Nowcasting US GDP growth rate

Geraci Ludovica

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1 Introduction

The scope of this assignment is to estimate different forecasting model of the growth rate of the US GDP, namely our variable y, through the high-frequency Index of Coincident Indicators (ci) which includes four indicators: nonfarm payroll employment, the unemployment rate, average hours worked in manufacturing and wages and salaries. This index is critical in forecasting US GDP growth because it tracks economic metrics that move in real-time with the overall economy and represents components of economic activity that are directly related to GDP.

The first model we see is the Bridge Model. Here, first of all, monthly indicators are predicted over the remainder of the quarter, on the basis of a univariate time series model, and then aggregated to obtain their quarterly correspondent values. Second, the aggregated values are used as regressors in the bridge equation to obtain forecasts of GDP growth. Next, we proceed to compute the MIDAS Model, which is closely related to the distributed lag (DL) model, but in this case the dependent variable y_t^L, sampled at a lower-frequency, is regressed on a distributed lag of x_t^H, which is sampled at a higher-frequency. Finally, we estimated the U-MIDAS model, which offers greater flexibility by estimating each lag individually, while MIDAS restricts them through lag structures to maintain simplicity and prevent overparameterization. Our findings revealed that the MIDAS model initially performed best with a minimal information set in October. However, as the information set expanded, the U-MIDAS model emerged as more accurate due to its ability to optimally select variable lags via the Gets approach, effectively minimizing forecast error without imposing rigid coefficient restrictions. Conversely, the Bridge and traditional MIDAS models, with their fixed and restrictive weighting methods, proved less flexible and ultimately less effective.

2 Benchmark model through ARMA model

Before estimating our forecasting models, we decide to estimate an ARMA model for the y^L variable, while pretending to be in October, in order to use it as a benchmark to compare our estimates to. For the lag selection, we relied on the Akaike Information Critierion through EViews, which resulted in an ARMA(4,2) Model. We expect the regression to be the least informative among the other forecasting methods. The table below presents the regression coefficients and statistics. The appendix contains the complete table. Figure 1 compares the actual and forecasted values of y^L while pretending to be in October. If we were in November we would obtain the same figure since we would still have only information up to the second quarter, whereas if we were in December we could rely also on information regarding the third quarter.

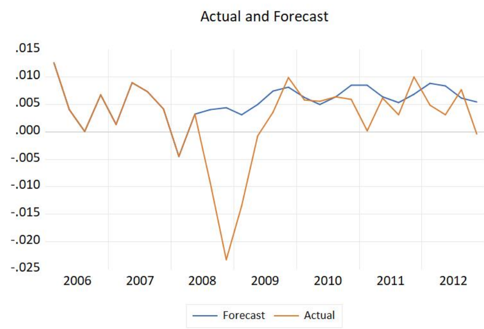


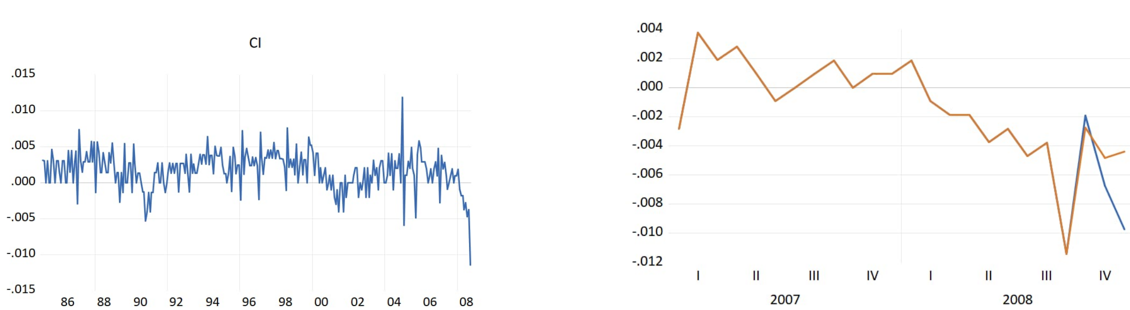
Figure 1: actual y^L vs forecasted y^L pretending to be in October

