

## *VIRTUAL PROTOTYPING*

# ***LECTURE 2***

## **Geometric Modeling: 3D Object Modeling & Knowledge Based Modeling**

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# Introduction

Modeling techniques are classified according to the type of representation and data structure.

- **Representation** is the set of methods used to describe the geometry.
- **Data structure** is how model information are stored in computer memory and on file.

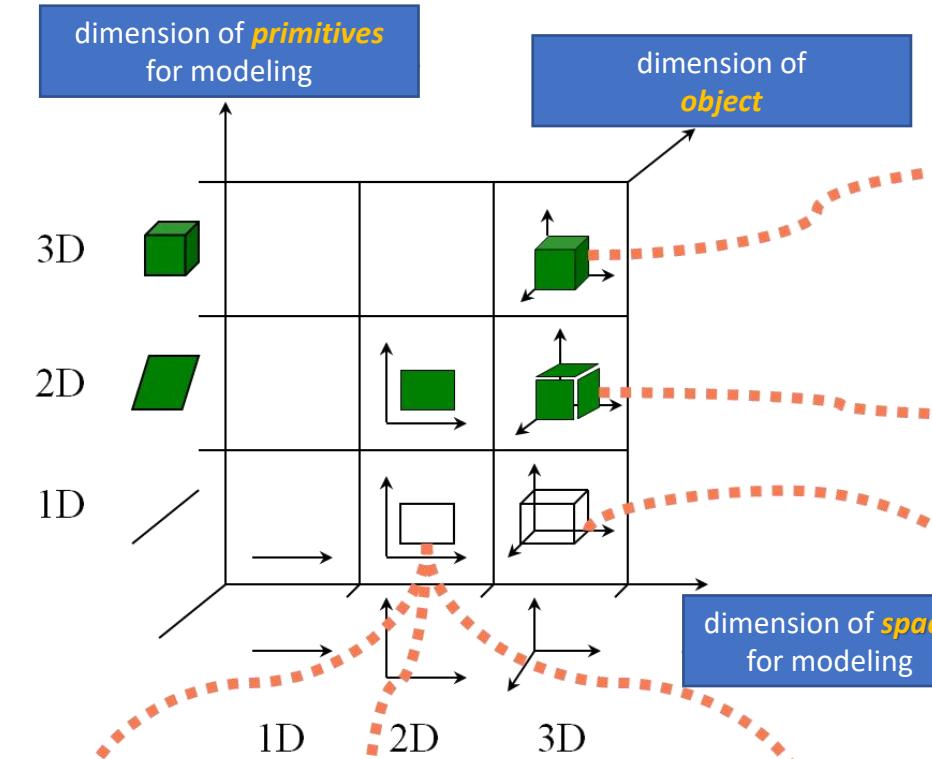
The type of representation and its data structure are a characterizing element of CAD software as it affects:

**Potentiality** (domain of possible geometries, modeling functions)

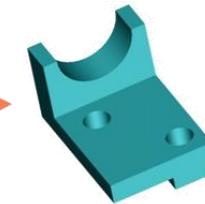
**Performance** (speed of execution of functions)



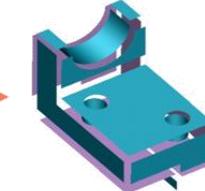
# 3D Object Modeling



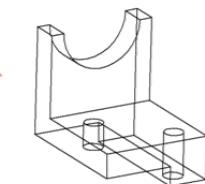
**CSG (Constructive Solid Geometry)**  
Solid primitives combined with procedural methods



**B-REP (Boundary Representation)**  
Representation of the external surface of the solid object

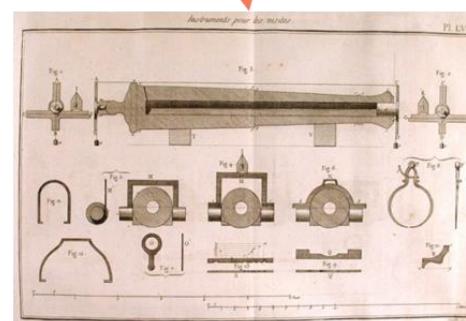
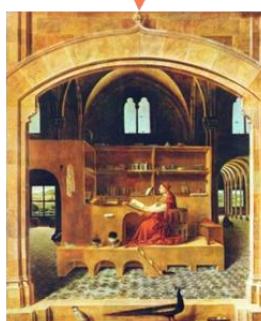
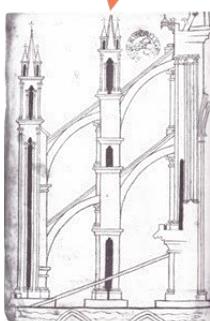


**WIREFRAME**  
Representation of the solid object through its edges



**Paintings:**

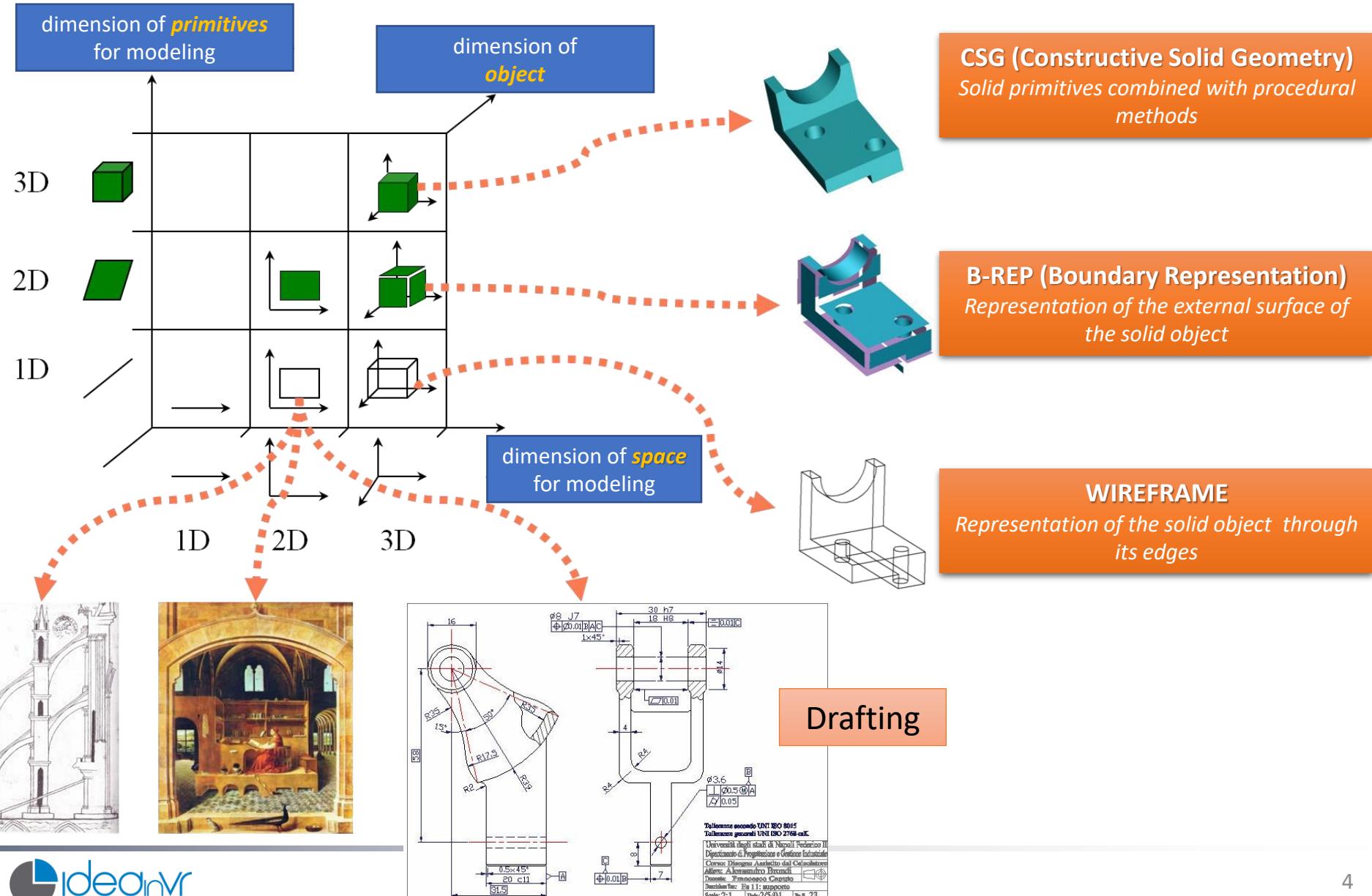
- No depth
- Perspective



**Drafting**



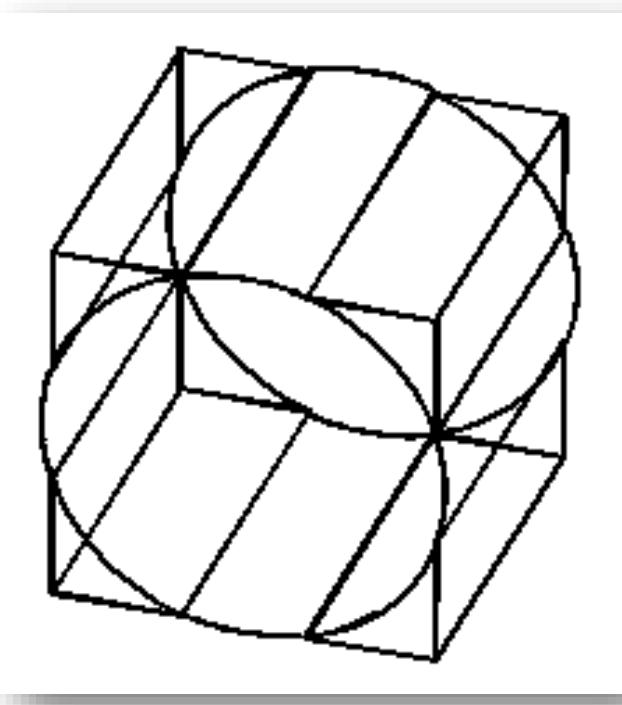
# 3D Object Modeling



DIPARTIMENTO DI  
INGEGNERIA  
INDUSTRIALE



# Wireframe



## Pro

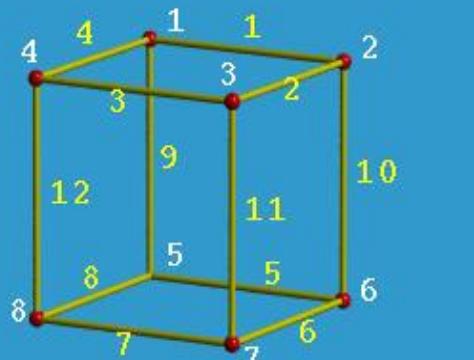
- They require few computational and graphical resources
- Easily implementable

## Cons

- "Poor geometry" (only visible and / or hidden edges).
- Complicated to read and interpret
- There is no explicit description of the boundary surfaces of the models.
- They cannot be interrogated on areas and volumes.
- Ambiguity in representation and in the transition from 3D to 2D.

# Wireframe

An object is specified by two tables: (1) Vertex Table (2) Edge Table.

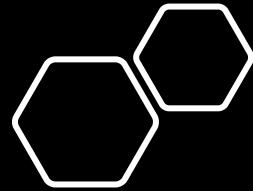


Vertex	X	Y	Z
1	1	1	1
2	1	-1	1
3	-1	-1	1
4	-1	1	1
5	1	1	-1
6	1	-1	-1
7	-1	-1	-1
8	-1	1	-1

The vertex table consists of three-dimensional coordinate values for each vertex with reference to the origin.

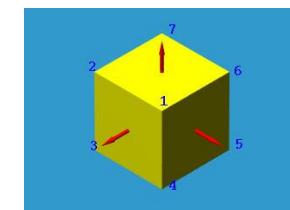
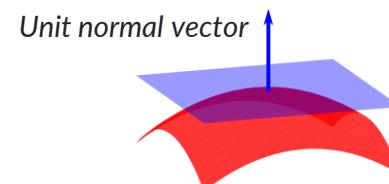
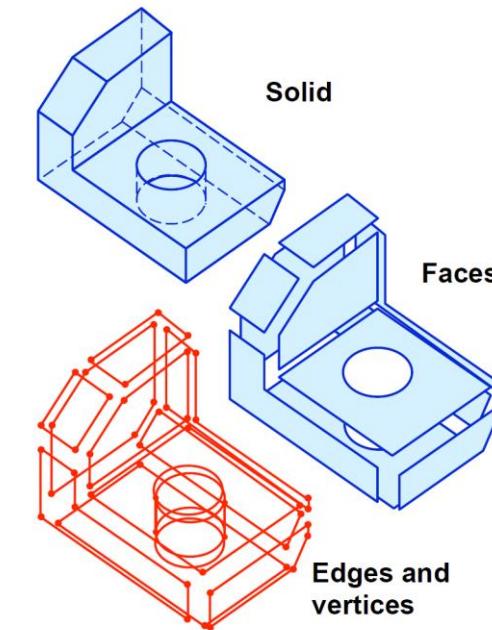
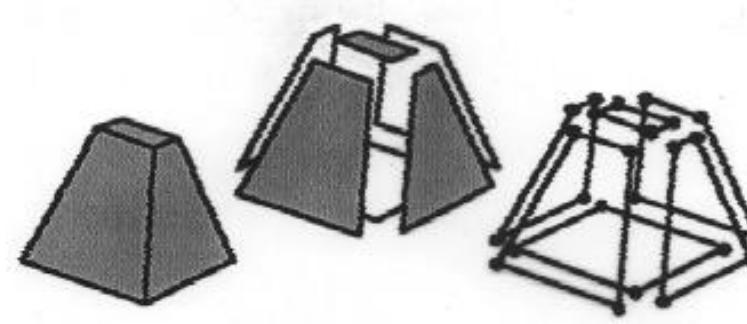
Edge	Start Vertex	End Vertex
1	1	2
2	2	3
3	3	4
4	4	1
5	5	6
6	6	7
7	7	8
8	8	5
9	1	5
10	2	6
11	3	7
12	4	8

Edge table specifies the start and end vertices for each edge.



# Boundary Representation

- The solid is defined indirectly through its limiting surfaces (faces) and their contiguity relations to each other
- Faces are represented as assemblies with edges and vertices
- The position of material is defined by the use of oriented geometry



*A normal to a surface at a point is the same as a normal to the tangent plane to the surface at the same point.*

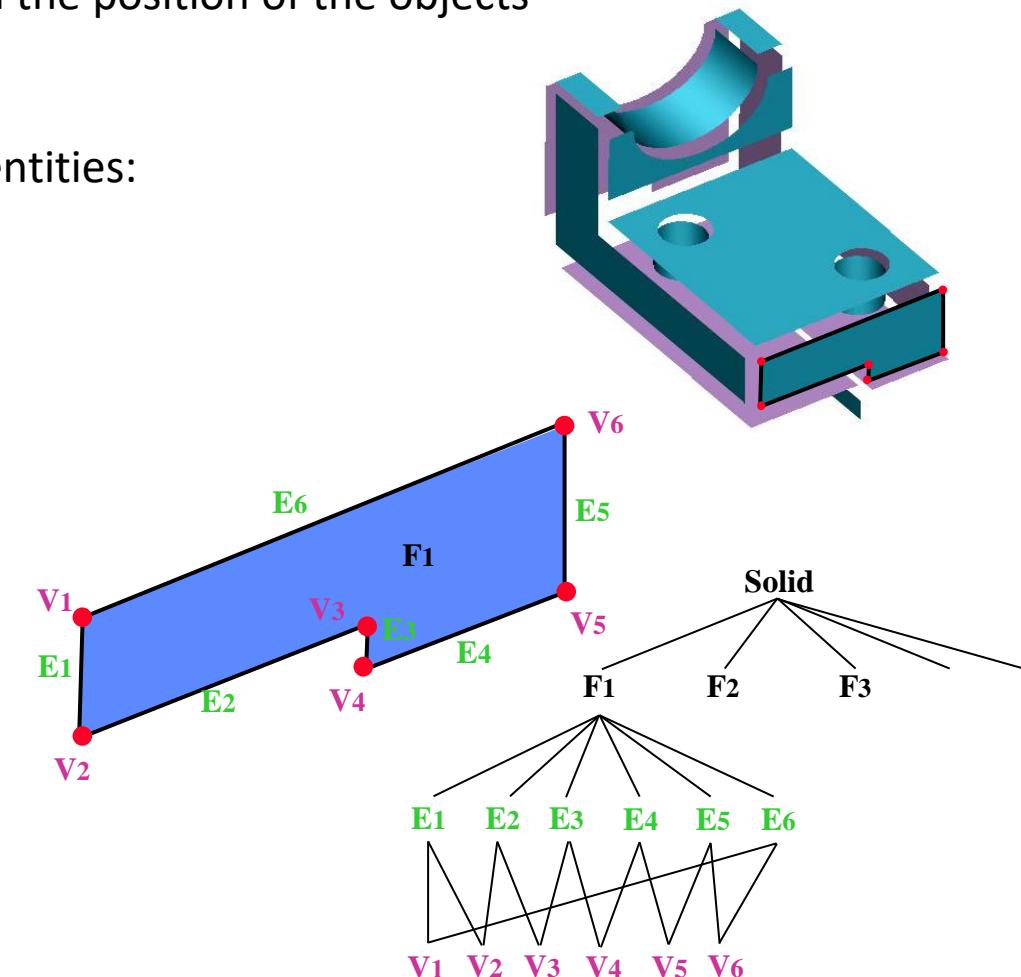
# Topological and geometrical entities

The **geometry** describes, by means of parameters, the shape, the size and the position of the objects:  
*coordinates of the vertices, radius, lengths and angles, etc.*

The **topology** describes the connections between the various geometric entities:  
*Vertices, edges and faces* are topological entities

- Geometrical Entities
  - Point (X,Y,Z)
  - Line (X,Y,Z) Start (X,Y,Z) End
  - Circle (X,Y,Z) Center & Diameter
  - Arc (X,Y,Z) Center , Radius, Start & End Angle or Point
- Topological entities
  - Vertex (edge1, edge2, etc..) – Point geom.
  - Edge (vertex1, vertex2) – Curve geom.
  - Face (edge1, edge2, etc) – surface geom.
  - Solid (Face1, Face2, etc.)

