Schiempflug Camera Calibration using Pin-hole Model

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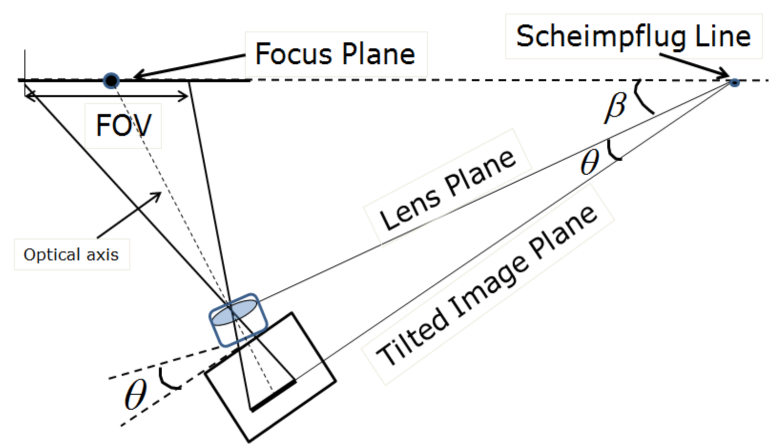

Figure 1: Scheimpflug System Set-up

Abstract

For any cameras to be in perfect focus, an object must be placed in a plane surface parallel to the image plane. However, having an image at sharp focus can be very difficult in the case

when the object of interest is located in an oblique plane to the camera (i.e object plane not parallel to the image plane). It is the case when one is interested in capturing a very large object such as the facade of a tall building, and a decorative cobblestone pathway. The camera can be aimed upward at an angle which makes it difficult to bring the complete object of interest in focus. Therefore, for a certain oblique plane to be in focus, the imaging set-up must obey "Scheimpflug's principle" (**?**). This principle calls for three planes to intersect in a common line:

- The oblique plane containing the observed objects,
- The image plane,
- The lens plane.

Scheimpflug principle as described in figure 1 must obey thin lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$, where f is the focal length, d_o and d_i are object and image distance with respect to the optical centre respectively. When the object of interest is located at an angle β to the lens plane, we tilt the lens plane at an angle θ so that the focus plane is brought to the object location.

For classical pin-hole model, the image plane is usually parallel to the lens plane $(O_c, [X_c, Y_c, Z_c])_c$ as shown in figure 2. However, for scheimpflug set-up, the image plane is tilted at a given angle θ around the axis parallel to the X_C axis but in O_I (see figure 2). This adjustment ensures that the image plane, the lens plane and the object plane intersect at a line parallel to X_c axis, denoted as "scheimpflug line".

We distinguish the following coordinate systems:

- The world coordinate system $(O_w[X_w, Y_w, Z_w])_w$,
- The camera/lens coordinate system $(O_c[X_c, Y_c, Z_c])_c$,

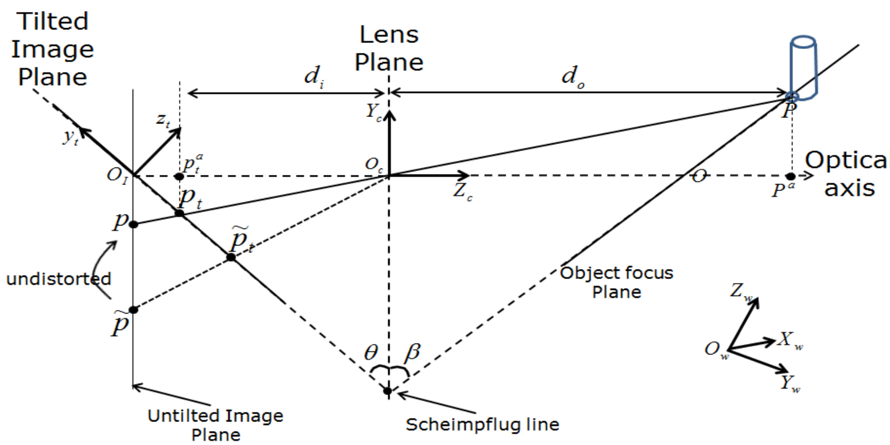

Figure 2: Scheimpflug camera model illustration.