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007193	0.001100	6.539578	0.0000
AR(1)	0.674126	0.136532	4.937509	0.0000
AR(2)	-0.658878	0.142633	-4.619408	0.0000
AR(3)	-0.107516	0.144692	-0.743072	0.4595
AR(4)	0.404316	0.098789	4.092713	0.0001
MA(1)	-0.532330	68.20145	-0.007805	0.9938
MA(2)	0.999998	256.0542	0.003905	0.9969
SIGMASQ	1.89E-05	0.002418	0.007825	0.9938

Table 1: ARMA(4,2) Model for y^L

3 ARMA specification for the x^H variable

In order to proceed with a Bridge approach, we first need to find the appropriate ARMA model specification for the variable *ci* (our x^H). After looking at how the variable behave in the estimation sample, we can say that it does not show any trend and looks stationary. Indeed, the ADF test confirms the stationarity and the correlogram shows that the correlation coefficient diminishes quickly over time, thus suggesting an ARMA structure (see Appendix 8.2). However, the proper order is unclear and hence we checked that the most frugal specification according to the BIC is an ARMA(1,2) (see Figure 3). After computing some diagnostic tests, we have the confirmation that we can rely on this specification to model our high-frequency variable within the Bridge approach, although there are some signs of skewness that violates the normality assumption as well as heteroskedasticity, probably due to some outliers.



(a) *ci* plot

(b) Actual/Forecast in October

Figure 2: Variable *ci* (left) and prediction with the ARMA specification in October (right)

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Best AR lag: 1 Best MA lag: 2 with BIC: −3087.37
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Figure 3: Best ARMA specification alghoritm

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001365	0.000681	2.006291	0.0458
AR(1)	0.932221	0.041990	22.20128	0.0000
MA(1)	-0.929649	0.061479	-15.12134	0.0000
MA(2)	0.262770	0.052995	4.958352	0.0000
SIGMASQ	4.99E-06	3.22E-07	15.48379	0.0000

Table 2: Estimated ARMA(1, 2) Model in October

4 BM models

4.1 Modeling Monthly Variables

In this section we compute the Bridge Equations of the model. Bridge equations are linear regressions that link (bridge) high-frequency variables, to low-frequency ones, providing some estimates of current and short-term developments in advance of the release. In terms of notation, $t = 1, ..., T$ indices the low-frequency time unit, and m is the number of times the higher sampling frequency appears in the same basic time unit (assumed fixed for simplicity). In our case, for quarterly GDP growth and monthly indicators as explanatory variables, m = 3.

Our Bridge Model will take the form of a static regression:

$$\hat{y}^L_{T+h} = \hat{a}_{T+h} + \hat{b}_T x^L_{T+h} \tag{1}$$

We don't have x^L_{T+h} available, but we have high frequency observations $x^H_{T+h-\frac{i}{m}}$. More specifically, before proceeding we recall that the information we have on the variables depend on the month that we are in, since publication delays make the availability of information different depending on the time. As a consequence, depending on the month we are in we will rely on the following conditions:

- Starting from 2008M10 (October), we only have information about ci up to 2008M9 (September), while about y he have until 2008Q2. As a consequence, the value for x^H from 2008M10 onward will be obtained as a forecast from the ARMA(1,2) model and it will be called $ci1$.
- In 2008M11 (November), we have information up to 2008M10, so the remaining observations of the quarter are replaced by the best forecasts from the selected ARMA models for x, estimated over the 1985 - 2008M10 time span, resulting in the variable $ci2$
- In 2008M12 (December) we have information up to 2008M11, and so in analogous manner as in the previous points we obtain $ci3$. Furthermore, we also have one more quarter (2008Q3) for our variable y^L.

If we want a flash estimate of y^L for 2008Q4, in 2008M10 we can rely on the information in $ci1$, in 2008M11 we can do the same with $ci2$, and similarly for 2008M12. A comparison of the various $ci1, ci2$ and $ci3$ with the actual data can give us a first intuition on the under-performing quality of the forecasts as a replacement of the missing observations (see Table 3) .

Date	**ci**	**ci1**	**ci2**	**ci3**
2008M10	-0.001918	-0.002764	-0.001918	-0.001918
2008M11	-0.006741	-0.004830	-0.004778	-0.006741
2008M12	-0.009709	-0.004410	-0.004143	-0.004277

Table 3: Table of Comparison

Next, we can finally apply the final step of the bridge approach, which only requires to aggregate all these variables for all the months in one quarter. Thanks to this operation, our workfile now contains the quarterly GDP growth rate variable y^L and a set of quarterly variables imported from the monthly series, each of them indicating which information is available when we are respectively in month 1, 2, or 3 of the fourth quarter of 2008, the period we are interested in.

