,
            y-p.y); }
       PT operator * (double c)  const { return PT(x*c, y*c );
            }
       PT operator / (double c)  const { return PT(x/c, y/c );
            }
       bool operator <(const PT &p) const {
```

```
            return x < p.x || (x == p.x && y < p.y);
        }
};

ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}


int steps = 600;

vector<PT> lower_hull, upper_hull;
int lower_hull_sz, upper_hull_sz;

bool leBorder = 0, riBorder = 0;

double func( double xx, double val )
{
        double ans1 = INF, ans2 = -INF, ans;
        for( int i = 0; i < lower_hull_sz-1; ++i ) {
                if( leBorder && (i == 0) ) continue;
                PT a = lower_hull[i], b = lower_hull[i+1];
                        // straight line passes through
                     points a and b
                double m = (a.y-b.y)/(a.x-b.x);     // slope of
                     the straight line; if the TL is strict,
                     then better precalculate all the slopes and
                     store them beforehand
                double c = a.y - a.x*(m);           // intercept
                     of the straight line; if the TL is strict,
                     then better precalculate all the intercepts
                     and store them beforehand
                double aa = m*xx;
                double bb = c;
                double cc = aa+bb;
                ans1 = min( ans1, cc );
        }
        for( int i = 0; i < upper_hull_sz-1; ++i ) {
                if( riBorder && (i == upper_hull_sz-2) )
                     continue;
                PT a = upper_hull[i], b = upper_hull[i+1];
                        // straight line passes through
                     points a and b
                double m = (a.y-b.y)/(a.x-b.x);     // slope of
                     the straight line; if the TL is strict,
                     then better precalculate all the slopes and
                     store them beforehand
                double c = a.y - a.x*(m);           // intercept
                     of the straight line; if the TL is strict,
                     then better precalculate all the intercepts
                     and store them beforehand
                double aa = m*xx;
                double bb = c;
                double cc = aa+bb;
                ans2 = max( ans2, cc );
        }
        ans = ans1-ans2;
        return ans;
}
```

```
bool Ternary_Search(double val)
{
        double lo = -INF, hi = INF, mid1, mid2;
        leBorder = 0, riBorder = 0;
        if( lower_hull[0].x == lower_hull[1].x ) lo =
             lower_hull[0].x+val, leBorder = 1;
        if( upper_hull[upper_hull_sz-2].x ==
             upper_hull[upper_hull_sz-1].x ) hi =
             upper_hull[upper_hull_sz-1].x-val, riBorder = 1;
        if( lo > hi ) return 0;
        for( int i = 0; i < steps; ++i ) {
                mid1 = (lo*2.0 + hi)/3.0;
                mid2 = (lo + 2.0*hi)/3.0;
                double ff1 = func(mid1, val);
                double ff2 = func(mid2, val);
                if( ff1 >= 0 || ff2 >= 0 ) return 1;
                if( ff1 > ff2 ) hi = mid2;
                else lo = mid1;
        }
        if( func(lo, val) >= 0 ) return 1;
        return 0;
}
```

5.3 ternary$_{s}$earch

```
double ternary_search(double l, double r) {
    double eps = 1e-9;              //set the error limit here
    while (r - l > eps) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1);      //evaluates the function at m1
        double f2 = f(m2);      //evaluates the function at m2
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l);                    //return the maximum of f(x)
         in [l, r]
}


double ternary_search( int l, int r) {
                //set the error limit here
    while (r - l <= 3) {
        int m1 = l + (r - l) / 3;
        int m2 = r - (r - l) / 3;
        int f1 = f(m1);      //evaluates the function at m1
        int f2 = f(m2);      //evaluates the function at m2
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }

    int ret = inf;
    for ( int i = l; i <= r; i++ ) tet = max ( ret, f(l) )
    return f(l);                    //return the maximum of f(x)
         in [l, r]
}
```

6 Graph

6.1 dijkstra

