SAPIENZA
UNIVERSITÀ DI ROMA

DIPARTIMENTO DI INFORMATICA, AUTOMAZIONE E GESTIONALE

A Rigidity-Based Decentralized Bearing Formation Controller for Groups of Quadrotor UAVs

CONTROL OF MULTI-AGENT SYSTEMS-MODULE B

Professor:
Andrea Cristofaro

Student:
Ludovica Cartolano
1796046

Academic Year 2022/2023

Contents

1 Introduction **2**

2 Preliminaries **4**

2.1 Agent Model 4

2.2 Directed Bearing Rigidity in $\mathbb{R}^3 \times \mathcal{S}^1$ 4

3 Decentralized Bearing Formation Control **6**

3.1 Rigidity-based control of bearing frameworks in $\mathbb{R}^3 \times \mathcal{S}^1$ 6

3.2 Rigidity-based localization of time-varying bearing frameworks in $\mathbb{R}^3 \times \mathcal{S}^1$ 8

3.3 Coordinated motions in the null-space of the bearing rigidity matrix . . 10

4 Simulation and Experimental Results **11**

4.1 Simulation Results 11

4.2 Experimental Results 13

5 Conclusions **16**

A PI-ACE Estimator **17**

References **18**

1 Introduction

Drone technology, now inexpensive and accessible, is continuously evolving and being put to several novel uses around the world. Initially known for their military use, drones are now being used to accomplish various other tasks such as shipping and delivery, disaster management, precision agriculture, search and rescue [?] ...

The paper [1] considers the problem of controlling formation of quadrotor UAVs equipped with on board cameras that can to measure relative bearings in their local body-frames with respect to their neighbouring UAVs.

Multi-robot systems are systems composed of multiple interacting dynamic units, biologically inspired from shimmering of honeybees or synchronising of fireflies [2].

These systems can behave in synchronous or in coordination: the first is open loop interaction, the control depends only on the state of each robot singularly; the second one is closed loop, each robot of the group takes decisions for itself depending on the neighbours position, velocity, ... so that they can cooperate [2].

In general groups of robots are called a flock of robots and the flock follows the movements of a leader which is controlled by a human [2].

Formation control involves the coordination of a team of agents (robots) in order to achieve some spatial arrangement while guaranteeing certain properties such as stability and performance [3].

A challenging scenario is decentralized formation control of mobile robots. Every agent must elaborate the gathered information based on only local sensing, to run its own controller while the information flows through the flock. The agents can only obtain relative measurements with respect to the other robots of the group within visibility [1].

Decentralised formation control is useful in "non-trivial" environments where centralized sensing facilities (such as GPS) are not available and they can only rely on their local skills [1]. More in general when there is lack of a common reference frame for measurements and control actions, limited sensing and communication power, limited available memory and limited computational power. In fact the controller complexity is related to the amount of information needed and if the whole state information is needed and so the complexity increases with the number of agents [4].

The theory behind the article [1] rely on formation rigidity theory that applies on different fields like relative distance measurements, relative bearing measurements and cooperative localization from local relative measurements. Rigidity is, in fact, necessary requirement for recovering from the available relative measurements of a consistent solution of the localization problem in a common shared frame.

When addressing visual-based formation control in GPS-denied environments, the assumptions of a common reference frame and of an indirect topology that is hard to realize because it is impossible to keep constant mutual visibility because of limited camera fov.

This is why the authors in [1] approach at the problem by leaving undirected sensing topology and common frame behind and exploit instead the the bearings and distance measurement among an arbitrary pair of robots.

The article [1] generalises a fully decentralised bearing formation controller that only requires presence of a directed bearing rigid topology.

The control strategy proposed in [1] has as goal the bearing formation stabilization and group collective steering which is achieved under the following assumptions:

(i) The quadrotors are assumed able to collect bearing measurements and to impose motion commands in their local body-frames.

(ii) The bearing measurements are not necessarily required to be reciprocal, and the resulting directed sensing topology has no special constraints.

(iii) A single but arbitrary quadrotor pair is additionally assumed able to measure its inter distance.

2 Preliminaries

Before getting into the heart of the paper [1], there will be a brief section on the essential background information.

2.1 Agent Model

Consider a group of N quadrotors UAVs equipped with onboard Inertial Measurements Units (IMUs) and calibrated cameras that are able to exchange data over a radio communication channel.

Let $\mathcal{W} : \{\boldsymbol{O}_{\mathcal{W}}, \boldsymbol{X}_{\mathcal{W}}, \boldsymbol{Y}_{\mathcal{W}}, \boldsymbol{Z}_{\mathcal{W}}\}$ be the world frame.

Consider the simplified kinematic model for the i-th quadrotor

$$\begin{pmatrix} \dot{p}_i \\ \dot{\psi}_i \end{pmatrix} = \begin{pmatrix} R_i & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_i \\ w_i \end{pmatrix} \tag{1}$$

with: $p_i \in \mathbb{R}^3$ is the robot's 3D position in the world frame, $\psi_i \in \mathcal{S}^1$ is the absolute yaw angle which is not considered as an available quantity and $R_i = R_z(\psi_i) \in SO(3)$ is the canonical rotation about the z-axis. $u_i \in \mathbb{R}^3$ is the *body-frame* linear velocity and $w_i \in \mathbb{R}$ is the yaw rate, those are assumed to be known and controllable.
Notice that he quadrotors do not share as a group a common reference frame .

