Logistics Project Report

Ludovico Lemma

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1 Description of the problem

A U.S. based company has to organize a distribution network. There is a set of operating terminals in Altus, Ardmore, Bartlesville, Duncan, Edmond, Enid, Lawton, Muskogee, Oklahoma City, Ponca City, Stillwater and Tulsa.
The managers decided to open two hubs in the cities with the already operating terminals. They face the problem of assigning each terminal to one of the two hubs by minimizing the transportation cost between the terminal, the hub and vice versa.
The transportation cost is 0,74 dollars per mile, so the overall cost of a terminal-hub connection can be defined by the expression:

$$c_{ij} = 2 \times 0.74 \times l_{ij}$$

i and j being the sets of terminals and hubs
l_{ij} being the distance between the terminal and the hub

The distances between the Terminals (and as a consequence the candidate location of the hubs) are known and they can be seen in Table 1.

	Altus	Ard.	Bartle.	Dunc.	Edmo.	Enid	Lawton	Musk.	Okla.	Ponca	Still.	Tulsa
Altus	0	169,8	291,8	88,2	153,9	208,2	54,2	274,2	141,1	245	209,2	248
Ard.	169,8	0	248,6	75,9	112,5	199	115,8	230,4	100,5	202,2	162,6	204,6
Bartle.	291,8	248,6	0	231,5	146	132,4	238,7	92,2	151,4	70,2	115	45,6
Dunc.	88,2	75,9	231,5	0	93,5	137,5	34,1	213,5	80,9	184,8	145,3	187,8
Edmo.	153,9	112,5	146	93,5	0	88,8	100,7	145,7	14,4	91,9	53	102,2
Enid	208,2	199	132,4	137,5	88,8	0	145	166,4	87,6	64,5	65,8	118,4
Lawton	54,2	115,8	238,7	34,1	100,7	145	0	220,6	88	191,9	152,5	194,9
Musk.	274,2	230,4	92,2	213,5	145,7	166,4	220,6	0	140,4	142,5	119,2	48,1
Okla.	141,1	100,5	151,4	80,9	14,4	87,6	88	140,4	0	104,7	66,6	107,6
Ponca	245	202,2	70,2	184,8	91,9	64,5	191,9	142,5	104,7	0	41,9	96,5
Still.	209,2	162,6	115	145,3	53	65,8	152,5	119,2	66,6	41,9	0	71,2
Tulsa	248	204,6	45,6	187,8	102,2	118,4	194,9	48,1	107,6	96,5	71,2	0

Table 1: Distanes (in miles) between the Terminals

The managers also wanted to compare a case in which it is minimized the maximum cost of a terminal-hub connection instead of minimizing the total costs.

2 Mathematical Formulation

The two proposed formulations require to minimize two different costs, both of them, though, are expressed here as linear functions of the distance. Therefore the two problems require in reality to minimize the total distance and the maximum distance, which in turn will minimize their respective costs by multiplying the resulting optimized distances by (2×0.74).

I'll provide in the following two subsections the most suitable models to do the computation, but as a first step, here I present the notation of the input data:

- I: Set of Terminals
- J: Set of Hubs
- $p = 2$: Number of facilities to locate
- l_{ij} $\forall i \in I$, $\forall j \in J$: Distances between Terminals and Hubs
- h_i $\forall i \in I$: Demand of each Terminal (I assume it is 1 as it is not explicitly stated, so I will treat all nodes equally, implementing an unweighted variation of the models I will present)

It is also important to consider that both problems state the equality of the two sets of Terminals and Hubs:

- $I = J = \{$Altus, Ardmore, Bartlesville, Duncan, Edmond, Enid, Lawton, Muskogee, Oklahoma City, Ponca City, Stillwater, Tulsa$\}$

Finally, we also need to declare two families of decision variables: x_j as a variable to determine if a hub is located in j, y_{ij} as a variable to determine if i is assigned to j.