Our work is based on the following theoretical formula for the bridge equation:

$$\hat{y}^L_{T+1|T+1-(m-i)/m} = \hat{a} + \hat{b}_T \left[\sum_{j=1}^{i} a_{m-j} x^H_{T+1-(m-j)/m} \right] + \hat{b}_T \left[\sum_{h=1}^{m-i} a_{m-(i+h)} \hat{\phi}_h \left(L^{1/m} \right) x^H_{T+1-(m-i-1)/m} \right]$$

$$\text{for} \quad i = 1, ..., m-1$$

where $i = 1, 2$. Hence, we can see in mathematical notation the collection of nowcasts, where for consecutive high-frequency observations we replaced the unknown regressor x^L_{T+h} with partial realizations of the high frequency observations and complemented with estimates of the missing one.

Following the equation, we are able to construct $x^L_{2008Q4|2008M10}$, $x^L_{2008Q4|2008M11}$ and $x^L_{2008Q4|2008M12}$.

4.2 Nowcasting

To proceed in our assignment, we estimate the quarterly bridge equation by referring to equation (1), and then using the estimated coefficients to produce flash estimates for y^L in each month of 2008Q4, using $x^L_{2008Q4|2008M10}$, $x^L_{2008Q4|2008M11}$ and $x^L_{2008Q4|2008M12}$. Table 4 shows the estimated coefficients for the model based on the information set available in October, which passes common diagnostic tests.

Dependent Variable: Y
Method: Least Squares
Sample: 1985Q1 2008Q2
Included observations: 94

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004241	0.000595	7.131507	0.0000
CI1*3	0.589610	0.079991	7.370962	0.0000

R-squared	0.371289	Mean dependent var	0.007348
Adjusted R-squared	0.364455	S.D. dependent var	0.005103
S.E. of regression	0.004068	Akaike info criterion	−8.150305
Sum squared resid	0.001522	Schwarz criterion	−8.096193
Log likelihood	385.0644	Hannan-Quinn criter.	−8.128448
F-statistic	54.33108	Durbin-Watson stat	2.448843
Prob(F-statistic)	0.000000		

Table 4: Estimated Bridge Equation with data available in October

To obtain the nowcasts of y^L in 2008Q4 according to the bridge approach, the coefficients are used to construct the flash estimates as:

$$\hat{y}^L_{2008Q4|2008M_j} = \hat{b}_1 + \hat{b}_2 x^L_{2008Q4|2008M_j} \qquad (\text{for } j = 10, 11, 12 \text{ and } x = ci)$$

BRIDGE	y^L	$\hat{y}^L_{2008Q4\|2008M10}$	$\hat{y}^L_{2008Q4\|2008M11}$	$\hat{y}^L_{2008Q4\|2008M12}$
2008Q4	-0.023276	-0.0028	-0.0022	-0.0037

Table 5: Forecasts for y^L in 2008Q4 obtained by the bridge approach

5 MIDAS models

MIDAS regressions are highly parameterized, reduced-form models that incorporate variables sampled at different frequencies. They capture the impact of higher-frequency explanatory variables using compact distributed lag polynomials. This approach helps to avoid an excessive number of parameters and mitigates problems associated with selecting the appropriate lag order.

In order to proceed with the computation of MIDAS models we start by assuming that we are in the first month of 2008Q4, i.e., in October, and we want to produce a flash estimate for y^L in 2008Q4 exploiting all the available information. The associated estimation sample in this case is 1985 - 2008M9. Note also that in October 2008 we only have data for y^L up to 2008Q2, hence the need to adjust the estimation sample accordingly. If we assume instead that we are in November, we have one additional month of indicators, but still only data for y^L up to 2008Q2. In December, we have indicators available up to 2008M11, and the final estimate for y^L is now available for 2008Q3.

A general formula for MIDAS models is:

$$y^L_{t+h} = a_h + b_h C\left(L^{1/m}; \theta_h\right) x^H_t + \varepsilon^L_{t+h}$$

The function $C(\cdot)$ represents the weighting scheme used and $y^L_{t+h} = y^L_{2008Q4|2008Mj}$, $x^H_t = ci_{2008Mj}$ where $j = 9, 10, 11$.