Bearing constraints keep a desired bearing vector (i.e. a set of angles) with respect to its neighbours which are defined by the agent relative bearing (body-frame bearing)

$$\beta_{ij} = R_i^T \frac{p_j - p_i}{||p_j - p_i||} \in \mathbb{S}^2 \tag{2}$$

is the relative bearing constraint from agent i pointing towards the center of mass of agent j in the body frame of agent i which can be typically measured by exploiting the onboard IMU.

2.2 Directed Bearing Rigidity in $\mathbb{R}^3 \times \mathcal{S}^1$

Remember some relevant definitions and properties of directed bearing formation and bearing rigidity in $\mathbb{R}^3 \times \mathcal{S}^1$.
According to [1]:

A Graph $\mathcal{G}$ is a mathematical structure made of nodes (vertices) and lines (edges) that connect each node in a certain configuration.

Directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1 \dots N\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and $e_k = (i, j) \in \mathcal{E}$ but $e_h = (j, i) \notin \mathcal{E}$.

Configuration of N agents Model (1) $q = (p, \psi) \in (\mathbb{R}^3 \times \mathcal{S}^1)^N$ and the configuration of the i-th agent $q_i = (p_i, \psi_i) \in \mathbb{R}^3 \times \mathcal{S}^1$, where $p_i : \mathcal{V} \to \mathbb{R}^3$ and $\psi_i : \mathcal{V} \to \mathcal{S}^1$ which map each vertex in $\mathcal{V}$ to a point $(p_i, \phi_i) \in \mathbb{R}^3 \times \mathcal{S}^1$.

A framework is the pair $(\mathcal{G}, q)$.

Consider two frameworks $(\mathcal{G}, q)$ and $(\mathcal{G}, q')$ are:

- *bearing equivalent* if $\beta_{ij}(q) = \beta_{ij}(q') \quad \forall\,(i,j) \in \mathcal{E}$ (the same constraints but not necessarily the same shape);

- *bearing congruent* if $\beta_{ij}(q) = \beta_{ij}(q') \quad \forall\, i,j \in \mathcal{V},\ i \neq j$ (if they have the same constraints over all the possible edges);

A framework $(\mathcal{G}, q)$ is said:

- *bearing rigid* if there exists a neighbourhood $\mathcal{U}$ of q such that any framework $(\mathcal{G}, q')$, $q' \in \mathcal{U}$ is both bearing equivalent and bearing congruent to $(\mathcal{G}, q)$.

- *globally bearing rigid* if $\mathcal{U} = (\mathbb{R}^3 \times \mathcal{S}^1)^N$, meaning that all frameworks which are bearing equivalent to $(\mathcal{G}, q)$ are also bearing congruent to $(\mathcal{G}, q)$.

A framework $(\mathcal{G}, q)$ non-rigid is also said *roto-flexible.*

A framework $(\mathcal{G}, q)$ is *minimally* rigid if it is rigid and the removal of any edge yields a roto-flexible framework.

Directed bearing function associated to a framework $(\mathcal{G}, q)$ is the map $\beta_{\mathcal{G}}(q)\ : (\mathbb{R}^3 \times \mathcal{S}^1)^N \to (\mathbb{S}^2)^{|\mathcal{E}|}$

$$\beta_{\mathcal{G}}(q) = \left[\beta_{e_1}^T \ \ldots\ \beta_{e_{|\mathcal{E}|}}^T\right]^T$$

with $e_i \in \mathcal{E}$ is used to represent a directed edge in the graph $\mathcal{G}$.

The *bearing rigidity matrix* is

$$\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q) = \frac{\partial \beta_{\mathcal{G}}(q)}{\partial q} \in \mathbb{R}^{3|\mathcal{E}| \times 4N} \tag{3}$$

The rigidity matrix is important since it establishes a link between agent motion and constraint variations.

Given these definitions and let $\mathcal{N}(\cdot)$ the null-space of a matrix. The null-space of the bearing rigidity matrix $\mathcal{N}(\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q))$ describes all the motions preserving the constraints.

The $(\mathcal{G}, q)$ is said *infinitesimally bearing rigid* at some position q if $\mathcal{N}(\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q)) = \mathcal{N}(\mathcal{B}_{\mathcal{K}_N}^{\mathcal{W}}(q))$ with $\mathcal{K}_N = N(N-1)$ being the complete directed graph. Otherwise $(\mathcal{G}, q)$ is said *infinitesimally roto-flexible.*

From the article [5] we know that *dim* $\mathcal{B}_{\mathcal{K}_N}^{\mathcal{W}}(q)) = 4N - 5$. In fact any rigid non-degenerate bearing formation defines a 5-dimensional manifold in a 4N-dimensional space. Therefore, only $4N - 5$ independent constraints are actually needed to determine a bearing formation.

3 Decentralized Bearing Formation Control

Consider now a bearing rigid framework $(\mathcal{G}, q)$ in $\mathbb{R}^3 \times \mathcal{S}^1$ consisting of N agents, each following the dynamics (1).
Let q_d be a desired configuration such that the framework $(\mathcal{G}, q_d)$ is bearing rigid.
Let $b_{\mathcal{G}}^d = \beta_{\mathcal{G}}(q_d) = (\beta_{e_1}^d \; ... \; \beta_{e_{|\mathcal{E}|}}^d)$ be the corresponding desired value for the *bearing function.*

The article [1] goal is to design a decentralized bearing formation controller able to accomplish two distinct objectives:

1. **Bearing formation stabilization**. By acting on the control inputs (u_i, w_i), the controller should drive the N agents towards a configuration q^* equivalent and congruent to q_d, such that $\beta_{\mathcal{G}}(q^*) = b_{\mathcal{G}}^d$. In other words one wants to ensure that $q(t)$ reaches the correct desired shape after taking into account any potential translation, vertical rotation, and scaling.

