A new distributed protocol for consensus of discrete-time systems

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Table of Contents

Preliminaries

Consensus is a fundamental concept in various fields and crucial in scenarios like multi-agent systems and distributed networks.

- To achieve a collective agreement, or the convergence to a common state, among interconnected entities.
- Suitable protocols needed to enable effective communication and state adjustments over time, particularly in situations with uncertainties, delays, or faults.

Introduction

A novel distributed protocol is proposed to achieve consensus within a discrete-time network comprising scalar agents [1]

- allowing agents to communicate and compute locally their states,
- offering the flexibility of assigning convergence rates arbitrarily,
- regardless of the network topology while focusing on scalability and efficiency in large networks.

Main approach

In discrete-time scalar dynamical agents this is performed by emulating the continuous-time counterpart.

- Given the agent dynamics

$$x_i(k+1) = x_i(k) + u_i(k) \tag{1}$$

  the usual coupling rule

$$u_i = -\kappa \sum_{j:v_j \in \mathcal{N}(v_i)} (x_i - x_j) \tag{2}$$

  - Convergence is guaranteed only if the coupling $\kappa > 0$ is sufficiently small w.r.t the network size [2].
  - A small κ leads to several problems related mostly to the speed of convergence, the stability of the system and the amount of information exchanged to reach consensus.

Table of Contents

- Unweighted directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $|\mathcal{V}| = N$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$
- Set of neighbors to a node $i \in \mathcal{V}$: $\mathcal{N}_i = j \in \mathcal{V} : (j, i) \in \mathcal{E}$
- Directed path from i to j:
  $i \rightarrow := (r, r+1) \in \mathcal{E} : \bigcup_{r=0}^{l-1}(r, r+1) \subseteq \mathcal{E}, 0 = i, l = j, l > 0$
- Reachable set from a node $i \in \mathcal{V}$: $R(i) := [i] \cup [j \in \mathcal{V} : i \rightarrow j]$
- Common part of $\mathcal{G}$: $\mathcal{C} = \mathcal{V} \setminus \cup_{i=1}^{\mu} \mathcal{H}_i$, cardinality c=$|\mathcal{C}|$
- Laplacian matrix $\mathcal{L} = D - A$, with $D \in \mathbb{R}^{n \times n}$ degree matrix and $A \in \mathbb{R}^{n \times n}$ adjacency matrix.
  $\mathcal{L}$ possesses one eigenvalue $\lambda = 0$ with both algebraic and geometric multiplicities coinciding with μ, the number of reaches of $\mathcal{G}$.

Consider a multi-agent system exchanging information via a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.

- Each vertex modelled as dynamical equation:

$$x_i^+ = x_i + u_i \tag{3}$$

  when coupled with the standard coupling rule

$$u_i = -\kappa_l \sum_{\nu_j \in \mathcal{N}(\nu_i)} (x_i - x_j) \tag{4}$$

  consensus is achieved only for κ_l small enough.
- Denote $\mathbf{x} = col(x_i, i = 1, \cdots n) \in \mathbb{R}^n$, $\mathbf{u} = col(u_i, i = 1, \cdots n) \in \mathbb{R}^n$
- The network dynamics

$$\mathbf{x}(t+1) = (I - \kappa_l \mathcal{L})\mathbf{x}(t) \tag{5}$$

  has dynamic matrix with one eigenvalue in $\lambda_d = 1$ and consensus is achieved if

$$\kappa_l \leq \frac{1}{\lambda_{max}}, \ \lambda_{max} = \max\{\lambda \in \sigma(\mathcal{L}) \mid \lambda > 0\} \tag{6}$$

Challenges

Accordingly to the previous work [2] :

1. if κ_l is not small enough consensus might be lost and the dynamics might diverge;

2. A decrease in κ_l as the network size grows affect both the convergence rate and the information exchange by agents to reach consensus;

3. Not desirable since the coupling strength cannot be fixed small a priori for both modelling and control reasons [2].
   - $\lambda_{\max}$ might not be known by all agents and, and the transient performances might not be acceptable.

To solve article [2] proposed a neighbour-based coupling protocol

- Realized via the average passive output;
- through feedback interconnection to guarantee consensus, only dictated by the communication graph and the initial condition, overcoming the need of small gains.

The new discrete-time coupling rule (”average coupling”)

$$u_i = -\kappa \sum_{j:v_j \in \mathcal{N}(v_i)} (y_i - y_j) \tag{7}$$

converge to a consensus $\forall\ \kappa > 0$.