These MIDAS models are all built considering a maximum level of lags of three and the chosen weighting scheme is the exponential Almon Lag, which takes its properties from the smooth polynomial Almon lag Functions that are used to reduce multicollinearity in the distributed lag literature.

$$c(k;\theta) = \frac{\exp(\theta_1 k + \cdots + \theta_Q k^Q)}{\sum_{k=0}^{K} \exp(\theta_1 k + \cdots + \theta_Q k^Q)} \tag{2}$$

Table 6 shows the estimates for the MIDAS parameters obtained with data available in October:

Dependent Variable: Y
Method: MIDAS
Sample (adjusted): 1985Q3 2008Q2
Included observations: 92 after adjustments
Method: Exp-Almon
Optimization method: initial OPG iterations followed by BFGS
Coefficient covariance computed using observed Hessian
Convergence achieved after 21 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.004695	0.000714	6.575085	0.0000
Page: MONTHLY Series: CI(-3) Lags: 3				
SLOPE	1.375427	0.291168	4.723819	0.0000
EXPPDL01	5.080611	2.795438	1.817465	0.0725
EXPPDL02	−1.247333	0.617114	−2.021235	0.0463
R-squared	0.229328	Mean dependent var		0.007314
Adjusted R-squared	0.203055	S.D. dependent var		0.005153
S.E. of regression	0.004600	Akaike info criterion		−7.883062
Sum squared resid	0.001862	Schwarz criterion		−7.773419
Log likelihood	366.6208	Hannan-Quinn criter.		−7.838809
Durbin-Watson stat	2.157154			
MONTHLY\CI(-3)	Lag	Coefficient		
	0	0.228703		
	1	0.872215		
	2	0.274509		

Table 6: MIDAS Model (in October)

In Appendix 8.4 it is possible to see the analogous figures for the months of November and December. Finally, through the different MIDAS models, we are able to compute the outcomes that we report in Table 7:

MIDAS	y^L	$\hat{y}^L_{2008Q4\|2008M10}$	$\hat{y}^L_{2008Q4\|2008M11}$	$\hat{y}^L_{2008Q4\|2008M12}$
2008Q4	-0.023276	-0.007181	-0.008741	-0.011898

Table 7: Nowcast comparison for y^L with MIDAS

6 U-MIDAS

Our last model is U-MIDAS, which is an unconstrained version of the MIDAS. Indeed, unlike MIDAS, this model does not impose restrictions on the lag coefficients. It estimates each coefficient independently, providing more flexibility but potentially increasing the number of parameters and the risk of overfitting, hence running against the idea that high-frequency data parameter proliferation has to be avoided.

However, in this case, we used the Gets (General-to-Specific) approach to identify the most influential lags, thereby avoiding the inclusion of all parameters. The Gets approach is a model selection algorithm that begins by estimating a general model with all possible variables and then progressively reduces the model by removing insignificant variables, ensuring that only the most relevant parameters remain in the final specification.

Our work is based on the following general theoretical formula:

$$y^L_{t+h} = a_h + \lambda_h y^L_t + c^0_h x^H_t + c^1_h x^H_{t-1/m} + c^2_h x^H_{t-2/m} + \cdots + c^{m\tilde{K}}_h x^H_{t-\tilde{K}} + \varepsilon^L_{t+h}, \tag{3}$$

For the estimation, it is reasonable to obtain the regression parameters via OLS. In Table 8 we report the estimates for when we are in October (the other months are illustrated in Appendix 8.5).

Dependent Variable: Y
Method: MIDAS
Sample (adjusted): 1986Q2 2008Q2
Included observations: 89 after adjustments
Method: Auto/Gets
1 MIDAS terms selected

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.005261	0.000652	8.071676	0.0000
Page: MONTHLY Series: CI(-3)				
LAG2	0.959092	0.210790	4.549981	0.0000

R-squared	0.192218	Mean dependent var	0.007194
Adjusted R-squared	0.182933	S.D. dependent var	0.005160
S.E. of regression	0.004664	Akaike info criterion	−7.875751
Sum squared resid	0.001892	Schwarz criterion	−7.819827
Log likelihood	352.4709	Hannan-Quinn criter.	−7.853210
Durbin-Watson stat	1.990649		

Table 8: U-MIDAS with Data available in October

Next, we show the comparison between the actual value assumed by y^L in 2008Q4 and its nowcasts obtained through the U-MIDAS approach while being in October, November and December.