2. The possibility to drive the agent group along the motion directions that do not affect the bearing rigidity function which are spanned by the null space of the rigidity matrix.

This way the formation controller will be decentralized and only based on information locally available of communicated by 1-hop neighbors.

Remember that rigidity is the most important property for formation control and localization.

3.1 Rigidity-based control of bearing frameworks in $\mathbb{R}^3 \times \mathcal{S}^1$

For rigidity-based bearing control one means the ability for the framework to get into a desired configuration. This concerns only the final shape and not its position in the space [4].

The k-th row block of the bearing rigidity matrix $\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}$ associated to
edge $e_k = (i, j)$ is

$$\begin{bmatrix} -0- & \underbrace{-\frac{P_{ij}R_i^T}{d_{ij}}}_{i} & -0- & \underbrace{\frac{P_{ij}R_i^T}{d_{ij}}}_{j} & -0- & ... & \underbrace{-S\beta_{ij}}_{3N+i} & -0- \end{bmatrix} \in \mathbb{R}^{3 \times 4N} \tag{4}$$

where, $d_{ij} = ||p_i - p_j||$, $P_{ij} = I_3 - \beta_{ij}\beta_{ij}^T$ is the orthogonal projector onto the orthogonal complement of β_{ij} and $S = \left[\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T\right]_\times$.

The bearing rigidity matrix $\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}$ relates changes in the bearing function $\beta_{\mathcal{G}}$ to the world-frame velocities $\dot{q} = (\dot{p}, \dot{\psi})$ of the framework

$$\dot{\beta}_{\mathcal{G}} = \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q) \begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} \tag{5}$$

Preposition 1. The null-space of the bearing rigidity matrix can be explicitly characterized as

$$\mathcal{N}(\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q)) = \mathbf{span}\left\{ \begin{bmatrix} \mathbf{1}_{N_3} \\ 0 \end{bmatrix}, \begin{bmatrix} p \\ 0 \end{bmatrix}, \begin{bmatrix} p^{\perp} \\ \mathbf{1}_N \end{bmatrix} \right\} = \left\{ n_1, n_2, n_3 \right\} \tag{6}$$

with $\mathbf{1}_N$ is a vector of all ones of dimension N, $\mathbf{1}_{N_3} = \mathbf{1}_N \otimes I_3$, $p^{\perp} = (I_N \otimes S)p$

Each null-space for $\mathcal{N}(\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q))$ represents three translations along the world axes, an expansion about $\mathbf{O}_{\mathcal{W}}$ and a rotation about a vertical axis passing through $\mathbf{O}_{\mathcal{W}}$.

Proof. The proof that vectors $\begin{bmatrix} \mathbf{1}_{N_3} \\ 0 \end{bmatrix}$ and $\begin{bmatrix} p \\ 0 \end{bmatrix}$ belong to $\mathcal{N}(\mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q))$ can be found in the other works [3] and [6]. The explicit expression for $\begin{bmatrix} p^{\perp} \\ \mathbf{1}_N \end{bmatrix}$ can be shown as follows: consider the k-th element of

$$\mathcal{B}_{\mathcal{G}}^{\mathcal{W}} \begin{bmatrix} p^{\perp} \\ \mathbf{1}_N \end{bmatrix} = \frac{P_{ij} R_i^T S(p_j - p_i)}{d_{ij}} - S\beta_{ij} =$$

$$= P_{ij} R_i^T S R_i \frac{R_i^T (p_j - p_i)}{d_{ij}} - S\beta_{ij} = P_{ij} S\beta_{ij} - S\beta_{ij} = S\beta_{ij} - S\beta_{ij} = 0$$

where the authors of [1] have used the properties $R_i^T S R_i = S$ and $P_{ij} S\beta_{ij} = S\beta_{ij}$ (because $S\beta_{ij} \perp \beta_{ij}$). □

One can define a body-frame rigidity matrix, such that it relates changes in the bearing function $\beta_{\mathcal{G}}$ in terms of the body-frame velocity inputs of the agent group $u = \begin{bmatrix} ... \; u_i^T \; ... \end{bmatrix}^T \in \mathbb{R}^{3N}$ and $w = \begin{bmatrix} ... \; w_i \; ... \end{bmatrix}^T \in \mathbb{R}^N$

$$\dot{\beta}_{\mathcal{G}} = \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(q) \begin{bmatrix} diag\{R_i\} & 0 \\ 0 & I_N \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} = \mathcal{B}_{\mathcal{G}}(q) \begin{bmatrix} u \\ w \end{bmatrix} \tag{7}$$

Follows the k-th row block of the body-frame bearing rigidity matrix $\mathcal{B}_{\mathcal{G}}(q)$ for edge $e_k = (i, j)$

$$\begin{bmatrix} -0- & \underbrace{-\frac{P_{ij}}{d_{ij}}}_{i} & -0- & \underbrace{\frac{P_{ij}\,{}^iR_j}{d_{ij}}}_{j} & -0- & ... & \underbrace{-S\beta_{ij}}_{3N+i} & -0- \end{bmatrix} \in \mathbb{R}^{3\times 4N} \tag{8}$$

with ${}^iR_j = R_z(\psi_{ij})$ with $\psi_{i,j} = \psi j - \psi_i$.