- In this way the consensus of the discrete-time network coincides with the one of the continuous-time counterpart, independently on κ.

The proposal in article [2] :

1. The convergence rate cannot be fixed arbitrarily as directly proportional to the coupling gain.
   - $\rightarrow$ Fix an arbitrarily fast convergence;

2. Cannot be implemented in a distributed manner;
   - $\rightarrow$ Distributed version of the protocol which allows to solve approximately the solution of the coupling input (7) in an arbitrary number of steps.

Primary Objective

The previous proposal in [2] solving part was centralised and could not assign arbitrary convergence rate to consensus.

- Main goal: design a local distributed control law

$$u_i(t) = \kappa\varphi_i(x_i(t), col(x_j(t), u_j(t-1), j \in \mathcal{N}_i))$$

ensuring all agents asymptotically converge to some consensus $x_s \in \mathbb{R}\ \forall \kappa > 0$ as $t \to \infty$ i.e.

$$\mathbf{x}(t) \to \mathbf{1}_N x_s \tag{8}$$

Time scale separation procedure

Two nested consensus processes over a time window of length γ:

- At step t, $u_i(t)$ is computed over a time window of length γ.
- At all steps $t + \tau$ (with $\tau = 1, \ldots, \gamma$), each agent computes an approximate solution $v_i(t, \tau)$ based on the available (local) information and then sends it to the neighbors.
- At step $t + \gamma$, the actual control is deduced as the result of the approximating consensus phase after γ steps, ($u_i(t) = v_i(t, \gamma)$).

$\rightarrow$ This ensures the enforcement of consensus for all values of γ, and the performance of the centralized implementation is regained as γ increases.

Table of Contents

3 Main Result

A Refined Centralised Consensus Protocol

3 Main Result

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First, redefine and extend the centralized algorithm from [2].

Theorem

Theorem 1

Consider a network of N discrete-time agents of the form (3) with communication digraph $\mathcal{G}$ with only one reach, i.e. the Laplacian $\mathcal{L}$ has a zero eigenvalue with multiplicity 1. Then, for all i $= \{1, \ldots, N\}$ the local control law

$$u = -\kappa(I_N + \kappa g\mathcal{L})^{-1}\mathcal{L}x \tag{9}$$

whose components are solutions to

$$u_i = -\kappa \sum_{j \in N_i} (y_i - y_j) \tag{10}$$

$$y_i = x_i + g u_i \tag{11}$$

guarantee consensus for all $\kappa > 0$ *and* $g \geq \frac{1}{2}$*; namely, for the network dynamics*

$$x(t+1) = \Theta_c(\kappa, g) x(t) \tag{12}$$

$$\Theta_c(\kappa, g) = (I_N + \kappa g \mathcal{L})^{-1} (I_N + \kappa (g-1) \mathcal{L}) \tag{13}$$

as $t \to \infty$*, one gets that (8) holds with*

$$x_s = v_1^T \mathbf{x}(0) \tag{14}$$

with $v_1^T \in \mathbb{R}^N : v_1^T \mathcal{L} = 0$ and $v_1^T \mathbf{1}_N = 1$.

Following the lines in [2](*Theorem 4.1*):

Proof: For all choices $\kappa, g \in \mathbb{R}$ all agents converge to the consensus: the eigenvalues of Θ_c are

$$\lambda_d^i(\kappa, g) = \frac{1 + \kappa(g-1)\lambda^i}{1 + \kappa g \lambda^i} \; , \forall \lambda^i \in \sigma\{\mathcal{L}\} \tag{15}$$

with an eigenvalues in $\lambda_d = 1$ and multiplicity 1 corresponding to $\lambda = 0$. All others in the unit circle if and only if $\kappa > 0$ and $g > \frac{1}{2}$.

- The associated center subspace from $\mathcal{V} = ker(\mathcal{L})$ is attractive, coinciding with the consensus subspace.

- Introducing

$$\begin{bmatrix} x_s \\ \mathbf{x}_r \end{bmatrix} = v_{\mathbf{1}}^T \mathbf{x} = \begin{bmatrix} V_0^T \\ V_r^T \end{bmatrix} \mathbf{x} \tag{16}$$

with $V^{-T} = Z = \begin{bmatrix} \mathbf{1}_n & Z_r \end{bmatrix}$, $V^\top \mathcal{L} Z = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda^r \end{pmatrix}$, $\Lambda^r = \mathrm{diag}\{\sigma\{\mathcal{L}\} \backslash \{0\}\}$

one gets that

$$V^\top \Theta_c(\kappa, g) Z = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda_d^r \end{pmatrix} \tag{17}$$

with $\Lambda_d^r(\kappa, g) = \mathrm{diag}\left\{\sigma\left\{\Theta_c(\kappa, g)\right\} \backslash \{1\}\right\}$.