Topological entities	SOLID	SHELL	FACE	LOOP	EDGE	VERTEX
Geometrical entities						
Topological entities						



# Topological consistency: The Eulero-Poincaré Formula

$$V - E + F = 2(S - H) + R$$

V: number of vertices

E: number of edges

F: number of faces

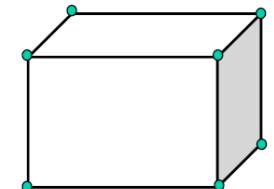
H: number of holes that penetrate the solid

S: number of shells (the solid is counted as a shell, so always  $S \geq 1$ )

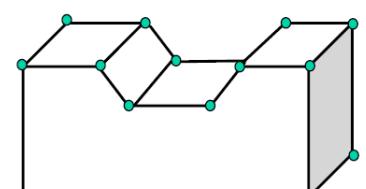
R: number of rings (closed loops on faces)



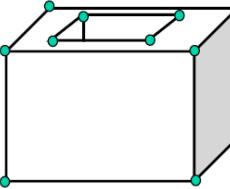
Leonhard Euler (1707 -1783) defined the fundamental rules governing the topology of solid bodies.



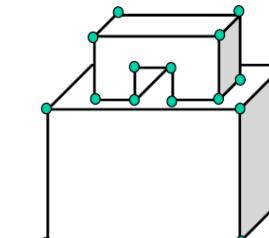
$V=8, E=12, F=6, S=1, H=0, R=0$



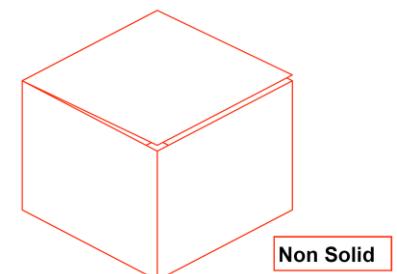
$V=16, E=24, F=10, S=1, H=0, R=0$



$V=16, E=24, F=10, S=1, H=1, R=2$



$V=24, E=36, F=14, S=1, H=1, R=2$



$V=10, E=15, F=6, S=1, H=0, R=0$

$$V-E+F=2(S-H)+R$$

$$10-15+6=2(1-0)+0$$

**1≠2**



# Topological consistency: The Eulero-Poincaré Formula

$$V - E + F = 2(S - H) + R$$

V: number of vertices

E: number of edges

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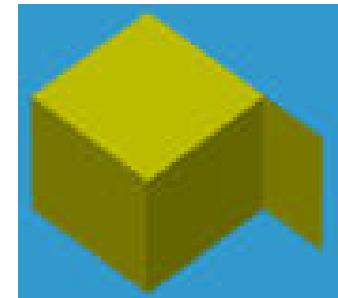
R: number of rings (closed loops on faces)



Leonhard Euler (1707-1783) defined the fundamental rules governing the topology of solid bodies.

The Euler-Poincaré formula is a necessary but not sufficient condition.

If the relationship is true, this does not mean that the solid is valid.



$$V=10, E=15, F=7, S=1, H=0, R=0$$

$$V-E+F=2(S-H)+R$$

$$10-15+7=2(1-0)+0$$

$$2=2$$

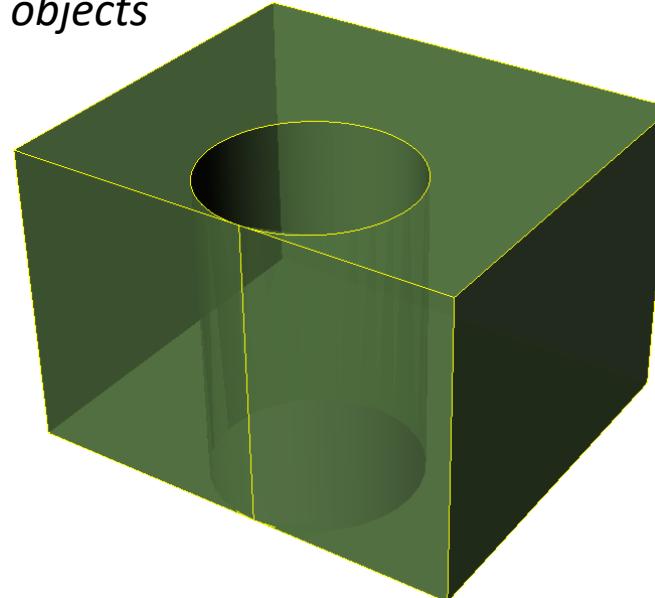
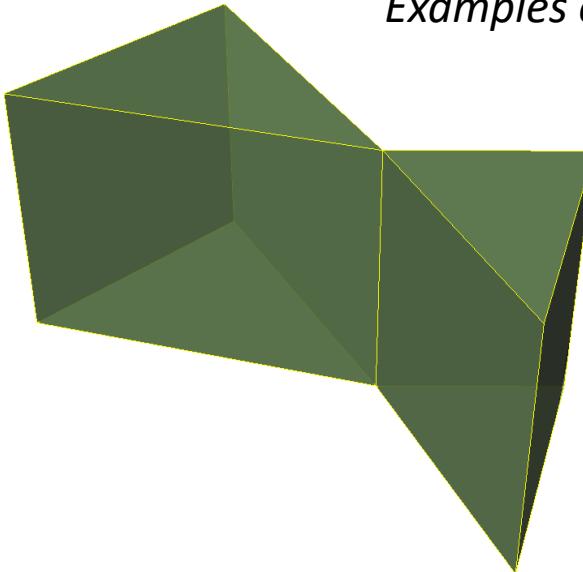


# MANIFOLD and NON-MANIFOLD objects

In manifold objects, each boundary unambiguously separates an inner region from an outer one

In manifold objects, edges can only belong to two faces

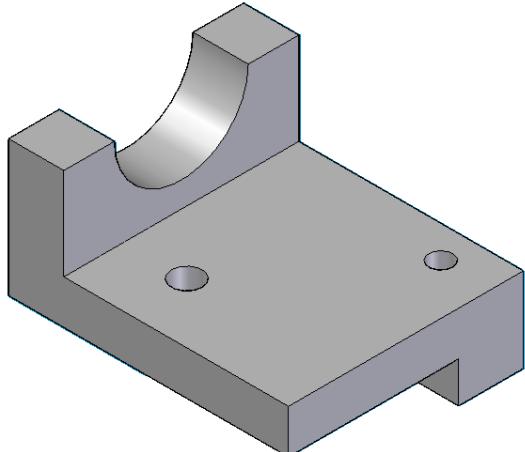
*Examples of non-manifold objects*



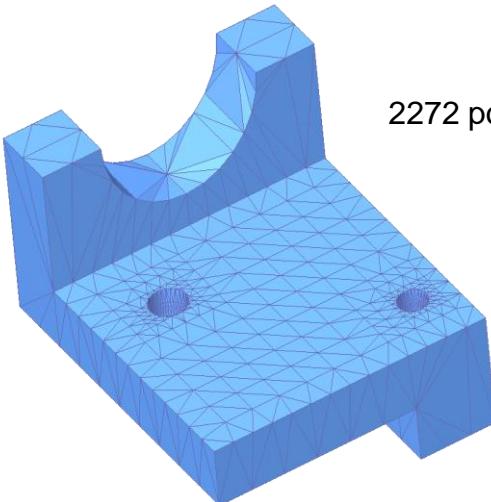
# Precise vs approximated geometry

A B-rep model is a precise representation of an object. A simplified representation (always based on "surface" information) can be obtained with a "faceting" process that creates a polygon mesh.

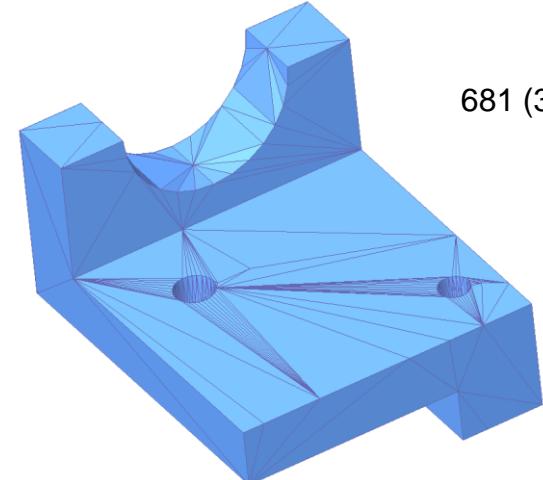
Precise geometry



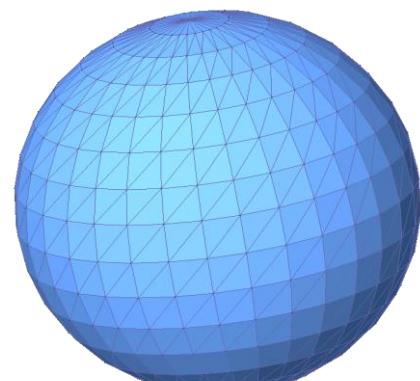
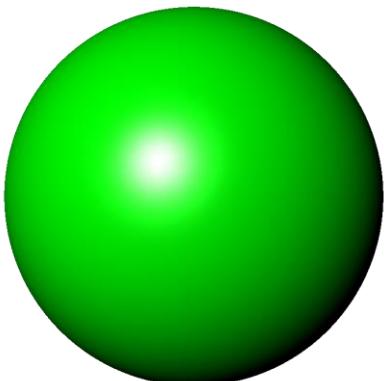
Approximated geometry



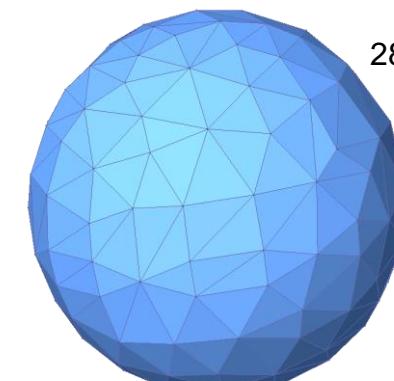
2272 pol.



681 (30%)



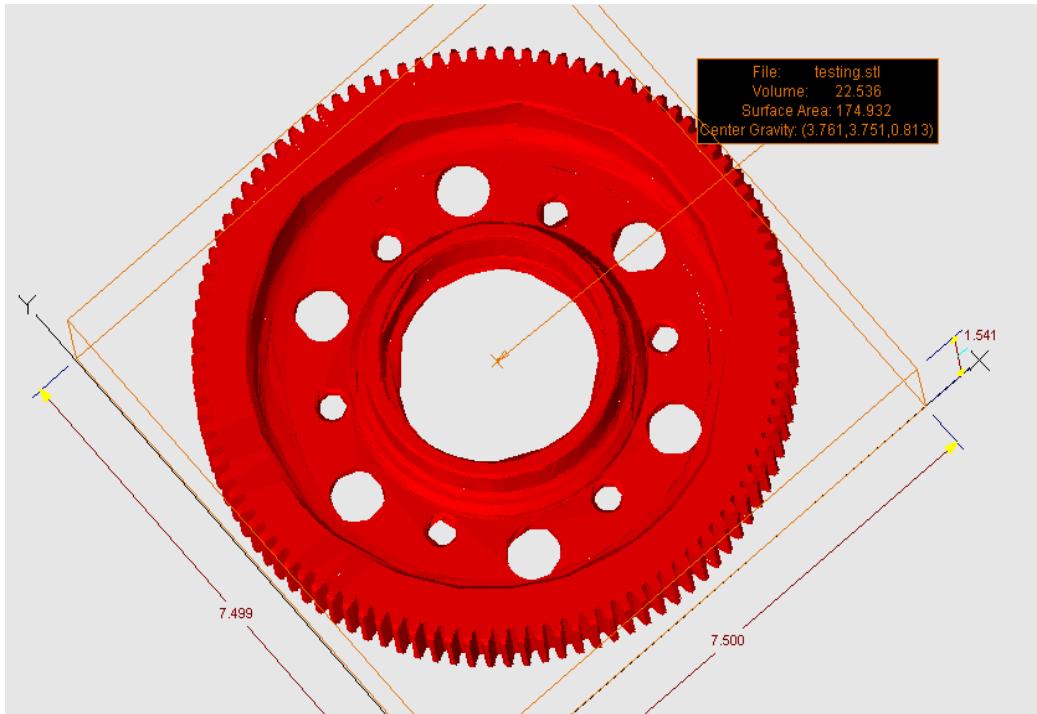
960 pol.



288 (30%)

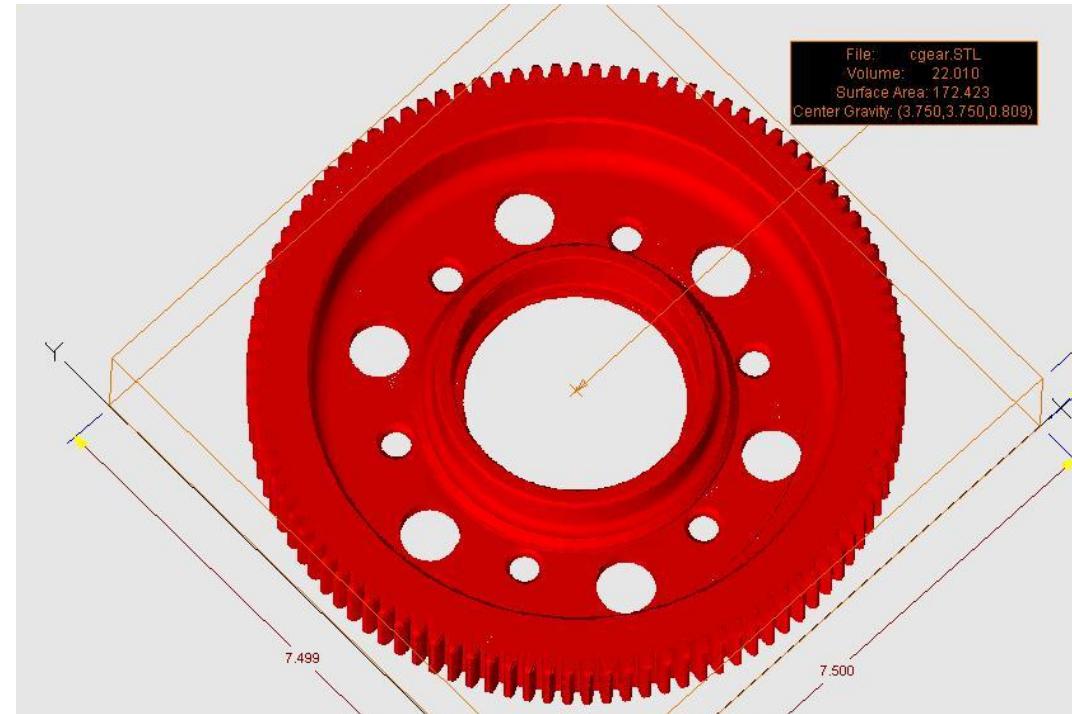
# Precise vs approximated geometry

The file size grows more than linearly with the number of triangles !!