```
const int INF = 2147483647;
const int MAX = 5005;
int D[MAX], N; // Keeps minimum distance to each node
vector<pair<int,int>> E[MAX]; // Adjacency list

void dijkstra()
{
    for(int i = 1; i <= N; i++) D[i] = INF;
    D[1] = 0;
    priority_queue<pair<int,int>,vector<pair<int,int>>,greater<pair<int,
         q;
    q.push({0,1});

    while(!q.empty())
    {
        pair<int,int> p = q.top();
        q.pop();

        int u = p.second, dist = p.first;
        if(dist > D[u]) continue;

        for(pair<int,int> pr : E[u])
        {
            int v = pr.first;
            int next_dist = dist + pr.second;

            if(next_dist < D[v])
            {
                D[v] = next_dist;
                q.push({next_dist,v});
            }
        }
    }
}
```

6.2 dynamic$_{c}$onnectivity

```
const int mx = 100100;
int n, m, par[mx], sz[mx];
bool ans[mx];
pii queries[mx];
vii t[mx*5];
map<pii, int> M;
stack<int> st;
```

```
void update(int cur, int s, int e, int l, int r, pii val) {
    if (s > r || e < l) return;
    if (l <= s && e <= r) {
        t[cur].PB(val);
        return;
    }
    int c1 = (cur << 1), c2 = c1 | 1, m = (s + e) >> 1;
    update(c1, s, m, l, r, val);
    update(c2, m + 1, e, l, r, val);
}

int Find(int u) { return (par[u] == u ? u : Find(par[u])); }

bool isSame(int u, int v) { return Find(u) == Find(v); }

bool makeAns(int i) {
    if (queries[i].F != -1) {
        return isSame(queries[i].F, queries[i].S);
    }
    return 0;
}

void Merge(pii edge) {
    int u = Find(edge.F), v = Find(edge.S);
    if (u == v) return;
    if (sz[u] < sz[v]) swap(u, v);
    sz[u] += sz[v];
    par[v] = u;
    st.push(v);
}

void rollback(int moment) {
    while (st.size() > moment) {
        int cur = st.top();
        st.pop();
        sz[Find(cur)] -= sz[cur];
        par[cur] = cur;
    }
}

void dfs(int cur, int s, int e) {
    if (s > e) return;
    int moment = st.size();
    for (pii edge : t[cur]) {
        Merge(edge);
    }
    if (s == e) ans[s] = makeAns(s);
    else {
        int c1 = (cur << 1), c2 = c1 | 1, m = (s + e) >> 1;
        dfs(c1, s, m);
        dfs(c2, m + 1, e);
    }
    rollback(moment);
}

int main() {
    optimize();
    cin >> n >> m;
    for (int i = 1; i <= n; ++i) {
        par[i] = i;
        sz[i] = 1;
    }
    for (int i = 1; i <= m; ++i) queries[i] = MP(-1, -1);
    for (int i = 1; i <= m; ++i) {
        string q;
        int u, v;
        cin >> q >> u >> v;
        if (u < v) swap(u, v);
        if (q == "conn") queries[i] = MP(u, v);
        else {
            if (q == "rem") {
                update(1, 1, m, M[MP(u, v)], i, MP(u, v));
                M.erase(MP(u, v));
            }
            else M[MP(u, v)] = i;
        }
    }
    for (auto it : M) update(1, 1, m, it.S, m, it.F);
    dfs(1, 1, m);
    for (int i = 1; i <= m; ++i) {
        if (queries[i].F != -1) {
            cout << (ans[i] ? "YES" : "NO") << endl;
        }
    }
    return 0;
}
```

6.3 HLD

```
const int mx = 2e5+123;
ll t[mx*3], a[mx], prop[3*mx];
bool vis[3*mx];

int baseArry[mx], basePos[mx], chainNO, chainHead[mx],
     parent[mx], level[mx], chainInd[mx], ptr, p[mx][40],
     sz[mx];
vii adj[mx];

void shift ( int id, int b, int e )
{
    int mid = ( b + e ) >> 1;
    t[id*2] += ( mid-b+1 * prop[id] );
    t[id*2+1] += ( e-(mid+1)+1 * prop[id] );

    prop[id*2] += prop[id];
    prop[id*2+1] += prop[id];

    vis[id*2] = vis[id*2+1] = 1;
    prop[id] = vis[id] = 0;
}

void init ( int id, int b, int e )
{
    if ( b == e ) {
        t[id] = baseArry[b];
        return;
    }
    int mid = ( b + e ) >> 1;

    init ( id*2, b, mid );
    init ( id*2+1, mid+1, e );

