# TCR

## TCR

## Contents

At the start of a contest, type this in a terminal:

```
printf "set nu sw=4 ts=4 sts=4 noet ai hls shellcmdflag=-ic\nsy on | colo slate" > .vimrc
printf "\nalias gsubmit='g++ -Wall -Wshadow -std=c++11'" >> .bashrc
printf "\nalias g11='gsubmit -DLOCAL -g'" >> .bashrc
. .bashrc
mkdir contest; cd contest
```

template.cpp

```cpp
#include<bits/stdc++.h>
using namespace std;

// Order statistics tree (if supported by judge!):
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<class TK, class TM>
using order_tree = tree<TK, TM, less<TK>, rb_tree_tag, tree_order_statistics_node_update>;
// iterator find_by_order(int r) (zero based)
// int      order_of_key(TK v)
template<class TV> using order_set = order_tree<TV, null_type>;

#define x first
#define y second
#define pb push_back
#define eb emplace_back
#define rep(i,a,b) for(auto i=(a);i!=(b); ++i)
#define all(v) (v).begin(), (v).end()
#define rs resize

typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
typedef vector<vi> vvi;
template<class T> using min_queue = priority_queue<T, vector<T>, greater<T>>;

const int INF = 2147483647; // (1 << 30) - 1 + (1 << 30)
const ll LLINF = (1LL << 62) - 1 + (1LL << 62); // = 9.223.372.036.854.775.807
const double PI = acos(-1.0);

#ifdef LOCAL
#define DBG(x) cerr << __LINE__ << ": " << #x << " = " << (x) << endl
#else
#define DBG(x)
const bool LOCAL = false;
#endif

void Log() { if(LOCAL) cerr << "\n\n"; }
template<class T, class... S>
void Log(T t, S... s) { if(LOCAL) cerr << t << "\t", Log(s...); }

// lambda-expression: [] (args) -> retType { body }
int main() {
    ios_base::sync_with_stdio(false); // fast IO
    cin.tie(NULL); // fast IO
    cerr << boolalpha; // print true/false
    (cout << fixed).precision(10); // adjust precision

    return 0;
}
```

Prime numbers: *982451653*, *81253449*, $10^3+\{-9,-3,9,13\}$, $10^6+\{-17,3,33\}$, $10^9+\{7,9,21,33,87\}$

## 0.1. De winnende aanpak.

- Goed slapen & een vroeg ritme hebben
- Genoeg drinken & eten voor en tijdens de wedstrijd
- Een lijst van alle problemen met info waar het over gaat, en wie het goed kan oplossen
- Ludo moet **ALLE** opgaves **goed** lezen
- Test de kleine voorbeeldgevallen
- Houd na 2 uur een pauze en overleg waar iedereen mee bezig is
- Maak zelf wat test-cases
- Typ de dingen uit de TCR, die je zeker nodig hebt, alvast in
- Als iemand niks te doen heeft, kan hij nodige dingen uit de TCR typen.
- We moeten ook een voorbeeld test-case voor TCR algoritmes hebben om te testen of het goed overgetypt is
- Bij geometrie moeten we om kunnen gaan met meerdere input manieren (voor bv. lijnen)
- Gebruik veel long long's

## 0.2. Wrong Answer.

(1) Print de oplossing om te debuggen! Kijk ook naar andere (mogelijk makkelijkere) problemen.
(2) Bedenk zelf test-cases met **randgevallen**!
(3) Controleer op **overflow** (gebruik **OVERAL** long long, long double).
*Kijk naar overflows in tussenantwoorden bij modulo.*
(4) Controleer de **precisie**.
(5) Controleer op **typo's**.
(6) Loop de voorbeeldinput accuraat langs.
(7) Controleer op off-by-one-errors (in indices of lus-grenzen)?

## 0.3. Detecting overflow.

These are GNU builtins, detect both over- and underflow. Returns a boolean upon failure, otherwise the result is present in `ref`. Follow the template:

```cpp
bool isOverflown = __builtin_[add|mul|sub]_overflow(a, b, \&res);
```

## 0.4. Covering problems.

*Minimum edge cover* $\Longleftrightarrow$ *Maximum independent set*

**Matching:** A set of edges without common vertices *(Maximum is the **largest** such set, maximal is a set which you cannot add more edges to without breaking the property)*.

**Minimum Vertex Cover:** A set vertices (cover) such that each edge in the graph is incident to at least one vertex of the set.

**Minimum Edge Cover:** A set of edges (cover) such that every vertex is incident to at least one edge of the set.

**Maximum Independent Set:** A set of vertices in a graph such that no two of them are adjacent.

**König's theorem:** In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

## 0.5. Game theory.

A game can be reduced to Nim if it is a finite impartial game. Nim and its variants include:

**Nim:** Let $X = \bigoplus_{i=1}^n x_i$, then $(x_i)_{i=1}^n$ is a winning position iff $X \neq 0$. Find a move by picking $k$ such that $x_k > x_k \oplus X$.

**Misère Nim:** Regular Nim, except that the last player to move *loses*. Play regular Nim until there is only one pile of size larger than 1, reduce it to 0 or 1 such that there is an odd number of piles.

**Staircase Nim:** Stones are moved down a staircase and only removed from the last pile. $(x_i)_{i=1}^n$ is an $L$-position if $(x_{2i-1})_{i=1}^{n/2}$ is (i.e. only look at odd-numbered piles).

