KyotoUx-009x (/github/ryo0921/KyotoUx-009x/tree/master)
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Stochastic Processes:
Data Analysis and Computer Simulation

Stochastic processes in the real world

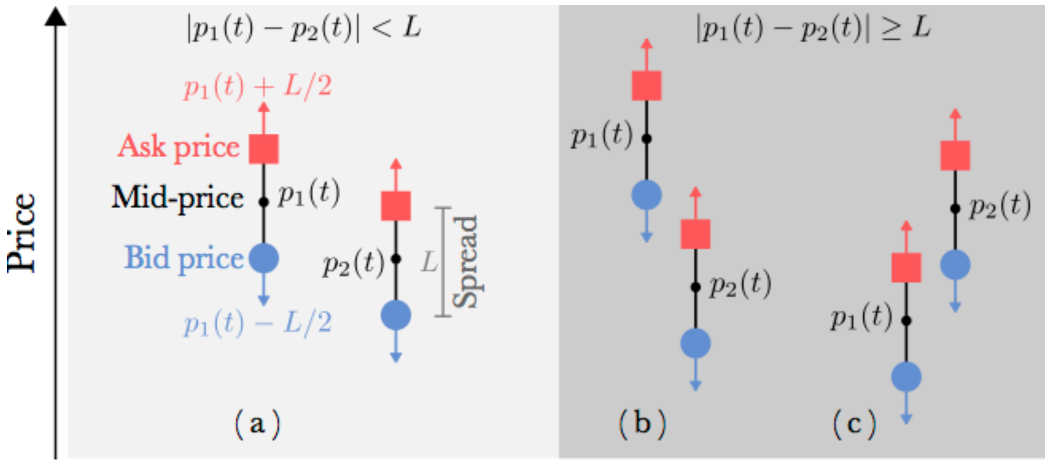
2. A Stochastic Dealer Model I

2.1. Preperation

In [1]:
```
% matplotlib inline
import numpy as np  # import numpy library as np
import math         # use mathematical functions defined by the C standa
import matplotlib.pyplot as plt # import pyplot library as plt
plt.style.use('ggplot') # use "ggplot" style for graphs
pltparams = {'legend.fontsize':16,'axes.labelsize':20,'axes.titlesize':2
             'xtick.labelsize':12,'ytick.labelsize':12,'figure.figsize':
plt.rcParams.update(pltparams)
```

In [2]:
```
# Logarithmic return of price time series
def logreturn(St,tau=1):
    return np.log(St[tau:])-np.log(St[0:-tau]) # Eq.(J2) : G_tau(t) = lc
# normalize data to have unit variance (<(x - <x>)^2> = 1)
def normalized(data):
    return ((data)/np.sqrt(np.var(data)))
# compute normalized probability distribution function
def pdf(data,bins=50):
    hist,edges=np.histogram(data[~np.isnan(data)],bins=bins,density=True
    edges   = (edges[:-1] + edges[1:])/2.0 # get bar center
    nonzero = hist > 0.0                   # non-zero points
    return edges[nonzero], hist[nonzero]
```