Error = 0.01 mm

**Triangles: 38,000**  
**File Size: 1.9 MB**



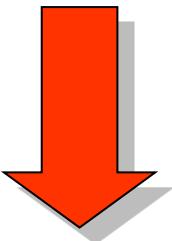
Error = 0.001 mm

**Triangles: 195,000**  
**File Size: 19.5 MB**



# Level of Detail - LoD

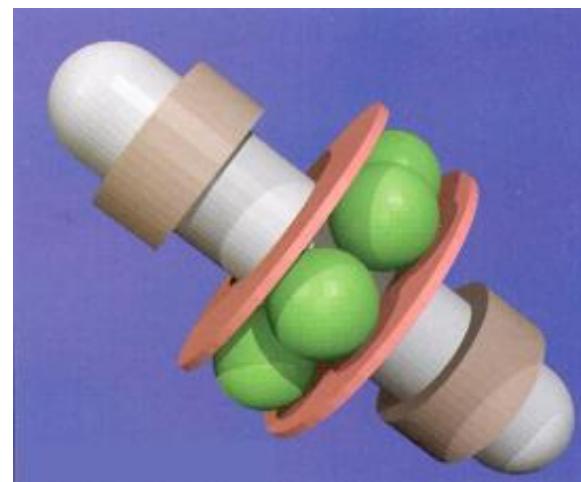
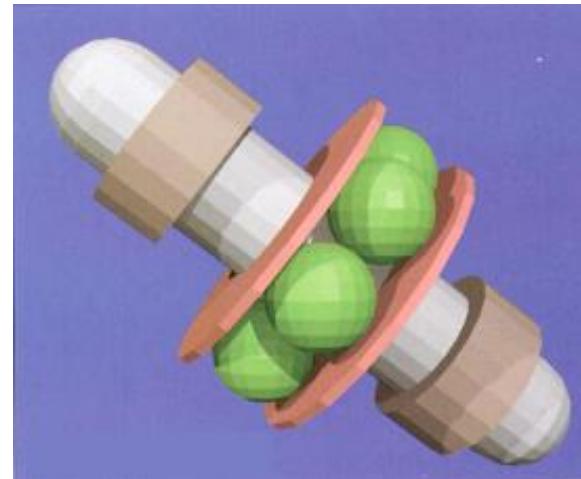
Less detail



More detail

But also:

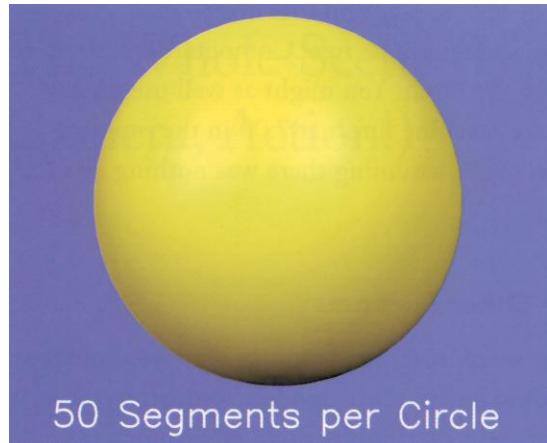
- + time for rendering
- + memory space



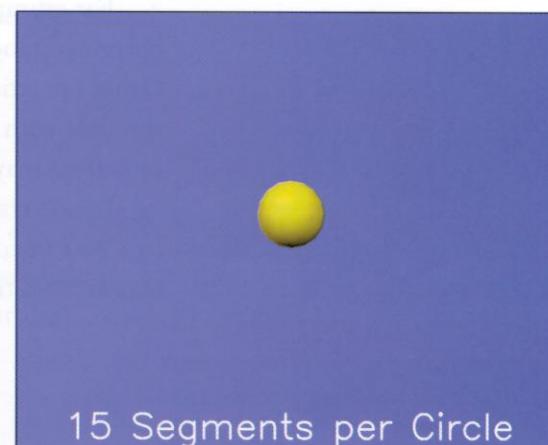
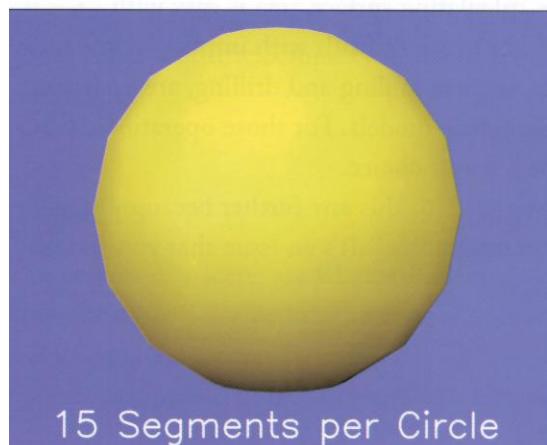
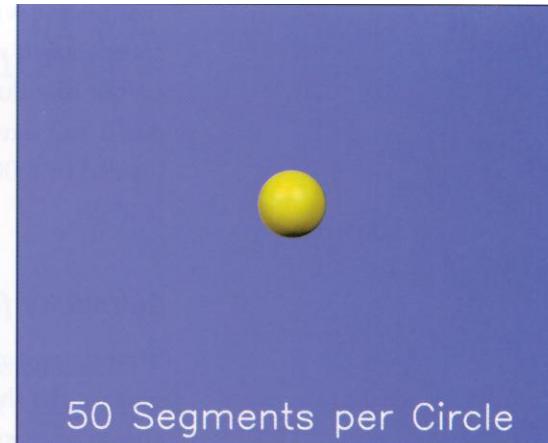
# Level of Detail - LoD

Distant or small objects  
on the screen require  
less detail

More detail

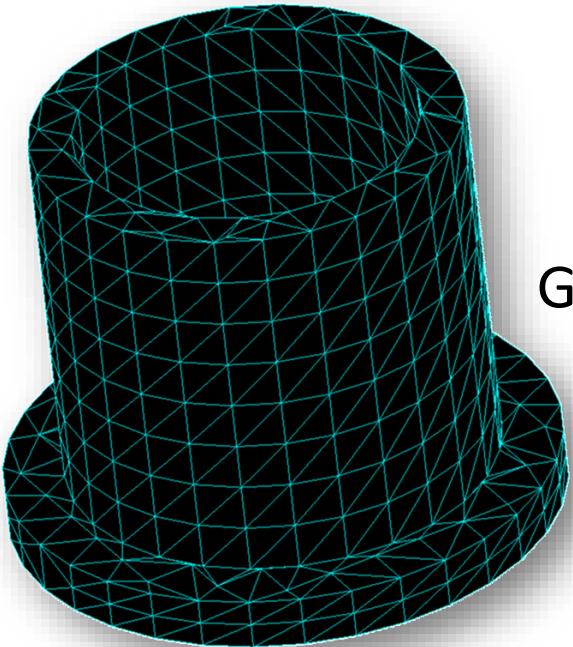


Less detail

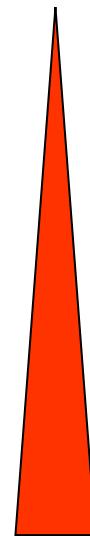
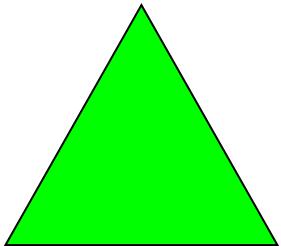


# Quality of tessellation

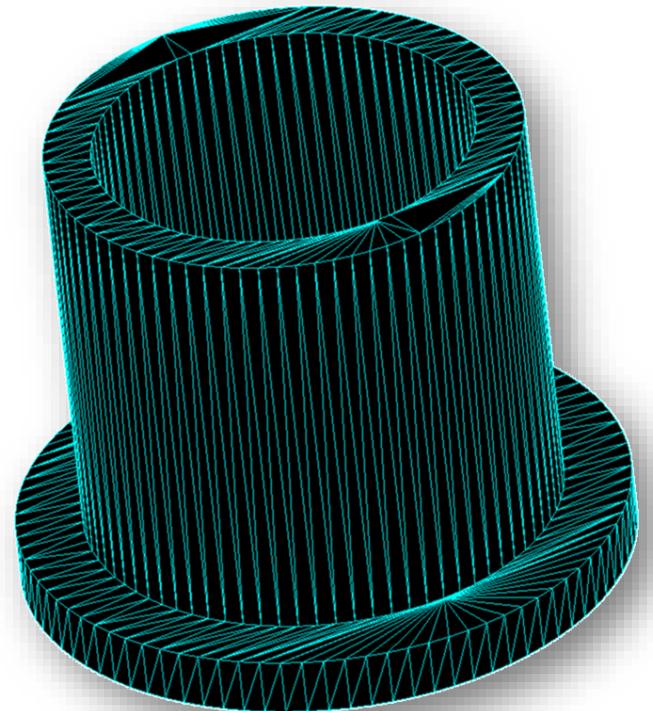
Nearly equilateral triangles indicate an optimized tessellation.



Good!



No Good!



# Classification of tessellation

Based on the type of data structure:

## Without connectivity

(e.g. reverse engineering or 3D scanning models)

- *For each polygon (triangles) the coordinates of the vertices and the normal (towards the outside) are stored*
- *The resulting data structure is very simple, but not compact (many vertices are repeated thousands of times !!)*
- *Universal as it is used by the standard .STL (Stereolithography by 3D labs) - ASCII or binary*

## With connectivity (shared vertices)

usually result from direct CAD modeling.

- *Objects are represented with vertices and their connectivity.*
- *This information is stored in tables:*
  - *Vertex table*
  - *Edge table*
  - *Polygon table*
- *Tables are saved in memory and on file.*



# Classification of tessellation

Based on the type of data structure:

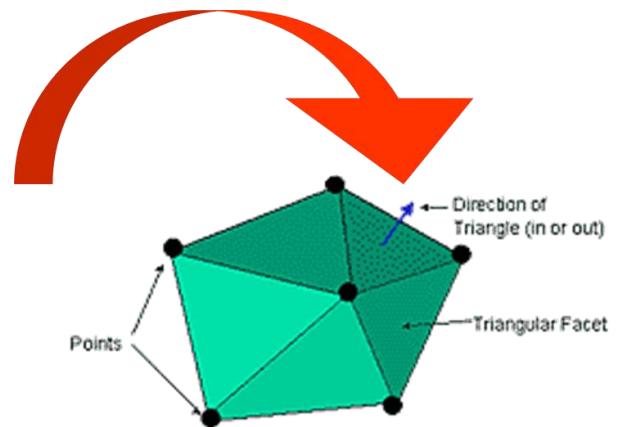
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*STL standard*

```
solid <part name>
facet normal -1 0 0
outer loop
    vertex 1 1 1
    vertex 1 1 2
    vertex 1 2 1
endloop
endfacet
facet normal 0 -1 0
outer loop
    vertex ...
endloop
endfacet
endsolid <part name>
```



Repeated for each triangle -> large files !!



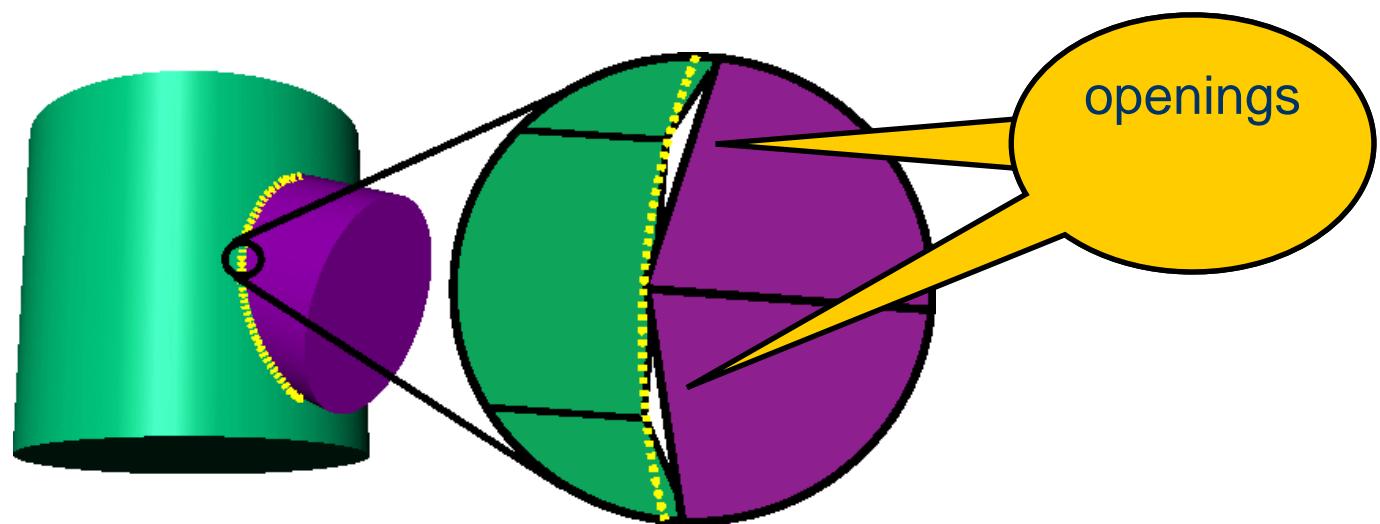
# Classification of tessellation

Based on the type of data structure:

## Without connectivity

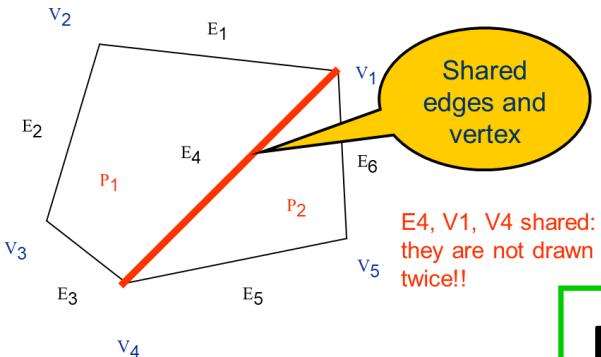
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# Classification of tessellation

Based on the type of data structure:



## Vertex Table

$V_1 : x_1, y_1, z_1$

$V_2 : x_2, y_2, z_2$

$V_3 : x_3, y_3, z_3$

$V_4 : x_4, y_4, z_4$

$V_5 : x_5, y_5, z_5$

## Edge Table

$E_1 : V_1, V_2$

$E_2 : V_2, V_3$

$E_3 : V_3, V_4$

$E_4 : V_4, V_1$

$E_5 : V_4, V_5$

$E_6 : V_5, V_1$

## Polygon table

$P_1 : E_1, E_2, E_3, E_4$

$P_2 : E_4, E_5, E_6$

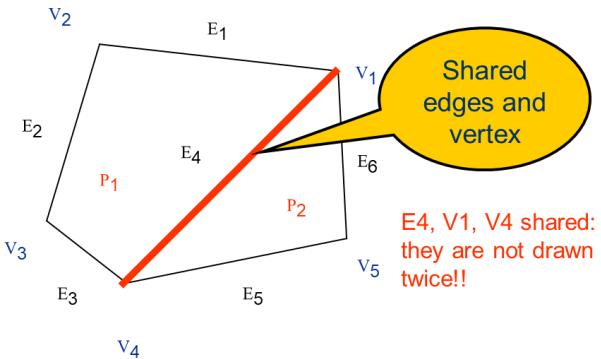
**With connectivity** (shared vertices)  
usually result from direct CAD modeling.

- *Objects are represented with vertices and their connectivity.*
- *This information is stored in tables:*
  - Vertex table
  - Edge table
  - Polygon table
- *Tables are saved in memory and on file.*



# Classification of tessellation

Based on the type of data structure:



- The vertices must belong to the end point of at least 2 edges
- Each edge must belong to at least one polygon
- All polygons are closed
- All polygons have at least one shared edge