    t[id] = t[id*2] + t[id*2+1];
}

void upd ( int id, int b, int e, int i, int j, ll val )
{
    if ( b > j || e < i ) return;
    if ( b >= i && e <= j ) {
        t[id] += ( val * e-b+1 );
        prop[id] += ( prop[id] * val );
        vis[id] = 1;
        return;
    }

    if ( vis[id] ) shift ( id, b, e );
    int mid = ( b + e ) >> 1;

    upd ( id*2, b, mid, i, j, val );
    upd ( id*2+1, mid+1, e, i, j, val );

    t[id] = t[id*2] + t[id*2+1];
}

ll ask ( int id, int b, int e, int i, int j )
{
    if ( b > j || e < i ) return 0;
    if ( b >= i && e <= j ) return t[id];

    if ( vis[id] ) shift ( id, b, e );
    int mid = ( b + e ) >> 1;

    ll ret1 = ask ( id*2, b, mid, i, j );
    ll ret2 = ask ( id*2+1, mid+1, e, i, j );

    return ret1 + ret2;
}

int dfs ( int u, int lev )
{
    int ret = 1;
    level[u] = lev;
    for ( auto v : adj[u] ) {
        if ( parent[u] != v.F ) {
            parent[v.F] = u;
            ret += dfs ( v.F, lev+1 );
        }
    }

    sz[u] = ret;
    return ret;
}


void HLD ( int u, int cost, int pU )
{
```

```
    if ( chainHead[chainNO] == -1 ) {
        chainHead[chainNO] = u;
    }

    chainInd[u] = chainNO;
    basePos[u] = ++ptr;
    baseArry[ptr] = cost;

    int m = -1, id = -1, c = -1;

    for ( auto v : adj[u] ) {
        if ( v.F != pU ) {
            if ( sz[v.F] > m ) {
                m = sz[v.F];
                id = v.F;
                c = v.S;
            }
        }
    }


    if ( id != -1 ) HLD ( id, c, u );

    for ( auto v : adj[u] ) {
        if ( v.F != pU && v.F != id ) {
            chainNO++;
            HLD ( v.F, v.S, u );
        }
    }
}

void preprocess ( int n )
{
    for ( int i = 1; i <= n; i++ ) p[i][0] = parent[i];

    for ( int j = 1; (1 << j) <= n; j++ ) {
        for ( int i = 1; i <= n; i++ ) {
            if ( p[i][j-1] != -1 ) p[i][j] = p[p[i][j-1]][j-1];
        }
    }
}

int LCA ( int u, int v )
{
    if ( level[u] < level[v] ) swap ( u, v );

    int dist = level[u] - level[v];

    int rise;
    while ( dist > 0 ) {
        rise = log2( dist );
        u = p[u][rise];
        dist -= ( 1 << rise );
    }

    if ( u == v ) return u;

    for ( int i = 20; i >= 0; i-- ) {
        if ( p[u][i] != p[v][i] && p[u][i] != -1 ) {
            u = p[u][i];
            v = p[v][i];
        }
    }

    return parent[u];
}


void query_upd ( int u, int v, ll val )
{
    if ( u == v ) return;
    int chainU, chainV = chainInd[v];

    while ( 1 ) {
        chainU = chainInd[u];
        if ( chainU == chainV ) {
            upd ( 1, 1, ptr, basePos[v]+1, basePos[u], val );
            break;
        }

        upd ( 1, 1, ptr, basePos[chainHead[chainU]],
              basePos[u], val );
        u = chainHead[chainU];
        u = parent[u];
    }

    return;
}


void queryUpd ( int u, int v, ll val )
{
    int lca = LCA ( u, v );
    query_upd ( u, lca, val );
    query_upd ( v, lca, val );
}



ll query_ask ( int u, int v )
{
    if ( u == v ) return 0;
    int chainU, chainV = chainInd[v];
    ll ans = 0;

    while ( 1 ) {
        chainU = chainInd[u];
        if ( chainU == chainV ) {
            ans += ask ( 1, 1, ptr, basePos[v]+1, basePos[u] );
            break;
        }

        ans += ask ( 1, 1, ptr, basePos[chainHead[chainU]],
              basePos[u] );
        u = chainHead[chainU];
        u = parent[u];
    }

    return ans;
}

ll queryAsk ( int u, int v )
{
    int lca = LCA ( u, v );
    return query_ask ( u, lca ) + query_ask ( v, lca );
}



int main()
{
    optimize();

    int n;

    ptr = 0, chainNO = 1;
    mem ( p, -1 );
    mem ( chainHead, -1 );


    dfs ( 1, 0 );
    HLD ( 1, 0, -1 );
    preprocess( n );
    init ( 1, 1, ptr );

    return 0;
}
```

6.4 LCA

```
int n, l;
vector<vector<int>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool is_ancestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
```