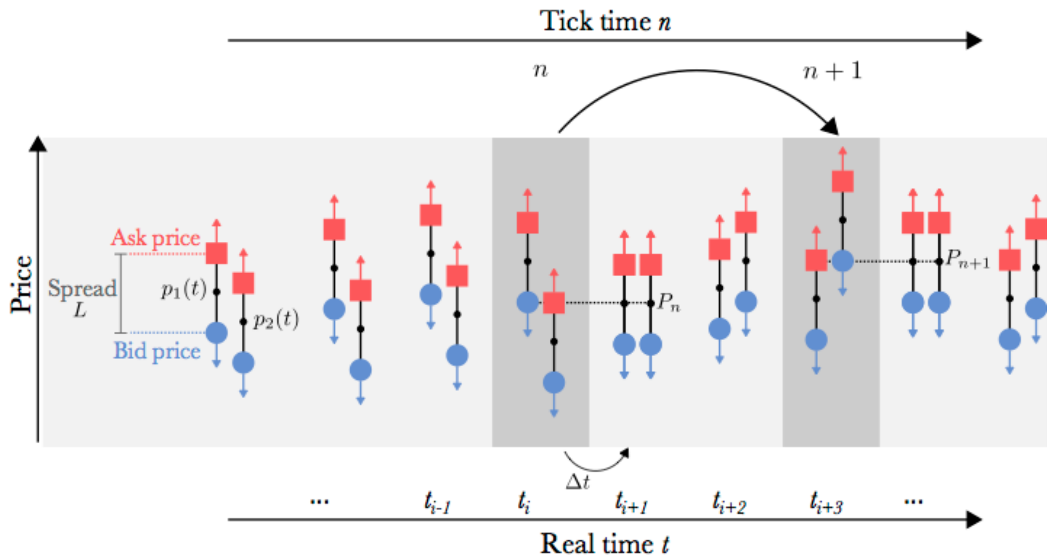
2.2. The Dealer Model



- K. Yamada, H. Takayasu, T. Ito and M. Takayasu, *Physical Revew E* **79**, 051120 (2009).
- Simple Stochastic model with only two dealers offering buying and selling options.
- The mid-price $p_i(t)$ of dealer i at time t is the average of his bid and ask price.

$$|p_i(t) - p_j(t)| \geq L : \text{Transaction Criterion} \qquad \text{(K1)}$$

2.3. Dynamics of the dealer model



- Prices carry out a 1D random walk in 'price' space until a transaction takes place.

$$p_i(t + \Delta t) = p_i(t) + cf_i(t), \qquad i = 1, 2 \qquad \text{(K2)}$$

$$f_i(t) = \begin{cases} +\Delta p & \text{probability} 1/2 \\ -\Delta p & \text{probability} 1/2 \end{cases}$$

- The "Market" price P of a transaction is given by the average of the mid-prices

$$P = (p_1 + p_2)/2 \qquad \text{(K3)}$$

2.4. Dealer model as a 2D random walk

- The dealer model can be understood as a standard 2D Random walk with absorbing boundaries.
- Perform a change in variables, from $p_1(t)$ and $p_2(t)$, to the price difference $D(t)$ and average $A(t)$

$$D(t) = p_1(t) - p_2(t) \tag{K4}$$

$$A(t) = \frac{1}{2}\left(p_1(t) + p_2(t)\right) \tag{K5}$$

- Dynamics of D and A describe a 2D random walk

$$D(t+\Delta t) = D(t) + \begin{cases} +2c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -2c\Delta p & \text{probability } 1/4 \end{cases} \tag{K6}$$