Notice that the body-frame rigidity matrix is no more a function of absolute yaw rotations ψ_i but of the relative orientations $\psi_j - \psi_i$ among neighboring agents.

Let $e_F(q) = b_{\mathcal{G}}^d - \beta_{\mathcal{G}}(q)$ is the bearing formation control *error* to be regulated to zero to solve formation stabilization.

To do that, one needs to minimize $||e_F||$ by implementing the scale-free controller based on the body-frame rigidity matrix

$$\begin{bmatrix} u \\ w \end{bmatrix} = k_c \begin{bmatrix} diag(d_{ij}) & 0 \\ 0 & I_N \end{bmatrix} \mathcal{B}_{\mathcal{G}}(q)^T b_{\mathcal{G}}^d, \quad k_c > 0 \tag{9}$$

the i-th agent velocity command

$$\begin{cases} u_i = -k_c \sum_{(i,j)\in\mathcal{E}} P_{ij}\beta_{ij}^d + k_c \sum_{(i,j)\in\mathcal{E}} {}^iR_j P_{ij}\beta_{ji}^d \\ w_i = k_c \sum_{(i,j)\in\mathcal{E}} \beta_{ij}^T S \beta_{ij}^d \end{cases} \tag{10}$$

The centroid $\overline{p} = \mathbf{1}_{N_3}^T p/N$ and scale $p^T p$ of the formation can be shown to be invariant under the action of (10). Moreover the action (10) has a decentralized structure which depends only on the interactions between $\mathcal{G}$ and relative quantities. The controller, however, requires communication among agents when $e_k(j,i) \in \mathcal{E}$ i.e., an agent i needs to receive β_{ji} and β_{ji}^d from agent j.

However, since the authors of [1] consider a bearing rigid framework, all the relative orientations among the agent pairs are univocally fixed by the existing inter-agent bearing constraints.

Furthermore, (10) needs access to the relative orientation among neighbouring pairs (${}^iR_j$) which is a quantity not available from direct measurements. To this, the authors of [1] detail a localization algorithm to obtain a *decentralized* estimation of ${}^iR_j$ in presence of a generic bearing rigid graph and of non-stationary agents.

3.2 Rigidity-based localization of time-varying bearing frameworks in $\mathbb{R}^3 \times \mathcal{S}^1$

For rigidity-based bearing localization of time-varying frameworks one means the ability for the framework to find a position in the space [4].

Remember that to access not available measurements is common to use estimation of said variables.

Let $\hat{q} = (\hat{p}, \hat{\psi})$ be an estimation of the configuration of N agents with Model (1).

Define then the bearing estimation error $e_L(q,\hat{q}) = \beta_{\mathcal{G}}(q) - \beta_{\mathcal{G}}(\hat{q})$. Assuming $\beta_{\mathcal{G}}(q)$ constant then the minimization of $||e_L||$ is obtained by following the negative gradient of

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{\psi}} \end{bmatrix} = k_e \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q})^T \beta_{\mathcal{G}}(q), \quad k_e > 0 \tag{11}$$

Under the action of (11) the estimation $\hat{q}(t)$ will converge towards a configuration equivalent to q. Since the framework $(\mathcal{G}, q)$ is bearing rigid then the configuration will be also congruent.

At convergence ($e_L = 0$), $\hat{q}$ will reach a configuration such that

$$\begin{cases} \hat{p} = s(I_N \otimes R_z(\bar{\psi}))p + \mathbf{1}_N \otimes t \\ \hat{\psi} = \psi + \mathbf{1}_N \bar{\psi} \end{cases} \tag{12}$$

$t \in \mathbb{R}^3$ is an arbitrary translation, $\bar{\psi} \in \mathcal{S}^1$ rotation angle, $s \in \mathbb{R}^+$ scaling factor.

The unknown ${}^iR_j$ can be replaced with the estimation ${}^i\hat{R}_j = R_z(\hat{\psi}_j - \hat{\psi}_i)$ by exchanging the two estimates of the yaw angles i and j.

The estimator (11) is fully decentralized and only requires $\beta_{\mathcal{G}}(q)$ which is considered constant while the inter-agent relative bearing will be time-varying under the action of controller (10). $\beta_{\mathcal{G}}(q(t))$ prevents the convergence of the estimation error; this can be addressed by adding a feed-forward term to (11)

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{\psi}} \end{bmatrix} = k_e \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q})^T \beta_{\mathcal{G}}(q) + \begin{bmatrix} diag\{R_z(\hat{\psi}_i)\} & 0 \\ 0 & I_N \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} \tag{13}$$

Preposition 2. If the initial estimation error $||e_L(t_0)||$ is small enough and s $= 1$ then the Model (13) will guarantee that $||e_L(t_0)|| \to 0$ in the case of time-varying bearings $\beta_{\mathcal{G}}(q(t))$ is not constant.

