Practicum 2:
Monte Carlo simulation for SPECT

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Practical information:

- Work in groups of two (same groups as last week)
- Hand in the notebook (in both .ipynb and .html format) via UFORA (UGent) or Canvas (VUB)
- Deadline: in two weeks (1 November at 23:59)
- For questions:
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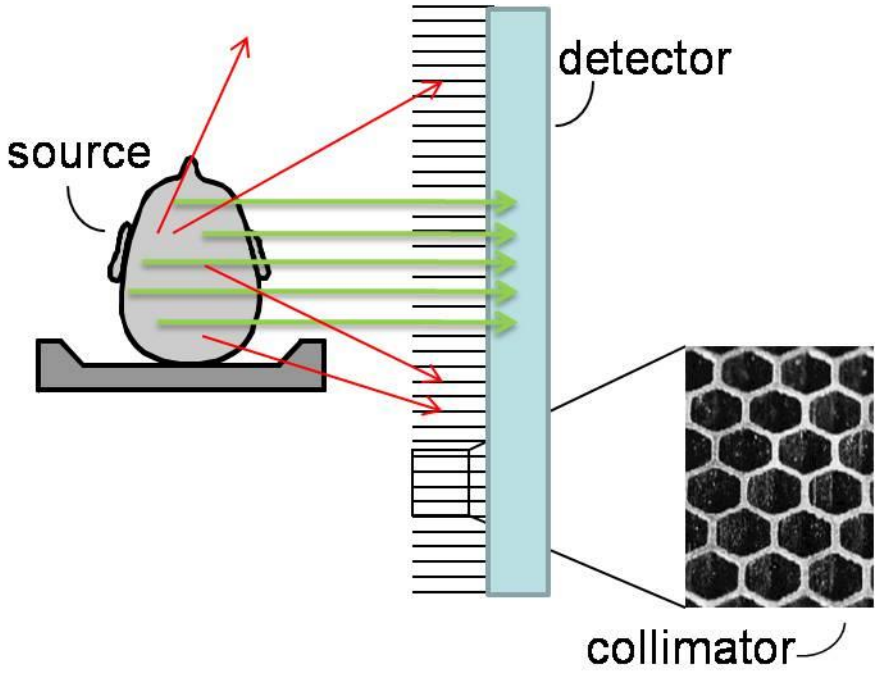
  - jens.maebe@ugent.be

Background: SPECT camera

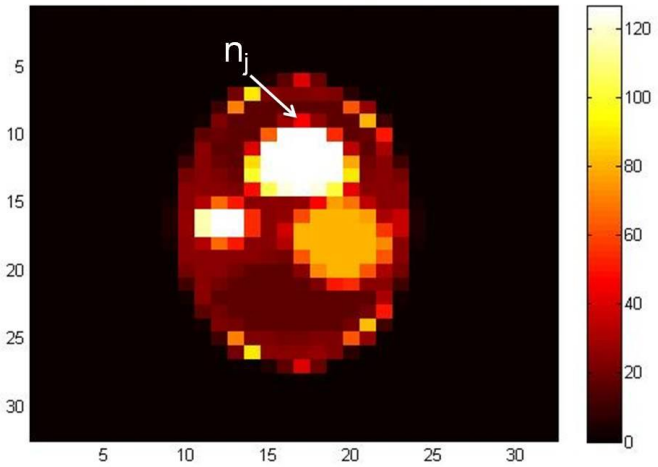


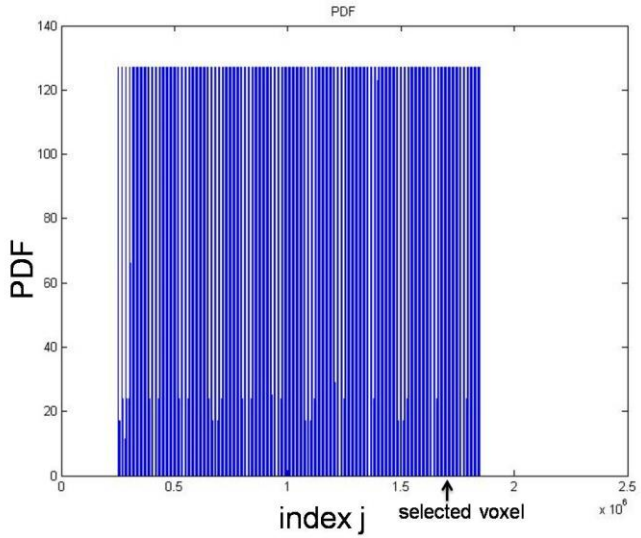
Monte Carlo simulation: part 1: sampling of the source