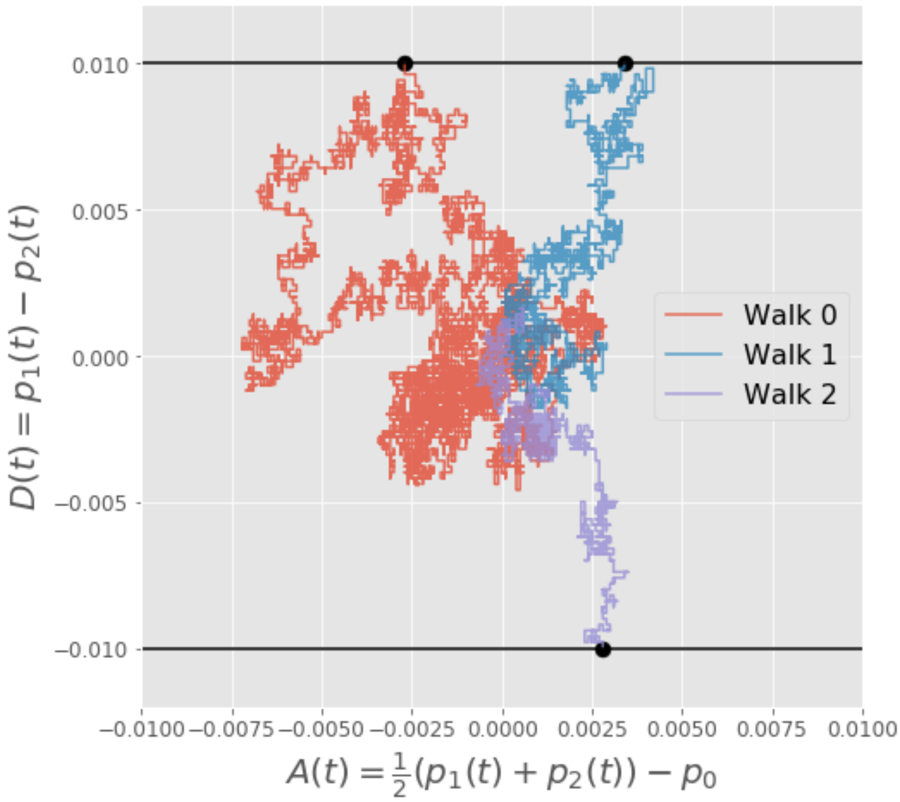
$$A(t+\Delta t) = A(t) + \begin{cases} +c\Delta p & \text{probability } 1/4 \\ 0 & \text{probability } 1/2 \\ -c\Delta p & \text{probability } 1/4 \end{cases} \tag{K7}$$

- When $D(t) = \pm L$ a transaction occurs and the random walk ends, the "particle" is absorbed by the boundary.

In [3]:
```
params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2} # define model paramet
def model1RW(params,p0):                          # simulate Random-Walk for 1 tra
    price = np.array([p0[0], p0[1]])    # initialize mid-prices for deal
    cdp   = params['c']*params['dp']    # define random step size
    Dt    = [price[0]-price[1]]         # initialize price difference as
    At    = [np.average(price)]         # initialize avg price as empy l
    while np.abs(price[0]-price[1]) < params['L']:
        price=price+np.random.choice([-cdp,cdp],size=2) # random walk st
        Dt.append(price[0]-price[1])
        At.append(np.average(price))
    return np.array(Dt),np.array(At)-At[0] # return difference array anc
```

In [4]:
```
np.random.seed(123456)
fig,ax=plt.subplots(figsize=(7.5,7.5),subplot_kw={'xlabel':r'$A(t) = \fr
p0 = [100.25, 100.25]
for i in range(3):
    Dt,At = model1RW(params, p0)
    ax.plot(At,Dt,alpha=0.8,label='Walk '+str(i)) #plot random walk traj
    ax.scatter(At[-1],Dt[-1],marker='o',s=80,color='k') #last point
    print('Walk ', i,' : number of steps = ',len(At),', price change = '
ax.plot([-0.01,0.03],[params['L'],params['L']],color='k') #top absorbing
ax.plot([-0.01,0.03],[-params['L'],-params['L']],color='k') #bottom absc
ax.set_ylim([-0.012, 0.012])
ax.set_xlim([-0.01, 0.01])
ax.legend(loc=5,framealpha=0.8)
plt.show()
```

```
Walk  0  : number of steps =  9248 , price change =  -0.00270000000009
Walk  1  : number of steps =  2201 , price change =  0.00340000000011
Walk  2  : number of steps =  1629 , price change =  0.00280000000009
```



2.5. Perform simulations

In [5]:

```
params={'L':0.01,'c':0.01,'dp':0.01,'dt':0.01**2} # define model paramet
def model1(params,p0,numt):                      # simulate dealer model for n
    mktprice = np.zeros(numt)                    # initialize array for market
    ticktime = np.zeros(numt,dtype=np.int)       # initialize array for tick t
    price    = np.array([p0[0], p0[1]])          # initailize dealer's mid-pri
    time,tick = 0,0                              # real time (t) and tick time
    cdp      = params['c']*params['dp']          # define random step size
    while tick < numt:                           # loop over ticks
        while np.abs(price[0]-price[1])< params['L']: # perform one RW f
            price=price+np.random.choice([-cdp,cdp],size=2) # random wal
            time += 1 # update time t
        price[:] = np.average(price)             # set mid-prices to new marke
        mktprice[tick] = price[0]                # save market price
        ticktime[tick] = time                    # save tick time
        tick += 1                                # updat ticks
    return ticktime,mktprice
```