Proof. The closed-loop dynamics of the estimation error

$$\dot{e}_L = \mathcal{B}_{\mathcal{G}}(q) \begin{bmatrix} u \\ w \end{bmatrix} - \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q}) \begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{\psi}} \end{bmatrix} \tag{14}$$

$$= -k_e \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q}) \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q})^T \beta_{\mathcal{G}} + (\mathcal{B}_{\mathcal{G}}(q) - \mathcal{B}_{\mathcal{G}}(\hat{q})) \begin{bmatrix} u \\ w \end{bmatrix} \tag{15}$$

The term $-k_e \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q}) \mathcal{B}_{\mathcal{G}}^{\mathcal{W}}(\hat{q})^T \beta_{\mathcal{G}}$ represents the nominal closed-loop dynamics of the constraint bearing case, the term $(\mathcal{B}_{\mathcal{G}}(q) - \mathcal{B}_{\mathcal{G}}(\hat{q})) \begin{bmatrix} u \\ w \end{bmatrix}$ is a perturbation due to the agent motion. From previous studies [6] it is known that the nominal closed-loop is asymptotically stable. Moreover, if it can be shown that the perturbation term is vanishing with respect to the estimation error e_L then the overall system is locally stable under mild conditions. Consider (8), when (12) holds and so $e_L = 0$, then ${}^iR_j = {}^i\hat{R}_j, \beta_{ij} = \hat{\beta}_{ij}$ and $d_{ij} = s\hat{d}_{ij}$. If $s = 1$ then $e_L \to 0 \Rightarrow (\mathcal{B}_{\mathcal{G}}(q) - \mathcal{B}_{\mathcal{G}}(\hat{q})) \to 0$. $\square$

The estimated formation $\hat{q}$ needs a correct scale ($s = 1$) to properly compensate for the effects of the agent motion in the estimation dynamics. Since the formation scale cannot be recovered from only bearing measurements, one can then consider the following augmented cost function

$$\frac{1}{2}(k_e e_L^T e_L + k_d(\hat{p}_{\iota\kappa}^T \hat{p}_{\iota\kappa} - d_{\iota\kappa}^2)^2), \quad k_d > 0 \tag{16}$$

with ι and κ represent a single pair of agents. (16) mean to enforce the constraint $||\hat{p}_{\iota\kappa}|| = ||\hat{p}_\iota - \hat{p}_\kappa|| = d_{\iota\kappa}$.

To minimize (16) one will complement the law (13) by adding the decentralized terms $\pm k_d(\hat{p}_{\iota\kappa}^T \hat{p}_{\iota\kappa} - d_{\iota\kappa}^2))\hat{p}_{\iota\kappa}$.

The First objective full filled.

3.3 Coordinated motions in the null-space of the bearing rigidity matrix

The fulfilment of the second objective is the implementation of the null-space motions spanned by (6).

It can be done by realizing the world-frame velocity $\dot{q}_s = n_1\nu + n_2\lambda + n_3\omega$ which imposes to the framework a common linear velocity $\nu \in \mathbb{R}^3$, an expansion rate $\lambda \in \mathbb{R}$ about $\boldsymbol{O}_\mathcal{W}$ and a coordinated rotation with angular speed ω about a vertical axis passing through $\boldsymbol{O}_\mathcal{W}$. The body-frame velocities (u_s, w_s) that will be added to the formation control inputs (u, w) in (9) are:

$$\begin{bmatrix} u_s \\ w_s \end{bmatrix} = \begin{bmatrix} diag(R_i^T) & 0 \\ 0 & I_N \end{bmatrix} \dot{q}_s \tag{17}$$

which realizes the second control objective.

It is interesting to implement an expansion rate and coordinated rotation about a specific point on the interest attached to the formation itself.

Implement these motions relative to the formation centroid $\bar{p} = \mathbf{1}_{N_3}^T p/N$ and it can be obtained by using as basis for $\mathcal{N}(\mathcal{B}_\mathcal{G}^\mathcal{W}(q))$ the set $\{n_1, n_2 - n_1\bar{p}, n_3 - n_1 S\bar{p}\}$ which results in i-th agent velocity commands

$$\begin{cases} u_{s_i} = R_i^T(\nu + \lambda(p_i - \hat{p}) + \omega S(p_i - \hat{p})) \\ w_{s_i} = w \end{cases} \tag{18}$$

but an actual implementation of (18) would require the non-available quantities $(p_i, \psi_i, \bar{p})$. To solve this it is enough to exploit the estimator and use the estimated quantities $(\hat{p}_i, \hat{\psi}_i)$ with $\hat{p} = \mathbf{1}_{N_3}^T \hat{p}/N$ obtained by using a filtering technique such as PI-ACE (19).

The knowledge of the correct scale factor ($s = 1$) in the estimated $\hat{q}$ is not required for the motions associated to the vectors n_1 and n_2 (translation and expansion) but it is required for the vector n_3 (rotation). In fact, n_1 does not depend on p and n_2 is homogeneous in p with its direction thus unaffected by any scaling of the agent positions; n_3 is not homogeneous in p. Also in this case, if a distance measurement is not available for fixing the scale of $\hat{q}$, the coordinated rotation will not be exactly implemented.

4 Simulation and Experimental Results

Follow a brief discussion on the simulation and experimental results that the authors in the article [1] have found.

4.1 Simulation Results

In the article [1] the authors consider $N = 6$ quadrotors. The robustness of the bearing controller is tested against the discrepancies between the nominal agent (1) and the actual quadrotor flight dynamics, as well as against noise and democratization in the measured bearings, which are sampled at 60 Hz for mimicking an actual onboard camera.