```
int lca(int u, int v)
{
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}
```

6.5 mst$_{k}$*ruskal*

```
struct edge {
    int u, v, w;
    bool operator<(const edge& p) const
    {
        return w < p.w;
    }
};

int par[MAXN], size[MAXN];
vector<edge> e;

int find_root(int i) { return (par[i] == i ? i : par[i] =
     find_root(par[i])); }

void unite(int u, int v) {
    u = find_root(u), v = find_root(v);
    if (u != v) {
        if (size[u] < size[v]) swap(u, v);
        par[v] = u;
        size[u] += size[v];
    }
}

int mst(int n) {
    sort(e.begin(), e.end());
    for (int i = 1; i <= n; i++) {
        par[i] = i;
        size[i] = 1;
    }
    int s = 0;
    for (int i = 0; i < (int)e.size(); i++) {
        int u = find_root(e[i].u);
        int v = find_root(e[i].v);
        if (u != v) {
            unite(u, v);
            s += e[i].w;
        }
    }
    return s;
}
```

7 Math

7.1 big$_{i}$*nteger*

```
#include <bits/stdc++.h>

using namespace std;

struct Bigint {
    // representations and structures
    string a; // to store the digits
    int sign; // sign = -1 for negative numbers, sign = 1
         otherwise

    // constructors
    Bigint() {} // default constructor

    Bigint( string b ) { (*this) = b; } // constructor for
         string

    // some helpful methods
    int size() { // returns number of digits
        return a.size();
    }

    Bigint inverseSign() { // changes the sign
        sign *= -1;
        return (*this);
    }

    Bigint normalize( int newSign ) { // removes leading 0,
         fixes sign
        for( int i = a.size() - 1; i > 0 && a[i] == '0'; i-- )
            a.erase(a.begin() + i);
        sign = ( a.size() == 1 && a[0] == '0' ) ? 1 : newSign;
        return (*this);
    }

    // assignment operator
    void operator = ( string b ) { // assigns a string to Bigint
        a = b[0] == '-' ? b.substr(1) : b;
        reverse( a.begin(), a.end() );
        this->normalize( b[0] == '-' ? -1 : 1 );
    }

    // conditional operators
    bool operator < ( const Bigint &b ) const { // less than
         operator
        if( sign != b.sign ) return sign < b.sign;
        if( a.size() != b.a.size() )
            return sign == 1 ? a.size() < b.a.size() : a.size()
                 > b.a.size();
        for( int i = a.size() - 1; i >= 0; i-- ) if( a[i] !=
             b.a[i] )
            return sign == 1 ? a[i] < b.a[i] : a[i] > b.a[i];
        return false;
    }

    bool operator == ( const Bigint &b ) const { // operator
         for equality
        return a == b.a && sign == b.sign;
    }

    // mathematical operators
    Bigint operator + ( Bigint b ) { // addition operator
         overloading
        if( sign != b.sign ) return (*this) - b.inverseSign();
        Bigint c;
        for(int i = 0, carry = 0; i<a.size() || i<b.size() ||
             carry; i++ ) {
            carry+=(i<a.size() ? a[i]-48 : 0)+(i<b.a.size() ?
                 b.a[i]-48 : 0);
            c.a += (carry % 10 + 48);
            carry /= 10;
        }
        return c.normalize(sign);
    }

    Bigint operator - ( Bigint b ) { // subtraction operator
         overloading
        if( sign != b.sign ) return (*this) + b.inverseSign();
        int s = sign; sign = b.sign = 1;
        if( (*this) < b ) return ((b -
             (*this)).inverseSign()).normalize(-s);
        Bigint c;
        for( int i = 0, borrow = 0; i < a.size(); i++ ) {
            borrow = a[i] - borrow - (i < b.size() ? b.a[i] :
                 48);
            c.a += borrow >= 0 ? borrow + 48 : borrow + 58;
            borrow = borrow >= 0 ? 0 : 1;
        }
        return c.normalize(s);
    }