- $\mathbf{x}_r$ is the orthogonal component converging to zero.
- When consensus is achieved one has $\mathbf{x}_r = 0$ and $\mathbf{x} = \mathbf{1}_n x_s$. $\square$

- **Remark 3.1**: The output (11) makes all agents Input-Feedforward Passive, namely s.t. for a storage function $S(x_i)$:

$$\Delta S(x_i) \leq u_i y_i - (g - \frac{1}{2})u_i^2 \tag{18}$$

As $g > \frac{1}{2},\ u_i \to y_i$ is strictly passive and passive for $g = \frac{1}{2}$.

- **Remark 3.2**: fixing $g = \frac{1}{2}$ the consensus in [2] is recovered but the convergence rate cannot be fixed arbitrarily small via $\kappa > 0$.
One cannot compute κ to make all eigenvalues of Θ_c (15) arbitrary closed to 0.

- **Remark 3.3**: fixing $g = 1$ one can choose arbitrarily $\kappa > 0$ independently on the size of the network. The eigenvalues of Θ_c (15) are close to 0 as κ increases.
  Thus, the trajectories of network dynamics (12) converge with a rate proportional to κ and one can pick $\kappa \to \infty$ with no knowledge of $\sigma\{\mathcal{L}\}$.

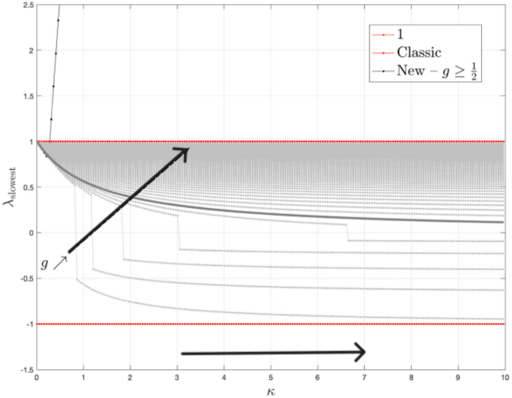

Fig. 1. Plot of the farthest eigenvalue of Θ_c from 0 for increasing values of $\kappa > 0$ and $g \geq \frac{1}{2}$.

Fig. (21): plot of the slowest eigenvalue of Θ_c for increasing values of $\kappa > 0$ and $g \geq \frac{1}{2}$.

Observations

The control (9) is implicitly defined and cannot be computed in a distributed manner [1].

- The i-th agent needs the input u_j of all its neighbors for computing the corresponding u_i so creating a bottleneck that cannot be solved locally.

$\rightarrow$ Consider a weighted Laplacian

$$W(\kappa, g) := (I_N + \kappa g \mathcal{L})^{-1} \tag{19}$$

which cannot be computed locally by each agent;
$\rightarrow$ The feedback (9) can be rewritten as

$$\boldsymbol{u} = -\kappa W(\kappa, g) \mathcal{L} \boldsymbol{x} \tag{20}$$

A distributed implementation of the new protocol

3 Main Result

- Challenges related to the implementation of the consensus protocol in a distributed (dynamic) manner.

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A distributed implementation of the new protocol

The protocol explores the use of a multi-rate controller to model information exchange and system evolution with coupling rule

$$u_i = \underbrace{-\frac{k}{1+g\kappa d_i}\sum_{j\in\mathcal{N}_i}(x_i - x_j)}_{1} + \underbrace{\frac{g\kappa}{1+g\kappa d_i}\sum_{j\in\mathcal{N}_i} u_j}_{2} \tag{21}$$

with $d_i = |\mathcal{N}_i|$.
The term

- 1. is immediately available at each time t,
- 2. can be approximated by a truncated fixed-point iteration with $\gamma \in \mathbb{N}$ steps.

Approximate γ steps implementation of (8) at node i.

1: At each time $t \geq 0$, send $x_i(t)$ to the neighbors.