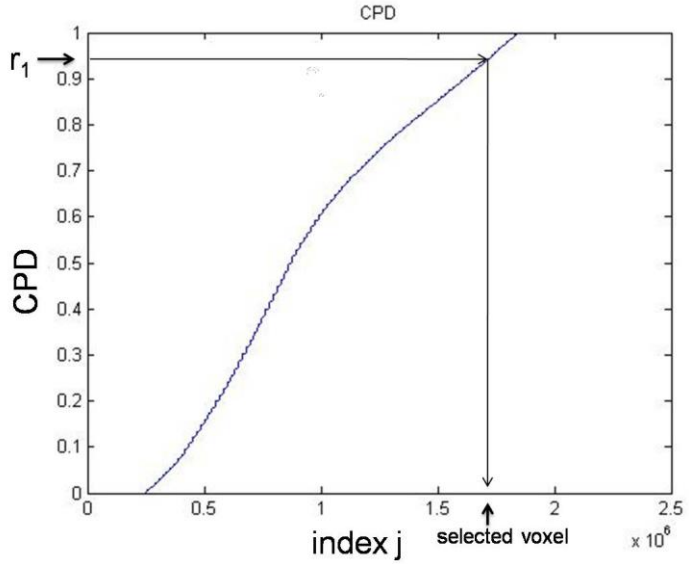
flatten

$$CPD(j) = \sum_{k=1}^{j} n_k$$ + normalize

activity distribution → probability density function (PDF) → cumulative probability density (CPD)







=> A random number r_1 ($0 < r_1 < 1$) can be used to sample the source

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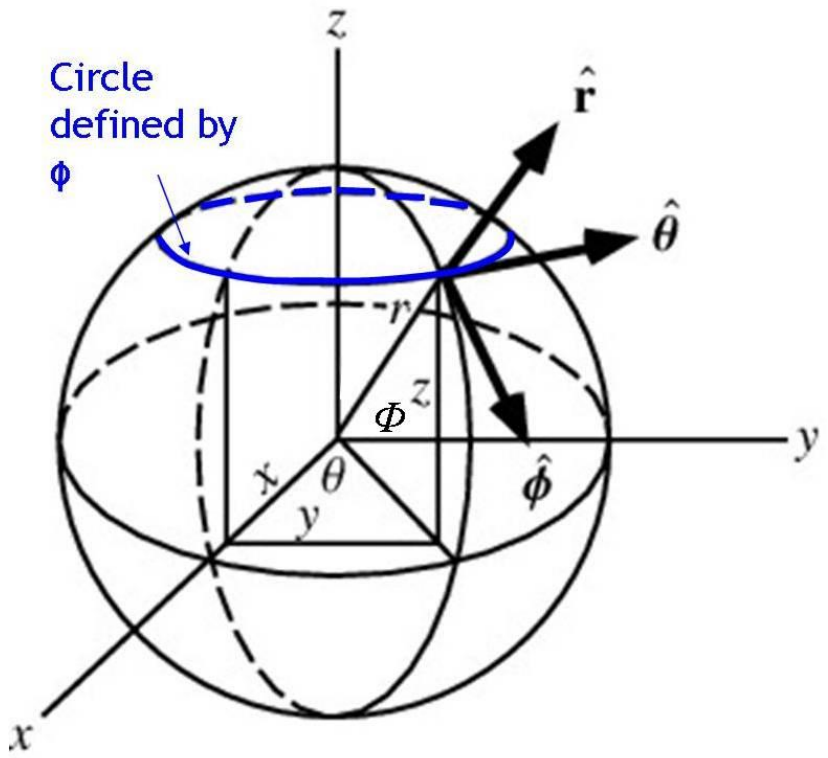
Monte Carlo simulation: part 2: simulating uniform emission



- uniform (constant) PDF for θ:

$$PDF(\theta) = \frac{1}{\theta_{max}}$$

- PDF for φ should be proportional to the circumference of the circle defined by φ:

$$PDF(\phi) = \text{circumference}(\phi) = 2\pi r \cos(\phi)$$

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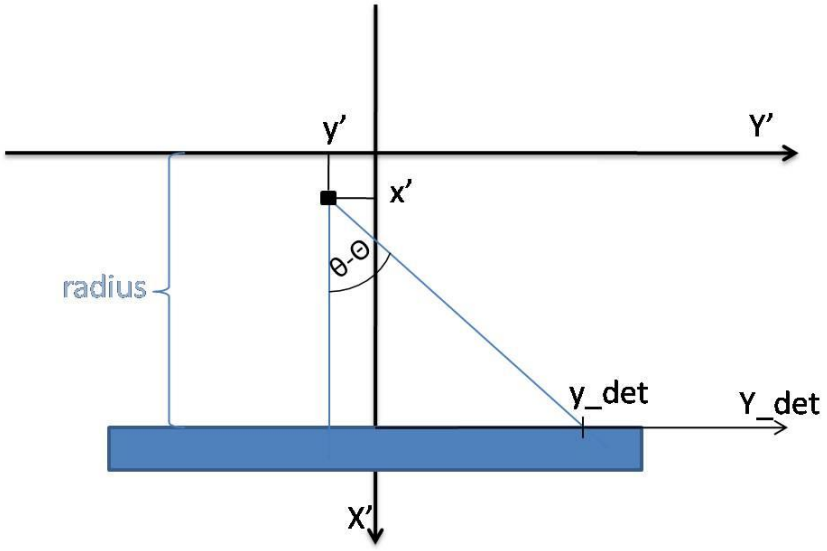
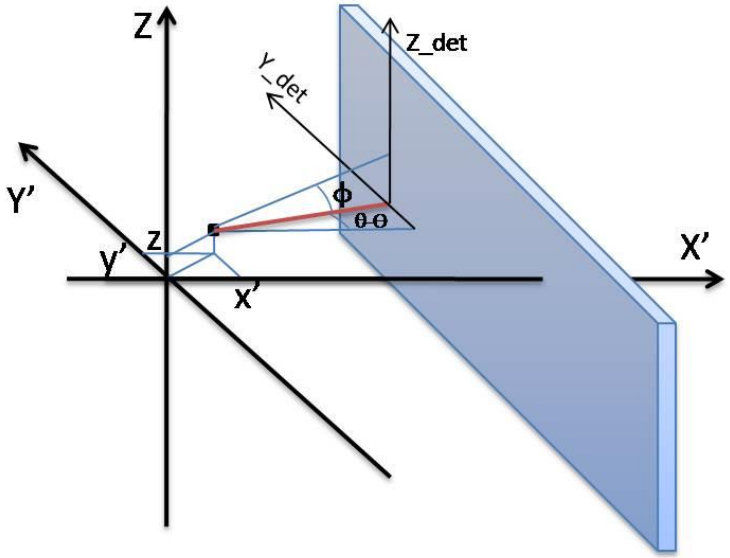
Monte Carlo simulation:
part 3a: intersection with detector and collimator



y_det=y'+ tan (θ-Θ).(radius-x')



z_det =z+((radius-x') tan(ϕ) /cos(θ-Θ))

Detector:

$$y_{det} = (radius - x')\tan(\theta - \Theta) + y'$$

$$z_{det} = (radius - x')\frac{\tan(\phi)}{\cos(\theta - \Theta)} + z$$

with

$$x' = x\cos(\Theta) + y\sin(\Theta)$$

$$y' = y\cos(\Theta) - x\sin(\Theta)$$

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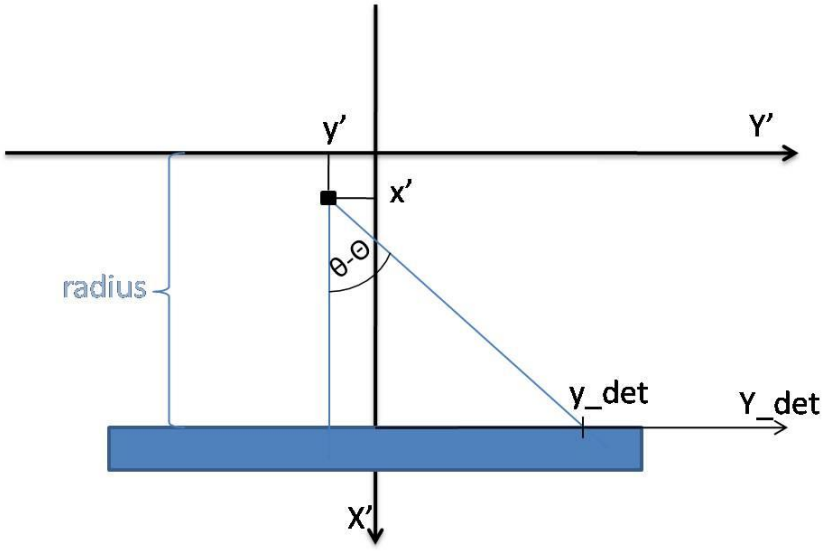
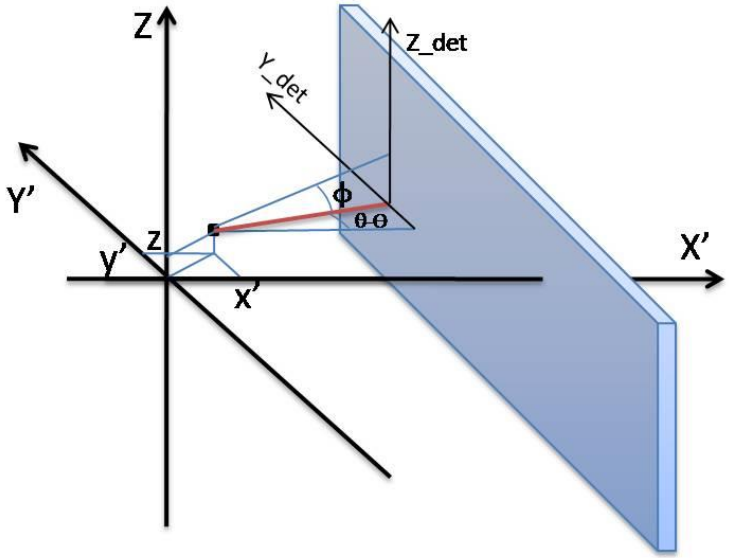
Monte Carlo simulation:
part 3a: intersection with detector and collimator