    Bigint operator * ( Bigint b ) { // multiplication operator
         overloading
        Bigint c("0");
        for( int i = 0, k = a[i] - 48; i < a.size(); i++, k =
             a[i] - 48 ) {
            while(k--) c = c + b; // ith digit is k, so, we add
                 k times
            b.a.insert(b.a.begin(), '0'); // multiplied by 10
        }
        return c.normalize(sign * b.sign);
    }

    Bigint operator / ( Bigint b ) { // division operator
         overloading
```

```
        if( b.size() == 1 && b.a[0] == '0' ) b.a[0] /= ( b.a[0]
             - 48 );
        Bigint c("0"), d;
        for( int j = 0; j < a.size(); j++ ) d.a += "0";
        int dSign = sign * b.sign; b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- ) {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) ) c = c - b, d.a[i]++;
        }
        return d.normalize(dSign);
    }

    Bigint operator % ( Bigint b ) { // modulo operator
         overloading
        if( b.size() == 1 && b.a[0] == '0' ) b.a[0] /= ( b.a[0]
             - 48 );
        Bigint c("0");
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- ) {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) ) c = c - b;
        }
        return c.normalize(sign);
    }

    // output method
    void print() {
        if( sign == -1 ) putchar('-');
        for( int i = a.size() - 1; i >= 0; i-- ) putchar(a[i]);
    }
};

int main() {
    Bigint a, b, c; // declared some Bigint variables

    //////////////////////////
    // taking Bigint input //
    //////////////////////////

    string input; // string to take input
    cin >> input; // take the Big integer as string
    a = input; // assign the string to Bigint a
    cin >> input; // take the Big integer as string
    b = input; // assign the string to Bigint b

    ////////////////////////////////
    // Using mathematical operators //
    ////////////////////////////////

    c = a + b; // adding a and b
    c.print(); // printing the Bigint
    puts(""); // newline
    c = a - b; // subtracting b from a
    c.print(); // printing the Bigint
    puts(""); // newline
    c = a * b; // multiplying a and b
    c.print(); // printing the Bigint
    puts(""); // newline
    c = a / b; // dividing a by b
    c.print(); // printing the Bigint
    puts(""); // newline
    c = a % b; // a modulo b
    c.print(); // printing the Bigint
    puts(""); // newline

    ////////////////////////////////
    // Using conditional operators //
    ////////////////////////////////

    if( a == b ) puts("equal"); // checking equality
    else puts("not equal");
    if( a < b ) puts("a is smaller than b"); // checking less
         than operator
    return 0;
}
```

7.2 bitwise$_{sieve}$

```
#define mx 100001010
long long a[mx / 64 + 200];
int prime[5800000];
int cnt = 0;


void sieveGen( int limit )
{
    limit += 100;
    int sq = sqrt ( limit );

        for (long long i = 3; i <= sq; i += 2) {
            if(!(a[i/64]&(1LL<<(i%64)))) {
                for(long long j = i * i; j <= limit; j +=
                     2 * i) {
                    a[j/64] |= (1LL<<(j%64));
                }
            }
        }

        prime[cnt++] = 2;
        for (long long i = 3; i <= limit; i += 2) {
            if(!(a[i / 64] & (1LL << (i % 64)))) {
                prime[cnt++] = i;
            }

        }

}
```

7.3 nominator$_{denominator}$

```
struct frac {
    ll n, d;

    frac(ll _n = 0, ll _d = 1) {
        if ( _d < 0 ) _n = -_n, _d = -_d;
        ll g = gcd ( abs(_n), _d );
        n = _n / g;
        d = _d / g;
    }

    friend frac operator + ( const frac &a, const frac &b ) {
        return frac ( ( a.n * b.d ) + ( b.n * a.d ), ( a.d *
             b.d ) );
    }

    friend frac operator - ( const frac &a, const frac &b ) {
        return frac ( ( a.n * b.d ) - ( b.n * a.d ), ( a.d *
             b.d ) );
    }