**With connectivity** (shared vertices)  
usually result from direct CAD modeling.

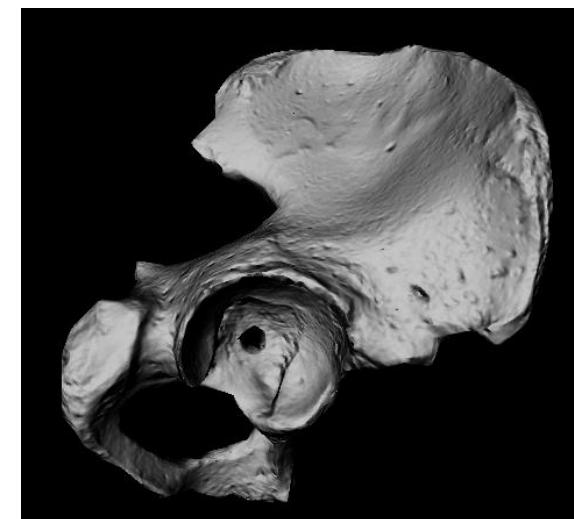
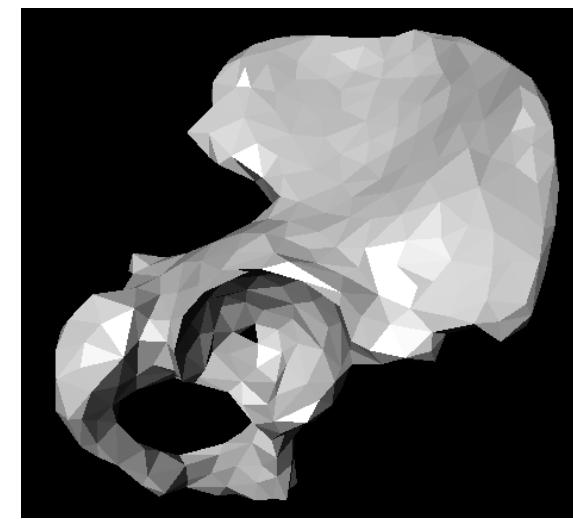
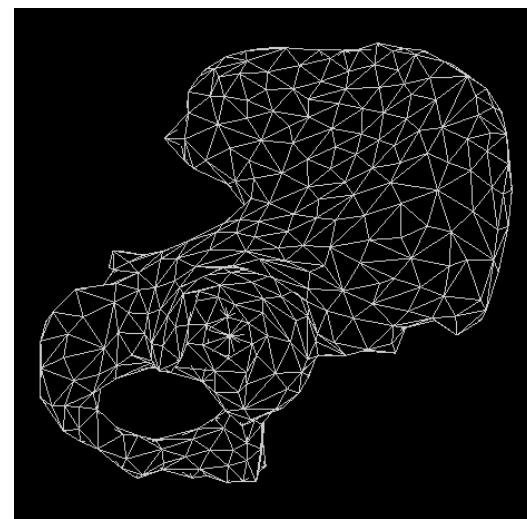
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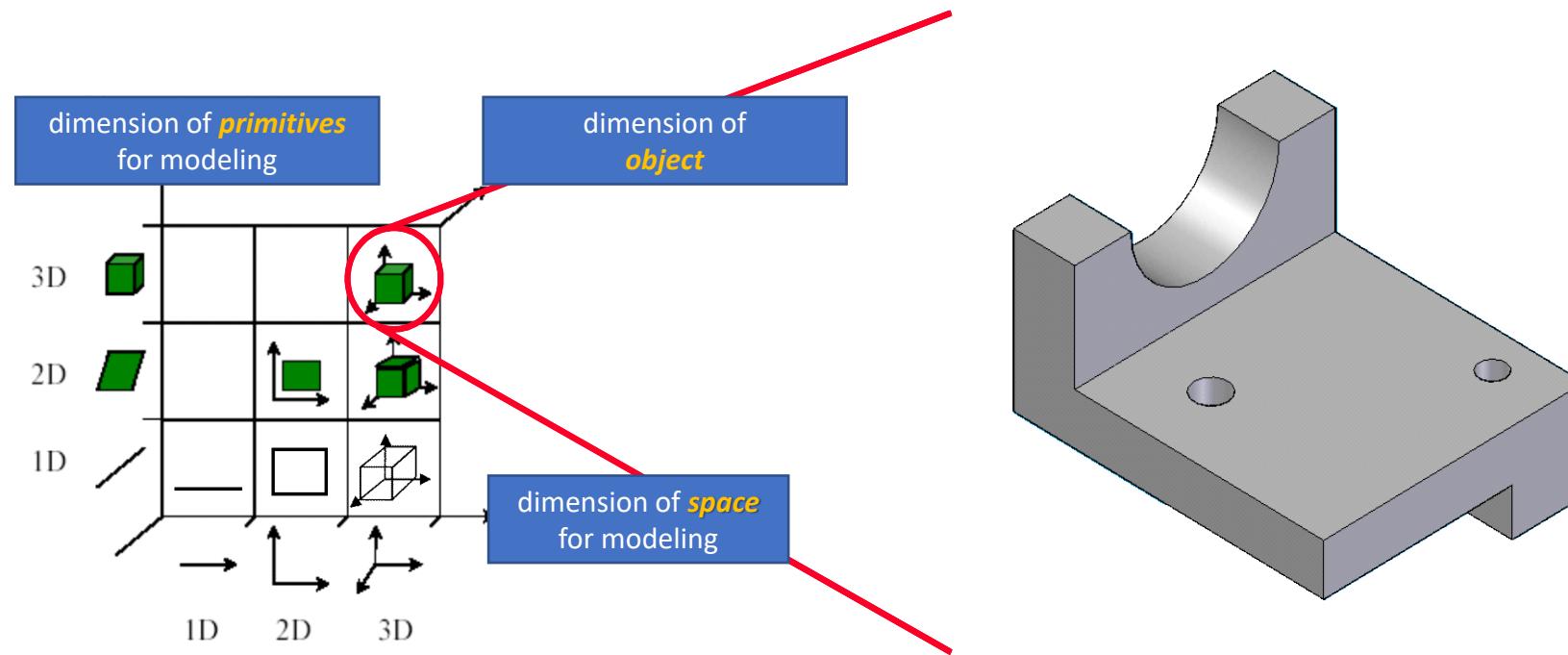
# Engineering applications of tessellated models

- **Visualization**: everything we see on a monitor is transformed into triangles (tessellation)
- **Reverse engineering** systems output (STL format)
- Input of **rapid prototyping** systems (eg stereolithography).
- Compact visualization of **3D models on the web** (vrml, 3dxml, java3D)
- Input for **VR software** (jt, iv, 3ds, vrml)

Research in the field of **video games** has led to the improvement of the aesthetic side, starting from very simple polygonal models (**texture, bump mapping**). The result sounds accurate but it isn't!



# 3D Modeling with solid elements

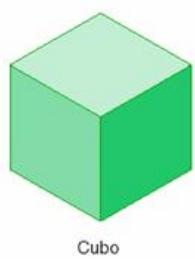


- Primitive Instancing
- Systems based on solid primitives and construction methods (C.S.G.)
- Decomposition based systems:
  - by spatial enumeration
  - Octree

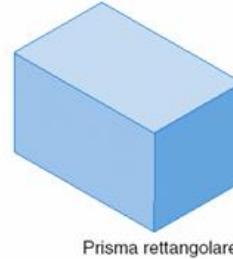


## Primitive Instancing

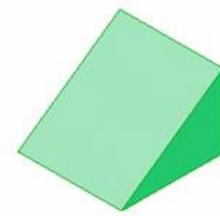
- Starting from simple solid primitives it is possible to obtain some solid objects of not complex shape.
- By combining the primitives in different ways
- The items obtained are very limited



Cubo



Prisma rettangolare



Prisma triangolare



Sfera



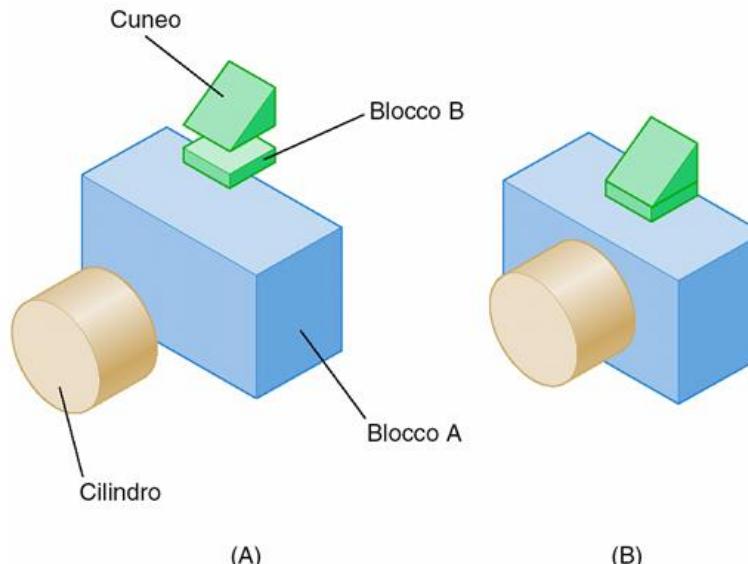
Cono



Toro



Cilindro



(A)

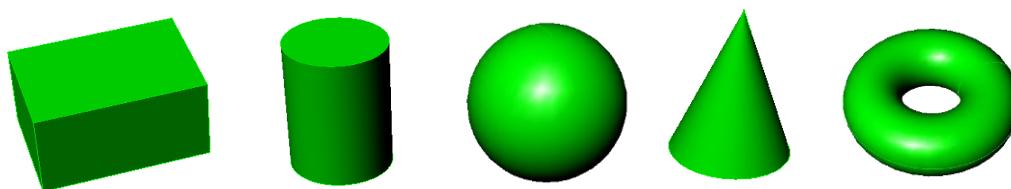
(B)

## C.S.G. Modeling

**C.S.G. = Constructive Solid Geometry**

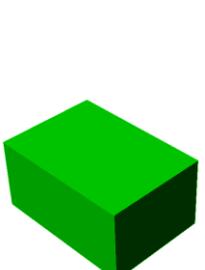
The solid is represented as a composition of parameterized instances of primitives of solids by means of Boolean operations and rigid motions.

Parallelepiped - Cylinder - Sphere - Cone - Torus

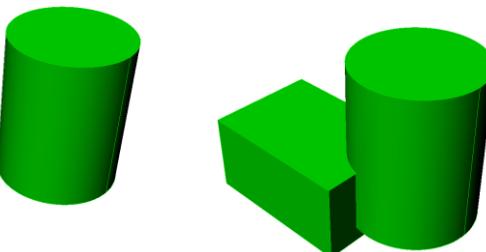


Primitives can usually be scaled, rotated, translated before being combined with the Boolean operators (Union - Subtraction - Intersection).

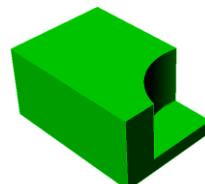
A    B



A + B



A - B



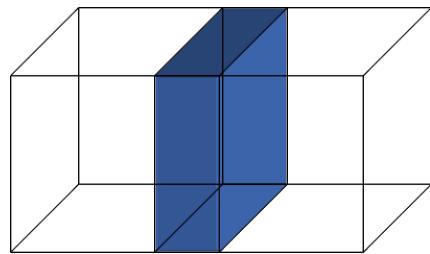
B - A



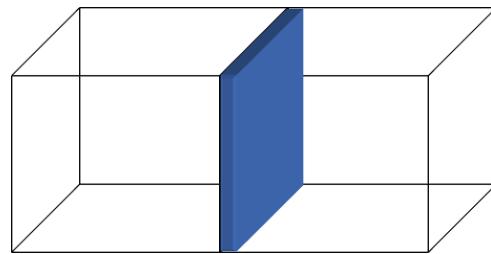
A ∩ B



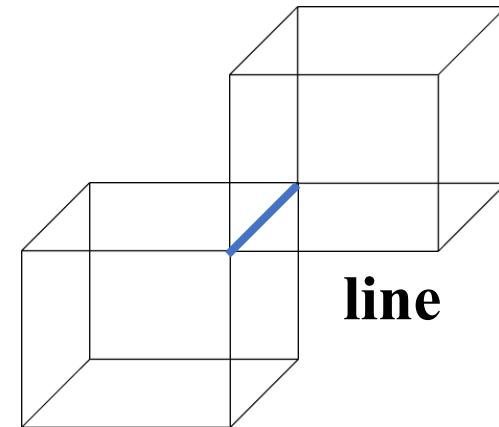
## *Boolean operations*



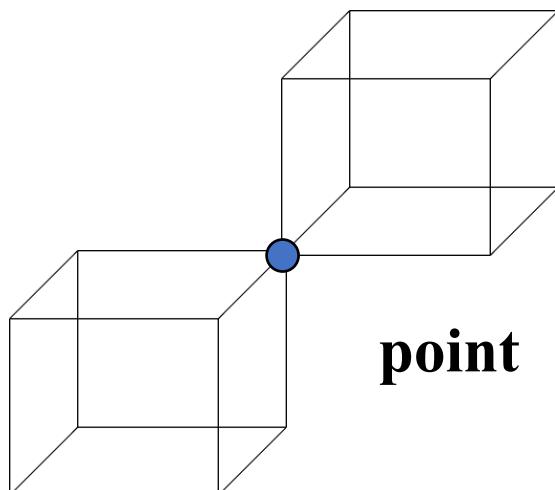
**solid**



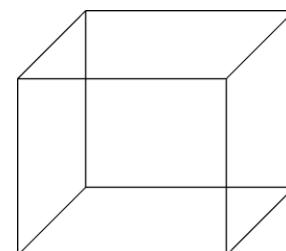
**plane**



**line**



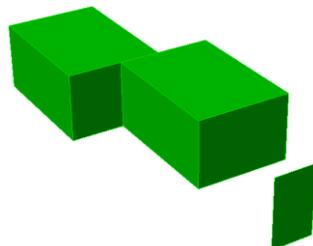
**point**



**null**

# *Boolean operations in C.S.G.*

Boolean operations on solids  
do not always result in a  
"solid"



$A \cap B$  is a surface!

Need for "regularized" Boolean operations

- 1) Carry out the "traditional" operation, which can generate a non-solid element (surface or curve).
- 2) Calculate the interior of the object obtained. All lower level entities are removed. The result is a solid without its boundary.
- 3) Calculate the closure of the object obtained in the previous step. This operation adds all the outline to the object.

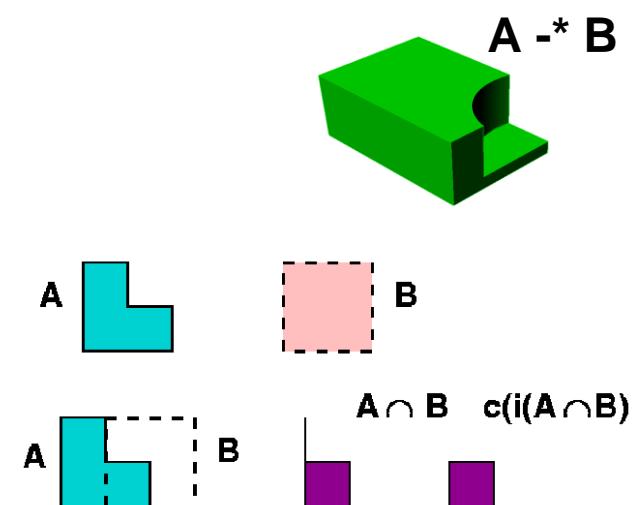
$$x^2 + y^2 + z^2 = 1$$

$$\text{Internal} \Rightarrow x^2 + y^2 + z^2 < 1$$

$$\text{Closure} \Rightarrow x^2 + y^2 + z^2 \leq 1$$

$$\text{External} \Rightarrow x^2 + y^2 + z^2 > 1$$

$A +^* B$	= closure (int ( $A + B$ ))
$A \cap^* B$	= closure (int ( $A \cap B$ ))
$A -^* B$	= closure (int ( $A - B$ ))



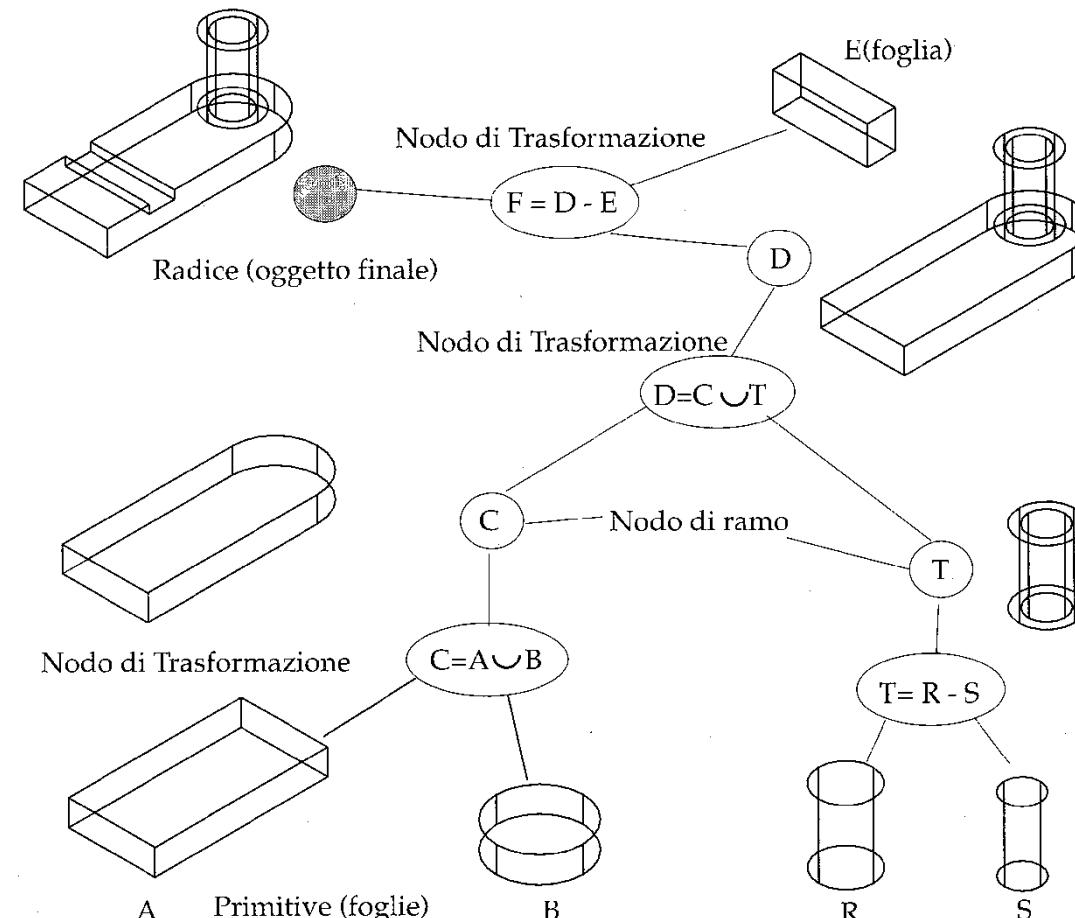
## C.S.G. Tree

The solid model is implicitly represented through a tree structure that reports the combination of Boolean operations and geometric transformations (translation, rotation, scale) on primitive solids.