2: Receive $x_j(t)$ from the neighbors, $j \in \mathcal{N}_i$, and compute, with $d_i = |\mathcal{N}_i|$,

$$v_i(t,0) = -\frac{\kappa}{1 + g\kappa d_i} \sum_{j \in \mathcal{N}_{(i)}} \left(x_i(t) - x_j(t)\right) \tag{17}$$

3: For $h = 0, \ldots, \gamma - 1$ do:

3.1: Send $v_i(t,h)$ to the neighbors

3.2: Compute

$$v_i(t,h+1) = v_i(t,0) + \frac{g\kappa}{1 + g\kappa d_i} \sum_{j \in \mathcal{N}_i} v_j(t,h) \tag{18}$$

4: Set $u_i(t) = v_i(t,\gamma)$.

Figure: Approximate distributed multi-step implementation of Eq.(10)-Eq.(11)

Important Results

3 Main Result

- The following theorems and lemmas prove convergence of distributed implementations under various values of γ, κ and g.

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Theorem

If the graph $\mathcal{G}$ has exactly one reach, then at each node i and time t the sequence $v_i(t, \gamma)$ (Eq.(18)) generated by the algorithm in Fig.(1) is such that, for all κ, $g > 0$ and $\gamma \in \mathbb{N}$,

$$\lim_{\gamma \to \infty} v_i(t, \gamma) = u_i(t), \tag{22}$$

with $u_i(t)$ is the i-th component of Eq.(9).

- with $g = 1$ it is possible to choose the convergence rate to the consensus.

Proof: Introduce the matrices

$$\tilde{D} = (I_n + \kappa g D)^{-1} \tag{23}$$

$$G = (I_n + \kappa g D)^{-1} \kappa g A = \kappa g \tilde{D} A \tag{24}$$

- G Schur and non negative.
- Gerschgorin criterion yields to $\rho(G) < 1 \; \forall \; \kappa, \; g$:

$$V(t, \gamma) = -\kappa (I_n + G + \cdots + G^{\gamma})(I + \kappa g D)^{-1} \mathcal{L} \mathbf{x}(t) \tag{25}$$

Since $\mathcal{L} = D - A$:

$$(I_n + \kappa g \mathcal{L}) = I_n + \kappa g D - \kappa g A = \tilde{D}^{-1}(I_n - \kappa g \tilde{D} A) = (I_n + \kappa g D)(I_n - G) \tag{26}$$

One gets,

$$V(t,\gamma) = -\kappa \sum_{h=0}^{\gamma} G^h (I_N + \kappa g \mathcal{L})^{-1} \mathcal{L} x(t)$$

Since

- when $\rho(G) < 1$ it holds that $(I_N - G)^{-1} = \sum_{i=0}^{\infty} G^i$

one concludes that, as $\gamma \to \infty$

$$V(t,\gamma) \to -\kappa (I_N + \kappa g \mathcal{L})^{-1} \mathcal{L} x(t)$$

which is exactly the centralized control Eq.(9) $\square$

- Vital for assessing stability and performance for the control system.

The following Lemmas extend and guarantee the consensus in distributed control

$v_i(t, \gamma) = u_i(t)$

in different scenarios involving the variation of γ, κ, and g.

Lemma

With the control law $v_i(t,\gamma) = u_i(t)$ *(Eq.(18)) generated by the algorithm in Fig.(1), the network dynamics (Eq.(1)) takes the form*

$$x(t+1) = \Theta_d(\kappa, g, \gamma)x(t) \tag{27}$$

with

$$\Theta_d(\kappa, g, \gamma) = I_N - \kappa(I_N - \mathrm{G}^{(\gamma+1)})\mathrm{W}(\kappa, g)\mathcal{L} \tag{28}$$

and $\mathrm{W}(\kappa, g)$ *in Eq.(19) is non-negative.*

Proof: Proof follows from Eq.(25)-Eq.(26) and $\sum_{h=0}^{\gamma} G^h = (I_N - G^{\gamma+1})(I_N - G)^{-1}$.
One gets

$$V(t,\gamma) = -\kappa(I - G^{\gamma+1})W(\kappa,g)\mathcal{L}x(t) \tag{29}$$

with G as in Eq.(24) and $W(\kappa,g)$ as in Eq.(19).
The cumulative agent dynamics Eq.(1) can be written as

$$x(t+1) = (I_N - \kappa)(I_N - G^{\gamma+1})W(\kappa,g)\mathcal{L}x(t) = \Theta_d(\kappa,g,\gamma)x(t) \tag{30}$$

Finally, Eq.(26) implies

$$W = \underbrace{(I_N - G)^{-1}}_{=\sum_{h=0}^{\infty} G^h \geq 0} \underbrace{\tilde{D}}_{\geq 0}$$

then $W \geq 0$ $\square$.