    friend frac operator * ( const frac &a, const frac &b ) {
        return frac ( ( a.n * b.n ), ( a.d * b.d ) );
    }

    friend frac operator / ( const frac &a, const frac &b ) {
        return frac ( a.n * b.d, b.n * a.d );
    }

    friend bool operator < ( const frac &a, const frac &b ) {
        frac ret = a - b;
        return ret.n < 0;
    }

    friend bool operator > ( const frac &a, const frac &b ) {
        frac ret = a - b;
        return ret.n >= 0;
    }

    friend void swap ( frac &a, frac &b ) {
        frac tmp = b;
        b = a;
        a = tmp;
    }

};



int main()
{

    frac f1 = frac ( 1, 2 ), f2 = frac( 2, 3 );
    frac ans;

    ans = f1 + f2;
    cout << ans.n << " " << ans.d << endl; ///7 6

    ans = f1 - f2;
    cout << ans.n << " " << ans.d << endl; ///-1 6

    ans = f1 * f2;
    cout << ans.n << " " << ans.d << endl; ///1 3
```

```
    ans = f1 / f2;
    cout << ans.n << " " << ans.d << endl; ///3 4

    swap ( f1, f2 );
    cout << f1.n << " " << f1.d << endl; ///2 3
    cout << f2.n << " " << f2.d << endl; ///1 2


    if ( f1 > f2 ) cout << "Greater\n";///Greater
    if ( f1 < f2 ) cout << "Smaller\n";///Condition is not true.

    swap( f1, f2 );
    if ( f1 > f2 ) cout << "Greater\n";///Condition is not true.
    if ( f1 < f2 ) cout << "Smaller\n";///Smaller

    return 0;
}
```

8 String

8.1 aho$_c$*orasick*

```
const int N = 1e4;

///beware! if k distinct patterns are given having sum of
     length m then size of ending array and oc array will
///be at most m.sqrt(m) ,But for similar patterns one must act
     with them differently
struct aho_corasick
{
       bool is_end[N];
       int link[N];          ///A suffix link for a vertex p is
            a edge that points to
                        ///the longest proper suffix of
                        ///the string corresponding to the
                             vertex p.
    int psz = 1;             ///tracks node numbers of the trie
       map<char, int> to[N]; ///tracks the next node
       vector<int> ending[N];        ///ending[i] stores the
            indexes of patterns which ends
                         ///at node i(from the trie)
       vector<int> oc[N];            ///oc[i] stores ending
            index of all occurrences of pattern[i]
                        ///so real
                             oc[i][j]=oc[i][j]-pattern[i].size()+1,0-indexed
       void clear()
       {
              for(int i = 0; i <= psz; i++)
                     is_end[i] = 0, link[i] = 0,
                          to[i].clear(),ending[i].clear(),oc[i].clear();

              psz = 1;
              is_end[0] = 1;
       }
       void faho_corasick() { clear(); }

       void add_word(string s,int idx)
       {
              int u = 0;
              for(char c: s)
              {
                     if(!to[u].count(c)) to[u][c] = psz++;
                     u = to[u][c];
              }

              is_end[u] = 1;
              ending[u].push_back(idx);
       }

    void populate(int cur)
    {
       /// merging the occurrences of patterns ending at cur
            node in the trie
       for(auto occ: ending[link[cur]])
           ending[cur].push_back(occ);
    }

    void populate(vector<int> &en, int cur)
    {
       /// occurrences of patterns in the given string
       for(auto idx: en)
       {
           oc[idx].push_back(cur);
       }
    }

       void push_links()
       {
              queue<int> q;
              int u, v, j;
              char c;

              q.push(0);
              link[0] = -1;

              while(!q.empty())
              {
                     u = q.front();
                     q.pop();

                     for(auto it: to[u])
                     {
                            v = it.second;
                            c = it.first;
                            j = link[u];

                            while(j != -1 && !to[j].count(c)) {
                        j = link[j];
                            }
                            if(j != -1) link[v] = to[j][c];
                            else link[v] = 0;