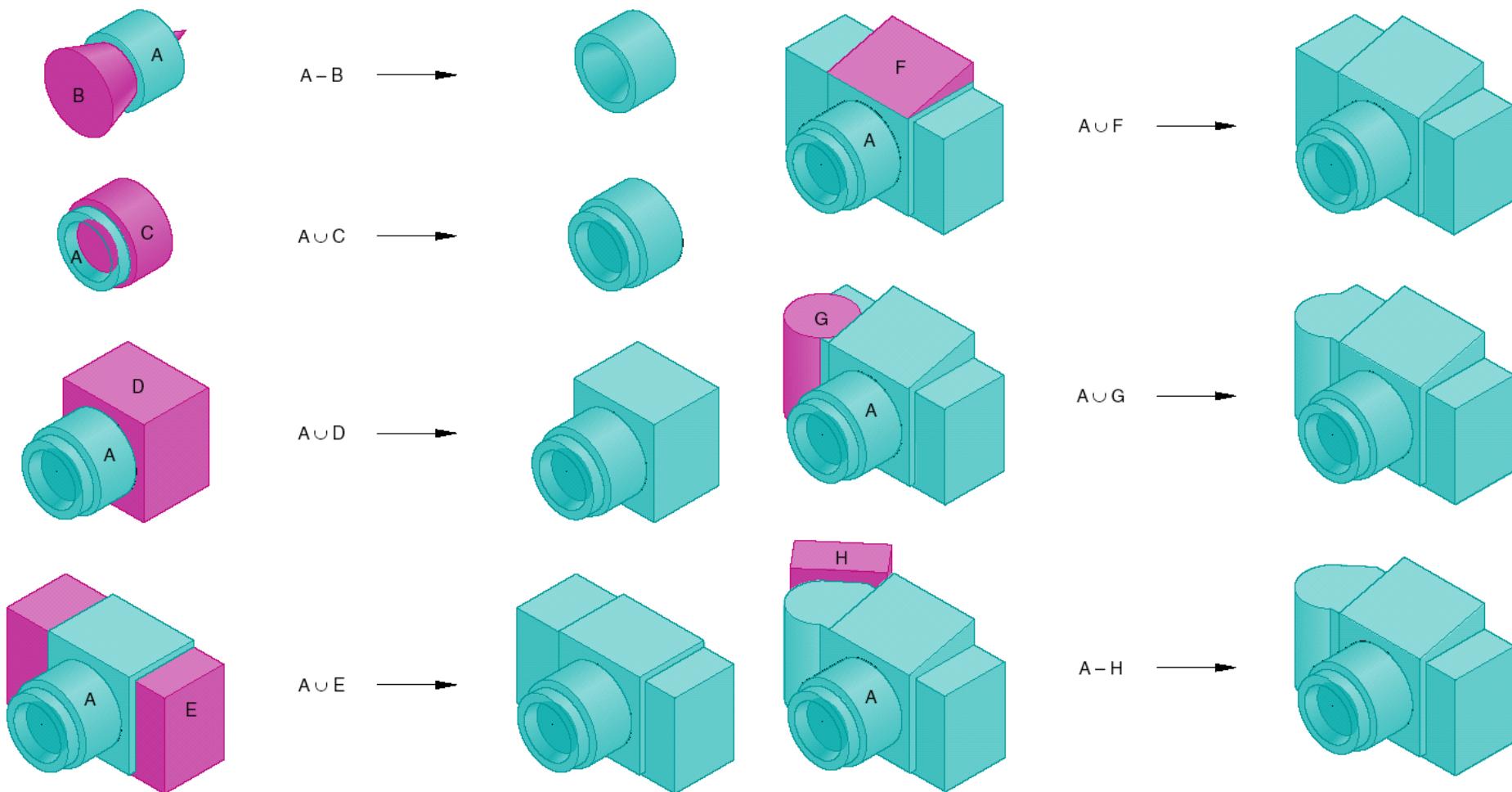
The solid is described as a sequence of operations.

The leaves of the CSG tree are the solid primitives and the nodes indicate the Boolean transformations.

Before carrying out a Boolean operation, it is necessary to position the primitives in space.



## C.S.G. Modeling example

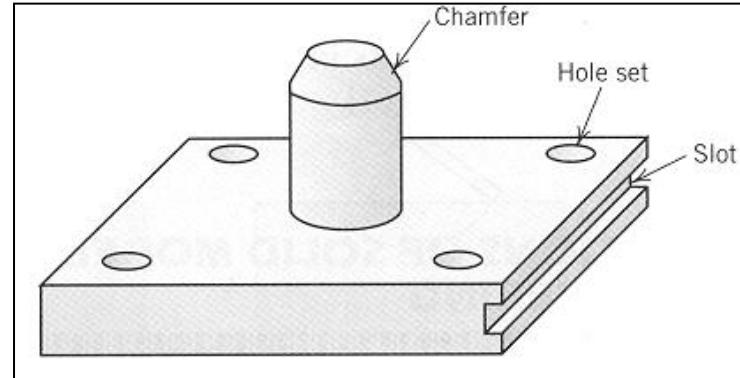


# 3D Knowledge Based Modeling

## Feature Based Parametric Modeling

Strong link between CAD and CAM

The manufacturing characteristics (features) are “embedded” in the CAD modeler

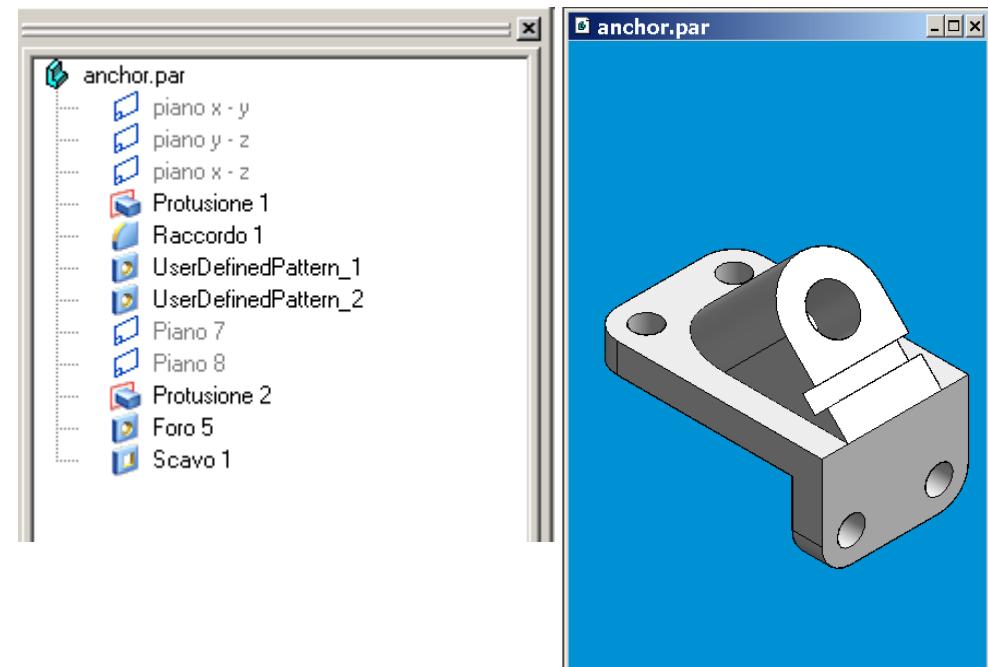


## Feature control by means of parameters: Parametric approach

The features are organized in a tree that represents the history of modeling

Most 3D CAD systems are feature-based

Feature-based modelers use some aspects of CSG and B-rep in a combined way



# 3D Knowledge Based Modeling

## Feature Based Parametric Modeling

- You start with a simple basic model and add features
- Features represent manufacturing “operations” such as holes, ribs, fillets, chamfers, cutouts, etc.
- The material can be added or subtracted, as in the CSG
- Features are not limited to just simple primitives but can be generated by extrusion, sliding, revolving, etc.
- The modeling history is stored in the form of a feature tree like the Boolean tree in the CSG



# 3D Knowledge Based Modeling

## Feature Based Parametric Modeling

The geometry is defined by first creating a 2D profile on a sketching plane which is then “extended” into 3D

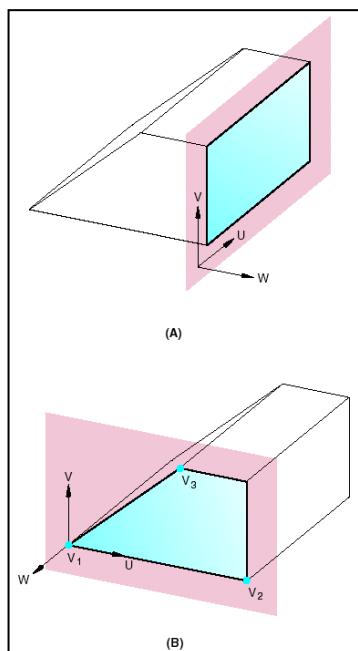
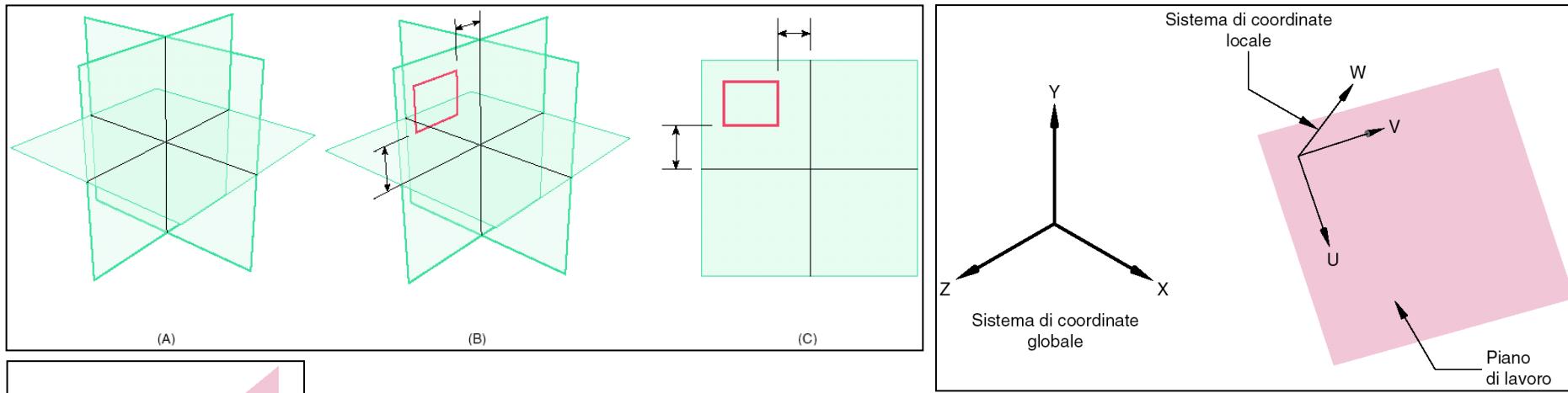
This 3D "extension" can take place by means of:

- *Extrusion (protrusion)*
- *Revolution*
- *Sweeping*
- *Lofting (blending) (passing through multiple profiles)*

These operations can add or subtract material



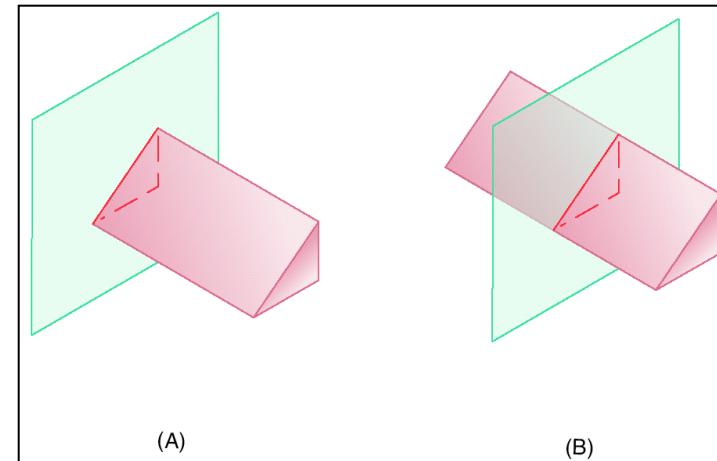
# Feature Based Parametric Modeling



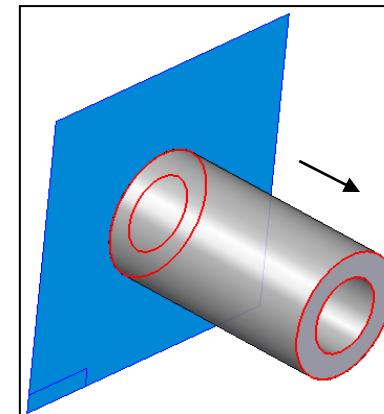
Choice of sketch plane:

- on a flat face
- for 3 points
- normal to a curve
- offset
- tangent
- angled
- ...

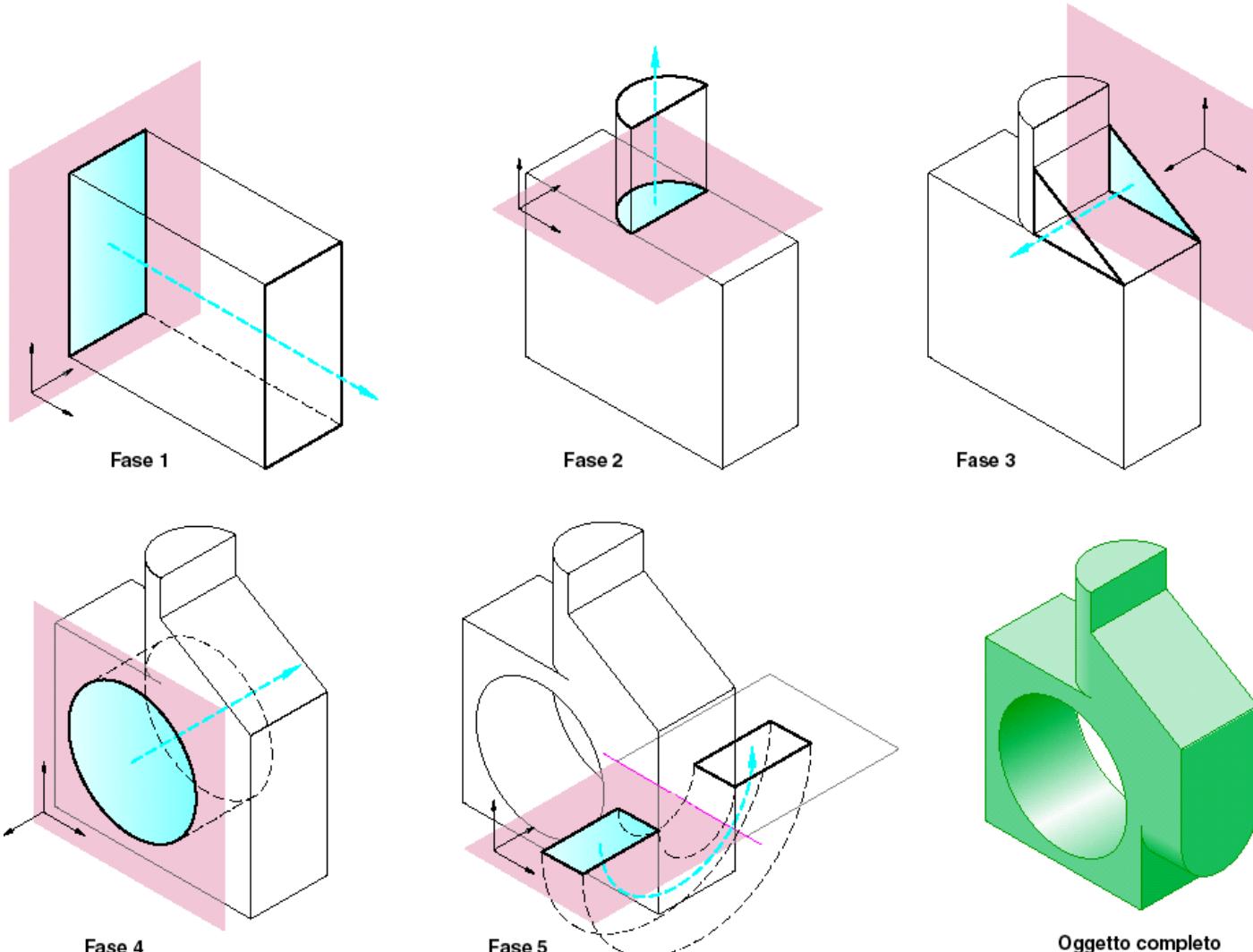
Unidirectional and bidirectional extrusion