**Moore's Nim$_k$:** The player may remove from at most $k$ piles (Nim $=$ Nim$_1$). Expand the piles in base 2, do a carry-less addition in base $k+1$ (i.e. the number of ones in each column should be divisible by $k+1$).

**Dim$^+$:** The number of removed stones must be a divisor of the pile size. The Sprague-Grundy function is $k+1$ where $2^k$ is the largest power of 2 dividing the pile size.

**Aliquot game:** Same as above, except the divisor should be proper (hence 1 is also a terminal state, but watch out for size 0 piles). Now the Sprague-Grundy function is just $k$.

**Nim (at most half):** Write $n+1 = 2^m y$ with $m$ maximal, then the Sprague-Grundy function of $n$ is $(y-1)/2$.

**Lasker's Nim:** Players may alternatively split a pile into two new non-empty piles. $g(4k+1) = 4k+1$, $g(4k+2) = 4k+2$, $g(4k+3) = 4k+4$, $g(4k+4) = 4k+3$ $(k \geq 0)$.

**Hackenbush on trees:** A tree with stalks $(x_i)_{i=1}^n$ may be replaced with a single stalk with length $\bigoplus_{i=1}^n x_i$.

A useful identity: $\bigoplus_{x=0}^{a-1} x = \{0, a-1, 1, a\}[a \bmod 4]$.

## 1. Mathematics

```cpp
int abs(int x) { return x > 0 ? x : -x; }
int sign(int x) { return (x > 0) - (x < 0); }

// greatest common divisor
ll gcd(ll a, ll b) { while (b) a %= b, swap(a, b); return a; };
// least common multiple
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }
ll mod(ll a, ll b) { return (a %= b) < 0 ? a + b : a; }

// safe multiplication (ab % m) for m <= 4e18 in O(log b)
ll mulmod(ll a, ll b, ll m) {
    ll r = 0;
    while (b) {
        if (b & 1) r = (r + a) % m;
        a = (a + a) % m;
        b >>= 1;
    }
    return r;
}

// safe exponentation (a^b % m) for m <= 2e9 in O(log b)
ll powmod(ll a, ll b, ll m) {
    ll r = 1;
    while (b) {
        if (b & 1) r = (r * a) % m; // r = mulmod(r, a, m);
        a = (a * a) % m; // a = mulmod(a, a, m);
        b >>= 1;
    }
    return r;
}

// returns x, y such that ax + by = gcd(a, b)
ll egcd(ll a, ll b, ll &x, ll &y) {
    ll xx = y = 0, yy = x = 1;
    while (b) {
        x -= a / b * xx; swap(x, xx);
        y -= a / b * yy; swap(y, yy);
        a %= b; swap(a, b);
    }
    return a;
}

// Chinese remainder theorem
const pll NO_SOLUTION(0, -1);
// Returns (u, v) such that x = u % v <=> x = a % n and x = b % m
pll crt(ll a, ll n, ll b, ll m) {
    ll s, t, d = egcd(n, m, s, t), nm = n * m;
    if (mod(a - b, d)) return NO_SOLUTION;
    return pll(mod(s * b * n + t * a * m, nm) / d, nm / d);
    /* when n, m > 10^6, avoid overflow:
    return pll(mod(mulmod(mulmod(s, b, nm), n, nm)
                    + mulmod(mulmod(t, a, nm), m, nm), nm) / d, nm / d);
                          */
}

// phi[i] = #{ 0 < j <= i | gcd(i, j) = 1 }
vi totient(int N) {
    vi phi(N);
    for (int i = 0; i < N; i++) phi[i] = i;
    for (int i = 2; i < N; i++)
        if (phi[i] == i)
            for (int j = i; j < N; j += i) phi[j] -= phi[j] / i;
    return phi;
}

// calculate nCk % p (p prime!)
ll lucas(ll n, ll k, ll p) {
    ll ans = 1;
    while (n) {
        ll np = n % p, kp = k % p;
        if (np < kp) return 0;
        ans = mod(ans * binom(np, kp), p); // (np C kp)
        n /= p; k /= p;
    }
    return ans;
}

// returns if n is prime for n < 3e24 ( > 2^64)
bool millerRabin(ll n)
{
    if (n < 2 || n % 2 == 0) return n == 2;
    ll d = n - 1, ad, s = 0, r;
    for (; d % 2 == 0; d /= 2) s++;
    for (int a : { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 }) {
        if (n == a) return true;
        if ((ad = powmod(a, d, n)) == 1) continue;
        for (r = 0; r < s && ad + 1 != n; r++)
            ad = mulmod(ad, ad, n);
        if (r == s) return false;
    }
    return true;
}
```

## 2. Datastructures

### 2.1. Standard segment tree $\mathcal{O}(\log n)$.

```cpp
typedef /* Tree element */ S;
const int n = 1 << 20;
S t[2 * n];

// required axiom: associativity
S combine(S l, S r) { return l + r; } // sum segment tree
S combine(S l, S r) { return max(l, r); } // max segment tree

void build() {
    for (int i = n; --i; ) t[i] = combine(t[2 * i], t[2 * i + 1]);
}

// set value v on position i
void update(int i, S v) {
    for (t[i += n] = v; i /= 2; ) t[i] = combine(t[2 * i], t[2 * i +
            1]);
}

// sum on interval [l, r)
S query(int l, int r) {
    S resL, resR;
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
        if (l & 1) resL = combine(resL, t[l++]);
        if (r & 1) resR = combine(t[--r], resR);
    }
    return combine(resL, resR);
}
```