                            q.push(v);
                            populate(v);
                     }
              }
       }

       void traverse(string s)
    {
       int n=s.size();
       int cur=0;///root

       for(int i=0;i<n;i++){
           char c=s[i];
           while(cur!=-1 && !to[cur].count(c)) cur=link[cur];
           if(cur!=-1) cur=to[cur][c];
           else cur=0;
           populate(ending[cur],i);
       }
    }
};

aho_corasick t;

int main()
{

    int T;
    cin >> T;
    for ( int tc = 1; tc <= T; tc++ ) {

        t.faho_corasick();
        string s;
        cin >> s;
        int q;
        cin >> q;


        for ( int k = 1; k <= q; k++ ) {
            string p;
            cin >> p;
            t.add_word( p, k );
        }

        t.push_links();
        t.traverse( s );


        for ( int i = 1; i <= q; i++ ) {
            cout << t.oc[i].size() << endl; /// Ending index of
                 patter i in s
            for ( auto u : t.oc[i] ) cout << u << " ";
            cout << endl;
        }
    }


    return 0;
}
```

8.2 aho$_c$orasick$_e$maxx

```
const int K = 26;

struct Vertex {
    int Next[K];
    bool leaf = 0;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];

    Vertex(int p = -1, char ch = '$') : p(p), pch(ch) {
        fill(begin(Next), end(Next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].Next[c] == -1) {
            t[v].Next[c] = t.size();
            t.push_back(Vertex(v, ch));
        }
        v = t[v].Next[c];
    }
    t[v].leaf = 1;
}

int go(int v, char ch);

int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].Next[c] != -1) t[v].go[c] = t[v].Next[c];
        else t[v].go[c] = (v == 0 ? 0 : go(get_link(v), ch));
    }
    return t[v].go[c];
}
```

8.3 hashing

```
struct simpleHash{
    vector<long long>p;
    vector<long long>h;

    long long base,mod,len;

    simpleHash(){}
    simpleHash(string &str, long long b, long long m){
        //0 base index array.
        base=b; mod=m; len=str.size();
        p.resize(len,1);
        h.resize(len+1,0);

        for(int i=1;i<len;i++)p[i]=(p[i-1]*base)%mod;
        for(int
              i=1;i<=len;i++)h[i]=(h[i-1]*base+(str[i-1]-'a'+3))%mod;
    }

    long long rangeHash(int l,int r){ //l and r inclusive
        return (h[r+1]-((h[l]*p[r-l+1])%mod)+mod)%mod;
    }
};


struct doubleHashing{
    simpleHash h1,h2;

    doubleHashing(string &str){
        h1=simpleHash(str,43, (long long)1e9+7);
        h2=simpleHash(str,97, (long long)1e9+7);
    }

    long long rangeHash(int l,int r){
        return (h1.rangeHash(l,r)<<32LL)^h2.rangeHash(l,r);
    }
};

//***Double Hashing***


int pw[123], hash_s[123];

int main()
{
    optimize();

    string s = "asdf";
    doubleHashing d = doubleHashing( s );
    cout << d.rangeHash( 0, sz ( s ) );

    ///Normal Hashing :

    int p = 31; /// Magical primes : 31, 41, 37
    pw[0] = 1;
    for ( int i = 1; i <= 12; i++ ) {
        pw[i] = ( p * pw[i-1] ) % MOD;
    }

    hash_s[0] = ( s[0] - 'a' )+1;
    for ( int i = 1; i < sz(s); i++ ) hash_s[i] = ( hash_s[i-1]
         + ( pw[i] * ( s[i] - 'a' + 1 ) ) ) % MOD;

    return 0;
}
```

8.4 manachers

```
void manachers(string s, vector<int> &d1, vector<int> &d2) {
    int n = s.size();
    d1.resize(n);
    d2.resize(n);
    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
        while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
              {
            k++;
        }
        d1[i] = k--;
        if (i + k > r) {
            l = i - k;
            r = i + k;
        }
    }
    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (0 <= i - k - 1 && i + k < n && s[i - k - 1] ==
              s[i + k]) {
            k++;
        }
        d2[i] = k--;
        if (i + k > r) {
            l = i - k - 1;
            r = i + k;
        }
    }
}
```

8.5 suffix$_a$rray