2.1 Minimizing the Total Cost (ILP Model)

As said, this can be reduced into a problem which requires to minimize the total distance, therefore, this optimization problem can be defined as an instance of the general p-median problem (Hakimi 1964-1965), hence it can be summarized in the following way:

- Decision Variable $x_j = \begin{cases} 1 \text{ if we decide to locate a hub in } j \\ 0 \text{ otherwise} \end{cases} \quad \forall j \in J$
- Decision Variable $y_{ij} = \begin{cases} 1 \text{ if terminal } i \text{ is assigned to hub } j \\ 0 \text{ otherwise} \end{cases} \quad \forall i \in I, \forall j \in J$
- Budget Constraint (to set the sum of chosen location x_j to the given p):

$$\sum_{j \in J} x_j = 2$$

- Allocation Constraint (to assign all i terminal to only 1 hub as stated in the problem):

$$\sum_{j \in J} y_{ij} = 1, \; \forall i \in I$$

- Linking Constraint (to create a terminal-hub connection only if a hub exist in j):

$$y_{ij} \leq x_j, \; \forall i \in I, \; \forall j \in J$$

- Objective Function (I assumed the demand of each terminal is 1, hence h_j doesn't appear):

$$(\min \sum_{i \in I} \sum_{j \in J} y_{ij} \times l_{ij}) \times (2 \times 0,74)$$

The explicitly extended constraints and objective function are indicate hereunto (the name of the cities is reduced to 2 letters):

- Budget Constraint:

$$x_{Al} + x_{Ar} + x_{Ba} + x_{Du} + x_{Ed} + x_{En} + x_{La} + x_{Mu} + x_{Ok} + x_{Po} + x_{St} + x_{Tu} = 2$$

- Allocation Constraints:

$$y_{AlAl} + y_{ArAl} + y_{BaAl} + y_{DuAl} + y_{EdAl} + y_{EnAl} + y_{LaAl} + y_{MuAl} + y_{OkAl} + y_{PoAl} + y_{StAl} + y_{TuAl} = 1$$
$$y_{AlAr} + y_{ArAr} + y_{BaAr} + y_{DuAr} + y_{EdAr} + y_{EnAr} + y_{LaAr} + y_{MuAr} + y_{OkAr} + y_{PoAr} + y_{StAr} + y_{TuAr} = 1$$
$$y_{AlBa} + y_{ArBa} + y_{BaBa} + y_{DuBa} + y_{EdBa} + y_{EnBa} + y_{LaBa} + y_{MuBa} + y_{OkBa} + y_{PoBa} + y_{StBa} + y_{TuBa} = 1$$
$$y_{AlDu} + y_{ArDu} + y_{BaDu} + y_{DuDu} + y_{EdDu} + y_{EnDu} + y_{LaDu} + y_{MuDu} + y_{OkDu} + y_{PoDu} + y_{StDu} + y_{TuDu} = 1$$
$$y_{AlEd} + y_{ArEd} + y_{BaEd} + y_{DuEd} + y_{EdEd} + y_{EnEd} + y_{LaEd} + y_{MuEd} + y_{OkEd} + y_{PoEd} + y_{StEd} + y_{TuEd} = 1$$
$$y_{AlEn} + y_{ArEn} + y_{BaEn} + y_{DuEn} + y_{EdEn} + y_{EnEn} + y_{LaEn} + y_{MuEn} + y_{OkEn} + y_{PoEn} + y_{StEn} + y_{TuEn} = 1$$
$$y_{AlLa} + y_{ArLa} + y_{BaLa} + y_{DuLa} + y_{EdLa} + y_{EnLa} + y_{LaLa} + y_{MuLa} + y_{OkLa} + y_{PoLa} + y_{StLa} + y_{TuLa} = 1$$
$$y_{AlMu} + y_{ArMu} + y_{BaMu} + y_{DuMu} + y_{EdMu} + y_{EnMu} + y_{LaMu} + y_{MuMu} + y_{OkMu} + y_{PoMu} + y_{StMu} + y_{TuMu} = 1$$
$$y_{AlOk} + y_{ArOk} + y_{BaOk} + y_{DuOk} + y_{EdOk} + y_{EnOk} + y_{LaOk} + y_{MuOk} + y_{OkOk} + y_{PoOk} + y_{StOk} + y_{TuOk} = 1$$
$$y_{AlPo} + y_{ArPo} + y_{BaPo} + y_{DuPo} + y_{EdPo} + y_{EnPo} + y_{LaPo} + y_{MuPo} + y_{OkPo} + y_{PoPo} + y_{StPo} + y_{TuPo} = 1$$
$$y_{AlSt} + y_{ArSt} + y_{BaSt} + y_{DuSt} + y_{EdSt} + y_{EnSt} + y_{LaSt} + y_{MuSt} + y_{OkSt} + y_{PoSt} + y_{StSt} + y_{TuSt} = 1$$
$$y_{AlTu} + y_{ArTu} + y_{BaTu} + y_{DuTu} + y_{EdTu} + y_{EnTu} + y_{LaTu} + y_{MuTu} + y_{OkTu} + y_{PoTu} + y_{StTu} + y_{TuTu} = 1$$