### 2.2. Binary Indexed Tree $\mathcal{O}(\log n)$. Use one-based indices $(i > 0)$!

```cpp
int bit[MAXN + 1];

// arr[i] += v
void update(int i, int v) {
    while (i <= MAXN) bit[i] += v, i += i & -i;
}

// returns sum of arr[i], where i: [1, i]
int query(int i) {
    int v = 0; while (i) v += bit[i], i -= i & -i; return v;
}
```

### 2.3. Disjoint-Set / Union-Find $\mathcal{O}(\alpha(n))$.

```cpp
int par[MAXN], rnk[MAXN];

void uf_init(int n) {
    fill_n(par, n, -1);
    fill_n(rnk, n, 0);
}

int uf_find(int v) {
    return par[v] < 0 ? v : par[v] = uf_find(par[v]);
}

void uf_union(int a, int b) {
    if ((a = uf_find(a)) == (b = uf_find(b))) return;
    if (rnk[a] < rnk[b]) swap(a, b);
    if (rnk[a] == rnk[b]) rnk[a]++;
    par[b] = a;
}
```

## 3. Graph Algorithms

### 3.1. Maximum matching $\mathcal{O}(nm)$. This problem could be solved with a flow algorithm like Dinic's algorithm which runs in $\mathcal{O}(\sqrt{V}E)$, too.

```cpp
const int sizeL = 1e4, sizeR = 1e4;

bool vis[sizeR];
int par[sizeR]; // par : R -> L
vi adj[sizeL]; // adj : L -> (N -> R)

bool match(int u) {
    for (int v : adj[u]) {
        if (vis[v]) continue;
        vis[v] = true;
        if (par[v] == -1 || match(par[v])) {
            par[v] = u;
            return true;
        }
    }
    return false;
}

// perfect matching iff ret == sizeL == sizeR
int maxmatch() {
    fill_n(par, sizeR, -1);
    int ret = 0;
    for (int i = 0; i < sizeL; i++) {
        fill_n(vis, sizeR, false);
        ret += match(i);
    }
    return ret;
}
```

### 3.2. Strongly Connected Components $\mathcal{O}(V + E)$.

```cpp
vvi adj, comps;
vi tidx, lnk, cnr, st;
vector<bool> vis;
int age, ncomps;

void tarjan(int v) {
    tidx[v] = lnk[v] = ++age;
    vis[v] = true;
    st.pb(v);

    for (int w : adj[v]) {
        if (!tidx[w]) tarjan(w), lnk[v] = min(lnk[v], lnk[w]);
        else if (vis[w]) lnk[v] = min(lnk[v], tidx[w]);
    }

    if (lnk[v] != tidx[v]) return;

    comps.pb(vi());
    int w;
    do {
        vis[w = st.back()] = false;
        cnr[w] = ncomps;
        comps.back().pb(w);
        st.pop_back();
    } while (w != v);
    ncomps++;
}

void findSCC(int n) {
    age = ncomps = 0;
    vis.assign(n, false);
    tidx.assign(n, 0);
    lnk.resize(n);
    cnr.resize(n);
    comps.clear();

    for (int i = 0; i < n; i++)
        if (tidx[i] == 0) tarjan(i);
}
```

### 3.2.1. *2-SAT* $\mathcal{O}(V+E)$. Include `findSCC`.

```cpp
void init2sat(int n) { adj.assign(2 * n, vi()); }

// vl, vr = true -> variable l, variable r should be negated.
void imply(int xl, bool vl, int xr, bool vr) {
    adj[2 * xl + vl].pb(2 * xr + vr);
    adj[2 * xr +!vr].pb(2 * xl +!vl);
}

void satOr(int xl, bool vl, int xr, bool vr) { imply(xl, !vl, xr, vr);
     }
void satConst(int x, bool v) { imply(x, !v, x, v); }
void satIff(int xl, bool vl, int xr, bool vr) {
    imply(xl, vl, xr, vr);
    imply(xr, vr, xl, vl);
}

bool solve2sat(int n, vector<bool> &sol) {
    findSCC(2 * n);
    for (int i = 0; i < n; i++)
        if (cnr[2 * i] == cnr[2 * i + 1]) return false;
    vector<bool> seen(n, false);
    sol.assign(n, false);
    for (vi &comp : comps) {
        for (int v : comp) {
            if (seen[v / 2]) continue;
            seen[v / 2] = true;
            sol[v / 2] = v & 1;
        }
    }
    return true;
}
```

#### 3.3. **Cycle Detection** $\mathcal{O}(V+E)$.

```cpp
vvi adj; // assumes bidirected graph, adjust accordingly

bool cycle_detection() {
    stack<int> s;
    vector<bool> vis(MAXN, false);
    vi par(MAXN, -1);
    s.push(0);
    vis[0] = true;
    while(!s.empty()) {
        int cur = s.top();
        s.pop();
        for(int i : adj[cur]) {
            if(vis[i] && par[cur] != i) return true;
            s.push(i);
            par[i] = cur;
            vis[i] = true;
        }
    }
    return false;
}
```

## 3.4. **Shortest path.**

### 3.4.1. *Dijkstra* $\mathcal{O}(E+V\log V)$.

#### 3.4.2. *Floyd-Warshall* $\mathcal{O}(V^3)$.

```cpp
int n = 100;
ll d[MAXN][MAXN];
for (int i = 0; i < n; i++) fill_n(d[i], n, 1e18);
// set direct distances from i to j in d[i][j] (and d[j][i])
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        for (int k = 0; k < n; k++)
            d[j][k] = min(d[j][k], d[j][i] + d[i][k]);
```