U-MIDAS	y^L	$\hat{y}^L_{2008Q4\|2008M10}$	$\hat{y}^L_{2008Q4\|2008M11}$	$\hat{y}^L_{2008Q4\|2008M12}$
2008Q4	-0.023276	-0.003598	-0.009216	-0.012050

Table 9: Comparison for y^L with UMIDAS

7 Conclusions

Model	y^L	$\hat{y}^L_{2008Q4\|2008M10}$	$\hat{y}^L_{2008Q4\|2008M11}$	$\hat{y}^L_{2008Q4\|2008M12}$
ARMA(4,2)	-0.023276	0.003362	0.003362	0.002862
BRIDGE	-0.023276	-0.002800	-0.002200	-0.003700
MIDAS	-0.023276	-0.007181	-0.008741	-0.011898
U-MIDAS	-0.023276	-0.003598	-0.009216	-0.012050

Table 10: Nowcast and forecast comparison for y^L using ARMA(4,2), BRIDGE, MIDAS and U-MIDAS approaches.

We clearly observe that all the three models that include the high-frequency indicator outperformed the benchmark model ARMA(4,2).

The inclusion of the Index of coincident indicators in the Bridge, MIDAS and U-MIDAS models allows them to leverage more immediate and potentially informative data, enhancing their predictive performance compared to the ARMA(4,2), which relies solely on the historical values of the variable y^L.

While we observe that forecasts improve in every model as the information set becomes larger, with the lightest set of information, i.e., in October, the model that tracks the real value of y^L in 2008Q4 most closely is the MIDAS. However, as soon as the information set becomes larger, the U-MIDAS model emerges as the one with the lowest forecast error. This is because U-MIDAS (in this case) successfully captures the optimal lags of the indicators, using a mechanism like the Gets approach, which ensures that the best path is followed during variable selection, leading to a model specification that minimizes the information criteria. This optimization takes into account both the accuracy of the model fit and the penalty associated with a large number of parameters.
Including too many lags and therefore too many parameters can lead to multicollinearity, which may inflate the variability of the estimates and offset any reduction in bias introduced by other explanatory variables. The U-MIDAS model, however, avoids imposing restrictions on the coefficients to be estimated, resulting in more accurate estimates, despite the potential computational and multicollinearity issues, which in our case are not particularly problematic thanks to the Gets algorithm.

On the other hand, in the Bridge model, the high-frequency observations are weighted by $\hat{b}_t\ a_m$, which are partly based on fixed time-aggregation weights a_m and estimated coefficients $\hat{b}_t$. Thus, the bridge weights are not fully estimated from the data compared to U-MIDAS. Furthermore, the functional polynomials in MIDAS also impose restrictions on the weights (such as the exponential Almon specification used in our case), and the effectiveness of the MIDAS model depends significantly on the appropriateness of the chosen functional form.

Given these considerations, and the fact that our analysis includes only one indicator, the restrictions imposed by both the Bridge and MIDAS models appear to be too rigid. This ultimately favors the model that estimates the coefficients freely and independently, namely U-MIDAS.

It is also worth noting that none of these models was able to capture the depth of the financial crisis in 2008, as all of them predicted a much milder recession than the one that actually occurred. This high-lights the difficulty in quantifying the scale of economic downturns, especially under rapidly evolving conditions, and it marks the fact that models should incorporate financial market signals typically not taken into account in more traditional macro forecasting models.

8 Appendix with graphs and tables

8.1 y^L Analysis

Dependent Variable: Y
Method: ARMA Maximum Likelihood (BFGS)
Date: 10/09/24 Time: 15:33
Sample: 1985Q1 2008Q2
Included observations: 94
Failure to improve objective (non-zero gradients) after 109 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007193	0.001100	6.539578	0.0000
AR(1)	0.674126	0.136532	4.937509	0.0000
AR(2)	-0.658878	0.142633	-4.619408	0.0000
AR(3)	-0.107516	0.144692	-0.743072	0.4595
AR(4)	0.404316	0.098789	4.092713	0.0001
MA(1)	-0.532330	68.20145	-0.007805	0.9938
MA(2)	0.999998	256.0542	0.003905	0.9969
SIGMASQ	$1.89E-05$	0.002418	0.007825	0.9938