- A simulation is performed if you run the cell below, but depending on your computer power it may take quite long time until it finishes with properly creating the simulation data "model1.txt".
- You may skip this cell and use pre-calculated simulation data "model1.txt" which can be downloaded from our website to continue further data analyses.

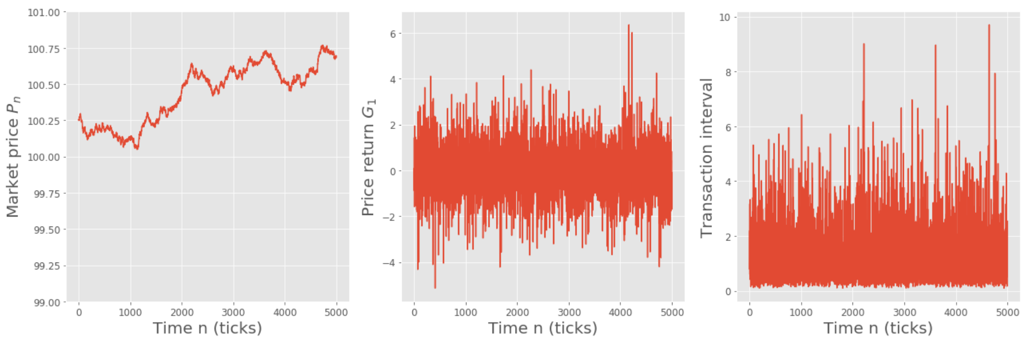
In [ ]:

```
np.random.seed(0)
ticktime,mktprice=model1(params,[100.25,100.25],5000)
np.savetxt('model1.txt',np.transpose([ticktime,mktprice]))
```

2.6. Analyses

In [6]:

```
ticktime,mktprice=np.loadtxt('model1.txt',unpack=True) # read saved data
timeinterval=normalized((ticktime[1:]-ticktime[0:-1])*params['dt']) # cc
dprice=normalized(logreturn(mktprice,1)) # compute logarithmic return of
fig,[ax,bx,cx]=plt.subplots(figsize=(18,6),ncols=3,subplot_kw={'xlabel':
ax.plot(mktprice)
ax.set_ylim(99,101)
ax.set_ylabel(r'Market price $P_n$')
bx.plot(dprice)
bx.set_ylabel(r'Price return $G_1$')
cx.plot(timeinterval)
cx.set_ylabel(r'Transaction interval')
fig.tight_layout() # get nice spacing between plots
plt.show()
```



In [7]:
```
fig,[ax,bx]=plt.subplots(figsize=(15,7.5),ncols=2,subplot_kw={'ylabel':r
edges,hist=pdf(np.abs(dprice),bins=25) # probability density of price ch
ax.plot(edges, hist, lw=3, label='Dealer Model')
x = np.linspace(0, 5)
ax.plot(x,2*np.exp(-x**2/2)/np.sqrt(2*np.pi),lw=6,ls='--',color='gray',a
ax.plot(x, 2*np.exp(-1.5*x),lw=6,color='k',ls='--',alpha=0.8,label=r'Exp
ax.set_xlabel(r'Absolute price return $G_1$')
ax.set_ylabel(r'Probability density')
ax.set_ylim([5e-4,1])
ax.semilogy()
ax.legend()
edges,hist=pdf(timeinterval,bins=100) # probability density of transacti
bx.plot(edges,hist, lw=2)
bx.set_xlabel(r'Transaction interval')
bx.set_ylabel(r'Probability distribution')
bx.semilogy()
plt.show()
```