y_det=y'+ tan (θ-Θ).(radius-x')



z_det =z+((radius-x') tan(ϕ) /cos(θ-Θ))

Collimator:

$$y_{coll} = (radius - height - x')\tan(\theta - \Theta) + y'$$

$$z_{coll} = (radius - height - x')\frac{\tan(\phi)}{\cos(\theta - \Theta)} + z$$

with

$$x' = x\cos(\Theta) + y\sin(\Theta)$$

$$y' = y\cos(\Theta) - x\sin(\Theta)$$

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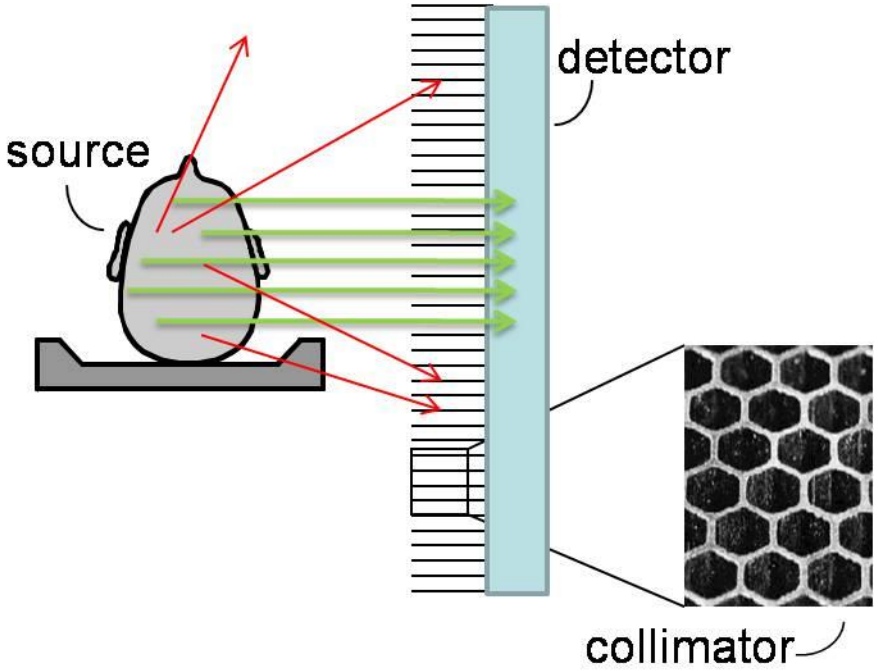
Monte Carlo simulation:
part 3b: Check if the photon passed the collimator



For a parallel collimator:
Simply check if (y_{det}, z_{det}) and (y_{coll}, z_{coll}) lie within the same hole

$$\left\lfloor \frac{y_{det}}{hole\_size} \right\rfloor == \left\lfloor \frac{y_{coll}}{hole\_size} \right\rfloor$$

$$\left\lfloor \frac{z_{det}}{hole\_size} \right\rfloor == \left\lfloor \frac{z_{coll}}{hole\_size} \right\rfloor$$

=> Sinogram can be obtained from the sampled values of y_{det}, z_{det} and Θ_{det}

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Practicum:

- Exercise 1:
  - Sample a given source and see how the obtained distribution changes for different numbers of samples.
- Exercise 2:
  - Sample the emission angles θ and φ and check the impact of this sampling on the unit sphere for different values of θ and φ.
- Exercise 3:
  - Perform a full Monte Carlo simulation for SPECT from start (a voxelized source) to finish (a 3D SPECT sinogram).