### 3.4.3. *Bellman Ford* $\mathcal{O}(VE)$. This is only useful if there are edges with weight $w_{ij} < 0$ in the graph.

```cpp
vector< pair<pii, ll> > edges; // ((from, to), weight)
vector<ll> dist;

// when undirected, add back edges
bool bellman_ford(int V, int source) {
    dist.assign(V, 1e18);
    dist[source] = 0;

    bool updated = true;
    int loops = 0;
    while (updated && loops < n) {
        updated = false;
        for (auto e : edges) {
            int alt = dist[e.x.x] + e.y;
            if (alt < dist[e.x.y]) {
                dist[e.x.y] = alt;
                updated = true;
            }
        }
    }
    return loops < n; // loops >= n: negative cycles
}
```

## 3.5. **Max-flow min-cut.**

#### 3.5.1. *Dinic's Algorithm* $\mathcal{O}(V^2E)$.

```cpp
struct edge {
    int to, rev;
    ll cap, flow;
    edge(int t, int r, ll c) : to(t), rev(r), cap(c), flow(0) {}
};

int s, t, level[MAXN]; // s = source, t = sink
vector<edge> g[MAXN];

void add_edge(int fr, int to, ll cap) {
    g[fr].pb(edge(to, g[to].size(), cap));
    g[to].pb(edge(fr, g[fr].size() - 1, 0));
}

bool dinic_bfs() {
    fill_n(level, MAXN, 0);
    level[s] = 1;

    queue<int> q;
    q.push(s);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (edge e : g[cur]) {
            if (level[e.to] == 0 && e.flow < e.cap) {
                level[e.to] = level[cur] + 1;
                q.push(e.to);
            }
        }
    }
    return level[t] != 0;
}

ll dinic_dfs(int cur, ll maxf) {
    if (cur == t) return maxf;

    ll f = 0;
    bool isSat = true;
    for (edge &e : g[cur]) {
        if (level[e.to] != level[cur] + 1 || e.flow >= e.cap)
            continue;
        ll df = dinic_dfs(e.to, min(maxf - f, e.cap - e.flow));
        f += df;
        e.flow += df;
        g[e.to][e.rev].flow -= df;
        isSat &= e.flow == e.cap;
        if (maxf == f) break;
    }
    if (isSat) level[cur] = 0;
    return f;
}

ll dinic_maxflow() {
    ll f = 0;
    while (dinic_bfs()) f += dinic_dfs(s, LLINF);
    return f;
}
```

## 3.6. **Min-cost max-flow.** Find the cheapest possible way of sending a certain amount of flow through a flow network.

```cpp
struct edge {
    // to, rev, flow, capacity, weight
    int t, r;
    ll f, c, w;
    edge(int _t, int _r, ll _c, ll _w) : t(_t), r(_r), f(0), c(_c), w(
        _w) {}
};

int n, par[MAXN];
vector<edge> adj[MAXN];
ll dist[MAXN];

bool findPath(int s, int t) {
    fill_n(dist, n, LLINF);
    fill_n(par, n, -1);

    priority_queue< pii, vector<pii>, greater<pii> > q;
    q.push(pii(dist[s] = 0, s));

    while (!q.empty()) {
        int d = q.top().x, v = q.top().y;
        q.pop();
        if (d > dist[v]) continue;

        for (edge e : adj[v]) {
            if (e.f < e.c && d + e.w < dist[e.t]) {
                q.push(pii(dist[e.t] = d + e.w, e.t));
                par[e.t] = e.r;
            }
        }
    }
    return dist[t] < INF;
}

pair<ll, ll> minCostMaxFlow(int s, int t) {
    ll cost = 0, flow = 0;
    while (findPath(s, t)) {
        ll f = INF, c = 0;
        int cur = t;
        while (cur != s) {
            const edge &rev = adj[cur][par[cur]], &e = adj[rev.t][rev.r
                ];
            f = min(f, e.c - e.f);
            cur = rev.t;
        }
        cur = t;
        while (cur != s) {
            edge &rev = adj[cur][par[cur]], &e = adj[rev.t][rev.r];
            c += e.w;
            e.f += f;
            rev.f -= f;
            cur = rev.t;
        }
        cost += f * c;
        flow += f;
    }
    return pair<ll, ll>(cost, flow);
}

inline void addEdge(int from, int to, ll cap, ll weight) {
    adj[from].pb(edge(to, adj[to].size(), cap, weight));
    adj[to].pb(edge(from, adj[from].size() - 1, 0, -weight));
}
```

## 3.7. **Minimal Spanning Tree.**

### 3.7.1. *Kruskal* $\mathcal{O}(E\log V)$.

## 4. String algorithms

### 4.1. Trie.

```
const int SIGMA = 26;

struct trie {
    bool word;
    trie **adj;

    trie() : word(false), adj(new trie*[SIGMA]) {
        for (int i = 0; i < SIGMA; i++) adj[i] = NULL;
    }

    void addWord(const string &str) {
        trie *cur = this;
        for (char ch : str) {
            int i = ch - 'a';
            if (!cur->adj[i]) cur->adj[i] = new trie();
            cur = cur->adj[i];
        }
        cur->word = true;
    }

    bool isWord(const string &str) {
        trie *cur = this;
        for (char ch : str) {
            int i = ch - 'a';
            if (!cur->adj[i]) return false;
            cur = cur->adj[i];
        }
        return cur->word;
    }
};
```

### 4.2. Z-algorithm $\mathcal{O}(n)$.

```
// z[i] = length of longest substring starting from s[i] which is also a prefix of s.
vi z_function(const string &s) {
    int n = (int) s.length();
    vi z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r) z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

### 4.3. Suffix array $\mathcal{O}(n \log^2 n)$.

This creates an array $P[0], P[1], \dots, P[n-1]$ such that the suffix $S[i \dots n]$ is the $P[i]^{th}$ suffix of $S$ when lexicographically sorted.