Lemma

When $\kappa g \geq 1$ and the graph contains only one reach, the control law $u_i = v_i(t, 0)$ (Eq.(18) i.e. with $\gamma = 0$) generated by the algorithm in Fig.(1), makes the agents converge to the same consensus value.

- **P.N.** Th.(1) and Lemma (4) in case of weakly connected digraphs with one reach $\mu = 1$ the consensus is guaranteed for $\gamma = \infty$ and $\gamma = 0$ consensus iterations.

Theorem 5: The Main Result

Theorem

If the graph $\mathcal{G}$ has exactly one reach, then the control $u_i(t) = v_i(t, \gamma)$ (Eq.(18)) generated by the algorithm in Fig.(1) makes the agents converge to the same consensus value $x_s \in \mathbb{R}$ $\forall\ \gamma \geq 0,\ \kappa > 0,\ g \geq 1.$

Proof of Theorem 5

Proof: From Lemma (3) one knows that the collective dynamics of the node is Eq.(27)-Eq.(28) it is rewritten as

$$\Theta_d(\kappa, g, \gamma) = I_N - \kappa W(\kappa, g)\mathcal{L} + \kappa G^{\gamma+1} W(\kappa, g)\mathcal{L} = \Theta_c(\kappa, g) + \kappa G^{\gamma+1} W(\kappa, g)\mathcal{L} \quad (31)$$

- Call $W = W(\kappa, g)$, to prove that

$$W = I_N - \kappa g W \mathcal{L} \quad (32)$$

$I_N - \kappa g W\mathcal{L} = W(W^{-1} - \kappa g \mathcal{L}) = W I_N = W$
$\rightarrow \kappa W \mathcal{L} = I_N - \frac{(I_N - W)}{g}$ and

$$\Theta_c(\kappa, g) = I_N - \kappa W \mathcal{L} = I_N - \frac{1}{g}(I_N - W) \quad (33)$$

- Replace Eq.(33) into Eq.(31) one gets

$$\Theta_d(\kappa, g) = I_N - \frac{1}{g} I_N + \frac{1}{g} G^{\gamma+1} + \frac{1}{g}(I_N - G^{\gamma+1})W \tag{34}$$

- Notice that for $g \geq 1$, $I_N - \frac{1}{g} I_N > 0$ and $G^{\gamma+1} > 0$
- Last,

$$(I_N - G^{\gamma+1})W = \sum_{h=0}^{\gamma} G^h \underbrace{(I_N - G)W}_{\text{from Eq.(26) } \tilde{D} \geq 0} = \sum_{h=0}^{\gamma} G^h \tilde{D} \geq 0 \tag{35}$$

- In conclusion $\Theta_d(\kappa, g, \gamma) \geq 0$ whenever $g \geq 1$, $\kappa > 0$ and $\gamma \geq 0$.
- Since $\Theta_d(\kappa, g, \gamma)\mathbf{1}_N = \mathbf{1}_N$ one gets that $\rho(\Theta_d(\kappa, g, \gamma)) = 1$, with one eigenvalue $\lambda_1 = 1$ which is on the unit circle and this guarantees consensus $\square$.

Observations

3 Main Result

- The distributed control Eq.(18) in Fig.(1) ensures convergence to the same consensus iterations in a network with one reach.
- The new protocol considers the network's connectivity (weakly connected or multiple reaches) and offers solutions that are adapted to various network structures, ensuring consensus regardless of the network's characteristics.

Table of Contents

Performance Evaluation

- The results of the suggested algorithm, both in centralized and distributed implementations, are presented across different networks.
- It will be demonstrated, using simulations, how the proposed distributed algorithm outperforms previous methods in terms of convergence time and efficiency, particularly in cases involving larger networks and/or directed graphs.
- The algorithm's performances are compared to the standard discrete-time protocol Eq.(4) while fixing the coupling gain at the largest permissible value ensuring condition Eq.(6).

Parameter Selection

- Authors of [1] discuss the importance of parameter selection in the protocol's performance. They suggest certain parameter settings (γ, κ and g) and the impact they have on achieving consensus in different network configurations.
- The evaluation of performances relies on the M$\%$-consensus settling time (t_s^M), which is the minimum steps needed for the network trajectories to reach M$\%$ of the consensus value.
- For Eq.(4) and Th.(1), t_s^M is an estimate of the minimum number of iterations that are required for consensus to be achieved.
- For algorithm in Fig.(1), from proof Lemma (3), Eq.(29), t_s^M is given by $(\gamma + 1)t_s^M$, with $\gamma \in \mathbb{N}$.
- Choose $\mathrm{M} = 10^{-1}$.