Multiple profile

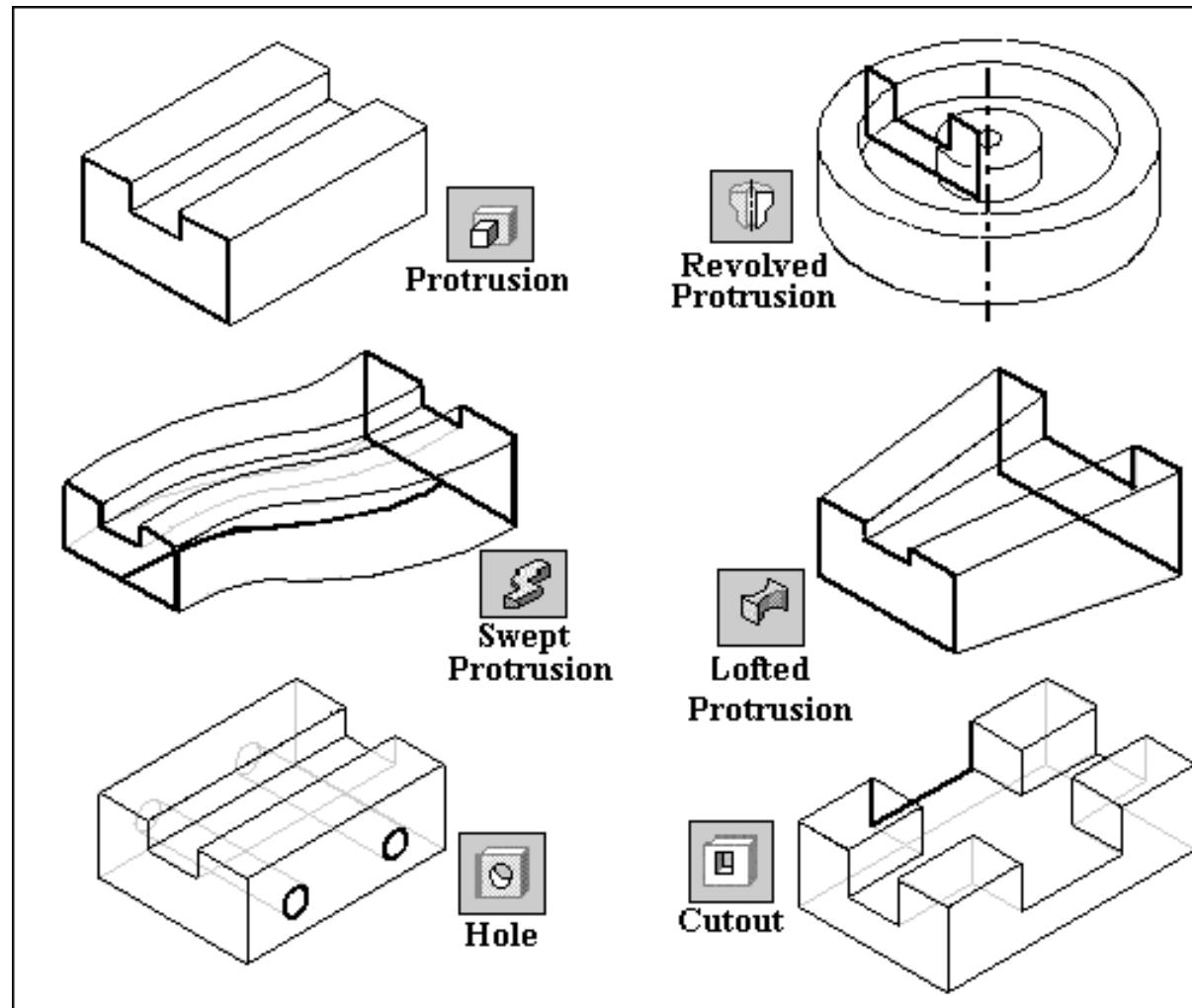


# Feature Based Parametric Modeling



# Feature Based Parametric Modeling

Main Modeling Tools

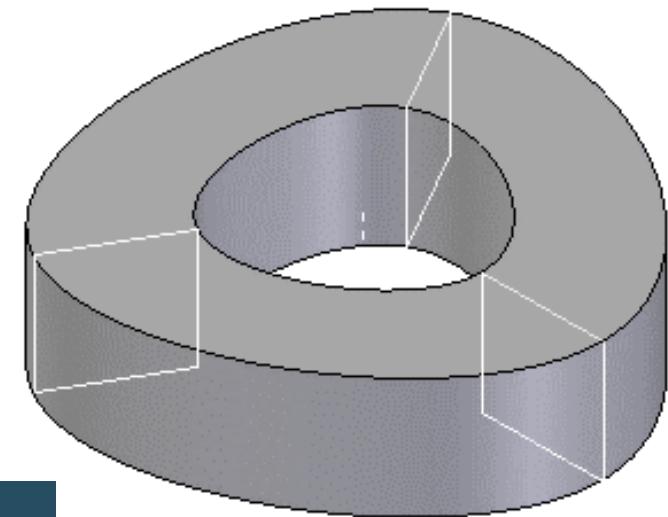
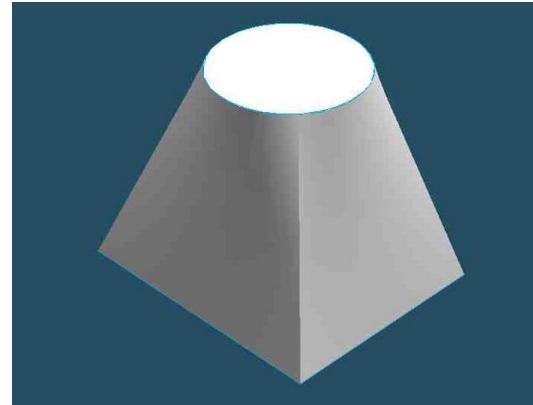
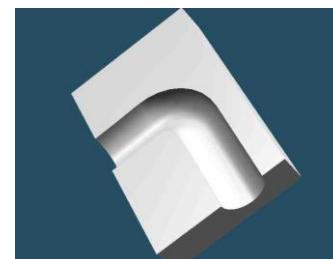
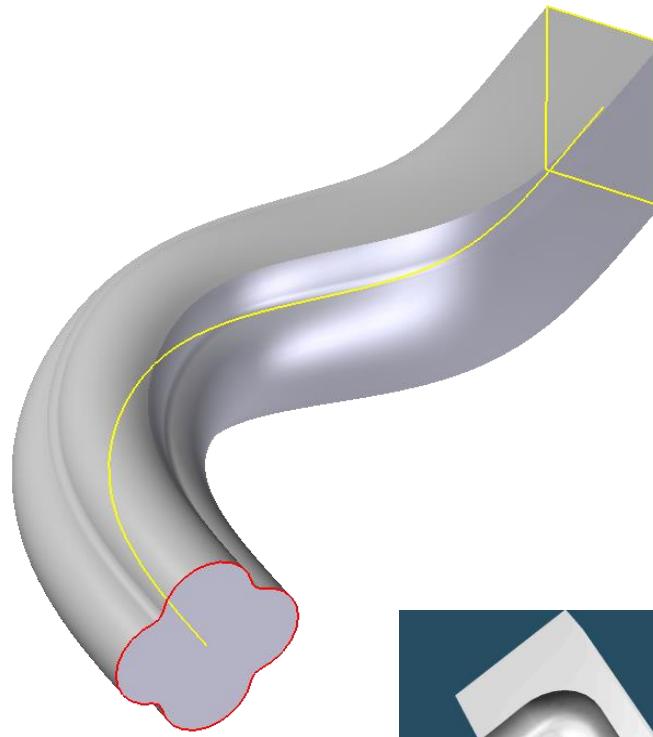


# Feature Based Parametric Modeling

Main Modeling Tools

## SWEET

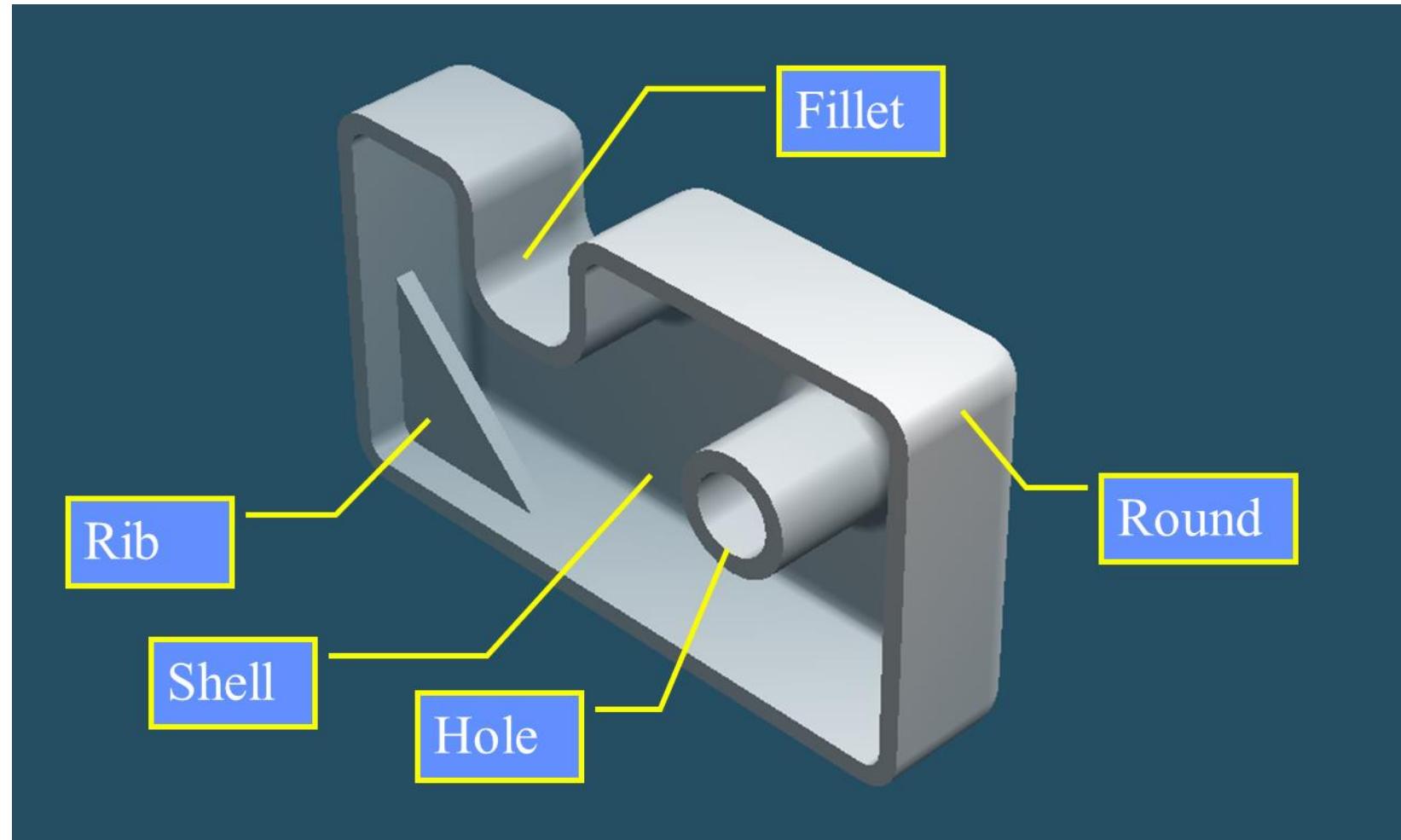
- Sliding profile
- Path curve (spines)
- Eventual guide curve
- Any additional passage profiles



# Feature Based Parametric Modeling

Main Modeling Tools

Fillets, chamfers  
Shells (emptying)  
Holes  
Ribs



# Feature Based Parametric Modeling

## Alternative modeling approaches

- There are usually many ways of modeling the same object
- A good approach takes a few steps and is easy to modify if necessary
- Think about which steps to use in modeling before starting!