- Linking Constraints:

$y_{AlAl} \leq x_{Al}$	$y_{ArAl} \leq x_{Al}$	$y_{BaAl} \leq x_{Al}$	$y_{DuAl} \leq x_{Al}$	$y_{EdAl} \leq x_{Al}$	$y_{EnAl} \leq x_{Al}$
$y_{LaAl} \leq x_{Al}$	$y_{MuAl} \leq x_{Al}$	$y_{OkAl} \leq x_{Al}$	$y_{PoAl} \leq x_{Al}$	$y_{StAl} \leq x_{Al}$	$y_{TuAl} \leq x_{Al}$
$y_{AlAr} \leq x_{Ar}$	$y_{ArAr} \leq x_{Ar}$	$y_{BaAr} \leq x_{Ar}$	$y_{DuAr} \leq x_{Ar}$	$y_{EdAr} \leq x_{Ar}$	$y_{EnAr} \leq x_{Ar}$
$y_{LaAr} \leq x_{Ar}$	$y_{MuAr} \leq x_{Ar}$	$y_{OkAr} \leq x_{Ar}$	$y_{PoAr} \leq x_{Ar}$	$y_{StAr} \leq x_{Ar}$	$y_{TuAr} \leq x_{Ar}$
$y_{AlBa} \leq x_{Ba}$	$y_{ArBa} \leq x_{Ba}$	$y_{BaBa} \leq x_{Ba}$	$y_{DuBa} \leq x_{Ba}$	$y_{EdBa} \leq x_{Ba}$	$y_{EnBa} \leq x_{Ba}$
$y_{LaBa} \leq x_{Ba}$	$y_{MuBa} \leq x_{Ba}$	$y_{OkBa} \leq x_{Ba}$	$y_{PoBa} \leq x_{Ba}$	$y_{StBa} \leq x_{Ba}$	$y_{TuBa} \leq x_{Ba}$
$y_{AlDu} \leq x_{Du}$	$y_{ArDu} \leq x_{Du}$	$y_{BaDu} \leq x_{Du}$	$y_{DuDu} \leq x_{Du}$	$y_{EdDu} \leq x_{Du}$	$y_{EnDu} \leq x_{Du}$
$y_{LaDu} \leq x_{Du}$	$y_{MuDu} \leq x_{Du}$	$y_{OkDu} \leq x_{Du}$	$y_{PoDu} \leq x_{Du}$	$y_{StDu} \leq x_{Du}$	$y_{TuDu} \leq x_{Du}$
$y_{AlEd} \leq x_{Ed}$	$y_{ArEd} \leq x_{Ed}$	$y_{BaEd} \leq x_{Ed}$	$y_{DuEd} \leq x_{Ed}$	$y_{EdEd} \leq x_{Ed}$	$y_{EnEd} \leq x_{Ed}$
$y_{LaEd} \leq x_{Ed}$	$y_{MuEd} \leq x_{Ed}$	$y_{OkEd} \leq x_{Ed}$	$y_{PoEd} \leq x_{Ed}$	$y_{StEd} \leq x_{Ed}$	$y_{TuEd} \leq x_{Ed}$
$y_{AlEn} \leq x_{En}$	$y_{ArEn} \leq x_{En}$	$y_{BaEn} \leq x_{En}$	$y_{DuEn} \leq x_{En}$	$y_{EdEn} \leq x_{En}$	$y_{EnEn} \leq x_{En}$
$y_{LaEn} \leq x_{En}$	$y_{MuEn} \leq x_{En}$	$y_{OkEn} \leq x_{En}$	$y_{PoEn} \leq x_{En}$	$y_{StEn} \leq x_{En}$	$y_{TuEn} \leq x_{En}$