R-squared	0.265546	Mean dependent var	0.007348
Adjusted R-squared	0.205765	S.D. dependent var	0.005103
S.E. of regression	0.004548	Akaike info criterion	-7.814435
Sum squared resid	0.001778	Schwarz criterion	-7.597985
Log likelihood	375.2785	Hannan-Quinn criter.	-7.727005
F-statistic	4.441977	Durbin-Watson stat	1.939012
Prob(F-statistic)	0.000296		

Inverted AR Roots	.70	.28+.96i	.28-.96i	-.58
Inverted MA Roots	.27+.96i	.27-.96i		

Table 11: ARMA(4,2) for y^L

8.2 *ci* Analysis

Null Hypothesis: CI has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.529616	0.0002
Test critical values:	1% level	-3.449857	
	5% level	-2.870031	
	10% level	-2.571363	

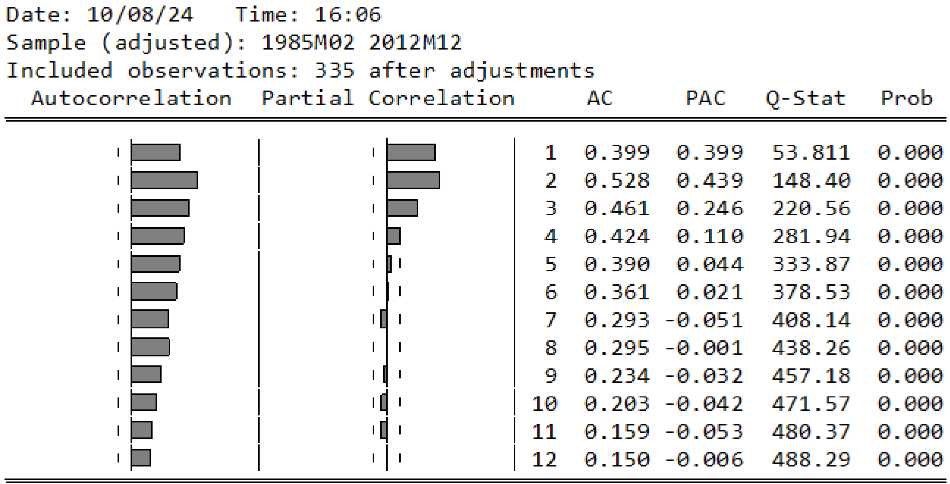
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CI)
Method: Least Squares
Date: 10/08/24 Time: 16:57
Sample (adjusted): 1985M05 2012M12
Included observations: 332 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CI(-1)	-0.251854	0.055602	-4.529616	0.0000
D(CI(-1))	-0.632816	0.063450	-9.973429	0.0000
D(CI(-2))	-0.246168	0.053592	-4.593374	0.0000
C	0.000363	0.000151	2.406795	0.0166

R-squared	0.471100	Mean dependent var	$5.75E-06$
Adjusted R-squared	0.466262	S.D. dependent var	0.003207
S.E. of regression	0.002343	Akaike info criterion	-9.263019
Sum squared resid	0.001800	Schwarz criterion	-9.217174
Log likelihood	1541.661	Hannan-Quinn criter.	-9.244736
F-statistic	97.38488	Durbin-Watson stat	2.050444
Prob(F-statistic)	0.000000		

Table 12: ADF unit-root test on *ci* (with 12 lags)



Date: 10/08/24 Time: 16:06
Sample (adjusted): 1985M02 2012M12
Included observations: 335 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.399	0.399	53.811	0.000
		2	0.528	0.439	148.40	0.000
		3	0.461	0.246	220.56	0.000
		4	0.424	0.110	281.94	0.000
		5	0.390	0.044	333.87	0.000
		6	0.361	0.021	378.53	0.000
		7	0.293	-0.051	408.14	0.000
		8	0.295	-0.001	438.26	0.000
		9	0.234	-0.032	457.18	0.000
		10	0.203	-0.042	471.57	0.000
		11	0.159	-0.053	480.37	0.000
		12	0.150	-0.006	488.29	0.000

Figure 4: Correlogram of ci (with 12 lags)