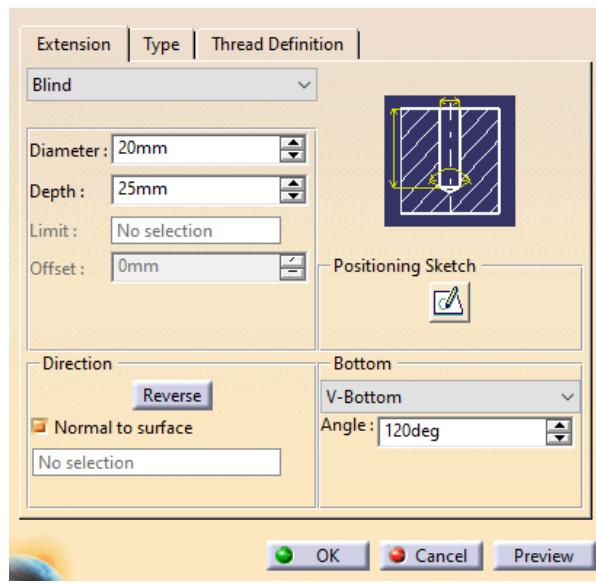
1<sup>st</sup> method



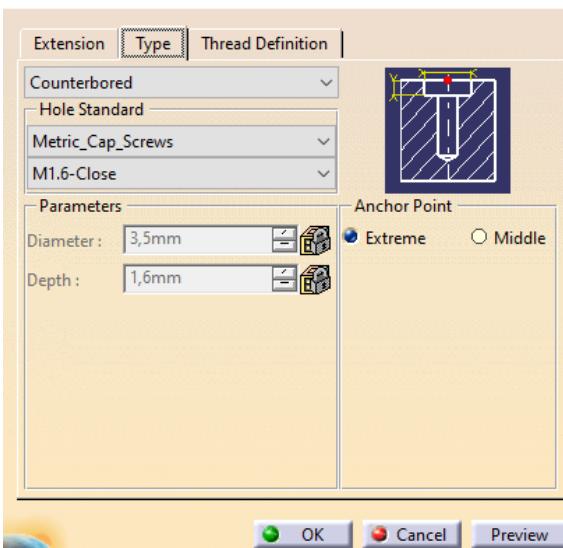
2<sup>nd</sup> method



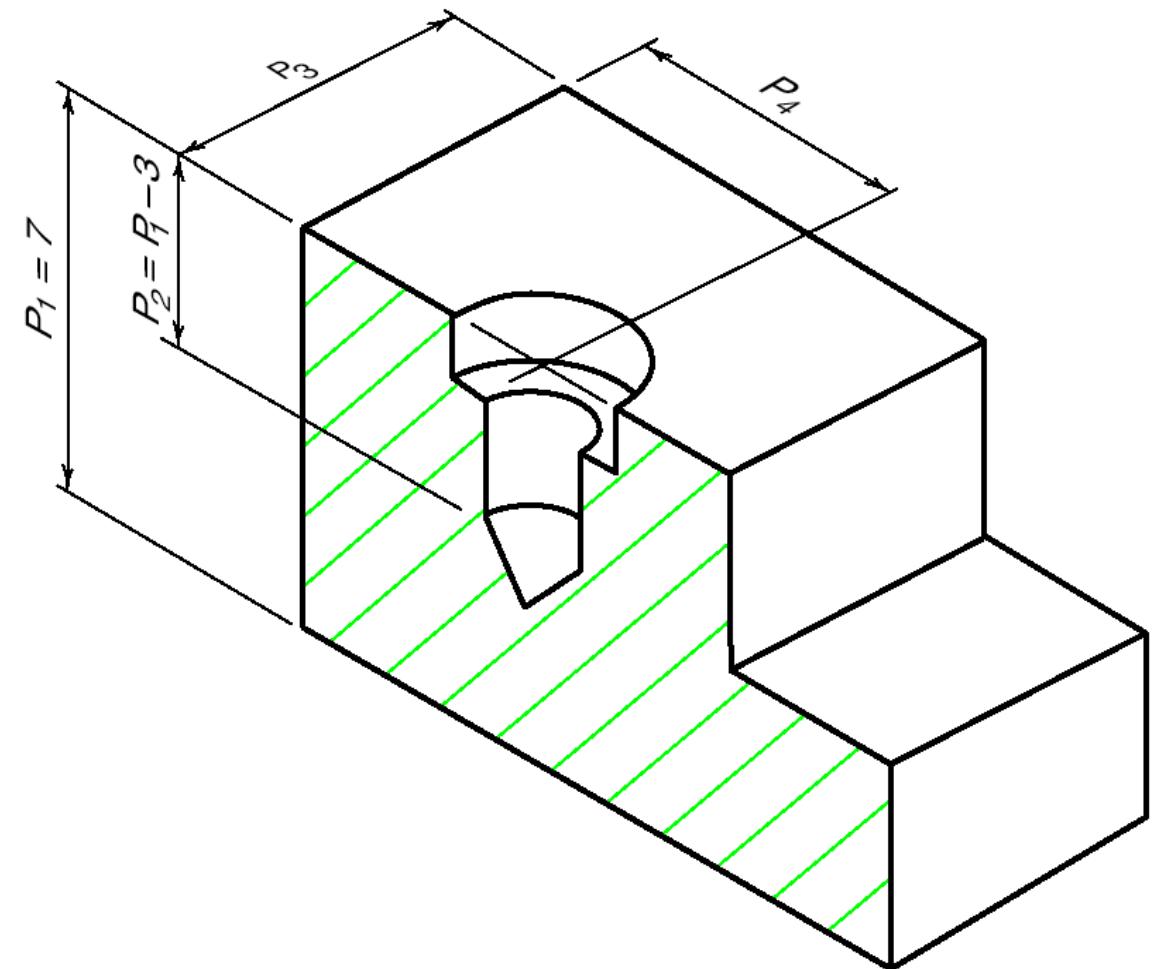
Hole Definition



Hole Definition



## Example of feature : counterbored hole

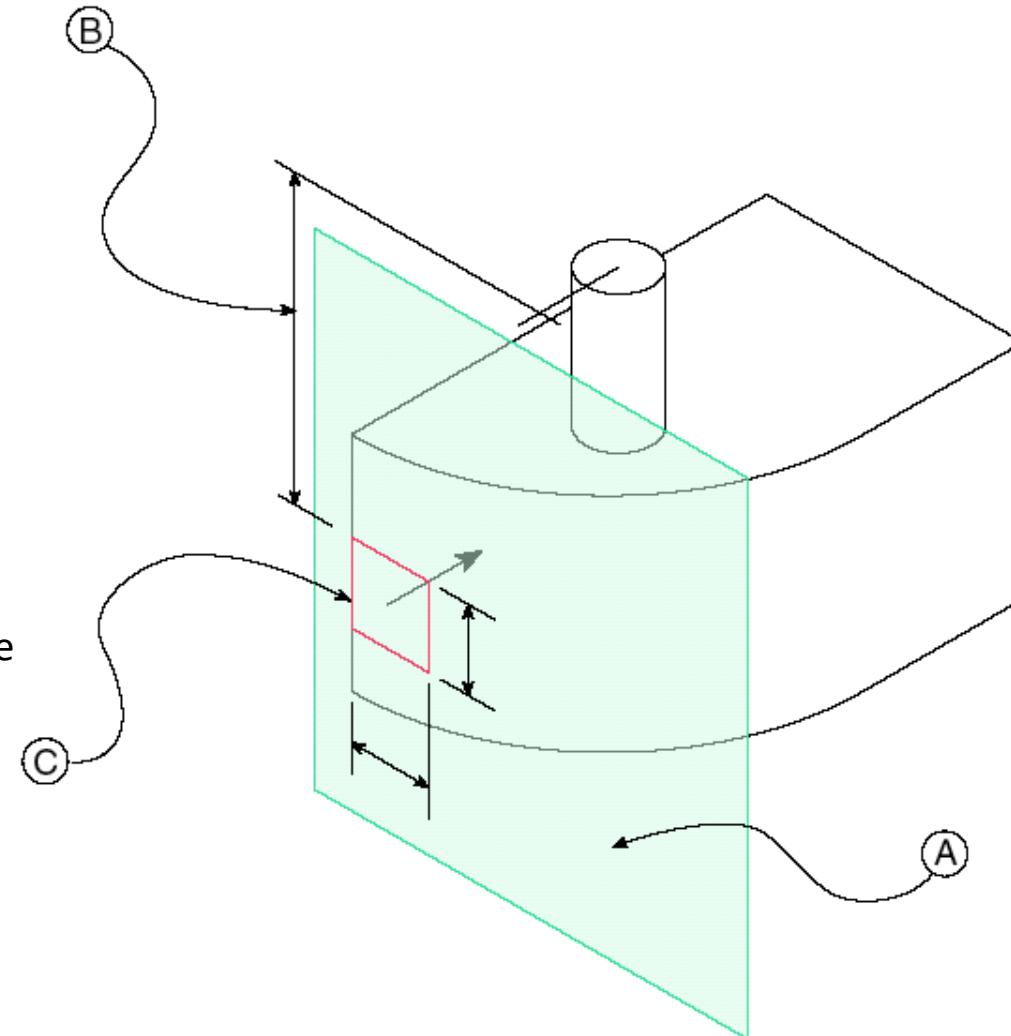


# Parent-child relationship

The "parent-child" relationship creates a close dependence of a feature (child) on those (parent) with which it has been geometrically constrained

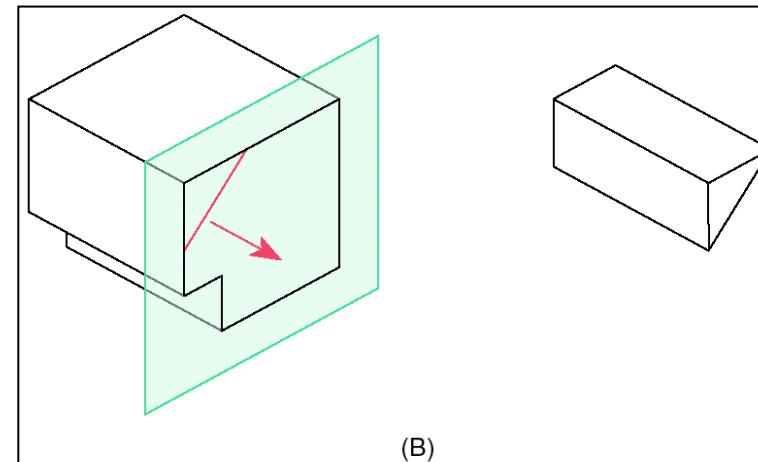
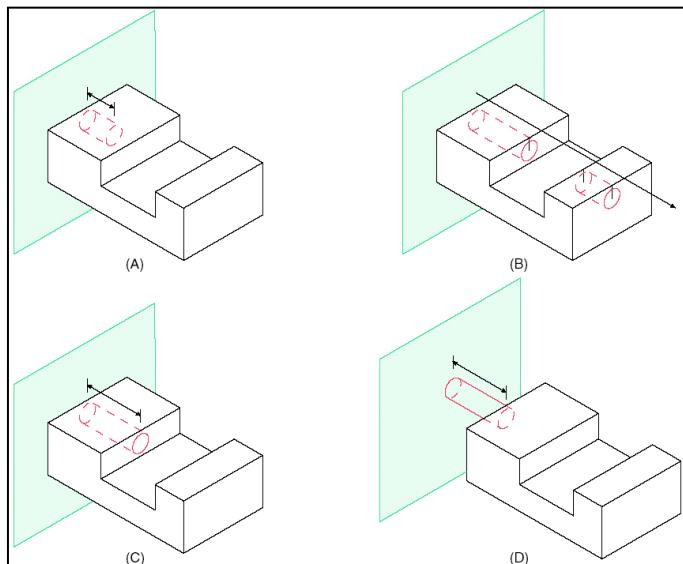
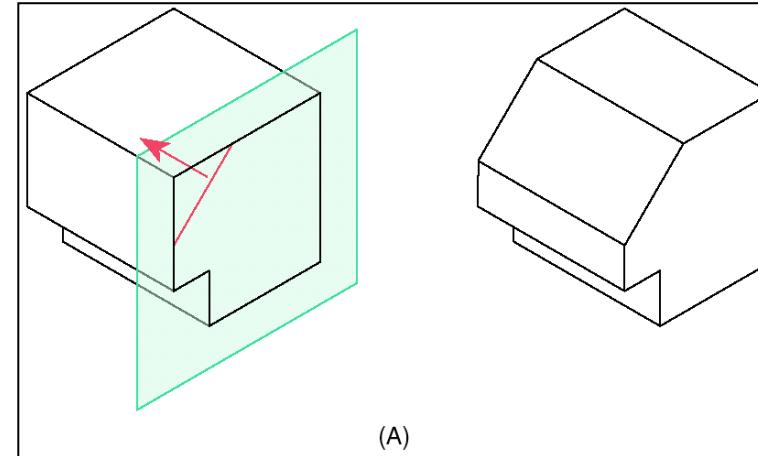
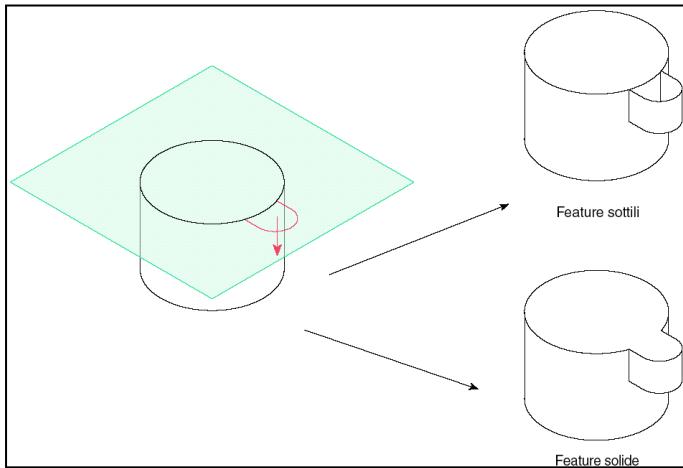
Parents of the new feature:

- A) the sketch plane
- B) The dimensional constraint with respect to an existing feature
- C) The overlap constraint to an existing feature



# Sketch

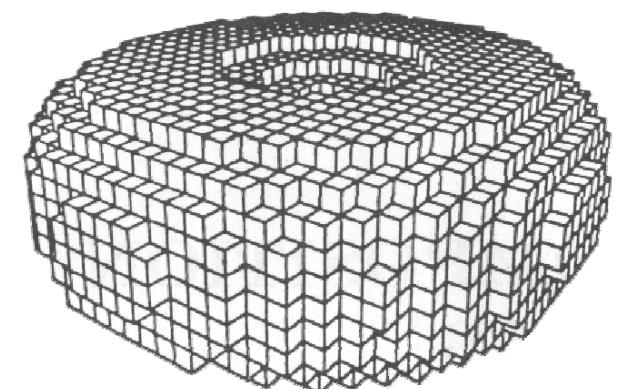
The profile can be constrained to existing geometry and can therefore also be non-closed



# Spatial enumeration

## Voxel Models

- A set of 3D adjacent cells “**approximating by filling**” the space occupied by the solid to be modelled
- Cells are usually cubic, not overlapping, of uniform size (voxel)
- Modeling is done by:
  - Defining a regular grid oriented along the 3 reference axis
  - Setting a TRUE value to all cells containing some material and FALSE to all the others
  - Indexing each cell by coordinates of a predefined vertex.



# Spatial enumeration

## OCTREE Models

Based on recursive subdivision in 8 cubic regions

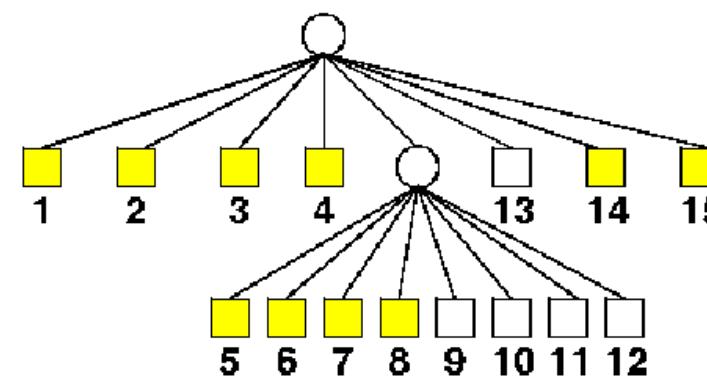
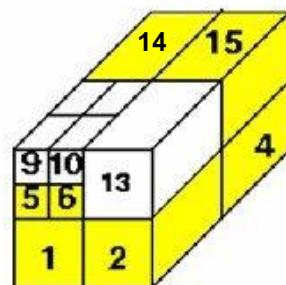
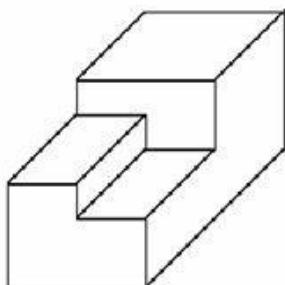
- Optimize memory space
- Improve accuracy

The model is a tree : each node ( father ) has 8 child

The space is recursively subdivided : at each step 8 child are generated

To any child a code is associated defining his status:

- Fully included into the solid
- Completely outside the solid
- Partially including the solid



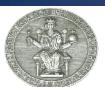
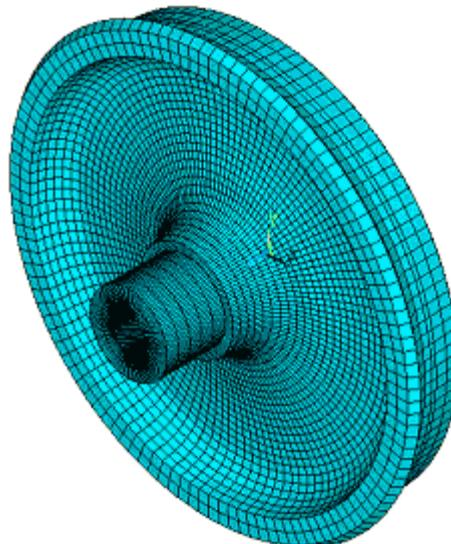
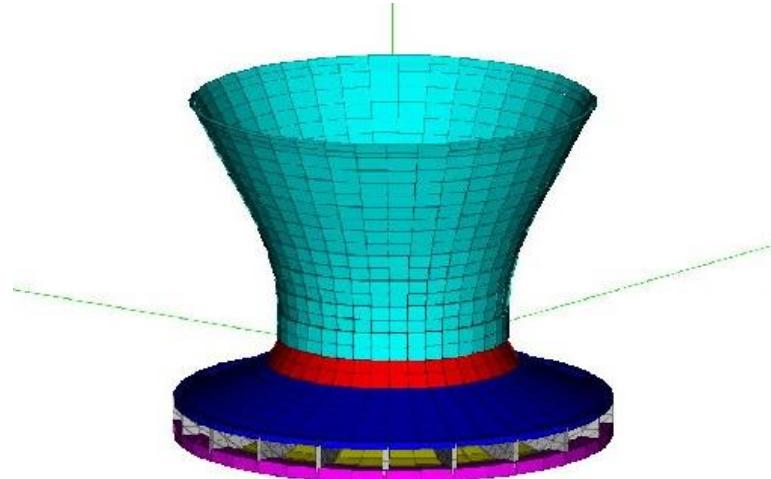
# Spatial enumeration

## Cellular Models

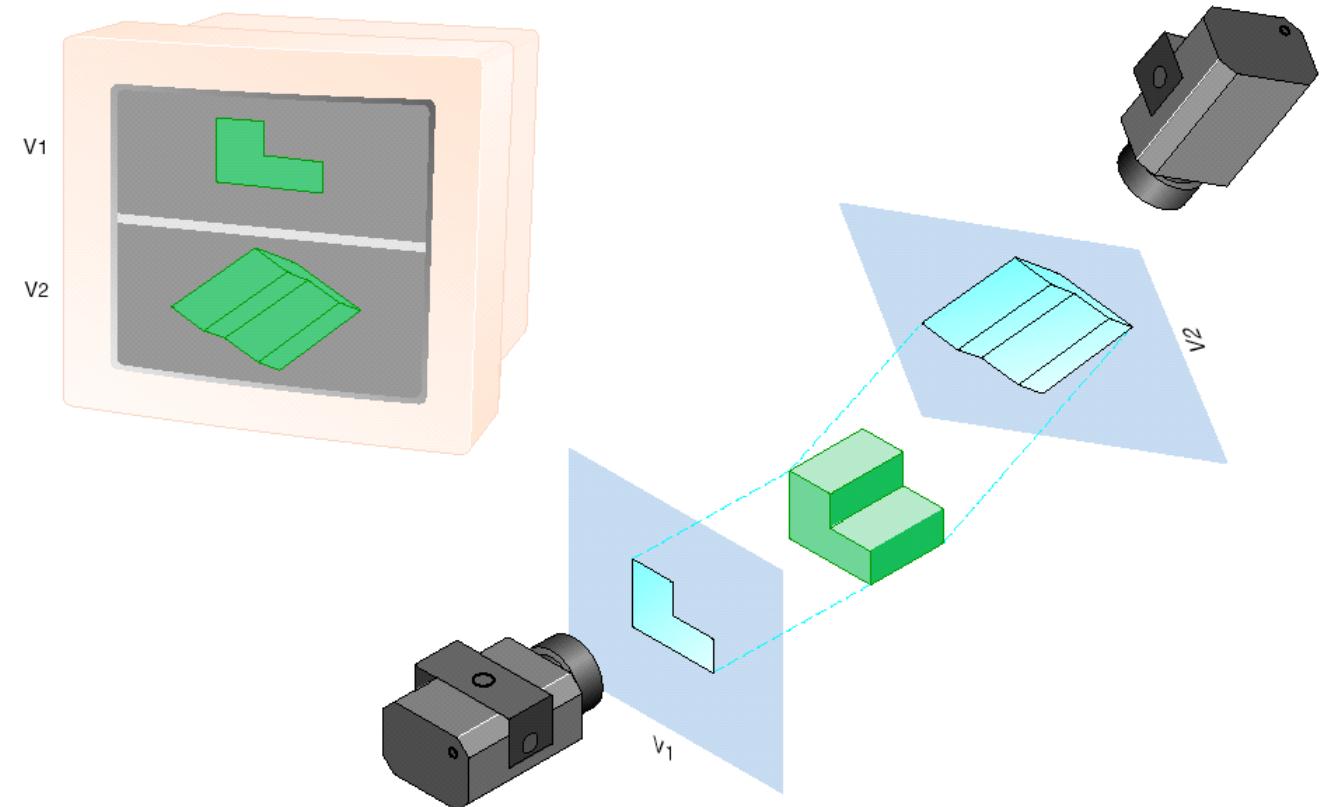
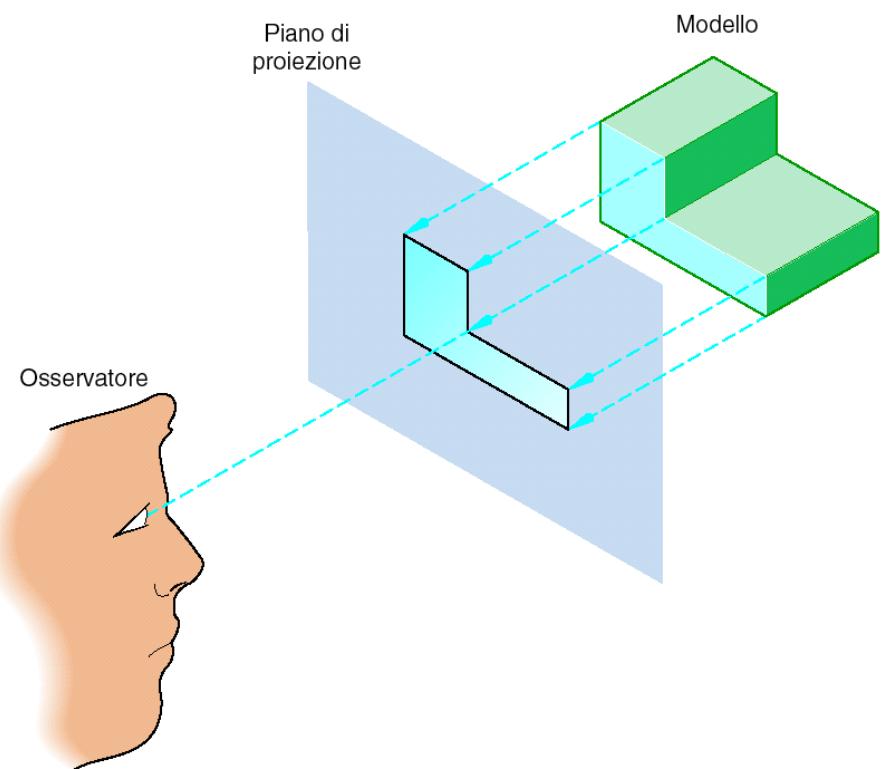
They decompose space by means of blocks of irregular form (tetrahedra, hexahedron) connected to each other.