$y_{AlLa} \leq x_{La}$	$y_{ArLa} \leq x_{La}$	$y_{BaLa} \leq x_{La}$	$y_{DuLa} \leq x_{La}$	$y_{EdLa} \leq x_{La}$	$y_{EnLa} \leq x_{La}$
$y_{LaLa} \leq x_{La}$	$y_{MuLa} \leq x_{La}$	$y_{OkLa} \leq x_{La}$	$y_{PoLa} \leq x_{La}$	$y_{StLa} \leq x_{La}$	$y_{TuLa} \leq x_{La}$
$y_{AlMu} \leq x_{Mu}$	$y_{ArMu} \leq x_{Mu}$	$y_{BaMu} \leq x_{Mu}$	$y_{DuMu} \leq x_{Mu}$	$y_{EdMu} \leq x_{Mu}$	$y_{EnMu} \leq x_{Mu}$
$y_{LaMu} \leq x_{Mu}$	$y_{MuMu} \leq x_{Mu}$	$y_{OkMu} \leq x_{Mu}$	$y_{PoMu} \leq x_{Mu}$	$y_{StMu} \leq x_{Mu}$	$y_{TuMu} \leq x_{Mu}$
$y_{AlOk} \leq x_{Ok}$	$y_{ArOk} \leq x_{Ok}$	$y_{BaOk} \leq x_{Ok}$	$y_{DuOk} \leq x_{Ok}$	$y_{EdOk} \leq x_{Ok}$	$y_{EnOk} \leq x_{Ok}$
$y_{LaOk} \leq x_{Ok}$	$y_{MuOk} \leq x_{Ok}$	$y_{OkOk} \leq x_{Ok}$	$y_{PoOk} \leq x_{Ok}$	$y_{StOk} \leq x_{Ok}$	$y_{TuOk} \leq x_{Ok}$
$y_{AlPo} \leq x_{Po}$	$y_{ArPo} \leq x_{Po}$	$y_{BaPo} \leq x_{Po}$	$y_{DuPo} \leq x_{Po}$	$y_{EdPo} \leq x_{Po}$	$y_{EnPo} \leq x_{Po}$
$y_{LaPo} \leq x_{Po}$	$y_{MuPo} \leq x_{Po}$	$y_{OkPo} \leq x_{Po}$	$y_{PoPo} \leq x_{Po}$	$y_{StPo} \leq x_{Po}$	$y_{TuPo} \leq x_{Po}$
$y_{AlSt} \leq x_{St}$	$y_{ArSt} \leq x_{St}$	$y_{BaSt} \leq x_{St}$	$y_{DuSt} \leq x_{St}$	$y_{EdSt} \leq x_{St}$	$y_{EnSt} \leq x_{St}$
$y_{LaSt} \leq x_{St}$	$y_{MuSt} \leq x_{St}$	$y_{OkSt} \leq x_{St}$	$y_{PoSt} \leq x_{St}$	$y_{StSt} \leq x_{St}$	$y_{TuSt} \leq x_{St}$
$y_{AlTu} \leq x_{Tu}$	$y_{ArTu} \leq x_{Tu}$	$y_{BaTu} \leq x_{Tu}$	$y_{DuTu} \leq x_{Tu}$	$y_{EdTu} \leq x_{Tu}$	$y_{EnTu} \leq x_{Tu}$
$y_{LaTu} \leq x_{Tu}$	$y_{MuTu} \leq x_{Tu}$	$y_{OkTu} \leq x_{Tu}$	$y_{PoTu} \leq x_{Tu}$	$y_{StTu} \leq x_{Tu}$	$y_{TuTu} \leq x_{Tu}$