```
typedef pair<pii, int> tii;

const int maxlogn = 17, maxn = 1 << maxlogn;

tii make_triple(int a, int b, int c) { return tii(pii(a, b), c); }

int p[maxlogn + 1][maxn];
tii L[maxn];

int suffixArray(string S) {
    int N = S.size(), stp = 1, cnt = 1;
    for (int i = 0; i < N; i++) p[0][i] = S[i];
    for (; cnt < N; stp++, cnt <<= 1) {
        for (int i = 0; i < N; i++) {
            L[i] = tii(pii(p[stp-1][i], i + cnt < N ? p[stp-1][i + cnt] : -1), i);
        }
        sort(L, L + N);
        for (int i = 0; i < N; i++) {
            p[stp][L[i].y] = i > 0 && L[i].x == L[i-1].x ? p[stp][L[i-1].y] : i;
        }
    }
    return stp - 1; // result is in p[stp - 1][0 .. (N - 1)]
}
```

### 4.4. Longest Common Subsequence $\mathcal{O}(n^2)$.

Substring: *consecutive characters* !!!

```
int dp[STR_SIZE][STR_SIZE]; // DP problem

int lcs(const string &w1, const string &w2) {
    int n1 = w1.size(), n2 = w2.size();
    for (int i = 0; i < n1; i++) {
        for (int j = 0; j < n2; j++) {
            if (i == 0 || j == 0) dp[i][j] = 0;
            else if (w1[i - 1] == w2[j - 1]) dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
        }
    }
    return dp[n1][n2];
}

// backtrace
string getLCS(const string &w1, const string &w2) {
    int i = w1.size(), j = w2.size();
    string ret = "";
    while (i > 0 && j > 0) {
        if (w1[i - 1] == w2[j - 1]) ret += w1[--i], j--;
        else if (dp[i][j - 1] > dp[i - 1][j]) j--;
        else i--;
    }
    reverse(ret.begin(), ret.end());
    return ret;
}
```

### 4.5. Levenshtein Distance $\mathcal{O}(n^2)$.

Also known as the 'Edit distance'.

```
int dp[MAX_SIZE][MAX_SIZE]; // DP problem

int levDist(const string &w1, const string &w2) {
    int n1 = w1.size(), n2 = w2.size();
    for (int i = 0; i <= n1; i++) dp[i][0] = i; // removal
    for (int j = 0; j <= n2; j++) dp[0][j] = j; // insertion
    for (int i = 1; i <= n1; i++)
        for (int j = 1; j <= n2; j++)
            dp[i][j] = min(
                1 + min(dp[i - 1][j], dp[i][j - 1]),
                dp[i - 1][j - 1] + (w1[i - 1] != w2[j - 1])
            );
    return dp[n1][n2];
}
```

### 4.6. Knuth-Morris-Pratt algorithm $\mathcal{O}(N + M)$.

```
int kmp_search(const string &word, const string &text) {
    int n = word.size();
    vi T(n + 1, 0);
    for (int i = 1, j = 0; i < n; ) {
        if (word[i] == word[j]) T[++i] = ++j; // match
        else if (j > 0) j = T[j]; // fallback
        else i++; // no match, keep zero
    }
    int matches = 0;
    for (int i = 0, j = 0; i < text.size(); ) {
        if (text[i] == word[j]) {
            i++;
            if (++j == n) { // match at interval [i - n, i)
                matches++;
                j = T[j];
            }
        } else if (j > 0) j = T[j];
        else i++;
    }
    return matches;
}
```

### 4.7. Aho-Corasick Algorithm $\mathcal{O}(N + \sum_{i=1}^{m} |S_i|)$.

All given P must be unique!

```
const int MAXP = 100, MAXLEN = 200, SIGMA = 26, MAXTRIE = MAXP * MAXLEN;

int nP;
string P[MAXP], S;

int pnr[MAXTRIE], to[MAXTRIE][SIGMA], sLink[MAXTRIE], dLink[MAXTRIE], nnodes;

void ahoCorasick() {
    fill_n(pnr, MAXTRIE, -1);
    for (int i = 0; i < MAXTRIE; i++) fill_n(to[i], SIGMA, 0);
    fill_n(sLink, MAXTRIE, 0);
    fill_n(dLink, MAXTRIE, 0);
    nnodes = 1;
    // STEP 1: MAKE A TREE
    for (int i = 0; i < nP; i++) {
        int cur = 0;
        for (char c : P[i]) {
            int i = c - 'a';
            if (to[cur][i] == 0) to[cur][i] = nnodes++;
            cur = to[cur][i];
        }
        pnr[cur] = i;
    }
    // STEP 2: CREATE SUFFIX_LINKS AND DICT_LINKS
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (int c = 0; c < SIGMA; c++) {
            if (to[cur][c]) {
                int sl = sLink[to[cur][c]] = cur == 0 ? 0 : to[sLink[cur]][c];
                // if all strings have equal length, remove this:
                dLink[to[cur][c]] = pnr[sl] >= 0 ? sl : dLink[sl];
                q.push(to[cur][c]);
            } else to[cur][c] = to[sLink[cur]][c];
        }
    }
    // STEP 3: TRAVERSE S
    for (int cur = 0, i = 0, n = S.size(); i < n; i++) {
        cur = to[cur][S[i] - 'a'];
        for (int hit = pnr[cur] >= 0 ? cur : dLink[cur]; hit; hit = dLink[hit]) {
            cerr << P[pnr[hit]] << " found at [" << (i + 1 - P[pnr[hit]].size()) << ", " << i << "]" << endl;
        }
    }
}
```

## 5. Geometry