Results: Undirect Network

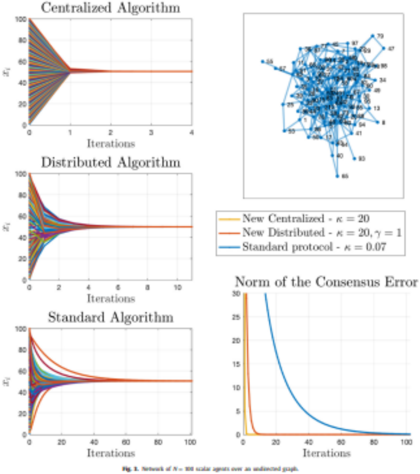
4 Simulations

- $N = 100$ agents,
- $\kappa = 10, \gamma = 1$ fixed.

$\rightarrow$ **Centralized algorithm** in Th.(1) converges in 3 iterations and $t_s^{10^{-1}} = 3$, despite the large number of agents.

$\rightarrow$ **Distributed implementation** in the algorithm in Fig.(1) converges with $t_s^{10^{-1}} = 10$ in 20 time steps.

$\rightarrow$ Outperforming the **standard consensus** algorithm Eq.(4) ($t_s^{10^{-1}} = 100$).



- $N = 15$ agents

$\longrightarrow$ **Centralized algorithm** in Th.(1) converges in 14 iterations, within 20 time steps with $\kappa = 1$ fixed.

$\longrightarrow$ **Distributed implementation** in algorithm in Fig.(1), with $\gamma = 1$, converges in 148 iterations ($t_s^{10^{-1}} = 74$),

$\longrightarrow$ Considerably better than the **standard algorithm** Eq.(4) with $t_s^{10^{-1}} = 298$.

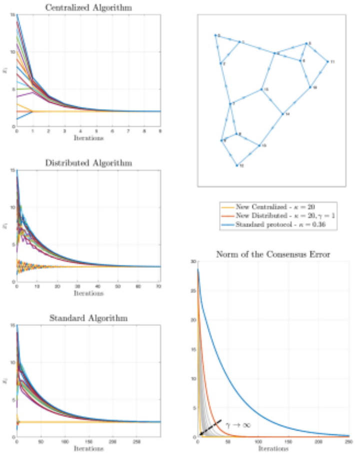


Fig. 4. Network of $N = 15$ scalar agents over a digraph.

Results: Directed Network

P.N. The distributed algorithm's performance improves as γ grows, approaching nominal performance (as in Eq.(9)) with slightly greater computational delays induced by the consensus steps.

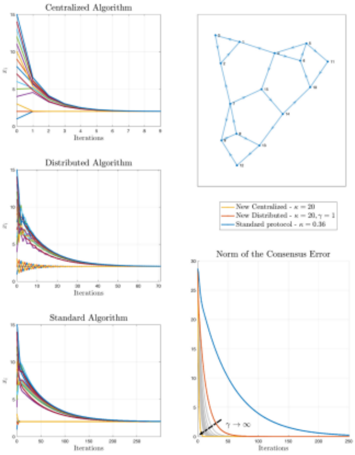


Fig. 4. Network of $N = 15$ scalar agents over a digraph.

- The proposed distributed algorithm in Fig.(1) consistently performs notably better than the standard consensus methods Eq.(4) across various network structures, proving its effectiveness even in scenarios with relatively small γ values like $\gamma = 1$ which is the worst-case scenario.
- The selection of κ as the minimum value ensuring the critically stable behavior of network dynamics and thus an attractive consensus.

Table of Contents

5 Conclusions and perspectives

Conclusions

- The protocol proposed in [2] has been generalised to enable convergence of consensus at an arbitrary rate, independently of the network's dimensions.
- This protocol has been adapted into a distributed implementation based on multi-rate forward computation spanning various consensus steps.

Perspectives

- Future direction of this work involves extending the protocol for multi-consensus scenarios within heterogeneous networks in discrete time, even when delays are present.
- Additionally, incorporating adaptive control strategies for individual agents to set individualised values for the weighting parameters in consensus equation Eq.(10).

Table of Contents

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A new distributed protocol for consensus of discrete-time systems

Thank you for listening!