- The untilted image coordinate system,$(O_I[x,y,z])_I$, where $z = 0$ for each point on the untilted image plane,
- The tilted image coordinate system $(O_I[x_t,y_t,z_t])_t$, where $z_t = 0$ for each point on the tilted image plane,
- The untilted pixel coordinate system $(O_p[u,v])_p$,
- The tilted pixel coordinate system, $(O_p[u_t,v_t])_p$

For a point P along the object focus plane (see figure 2), lets denote $d_i = \|\vec{O_c p_t^a}\|$ and $d_o = \|\vec{O_c P^a}\|$ the image and object distance from the centre of the lens along the optical axis, where p_t^a and P^a are the projections of p_t and P to the optical axis. The thin lens equations still hold for schiempflug set-up so that $\frac{1}{f} = \frac{1}{do} + \frac{1}{di}$. This is possible for any projected points on the tilted image plane from the object focus plane.

In order to determine the 3D coordinates of world point P from its projected point p_t in the tilted image plane, we need to calibrate the used camera. Therefore, we have proposed a schiempflug calibration method based on pin-hole model assumptions.

0.1 Image formation for single scheimpflug angle

Using homogeneous coordinate representation assuming no optical distortions, an object point $P(X_w, Y_w, Z_w, 1)_w$ is first projected to the point $p(x, y, 0, 1)_I$ in the untilted image plane as

$$\lambda\, p = \mathcal{K}_f \left[\mathcal{R} \mid \mathcal{T}\right] P \tag{1}$$

where intrinsic parameter $\mathcal{K}_f = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and extrinsic $\mathcal{R}$, and $\mathcal{T}$ [I].

In order to project the point P onto the untilted image plane, we first need to simplify the optical distortion. If we consider radial distortions only, we can express the coordinates of the undistorted point $p(x, y, 0)_I$ from the coordinates of the distorted one $\tilde{p}(\tilde{x}, \tilde{y}, 0)_I$ in the same untilted plane as,

$$\begin{aligned} \tilde{p}(\tilde{x}, \tilde{y}, 0)_I^T &= p(x, y, 0)_I^T + \delta_{(x,y,0)} \\ where\ \delta_{(x,y,0)} &= (\delta_x, \delta_y, 0) \end{aligned} \tag{2}$$

Now, let us model image formation on the tilted image plane. Assuming optical distortion, a ray originating from the the optical center O_c of the camera intersects the tilted plane at $\tilde{p}_t(\tilde{x}_t, \tilde{y}_t, 0)_t$ and the untilted at $\tilde{p}(\tilde{x}, \tilde{y}, 0)_I$ respectively. Without abuse of notations, we represent a distorted point $\tilde{p}_t(\tilde{x}_t, \tilde{y}_t, 0)_t$ in the tilted plane as $\tilde{p}_c^t(\tilde{X}_c, \tilde{Y}_c, \tilde{Z}_c)_c$ in the camera coordinate system, likewise point $\tilde{p}(\tilde{x}, \tilde{y}, 0)_I$ in the untilted image plane as $\tilde{p}_c(\tilde{X}_c, \tilde{Y}_c, \tilde{Z}_c = f)_c$. Therefore, the camera coordinate points $\tilde{p}_c(\tilde{X}_c, \tilde{Y}_c, \tilde{Z}_c = f)_c$ can be related to the untilted coordinate points $\tilde{p}(\tilde{x}, \tilde{y}, 0)_I$ as,

$$\begin{aligned} \tilde{X}_c &= \tilde{x} \\ \tilde{Y}_c &= \tilde{y} \\ \tilde{Z}_c &= f \end{aligned} \tag{3}$$

We have formulate a projective transformation $\mathcal{K}_c^t$ under scheimpflug model. We first applied this matrix $\mathcal{K}_c^t$ of the distorted point $\tilde{p}_c(\tilde{X}_c, \tilde{Y}_c, f)_c$ to $\tilde{p}_c^t(\tilde{X}_c, \tilde{Y}_c, \tilde{Z}_c)_c$ as shown below:

$$\tilde{p}_c^t = \mathcal{K}_c^t \tilde{p}_c, \text{ where } \mathcal{K}_c^t = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \\ 0 & -tan\,\theta & 1 \end{bmatrix} \tag{4}$$