```
const double EPS = 1e-7, PI = acos(-1.0);

typedef long long NUM; // EITHER double OR long long
typedef pair<NUM, NUM> pt;
#define x first
#define y second

pt operator+(pt p, pt q) { return pt(p.x + q.x, p.y + q.y); }
pt operator-(pt p, pt q) { return pt(p.x - q.x, p.y - q.y); }

pt& operator+=(pt &p, pt q) { return p = p + q; }
pt& operator-=(pt &p, pt q) { return p = p - q; }

pt operator*(pt p, NUM l) { return pt(p.x * l, p.y * l); }
pt operator/(pt p, NUM l) { return pt(p.x / l, p.y / l); }

NUM operator*(pt p, pt q) { return p.x * q.x + p.y * q.y; }
NUM operator^(pt p, pt q) { return p.x * q.y - p.y * q.x; }

istream& operator>>(istream &in, pt &p) { return in >> p.x >> p.y; }
ostream& operator<<(ostream &out, pt p) { return out << '(' << p.x << ", " << p.y << ')'; }

NUM lenSq(pt p) { return p * p; }
NUM lenSq(pt p, pt q) { return lenSq(p - q); }
double len(pt p) { return hypot(p.x, p.y); } // more overflow safe
double len(pt p, pt q) { return len(p - q); }
```

```
typedef pt frac;
typedef pair<double, double> vec;
vec getvec(pt p, pt dp, frac t) { return vec(p.x + 1. * dp.x * t.x / t.y, p.y + 1. * dp.y * t.x / t.y); }

// square distance from pt a to line bc
frac distPtLineSq(pt a, pt b, pt c) {
    a -= b, c -= b;
    return frac((a ^ c) * (a ^ c), c * c);
}

// square distance from pt a to linesegment bc
frac distPtSegmentSq(pt a, pt b, pt c) {
    a -= b; c -= b;
    NUM dot = a * c, len = c * c;
    if (dot <= 0) return frac(a * a, 1);
    if (dot >= len) return frac((a - c) * (a - c), 1);
    return frac(a * a * len - dot * dot, len);
}

// projects pt a onto linesegment bc
frac proj(pt a, pt b, pt c) { return frac((a - b) * (c - b), (c - b) * (c - b)); }
vec projv(pt a, pt b, pt c) { return getvec(b, c - b, proj(a, b, c)); }

bool collinear(pt a, pt b, pt c) { return ((a - b) ^ (a - c)) == 0; }

bool pointOnSegment(pt a, pt b, pt c) {
    NUM dot = (a - b) * (c - b), len = (c - b) * (c - b);
    return collinear(a, b, c) && 0 <= dot && dot <= len;
}

// true => 1 intersection, false => parallel, so 0 or \infty solutions
bool linesIntersect(pt a, pt b, pt c, pt d) { return ((a - b) ^ (c - d)) != 0; }
vec lineLineIntersection(pt a, pt b, pt c, pt d) {
    double det = (a - b) ^ (c - d);
    pt ret = (c - d) * (a ^ b) - (a - b) * (c ^ d);
    return vec(ret.x / det, ret.y / det);
}

// dp, dq are directions from p, q
// intersection at p + t_i dp, for 0 <= i < return value
int segmentIntersection(pt p, pt dp, pt q, pt dq, frac &t0, frac &t1)
{
    if (dp * dp == 0) swap(p, q), swap(dp, dq); // dq = 0
    if (dp * dp == 0) { t0 = t1 = frac(0, 1); return p == q; } // dp = dq = 0

    pt dpq = (q - p);
    NUM c = dp ^ dq, c0 = dpq ^ dp, c1 = dpq ^ dq;
    if (c == 0) { // parallel, dp > 0, dq >= 0
        if (c0 != 0) return 0; // not collinear
        NUM v0 = dpq * dp, v1 = v0 + dq * dp, dp2 = dp * dp;
        if (v1 < v0) swap(v0, v1);
        t0 = frac(v0 = max(v0, (NUM) 0), dp2);
        t1 = frac(v1 = min(v1, dp2), dp2);
        return (v0 <= v1) + (v0 < v1);
    } else if (c < 0) c = -c, c0 = -c0, c1 = -c1;
    t0 = t1 = frac(c1, c);
    return 0 <= min(c0, c1) && max(c0, c1) <= c;
}

// Returns TWICE the area of a polygon to keep it an integer
NUM polygonTwiceArea(const vector<pt> &pts) {
    NUM area = 0;
    for (int N = pts.size(), i = 0, j = N - 1; i < N; j = i++)
        area += pts[i] ^ pts[j];
    return abs(area); // area < 0 <=> pts ccw
}

bool pointInPolygon(pt p, const vector<pt> &pts) {
    double sum = 0;
    for (int N = pts.size(), i = 0, j = N - 1; i < N; j = i++) {
        if (pointOnSegment(p, pts[i], pts[j])) return true; // boundary
        double angle = acos((pts[i] - p) * (pts[j] - p) / len(pts[i], p) / len(pts[j], p));
        sum += ((pts[i] - p) ^ (pts[j] - p)) < 0 ? angle : -angle;
    }
    return abs(abs(sum) - 2 * PI) < EPS;
}
```

### 5.1. **Convex Hull** $\mathcal{O}(n \log n)$.