Table 2: Linking constraints

- Objective Function:

$$
\begin{array}{llllllllll}
\min (& 0y_{AlAl} & + & 169,8y_{ArAl} & + & 291,8y_{BaAl} & + & 88,2y_{DuAl} & + & \\
 & 153,9y_{EdAl} & + & 208,2y_{EnAl} & + & 54,2y_{LaAl} & + & 274,2y_{MuAl} & + & \\
 & 141,1y_{OkAl} & + & 245y_{PoAl} & + & 209,2y_{StAl} & + & 248y_{TuAl} & + & \\
 & 169,8y_{AlAr} & + & 0y_{ArAr} & + & 248,6y_{BaAr} & + & 75,9y_{DuAr} & + & \\
 & 112,5y_{EdAr} & + & 199y_{EnAr} & + & 115,8y_{LaAr} & + & 230,4y_{MuAr} & + & \\
 & 100,5y_{OkAr} & + & 202,2y_{PoAr} & + & 162,6y_{StAr} & + & 204,6y_{TuAr} & + & \\
 & 291,8y_{AlBa} & + & 248,6y_{ArBa} & + & 0y_{BaBa} & + & 231,5y_{DuBa} & + & \\
 & 146y_{EdBa} & + & 132,4y_{EnBa} & + & 238,7y_{LaBa} & + & 92,2y_{MuBa} & + & \\
 & 151,4y_{OkBa} & + & 70,2y_{PoBa} & + & 115y_{StBa} & + & 45,6y_{TuBa} & + & \\
 & 88,2y_{AlDu} & + & 75,9y_{ArDu} & + & 231,5y_{BaDu} & + & 0y_{DuDu} & + & \\
 & 93,5y_{EdDu} & + & 137,5y_{EnDu} & + & 34,1y_{LaDu} & + & 213,5y_{MuDu} & + & \\
 & 80,9y_{OkDu} & + & 184,8y_{PoDu} & + & 145,3y_{StDu} & + & 187,8y_{TuDu} & + & \\
 & 153,9y_{AlEd} & + & 112,5y_{ArEd} & + & 146y_{BaEd} & + & 93,5y_{DuEd} & + & \\
 & 0y_{EdEd} & + & 88,8y_{EnEd} & + & 100,7y_{LaEd} & + & 145,7y_{MuEd} & + & \\
 & 14,4y_{OkEd} & + & 91,9y_{PoEd} & + & 53y_{StEd} & + & 102,2y_{TuEd} & + & \\
 & 208,2y_{AlEn} & + & 199y_{ArEn} & + & 132,4y_{BaEn} & + & 137,5y_{DuEn} & + & \\
 & 88,8y_{EdEn} & + & 0y_{EnEn} & + & 145y_{LaEn} & + & 166,4y_{MuEn} & + & \\
 & 87,6y_{OkEn} & + & 64,5y_{PoEn} & + & 65,8y_{StEn} & + & 118,4y_{TuEn} & + & \\
 & 54,2y_{AlLa} & + & 115,8y_{ArLa} & + & 238,7y_{BaLa} & + & 34,1y_{DuLa} & + & \\
 & 100,7y_{EdLa} & + & 145y_{EnLa} & + & 0y_{LaLa} & + & 220,6y_{MuLa} & + & \\
 & 88y_{OkLa} & + & 191,9y_{PoLa} & + & 152,5y_{StLa} & + & 194,9y_{TuLa} & + & \\
 & 274,2y_{AlMu} & + & 230,4y_{ArMu} & + & 92,2y_{BaMu} & + & 213,5y_{DuMu} & + & \\
 & 145,7y_{EdMu} & + & 166,4y_{EnMu} & + & 220,6y_{LaMu} & + & 0y_{MuMu} & + & \\