Then we express the camera coordinates of the point $\tilde{p}_c^t(\tilde{X}_c, \tilde{Y}_c, \tilde{Z}_c)_c$ in the tilted image plane (i.e to have $\tilde{p}_t(\tilde{x}_t, \tilde{y}_t, 0)_t$). This is done by applying rotation $\mathcal{R}_s$ with respect to the X-axis parametrised by angle θ, preceded by a translation vector $\mathcal{T}_s$ (from O_c to O_I). Finally we have our $\mathcal{K}_s$ matrix as below,

$$\begin{aligned} \mathcal{K}_s = \begin{bmatrix} \mathcal{R}_s & \mathcal{T}_s \\ 0^T & 1 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\,\theta & sin\,\theta & 0 \\ 0 & -sin\,\theta & cos\,\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \mathcal{K}_s &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\,\theta & sin\,\theta & -f\,sin\,\theta \\ 0 & -sin\,\theta & cos\,\theta & -f\,cos\,\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{5}$$

Note that the two steps described earlier complete the scheimpflug image formation (”Intrinsic formation”). Hence, the point $\tilde{p}_t(\tilde{x}_t, \tilde{y}_t, 0)_t$ on the tilted image plane is deduced from $\tilde{p}_c(\tilde{X}_c, \tilde{Y}_c, f)_c$ in the camera coordinate system as below

$$\lambda \tilde{p}_t = \mathcal{K}_s \mathcal{K}_c^t \tilde{p}_c \tag{6}$$

A final conversion in pixel coordinates is effected with $\mathcal{K}_b$ matrix where s_x, s_y, s_z are the pixel sizes and (u_0, v_0) are the principal point coordinates in pixel unit. The pixel size s_z can be changed to one since it will finally be cancelled out in the equation later on.

$$\mathcal{K}_b = \begin{bmatrix} 1/s_x & 0 & 0 & u_0 \\ 0 & 1/s_y & 0 & v_0 \\ 0 & 0 & 1/s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Here is the full equation that that relates points $\tilde{p_c}(\tilde{X_c}, \tilde{Y_c}, f)_c$ to $\tilde{p_t}(\tilde{u_t}, \tilde{v_t})_p$ in pixel coordinates:

$$\lambda \tilde{p_t} = \mathcal{K}_b \mathcal{K}_s \mathcal{K}_c^t \tilde{p_c} \tag{8}$$

$$\lambda \begin{bmatrix} \tilde{u_t} \\ \tilde{v_t} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/s_x & 0 & 0 & u_0 \\ 0 & 1/s_y & 0 & v_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\ \theta & sin\ \theta & -f\ sin\ \theta \\ 0 & -sin\ \theta & cos\ \theta & -f\ cos\ \theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \\ 0 & -tan\ \theta & 1 \end{bmatrix} \begin{bmatrix} \tilde{X_c} \\ \tilde{Y_c} \\ f \end{bmatrix} \tag{9}$$

Finally, to obtain $p_t(u_t, v_t)_p$ from $P(X_c, Y_c, Z_c)_c$ under pin-hole model assumption, we have to apply successively equation 1, 2 and 9. Therefore, the full scheimpflug pin-hole model is shown thus,

$$\begin{aligned} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} &= \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \\ \lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} &= \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} \\ \lambda \begin{bmatrix} u_t \\ v_t \\ 1 \end{bmatrix} &= \begin{bmatrix} 1/s_x & 0 & 0 & u_0 \\ 0 & 1/s_y & 0 & v_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\ \theta & sin\ \theta & -f\ sin\ \theta \\ 0 & -sin\ \theta & cos\ \theta & -f\ cos\ \theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \\ 0 & -tan\ \theta & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} \end{aligned} \tag{10}$$

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References

1. Zhengyou Zhang, *A Flexible New Technique for Camera Calibration* (IEEE Trans. Pattern Anal. Mach. Intell., Pages 1330–1334, November, 2000).