```
// points are given by: pts[ret[0]], pts[ret[1]], ... pts[ret[ret.size()-1]]
vi convexHull(const vector<pt> &pts) {
    if (pts.empty()) return vi();
    vi ret;
    // find one outer point:
    int fsti = 0, n = pts.size();
    pt fstpt = pts[0];
    for(int i = n; i--; ) {
        if (pts[i] < fstpt) fstpt = pts[fsti = i];
    }
    ret.pb(fsti);
    pt refr = pts[fsti];

    vi ord; // index into pts
    for (int i = n; i--; ) {
        if (pts[i] != refr) ord.pb(i);
    }
    sort(ord.begin(), ord.end(), [&pts, &refr] (int a, int b) -> bool {
        NUM cross = (pts[a] - refr) ^ (pts[b] - refr);
        return cross != 0 ? cross > 0 : lenSq(refr, pts[a]) < lenSq(refr, pts[b]);
    });
    for (int i : ord) {
        // NOTE: > INCLUDES points on the hull-line, >= EXCLUDES
        while (ret.size() > 1 &&
                ((pts[ret[ret.size()-2]]-pts[ret.back()]) ^ (pts[i]-pts[ret.back()])) >= 0)
            ret.pop_back();
        ret.pb(i);
    }
    return ret;
}
```

### 5.2. **Rotating Calipers** $\mathcal{O}(n)$.

Finds the longest distance between two points in a convex hull.

```
NUM rotatingCalipers(vector<pt> &hull) {
    int n = hull.size(), a = 0, b = 1;
    if (n <= 1) return 0.0;
    while (((hull[1] - hull[0]) ^ (hull[(b + 1) % n] - hull[b])) > 0) b++;
    NUM ret = 0.0;
    while (a < n) {
        ret = max(ret, lenSq(hull[a], hull[b]));
        if (((hull[(a + 1) % n] - hull[a % n]) ^ (hull[(b + 1) % n] - hull[b])) <= 0) a++;
        else if (++b == n) b = 0;
    }
    return ret;
}
```

### 5.3. **Closest points** $\mathcal{O}(n \log n)$.

```
int n;
pt pts[maxn];

struct byY {
    bool operator()(int a, int b) const { return pts[a].y < pts[b].y; }
};

inline NUM dist(pii p) {
    return hypot(pts[p.x].x - pts[p.y].x, pts[p.x].y - pts[p.y].y);
}

pii minpt(pii p1, pii p2) {
    return (dist(p1) < dist(p2)) ? p1 : p2;
}

// closest pts (by index) inside pts[l ... r], with sorted y values in ys
pii closest(int l, int r, vi &ys) {
    if (r - l == 2) { // don't assume 1 here.
        ys = { l, l + 1 };
        return pii(l, l + 1);
    } else if (r - l == 3) { // brute-force
        ys = { l, l + 1, l + 2 };
        sort(ys.begin(), ys.end(), byY());
        return minpt(pii(l, l + 1), minpt(pii(l, l + 2), pii(l + 1, l + 2)));
    }
    int m = (l + r) / 2;
    vi yl, yr;
    pii delta = minpt(closest(l, m, yl), closest(m, r, yr));
    NUM ddelta = dist(delta), xm = .5 * (pts[m-1].x + pts[m].x);
    merge(yl.begin(), yl.end(), yr.begin(), yr.end(), back_inserter(ys), byY());
    deque<int> q;
    for (int i : ys) {
        if (abs(pts[i].x - xm) <= ddelta) {
            for (int j : q) delta = minpt(delta, pii(i, j));
            q.pb(i);
            if (q.size() > 8) q.pop_front(); // magic from Introduction to Algorithms.
        }
    }
    return delta;
}
```

## 6. Miscellaneous

### 6.1. **Binary search** $\mathcal{O}(\log(hi - lo))$.

```
bool test(int n);

int search(int lo, int hi) {
    // assert(test(lo) && !test(hi));
    while (hi - lo > 1) {
        int m = (lo + hi) / 2;
        (test(m) ? lo : hi) = m;
    }
    // assert(test(lo) && !test(hi));
    return lo;
}
```

### 6.2. **Fast Fourier Transform** $\mathcal{O}(n \log n)$.

Given two polynomials $A(x) = a_0 + a_1 x + \cdots + a_{n/2} x^{n/2}$ and $B(x) = b_0 + b_1 x + \cdots + b_{n/2} x^{n/2}$, FFT calculates all coefficients of $C(x) = A(x) \cdot B(x) = c_0 + c_1 x + \ldots c_n x^n$, with $c_i = \sum_{j=0}^{i} a_j b_{i-j}$.