Dependent Variable: CI
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 1985M02 2008M09
Included observations: 284
Convergence achieved after 20 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001365	0.000681	2.006291	0.0458
AR(1)	0.932221	0.041990	22.20128	0.0000
MA(1)	−0.929649	0.061479	−15.12134	0.0000
MA(2)	0.262770	0.052995	4.958352	0.0000
SIGMASQ	$4.99E-06$	$3.22E-07$	15.48379	0.0000

R-squared	0.249768	Mean dependent var	0.001663
Adjusted R-squared	0.239012	S.D. dependent var	0.002584
S.E. of regression	0.002254	Akaike info criterion	−9.331720
Sum squared resid	0.001417	Schwarz criterion	−9.267478
Log likelihood	1330.104	Hannan-Quinn criter.	−9.305964
F-statistic	23.22127	Durbin-Watson stat	1.952389
Prob(F-statistic)	0.000000		

Inverted AR Roots	.93	
Inverted MA Roots	.46+.22i	.46-.22i

Table 13: ARMA(1,2) of ci estimated in October

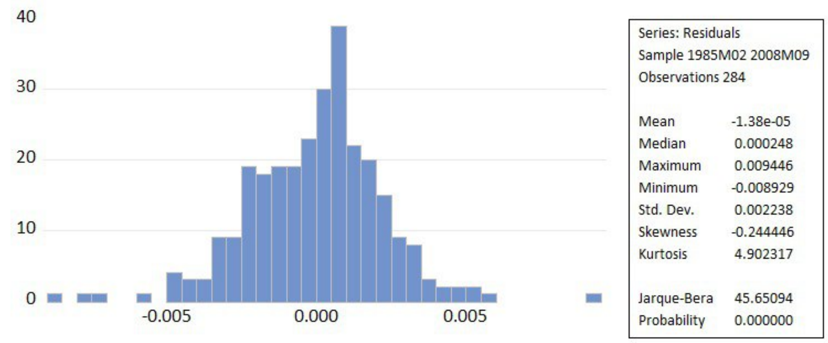


Figure 5: Normality test for ARMA model of *ci*

Heteroskedasticity Test: ARCH

F-statistic	22.26739	Prob. F(1,281)	0.0000
Obs*R-squared	20.77926	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESIDˆ2
Method: Least Squares
Sample (adjusted): 1985M03 2008M09
Included observations: 283 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	$3.57E-06$	$6.44E-07$	5.541981	0.0000
RESIDˆ2(-1)	0.303431	0.064302	4.718834	0.0000

R-squared	0.073425	Mean dependent var	$5.00E-06$
Adjusted R-squared	0.070128	S.D. dependent var	$9.90E-06$
S.E. of regression	$9.54E-06$	Akaike info criterion	-20.27411
Sum squared resid	$2.56E-08$	Schwarz criterion	-20.24834
Log likelihood	2870.786	Hannan-Quinn criter.	-20.26378
F-statistic	22.26739	Durbin-Watson stat	1.763072
Prob(F-statistic)	0.000004		

Table 14: ARCH test for *ci*

8.3 Bridge Model

Dependent Variable: Y
Method: Least Squares
Sample: 1985Q1 2008Q2
Included observations: 94

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004241	0.000595	7.131507	0.0000
CI2*3	0.589610	0.079991	7.370962	0.0000
R-squared	0.371289	Mean dependent var		0.007348
Adjusted R-squared	0.364455	S.D. dependent var		0.005103
S.E. of regression	0.004068	Akaike info criterion		−8.150305
Sum squared resid	0.001522	Schwarz criterion		−8.096193
Log likelihood	385.0644	Hannan-Quinn criter.		−8.128448
F-statistic	54.33108	Durbin-Watson stat		2.448843
Prob(F-statistic)	0.000000			

Table 15: Bridge Model in November

Dependent Variable: Y
Method: Least Squares
Sample: 1985Q1 2008Q3
Included observations: 95

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004151	0.000548	7.574266	0.0000
CI3*3	0.603752	0.071432	8.452074	0.0000
R-squared	0.434436	Mean dependent var		0.007172
Adjusted R-squared	0.428354	S.D. dependent var		0.005356
S.E. of regression	0.004050	Akaike info criterion		−8.159597
Sum squared resid	0.001525	Schwarz criterion		−8.105831
Log likelihood	389.5809	Hannan-Quinn criter.		−8.137872
F-statistic	71.43756	Durbin-Watson stat		2.479536
Prob(F-statistic)	0.000000			