 & 140,4y_{OkMu} & + & 142,5y_{PoMu} & + & 119,2y_{StMu} & + & 48,1y_{TuMu} & + & \\
 & 141,1y_{AlOk} & + & 100,5y_{ArOk} & + & 151,4y_{BaOk} & + & 80,9y_{DuOk} & + & \\
 & 14,4y_{EdOk} & + & 87,6y_{EnOk} & + & 88y_{LaOk} & + & 140,4y_{MuOk} & + & \\
 & 0y_{OkOk} & + & 104,7y_{PoOk} & + & 66,6y_{StOk} & + & 107,6y_{TuOk} & + & \\
 & 245y_{AlPo} & + & 202,2y_{ArPo} & + & 70,2y_{BaPo} & + & 184,8y_{DuPo} & + & \\
 & 91,9y_{EdPo} & + & 64,5y_{EnPo} & + & 191,9y_{LaPo} & + & 142,5y_{MuPo} & + & \\
 & 104,7y_{OkPo} & + & 0y_{PoPo} & + & 41,9y_{StPo} & + & 96,5y_{TuPo} & + & \\
 & 209,2y_{AlSt} & + & 162,6y_{ArSt} & + & 115y_{BaSt} & + & 145,3y_{DuSt} & + & \\
 & 53y_{EdSt} & + & 65,8y_{EnSt} & + & 152,5y_{LaSt} & + & 119,2y_{MuSt} & + & \\
 & 66,6y_{OkSt} & + & 41,9y_{PoSt} & + & 0y_{StSt} & + & 71,2y_{TuSt} & + & \\
 & 248y_{AlTu} & + & 204,6y_{ArTu} & + & 45,6y_{BaTu} & + & 187,8y_{DuTu} & + & \\
 & 102,2y_{EdTu} & + & 118,4y_{EnTu} & + & 194,9y_{LaTu} & + & 48,1y_{MuTu} & + & \\
 & 107,6y_{OkTu} & + & 96,5y_{PoTu} & + & 71,2y_{StTu} & + & 0y_{TuTu} &) \times & (2 \times 0,74)
\end{array}
$$

2.2 Minimizing the Maximum Cost (ILP Model)

While with the previous computation we had to minimize the total cost sustainable, here it is requested to minimize the maximum possible cost of a terminal-hub connection. This problem can be defined as an instance of the p-center problem (Hakimi 1964-1965).
To solve this problem and minimize the maximum possible distance of a terminal-hub connection, it is necessary to introduce just a few changes to the one presented before; namely a new variable, a new family of constraints to set it as the upper-bound of all possible distances, and finally as for the only thing it must be completely revised from the previous model, this will be the variable that must be minimized, so:

- w: This is the new variable, it will indicate the maximum distance between a terminal and its assigned hub