```
typedef complex<double> cpx;
const int logmaxn = 20, maxn = 1 << logmaxn;

cpx a[maxn] = {}, b[maxn] = {}, c[maxn];

void fft(cpx *src, cpx *dest) {
    for (int i = 0, rep = 0; i < maxn; i++, rep = 0) {
        for (int j = i, k = logmaxn; k--; j >>= 1) rep = (rep << 1) | (j & 1);
        dest[rep] = src[i];
    }
    for (int s = 1, m = 1; m <= maxn; s++, m *= 2) {
        cpx r = exp(cpx(0, 2.0 * PI / m));
        for (int k = 0; k < maxn; k += m) {
            cpx cr(1.0, 0.0);
            for (int j = 0; j < m / 2; j++) {
                cpx t = cr * dest[k + j + m / 2];
                dest[k + j + m / 2] = dest[k + j] - t;
                dest[k + j] += t;
                cr *= r;
            }
        }
    }
}

void multiply() {
    fft(a, c);
    fft(b, a);
    for (int i = 0; i < maxn; i++) b[i] = conj(a[i] * c[i]);
    fft(b, c);
    for (int i = 0; i < maxn; i++) c[i] = conj(c[i]) / (1.0 * maxn);
}
```

### 6.3. Minimum Assignment (Hungarian Algorithm) $\mathcal{O}(n^3)$.

```cpp
int a[MAXN + 1][MAXM + 1]; // matrix, 1-based

int minimum_assignment(int n, int m) { // n rows, m columns
    vi u(n + 1), v(m + 1), p(m + 1), way(m + 1);

    for (int i = 1; i <= n; i++) {
        p[0] = i;
        int j0 = 0;
        vi minv(m + 1, INF);
        vector<char> used(m + 1, false);
        do {
            used[j0] = true;
            int i0 = p[j0], delta = INF, j1;
            for (int j = 1; j <= m; j++)
                if (!used[j]) {
                    int cur = a[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
            for (int j = 0; j <= m; j++) {
                if(used[j]) u[p[j]] += delta, v[j] -= delta;
                else minv[j] -= delta;
            }
            j0 = j1;
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }

    // column j is assigned to row p[j]
    // for (int j = 1; j <= m; ++ j) ans[p[j]] = j;
    return -v[0];
}
```

### 6.4. Partial linear equation solver $\mathcal{O}(N^3)$. «««< HEAD

```cpp
typedef double NUM;

#define MAXN 110
#define EPS 1e-5

NUM mat[MAXN][MAXN + 1], vals[MAXN];
bool hasval[MAXN];

bool is_zero(NUM a) { return -EPS < a && a < EPS; }
bool eq(NUM a, NUM b) { return is_zero(a - b); }

int solvemat(int n)
{
    for(int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) cin >> mat[i][j];
    for (int i = 0; i < n; i++) cin >> mat[i][n];

    int pivrow = 0, pivcol = 0;
    while (pivcol < n) {
        int r = pivrow, c;
        while (r < n && is_zero(mat[r][pivcol])) r++;
        if (r == n) { pivcol++; continue; }

        for (c = 0; c <= n; c++) swap(mat[pivrow][c], mat[r][c]);

        r = pivrow++; c = pivcol++;
        NUM div = mat[r][c];
        for (int col = c; col <= n; col++) mat[r][col] /= div;
        for (int row = 0; row < n; row++) {
            if (row == r) continue;
            NUM times = -mat[row][c];
            for (int col = c; col <= n; col++) mat[row][col] += times *
                mat[r][col];
        }
    }
    // now mat is in RREF
    for (int r = pivrow; r < n; r++)
        if (!is_zero(mat[r][n])) return 0;

    fill_n(hasval, n, false);
    for (int col = 0, row; col < n; col++) {
        hasval[col] = !is_zero(mat[row][col]);
        if (!hasval[col]) continue;
        for (int c = col + 1; c < n; c++) {
            if (!is_zero(mat[row][c])) hasval[col] = false;
        }
        if (hasval[col]) vals[col] = mat[row][n];
        row++;
    }

    for (int i = 0; i < n; i++)
        if (!hasval[i]) return 2;
    return 1;
}
```

```
=======
```

```
typedef double NUM;

#define MAXN 110
#define EPS 1e-5

NUM mat[MAXN][MAXN + 1], vals[MAXN];
bool hasval[MAXN];

bool is_zero(NUM a) { return -EPS < a && a < EPS; }
bool eq(NUM a, NUM b) { return is_zero(a - b); }

int solvemat(int n)
{
    for(int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) cin >> mat[i][j];
    for (int i = 0; i < n; i++) cin >> mat[i][n];

    int pivrow = 0, pivcol = 0;
    while (pivcol < n) {
        int r = pivrow, c;
        while (r < n && is_zero(mat[r][pivcol])) r++;
        if (r == n) { pivcol++; continue; }

        for (c = 0; c <= n; c++) swap(mat[pivrow][c], mat[r][c]);

        r = pivrow++; c = pivcol++;
        NUM div = mat[r][c];
        for (int col = c; col <= n; col++) mat[r][col] /= div;
        for (int row = 0; row < n; row++) {
            if (row == r) continue;
            NUM times = -mat[row][c];
            for (int col = c; col <= n; col++) mat[row][col] += times * mat[r][col];
        }
    }
    // now mat is in RREF
    for (int r = pivrow; r < n; r++)
        if (!is_zero(mat[r][n])) return 0;

    fill_n(hasval, n, false);
    for (int col = 0, row; col < n; col++) {
        hasval[col] = !is_zero(mat[row][col]);
        if (!hasval[col]) continue;
        for (int c = col + 1; c < n; c++) {
            if (!is_zero(mat[row][c])) hasval[col] = false;
        }
        if (hasval[col]) vals[col] = mat[row][n];
        row++;
    }

    for (int i = 0; i < n; i++)
        if (!hasval[i]) return 2;
    return 1;
}
```

»»»> 6f9e7f0f4b6e0690dfea4b352511edebd24ac35b