- $q_(t_0)$ initial configuration,
- $\hat{q}(t_0)$ estimated configuration,
- q_d desired configuration

$q_(t_0)$ and $\hat{q}(t_0)$ where generated by adding to q_d a uniformly distributed random perturbation of 1 minute amplitude for the position and an orientation of 120 degree.

- $\mathcal{G}$ is a directed graph,
- $|\mathcal{E}| = 20$ directed edges.

$\mathcal{G}$ was randomly generated under the constraint pf guaranteeing bearing rigidity at $q(t_0)$, $\hat{q}(t_0)$ and q_d.

- Gain in equation (10), $k_c = 1$;
- Gain in equation (13), $k_e = 5$;
- Gain in equation (16), $k_d = 10$.

In all simulations in the article [1], the null-space velocity commands (18) are activated only after convergence to the desired formation $b_{\mathcal{G}}^d$ (desired value for the *bearing function*). The five coordinated motions were first actuated one at the time and then all together.

Fig.(1) and Fig.(2) report the simulation results.

Bottom Fig.(1.(a)) shows the behaviour of the bearing control error $||e_F(t)||$ in solid blue line and also the behaviour of the estimation error $||e_L(t)||$ in solid red line.

Top Fig.(1.(a)) depicts the five null-space velocity commands (ν, λ, w) in (18).

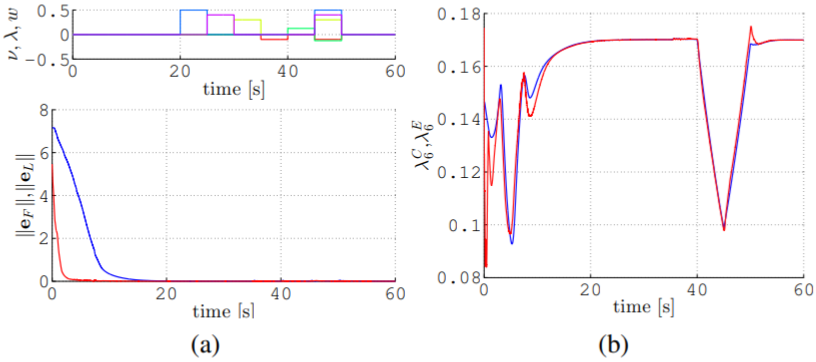


Figure 1: Results of the simulation. (a)-top: behavior of the five null-space motion commands $\nu(t)$ (blue, purple, yellow) $\lambda(t)$ (green) and $w(t)$ (red). (a)-bottom: behavior of the bearing control error $||e_F(t)$ and of the localization control error $e_L(t)$. (b): behavior of the rigidity eigenvalues $\lambda_6^C(t)$ (control – blue) and $\lambda_6^E(t)$ (estimation – red)

In Fig.(1) it is possible to verify how:

(i) both the control and estimation bearing errors converge to zero, despite their initial large value and despite the presence of time-varying bearings $\beta(q(t))$;

(ii) the implementation of the coordinated motions in (18) has no disturbing effect on the bearing errors, as expected.

Let $\lambda_6(q) \geq 0$ represent the sixth smallest eigenvalue of the square matrix $(\mathcal{B}_\mathcal{G}^\mathcal{W}(q))^T\mathcal{B}_\mathcal{G}^\mathcal{W})$ which is called rigidity eigenvalue [7]. Since for an infinitesimal rigid framework $rank(\mathcal{B}_\mathcal{G}^\mathcal{W}) = 4N - 5$, the quantity $\lambda_6(q)$ can be taken as a measure of the framework bearing rigidity [7]:

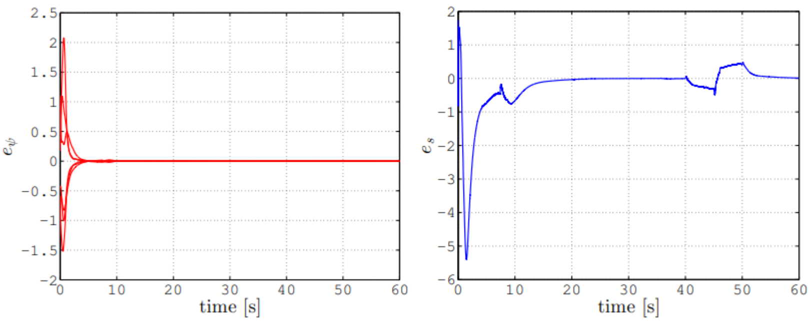
$$\begin{cases} \lambda_6(q) > 0 & iff\ (\mathcal{G}, q)\ is\ infinitesimal\ rigid \\ \lambda_6(q) = 0 & otherwise \end{cases}$$

Fig.(1.(b)) reports the behaviour of $\lambda_6^C(t) = \lambda_6((q)t)$ and $\lambda_6^E(t) = \lambda_6((\hat{q})t)$ which is the rigidity measures for the "control" framework $(\mathcal{G}, q)$ and the estimation framework $(\mathcal{G}, \hat{q})$.

One can then check how both frameworks remained rigid during motion, thus confirming congruency between q and q_d, which is the correct agent formation, and between q and $\hat{q}$, which is the correct agent localization.