- Additional Constraint (to set w as the upper-bound distance of an existing connection to a hub, that is $y_{ij} = 1$, for all terminals. I assumed h_i to be 1 as previously explained):

$$\sum_{j \in J} y_{ij} \times l_{ij} \leq w, \forall i \in I$$

- The new objective function, to get the minimized maximum cost:

$$(\min w) \times (2 \times 0,74)$$

Here I expanded the additional constraint which sets w as the upper-bound distance to minimize:

$$0y_{AlAl} + 169,8y_{AlAr} + 291,8y_{AlBa} + 88,2y_{AlDu} + 153,9y_{AlEd} + 208,2y_{AlEn} + 54,2y_{AlLa} + 274,2y_{AlMu} + 141,1y_{AlOk} + 245y_{AlPo} + 209,2y_{AlSt} + 248y_{AlTu} \leq \text{w}$$

$$169,8y_{ArAl} + 0y_{ArAr} + 248,6y_{ArBa} + 75,9y_{ArDu} + 112,5y_{ArEd} + 199y_{ArEn} + 115,8y_{ArLa} + 230,4y_{ArMu} + 100,5y_{ArOk} + 202,2y_{ArPo} + 162,6y_{ArSt} + 204,6y_{ArTu} \leq \text{w}$$

$$291,8y_{BaAl} + 248,6y_{BaAr} + 0y_{BaBa} + 231,5y_{BaDu} + 146y_{BaEd} + 132,4y_{BaEn} + 238,7y_{BaLa} + 92,2y_{BaMu} + 151,4y_{BaOk} + 70,2y_{BaPo} + 115y_{BaSt} + 45,6y_{BaTu} \leq \text{w}$$

$$88,2y_{DuAl} + 75,9y_{DuAr} + 231,5y_{DuBa} + 0y_{DuDu} + 93,5y_{DuEd} + 137,5y_{DuEn} + 34,1y_{DuLa} + 213,5y_{DuMu} + 80,9y_{DuOk} + 184,8y_{DuPo} + 145,3y_{DuSt} + 187,8y_{DuTu} \leq \text{w}$$

$$153,9y_{EdAl} + 112,5y_{EdAr} + 146y_{EdBa} + 93,5y_{EdDu} + 0y_{EdEd} + 88,8y_{EdEn} + 100,7y_{EdLa} + 145,7y_{EdMu} + 14,4y_{EdOk} + 91,9y_{EdPo} + 53y_{EdSt} + 102,2y_{EdTu} \leq \text{w}$$

$$208,2y_{EnAl} + 199y_{EnAr} + 132,4y_{EnBa} + 137,5y_{EnDu} + 88,8y_{EnEd} + 0y_{EnEn} + 145y_{EnLa} + 166,4y_{EnMu} + 87,6y_{EnOk} + 64,5y_{EnPo} + 65,8y_{EnSt} + 118,4y_{EnTu} \leq \text{w}$$

$$54,2y_{LaAl} + 115,8y_{LaAr} + 238,7y_{LaBa} + 34,1y_{LaDu} + 100,7y_{LaEd} + 145y_{LaEn} + 0y_{LaLa} + 220,6y_{LaMu} + 88y_{LaOk} + 191,9y_{LaPo} + 152,5y_{LaSt} + 194,9y_{LaTu} \leq \text{w}$$

$$274,2y_{MuAl} + 230,4y_{MuAr} + 92,2y_{MuBa} + 213,5y_{MuDu} + 145,7y_{MuEd} + 166,4y_{MuEn} + 220,6y_{MuLa} + 0y_{MuMu} + 140,4y_{MuOk} + 142,5y_{MuPo} + 119,2y_{MuSt} + 48,1y_{MuTu} \leq \text{w}$$

$$141,1y_{OkAl} + 100,5y_{OkAr} + 151,4y_{OkBa} + 80,9y_{OkDu} + 14,4y_{OkEd} + 87,6y_{OkEn} + 88y_{OkLa} + 140,4y_{OkMu} + 0y_{OkOk} + 104,7y_{OkPo} + 66,6y_{OkSt} + 107,6y_{OkTu} \leq \text{w}$$