```
vector<int> sort_cyclic_shifts(string const& s) {
    int n = s.size();
    const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]]) classes++;
        c[p[i]] = classes - 1;
    }

    vector<int> pn(n), cn(n);
```

```
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++) cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--) p[--cnt[c[pn[i]]]] =
             pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) %
                 n]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 <<
                 h)) % n]};
            if (cur != prev) ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}

vector<int> suffix_array_construction(string s) {
    s += "$";
    vector<int> sorted_shifts = sort_cyclic_shifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}

vector<int> lcp_construction(string const& s, vector<int>
     const& p) {
    int n = s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++) rank[p[i]] = i;

    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k]) k++;
        lcp[rank[i]] = k;
        if (k) k--;
    }
    return lcp;
}
```

9 template

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<vi> vvi;
typedef vector<vl> vvl;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<pii> vii;
typedef vector<pll> vll;

#define endl '\n'
#define PB push_back
#define F first
#define S second
#define all(a) (a).begin(), (a).end()
#define rall(a) (a).rbegin(), (a).rend()
#define sz(x) (int) x.size()

const double PI = acos(-1);
const double eps = 1e-9;
const int inf = 2000000000;
const ll infLL = 9000000000000000000;
#define MOD 1000000007

#define mem(a, b) memset(a, b, sizeof(a))
#define sqr(a) ((a) * (a))

#define optimize() ios_base::sync_with_stdio(0); cin.tie(0);
     cout.tie(0);
#define fraction() cout.unsetf(ios::floatfield);
     cout.precision(10); cout.setf(ios::fixed,
     ios::floatfield);
#define file() freopen("input.txt", "r", stdin);
     freopen("output.txt", "w", stdout);

//debug
template<typename F,typename
     S>ostream&operator<<(ostream&os,const pair<F,S>&p){return
     os<<"("<<p.first<<", "<<p.second<<")";}
template<typename T>ostream&operator<<(ostream&os,const
     vector<T>&v){os<<"{";for(auto
     it=v.begin();it!=v.end();++it){if(it!=v.begin())os<<",
     ";os<<*it;}return os<<"}";}
template<typename T>ostream&operator<<(ostream&os,const
     set<T>&v){os<<"[";for(auto
     it=v.begin();it!=v.end();++it){if(it!=v.begin())os<<",";os<<*it;}re
     os<<"]";}
template<typename T>ostream&operator<<(ostream&os,const
     multiset<T>&v) {os<<"[";for(auto
     it=v.begin();it!=v.end();++it){if(it!=v.begin())os<<",
     ";os<<*it;}return os<<"]";}
template<typename F,typename
     S>ostream&operator<<(ostream&os,const
     map<F,S>&v){os<<"[";for(auto
     it=v.begin();it!=v.end();++it){if(it!=v.begin())os<<",
     ";os<<it->first<<" = "<<it->second;}return os<<"]";}
#define dbg(args...) do {cerr << #args << " : "; faltu(args); }
     while(0)
void faltu(){cerr << endl;}
template<typename T>void faltu(T a[],int n){for(int
     i=0;i<n;++i)cerr<<a[i]<<' ';cerr<<endl;}
template<typename T,typename...hello>void faltu(T arg,const
     hello&...rest){cerr<<arg<<' ';faltu(rest...);}

inline bool checkBit(ll n, int i) { return n & (1LL << i); }
inline ll setBit(ll n, int i) { return n | (1LL << i); }
inline ll resetBit(ll n, int i) { return n & (~(1LL << i)); }

inline void normal(ll &a) { a %= MOD; (a < 0) && (a += MOD); }
inline ll modMul(ll a, ll b) { a %= MOD; b %= MOD; normal(a);
     normal(b); return (a * b) % MOD; }
inline ll modAdd(ll a, ll b) { a %= MOD; b %= MOD; normal(a);
     normal(b); return (a + b) % MOD; }
inline ll modSub(ll a, ll b) { a %= MOD; b %= MOD; normal(a);
     normal(b); a -= b; normal(a); return a; }
inline ll modPow(ll b, ll p) { ll r = 1LL; while (p) { if (p &
     1) r = modMul(r, b); b = modMul(b, b); p >>= 1; } return
     r; }
inline ll modInverse(ll a) { return modPow(a, MOD - 2); }
inline ll modDiv(ll a, ll b) { return modMul(a, modInverse(b));
     }

int main() {
    optimize();
    // ...
    return 0;
}
```