As an additional measure of the localization performance as considered the quantity $e_\psi = (I_N - \mathbf{1}_N\mathbf{1}_N^T/N)(\psi - \hat{\psi})$. e_ψ represents the disagreement between the orientation estimation error and its mean value. In Fig.(2.(a)) e_ψ should vanish in presence of a correct localization. A correct localization implies that $\hat{\psi}(t) \to \psi(t) + \mathbf{1}_N\bar{\psi}$. A converging $e_\psi(t)$ then allows to correctly compute the missing terms ${}^iR_j$ in the bearing controller (10).

Fig.(2.(b)) depicts the behaviour of the formation scale error $e_s(t) = ||p(t) - \mathbf{1}_N \otimes \bar{p}(t)|| - \hat{p}(t) - \mathbf{1}_N \otimes \hat{\bar{p}}(t)||$ which converges to zero as expected from (16).



(a) (b)

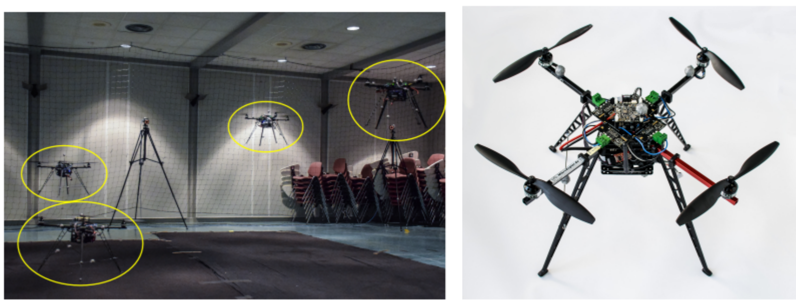
Figure 2: Results of the first simulation. (a): behavior of the orientation estimation error $e_{\psi}(t)$. (b): behavior of the formation scale error $e_s(t)$

Notice that it is worth nothing that the distortions in Fig.(2) are mainly due to the higher-order quadrotor dynamics neglected by the model (1). The said model introduced an unmodelled lag between commanded and actual velocities.

The proposed control strategy is nevertheless robust enough for coping with these model inaccuracies.

4.2 Experimental Results

In the article [1] the experiment conducted by the authors validate the formation controller described in Ch.(3).



(a) (b)

Figure 3: The flying arena (a) and our quadrotor platform (b)

For the purpose of the practical experiment the authors have used a group of four quadrotor UAVs like the one in Fig.(3.(b)). A Vicon motion capture system was employed for reconstructing the body-frame bearing measurements β_{ij} that would have been obtained by onboard camera running at 60 Hz. See Fig.(3.(a)) for reference of the place where the flight was performed.

The experiment followed a pattern similar to the previous simulation result:

(i) regulation towards a desired bearing formation,

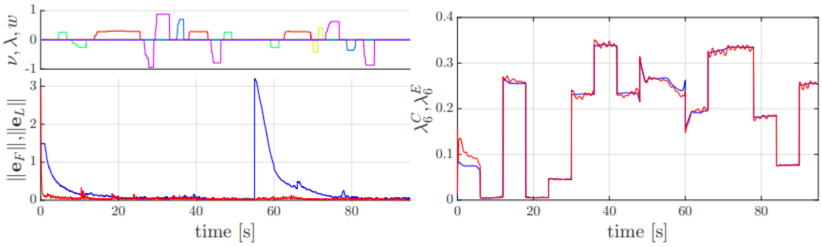
(ii) actuation of the null-space motions (18),

(iii) regulation towards a different desired bearing formation,

(iv) actuation of the null-space motions (18).

Additionally, one implements every 6 seconds, a random switch among all the possible rigid topologies for the sensing graph in order to show robustness of our approach also against possible topology changes during motion.

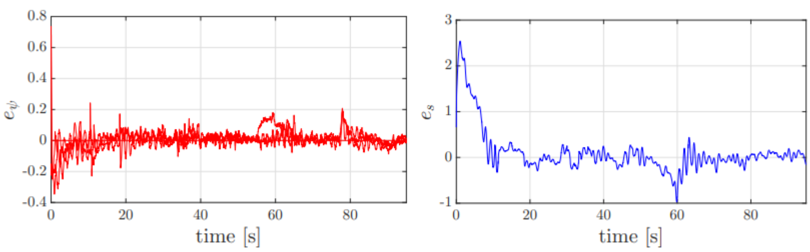
Fig.(4) and Fig.(5) report the results of the experiment.



(a) (b)

Figure 4: Results of the experiment. (a)-top: behavior of the five null-space motion commands $\nu(t)$ (blue, purple, yellow) $\lambda(t)$ (green) and w(t) (red). (a)-bottom: behavior of the bearing control error $||e_F(t)||$ and of the localization control error $||e_L(t)||$. (b): behavior of the rigidity eigenvalues $\lambda_6^C(t)$ (control – blue) and $\lambda_6^E(t)$ (estimation – red).

In Fig.(4.(a)), the UAV formation starts far from the desired configuration but, after 20 seconds $||e_F(t)||$ drops below 4% of its initial value. On the other hand, $||e_L(t)||$ is quite fast even though $\hat{q}(t_0)$ was generated by adding to the real $q(t_0)$ a uniformly distributed random perturbation of amplitude 1.5 meters for $p(t_0)$ and 80 degrees for orientations $\psi(t_0)$.



(a) (b)

Figure 5: Results of the experiment. (a): behavior of the orientation estimation error $e_\psi(t)$. (b): behavior of the formation scale error $e_s(t)$.