$$245y_{PoAl} + 202,2y_{PoAr} + 70,2y_{PoBa} + 184,8y_{PoDu} + 91,9y_{PoEd} + 64,5y_{PoEn} + 191,9y_{PoLa} + 142,5y_{PoMu} + 104,7y_{PoOk} + 0y_{PoPo} + 41,9y_{PoSt} + 96,5y_{PoTu} \leq \text{w}$$

$$209,2y_{StAl} + 162,6y_{StAr} + 115y_{StBa} + 145,3y_{StDu} + 53y_{StEd} + 65,8y_{StEn} + 152,5y_{StLa} + 119,2y_{StMu} + 66,6y_{StOk} + 41,9y_{StPo} + 0y_{StSt} + 71,2y_{StTu} \leq \text{w}$$

$$248y_{TuAl} + 204,6y_{TuAr} + 45,6y_{TuBa} + 187,8y_{TuDu} + 102,2y_{TuEd} + 118,4y_{TuEn} + 194,9y_{TuLa} + 48,1y_{TuMu} + 107,6y_{TuOk} + 96,5y_{TuPo} + 71,2y_{TuSt} + 0y_{TuTu} \leq \text{w}$$

3 Solutions

In order to compute the solutions, the previous two models were implemented with AMPL and solved with the CPLEX solver. The files are referenced hereinafter in each subsection.

3.1 Optimal Total Cost

The files for this solution are denominated "logistics-tot.dat" and "logistics-tot.mod".
The solver found an optimal solution with a total cost of $1081,73$ \$, the related total distance is 730,9 miles. The corresponding Hubs to open are Duncan and Stillwater. The terminals were assigned in the following way:

- Altus, Ardmore, Duncan and Lawton were assigned to the Duncan hub.
- Bartlesville, Edmond, Enid, Muskogee, Oklahoma City, Ponca City, Stilwater and Tulsa were assigned to the Stillwater hub.

3.2 Optimal Maximum Cost

The files for this solution are denominated "logistics-max.dat" and "logistics-max.mod"
The solver found an optimal solution with a maximum cost of $175,23$ \$, the related maximum distance is 118,4 miles. The corresponding Hubs that should be open in this cas would be Duncan and Tulsa. The terminals would be assigned in the following way:

- Altus, Ardmore, Duncan, Edmond, Lawton and Oklahoma City would be assigned to the Duncan hub.
- Bartlesville, Enid (were we would sustain the maximum cost), Muskogee, Ponca City, Stilwater and Tulsa would be assigned to the Tulsa hub.

3.3 Comparison between the two cases

To better understand the differences I evidenced in Table 3 the relevant costs (by multiplying the distances in Table 1 with the cost factors).

	Duncan	Stillwater	Tulsa
Altus	***130,536***	309,616	367,04
Ardmore	***112,332***	240,648	302,808
Bartlesville	342,62	**170,2**	*67,488*
Duncan	***0***	215,044	277,944
Edmond	*138,38*	**78,44**	151,256
Enid	203,5	**97,384**	*175,232*
Lawton	***50,468***	225,7	288,452
Muskogee	315,98	**176,416**	*71,188*
Oklahoma City	*119,732*	**98,568**	159,248
Ponca City	273,504	**62,012**	*142,82*
Stillwater	215,044	**0**	*105,376*
Tulsa	277,944	**105,376**	*0*

Table 3: Relevant terminal-hub costs (1st solution in bold, 2nd solution in italics)

As it can be seen, by opening in Duncan and Tulsa with the respective terminal-hub connections identified in the previous subsection, we get the total cost of $1113,55$ \$ with an increase of $31,82$ \$ to the case of the first solution ($1081,73$ \$).
Instead, for the first case the maximum cost would be sustained for the Muskogee-Stillwater connection with a cost of $176,41$ \$, with a slight increase of $1,18$ \$ from the Enid-Tulsa connection of the second case ($175,23$ \$).