Table 16: Bridge Model in December

8.4 MIDAS Model

Dependent Variable: Y
Method: MIDAS
Sample (adjusted): 1985Q2 2008Q2
Included observations: 93 after adjustments
Method: Exp-Almon
Optimization method: initial OPG iterations followed by BFGS
Coefficient covariance computed using observed Hessian
Convergence achieved after 29 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.004256	0.000654	6.502954	0.0000
Page: MONTHLY Series: CI(-2) Lags: 3				
SLOPE	1.816679	0.270689	6.711308	0.0000
EXPPDL01	−1.760641	1.969434	−0.893983	0.3737
EXPPDL02	0.379129	0.520766	0.728021	0.4685
R-squared	0.348927	Mean dependent var		0.007326
Adjusted R-squared	0.326981	S.D. dependent var		0.005126
S.E. of regression	0.004205	Akaike info criterion		−8.062938
Sum squared resid	0.001574	Schwarz criterion		−7.954009
Log likelihood	378.9266	Hannan-Quinn criter.		−8.018956
Durbin-Watson stat	2.259496			
MONTHLY\CI(-2)	Lag	Coefficient		
	0	0.845009		
	1	0.453090		
	2	0.518580		

Table 17: MIDAS Model in November

Dependent Variable: Y
Method: MIDAS
Sample (adjusted): 1985Q2 2008Q3
Included observations: 94 after adjustments
Method: Exp-Almon
Optimization method: initial OPG iterations followed by BFGS
Coefficient covariance computed using observed Hessian
Convergence achieved after 16 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.003662	0.000612	5.983006	0.0000
Page: MONTHLY Series: CI(-1) Lags: 3				
SLOPE	2.166941	0.259943	8.336201	0.0000
EXPPDL01	2.093250	1.071342	1.953858	0.0538
EXPPDL02	−0.574563	0.276147	−2.080646	0.0403

R-squared	0.447602	Mean dependent var	0.007149
Adjusted R-squared	0.429189	S.D. dependent var	0.005380
S.E. of regression	0.004065	Akaike info criterion	−8.131418
Sum squared resid	0.001487	Schwarz criterion	−8.023192
Log likelihood	386.1766	Hannan-Quinn criter.	−8.087703
Durbin-Watson stat	2.442174		

MONTHLY\CI(-1)	Lag	Coefficient
	0	0.696599
	1	1.008047
	2	0.462295

Table 18: MIDAS model in December

8.5 U-MIDAS Model

Dependent Variable: Y
Method: MIDAS
Sample (adjusted): 1986Q1 2008Q2
Included observations: 90 after adjustments
Method: Auto/Gets
3 MIDAS terms selected

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.004086	0.000646	6.328801	0.0000
Page: MONTHLY Series: CI(-2)				
LAG1	0.891805	0.198396	4.495077	0.0000
LAG2	0.505107	0.190835	2.646824	0.0097
LAG3	0.462970	0.214222	2.161167	0.0335

R-squared	0.371665	Mean dependent var	0.007220
Adjusted R-squared	0.349746	S.D. dependent var	0.005137
S.E. of regression	0.004142	Akaike info criterion	−8.091831
Sum squared resid	0.001475	Schwarz criterion	−7.980729
Log likelihood	368.1324	Hannan-Quinn criter.	−8.047028
Durbin-Watson stat	2.285893		

Table 19: U-MIDAS Model in November

Dependent Variable: Y
Method: MIDAS
Date: 10/08/24 Time: 12:37
Sample (adjusted): 1986Q1 2008Q3
Included observations: 91 after adjustments
Method: Auto/Gets
3 MIDAS terms selected

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.003542	0.000606	5.847621	0.0000
Page: MONTHLY Series: CI(-1)				
LAG1	0.631506	0.182526	3.459818	0.0008
LAG2	1.039732	0.174441	5.960349	0.0000
LAG3	0.508563	0.179194	2.838059	0.0056

R-squared	0.463065	Mean dependent var	0.007038
Adjusted R-squared	0.444550	S.D. dependent var	0.005394
S.E. of regression	0.004020	Akaike info criterion	−8.151938
Sum squared resid	0.001406	Schwarz criterion	−8.041570
Log likelihood	374.9132	Hannan-Quinn criter.	−8.107411
Durbin-Watson stat	2.464888		

Table 20: U-MIDAS Model in December