In Fig.(5), where the orientation estimate error $e_\psi(t)$ and the formation scale error $e_s(t)$ are displayed, convergence of the estimated $\hat{q}(t)$ towards a configuration congruent with $q(t)$ (with the correct scale) can also be seen.

One can then verify how a consistent estimation of the orientations $\hat{\psi}$ and of the formation scale s could be obtained despite the non-idealities present in any real implementation.

The underlying graph $\mathcal{G}$ switches randomly at every 6 seconds across all the possible rigid topologies for the $N = 4$ quadrotors; in particular, one allows switches among graphs with $|\mathcal{E}| \in \{6, 7, ..., 12\}$.

In fact in Fig.(4.(b)) shows the behaviour of the rigidity measures $\lambda_6^C(t)$ and $\lambda_6^E(t)$ which "jump" at every 6 seconds because of the topology switches. Nevertheless, these topology changes, and the associated increases/decreases of he control/estimation framework rigidity, did not negatively affected the overall performance of the proposed bearing control strategy.

5 Conclusions

In the paper [1] the authors have considered the problem of devising a decentralized control strategy to control a group of quadrotor UAVs able to measure relative bearings in their objectives:

(i) stabilization of the quadrotor formation towards a desired bearing configuration;

(ii) steering of the whole formation along the motion along the motion directions in the null-space of the bearing rigidity matrix.

The article [1] represents a significant extension, combination and generalisation of the strategy reported in previous works (such as [3] and [6]) which rely on a much more constrained design of the agent group. In particular in the case of non-stationary agents, together with a full explicit characterization of the null-space of the bearing rigidity matrix which does not require a special topology for the interaction graph.

The bearing controller (10), the localization algorithm (13) and the null-space motions (18) have the same decentralized expression for all the agents only as a function of the measured bearings and body-frame linear/angular velocities. The only exception is the inclusion of the distance measurement $d_{\iota\kappa}$ in the augmented cost function (16) which adds an additional control term to the agents of a single pair.

If a correct formation scale is assumed then the distance measurement $d_{\iota\kappa}$ can be neglected. Indeed, the unknown robot inter-distances could be estimated only by processing the measured inter-robot bearing and the known robot own motions by using a PI-ACE filter.

The practical impact of this method which requires direct bearing rigidity for the robot formation is the complexity shift from quadratic to linear. In fact, minimal rigidity requires presence of $|\mathcal{E}| = O(N)$ number of edges, instead of $N(N-1) = O(N^2)$ in the case of complete directed graph $\mathcal{K}_N$.

Moreover, the authors of [1] have a developed a decentralised bearing controller able to meet the two control objectives without the need of a common reference frame for the agent group nor the equipment of reciprocal bearing measurements (an undirected measurement topology).

Last, the authors reported simulation and experimental result that perfectly endorse the issues discussed in the article [1].

A PI-ACE Estimator

PI-ACE (Proportional/Integral - Average Consensus Estimator) is a mathematical tool which can estimate quantities in a decentralised way.

$$\begin{cases} \dot{z}^i = \gamma(\alpha^i - z^i) - K_P \sum_{j \in \mathcal{N}_\rangle} (z^i - z^j) + K_I \sum_{j \in \mathcal{N}_\rangle} (w^i - w^j) \\ \dot{w}^i = -K_I \sum_{j \in \mathcal{N}_\rangle} (z^i - z^j) \end{cases} \tag{19}$$

The quantities (z^i, w^i) are PI-ACE states and $\alpha_i = \{\hat{v}_{2_i}, \hat{v}^2_{2_i}\}$ are the external signals of which a moving average is taken in a decentralized way, with $\hat{v}_{2_i}$ is a vector and $\hat{v}^2_{2_i}$ is scalar. Everything in this filter is driven by the $(a^i - z^i)$ mismatch.

In the case of the study in article [1], PI-ACE allows every agent to distributively build an estimation converging to the average $\bar{x}(t) = \sum_{i=1}^N x_i(t)/N$ with a tunable dynamics that can be made faster than the underlying dynamics of each agent in the system.

References

[1] F. Schiano A. Franchi D. Zelazo and P. R. Giordano. A rigidity-based decentralized bearing formation controller for groups of quadrotor uavs. *2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 5099–5106, 2016.

[2] Andrea Critofaro and Paolo Robuffo Giordano. Analysis and control of multi-robot systems-introduction. ., 2023.

[3] Daniel Zelazo Paolo Robuffo Giordano Antonio Franchi. Bearing-only formation control using an se(2) rigidity theory. *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 6121–6126, 2015.

[4] Andrea Critofaro and Paolo Robuffo Giordano. Analysis and control of multi-robot systems-elements of graph theory. ., 2023.

[5] Ryll M Bülthoff HH Franchi A Masone C Grabe V and Giordano PR. Modeling and control of uav bearing formations with bilateral high-level steering. *The International Journal of Robotics Research*, pages 1504–1525, 2012.

[6] A. Franchi D. Zelazo and P. R. Giordano. Rigidity theory in se(2) for unscaled relative position estimation using only bearing measurements. *2014 European Control Conference (ECC), Strasbourg, France*, pages 2703–2708, 2014.

[7] Andrea Critofaro and Paolo Robuffo Giordano. Analysis and control of multi-robot systems-formation control of multiple robots. ., 2023.

[8] Naveen Joshi. 10 stunning applications of drone technology.