Example: finite element models.

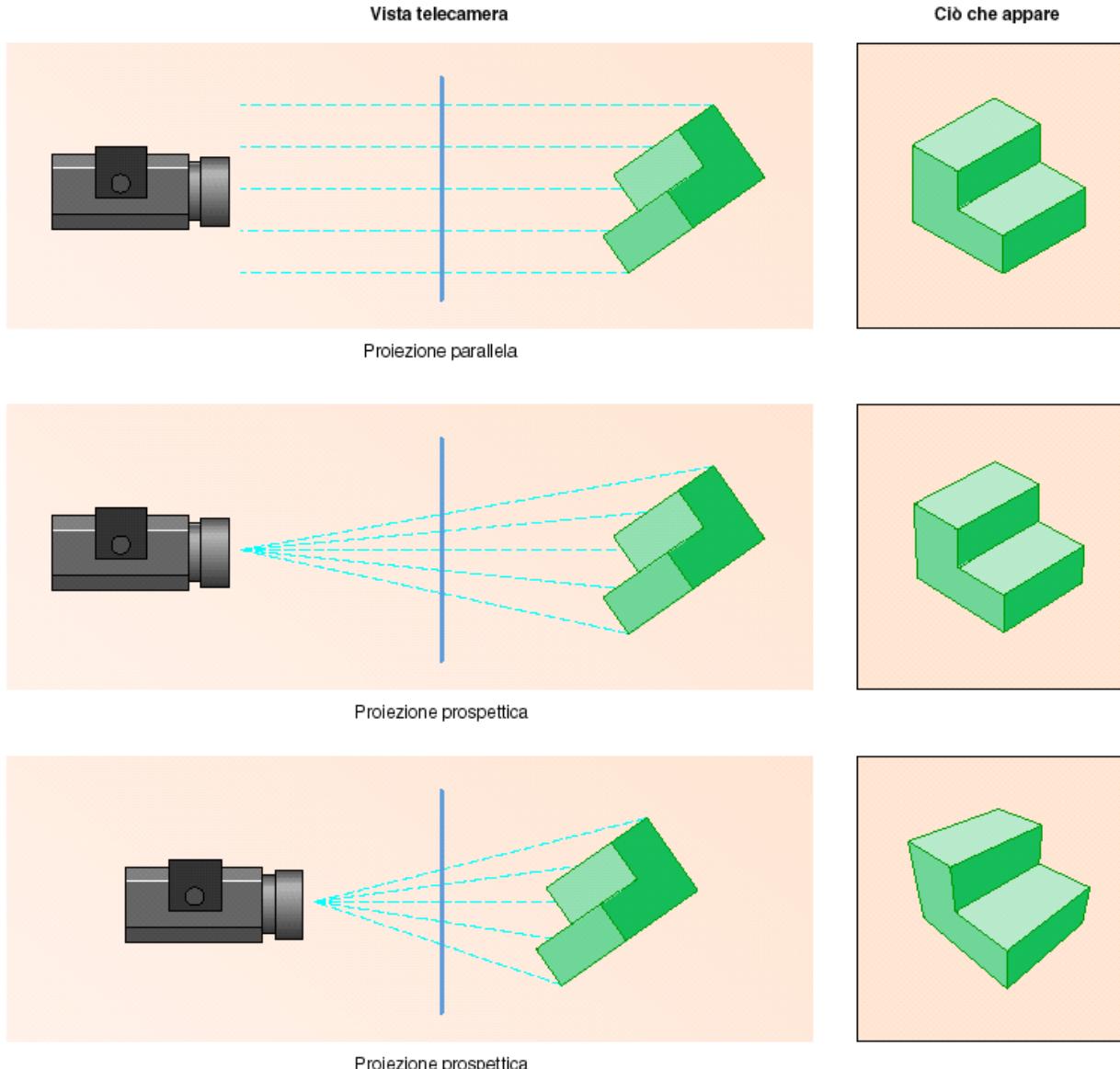
The cells are connected to each other at the vertices



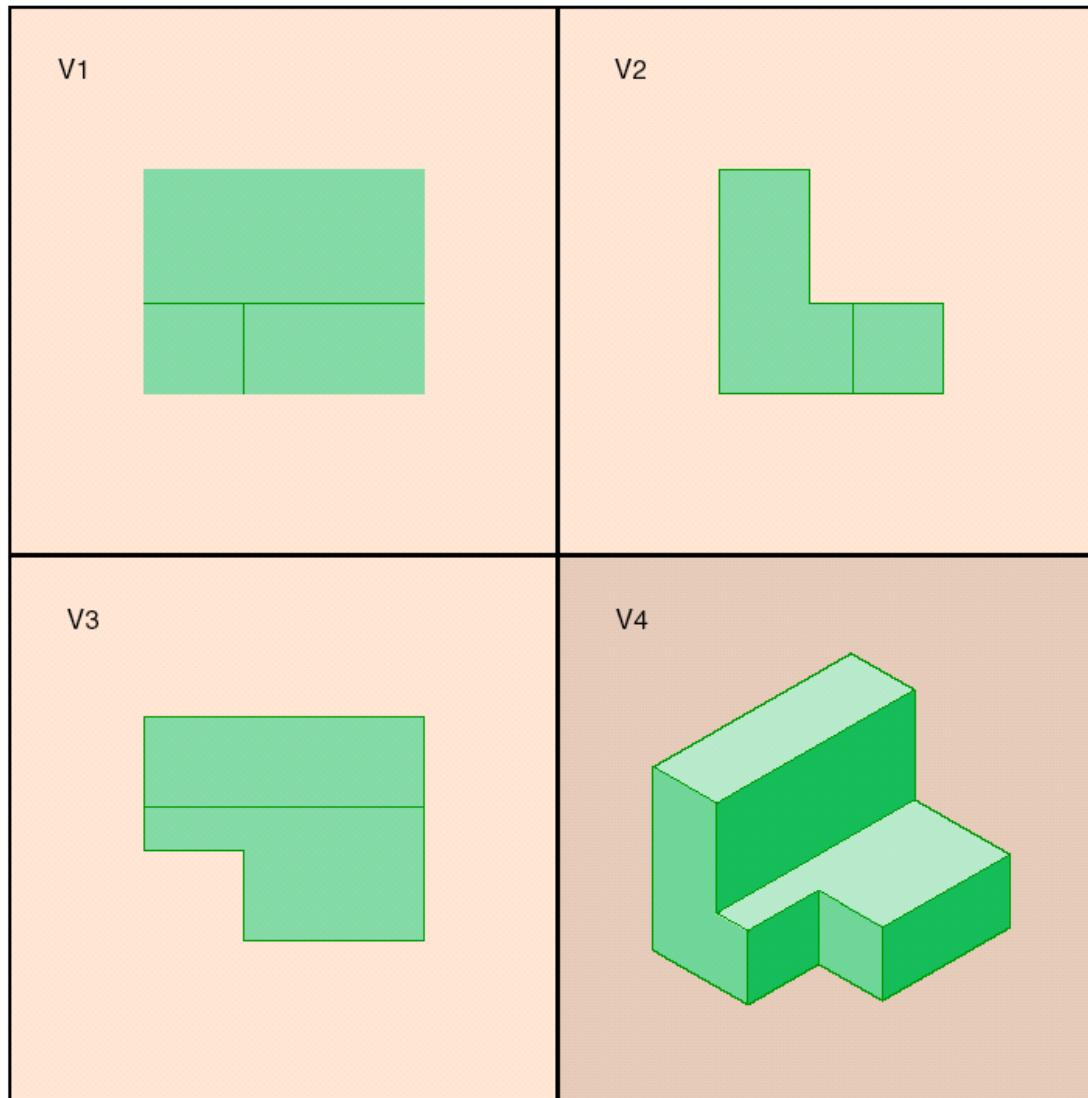
# Display of the model on the screen



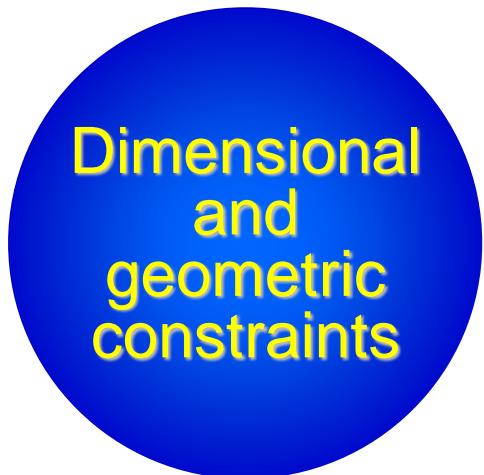
# Display of the model on the screen



# Display of the model on the screen



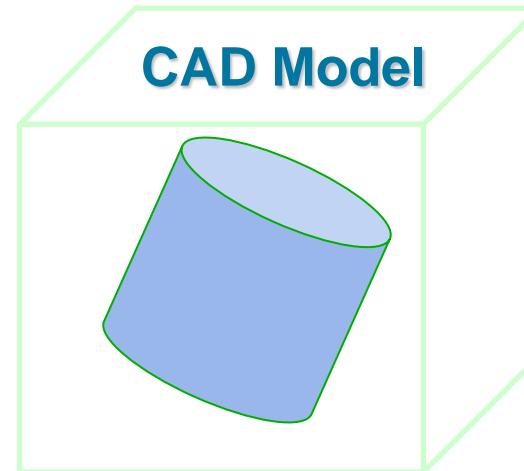
# Parametric vs Variational approaches



**Sketch  
definition**

**Parametric  
approach**

- $N_{\text{unknown}} = N_{\text{equations}}$
- Unique solution
- equations are solved sequentially



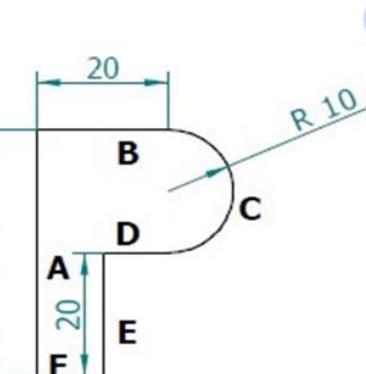
**Variational  
approach**

- $N_{\text{unknown}} > N_{\text{equations}}$
- Infinite solutions
- equations are solved simultaneously

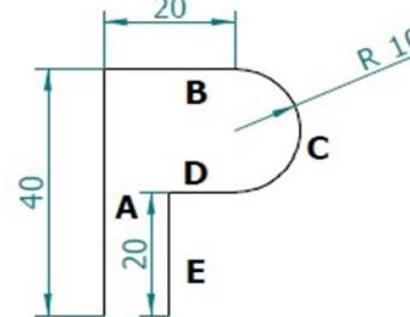
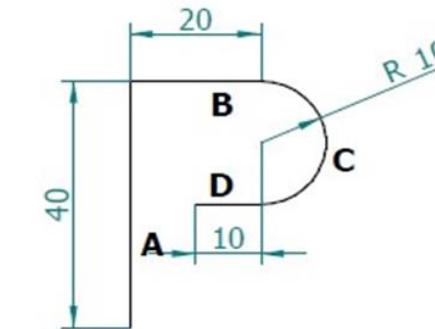
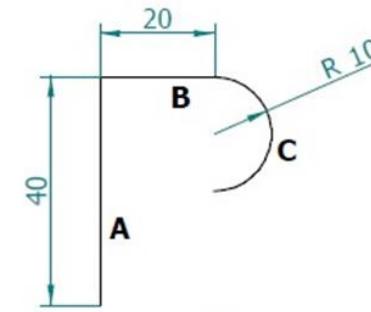
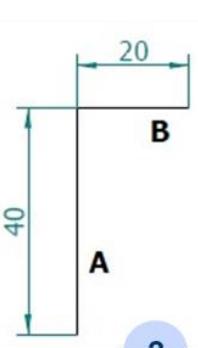


# Parametric vs Variational approaches

## Parametric approach

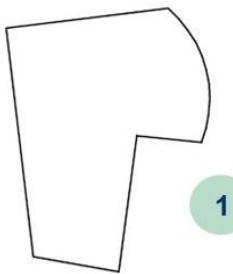


Sequential construction of  
geometric entities, parameters and relations

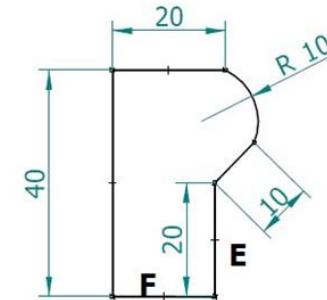
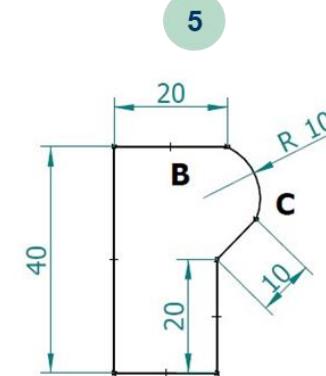
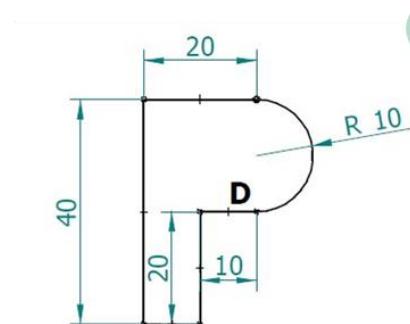
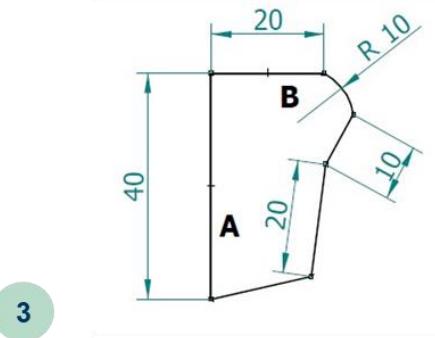
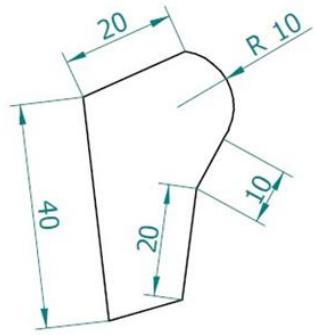


# Parametric vs Variational approaches

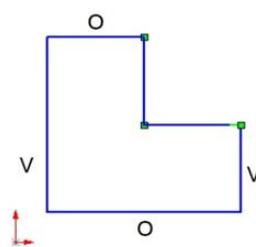
## Variational approach



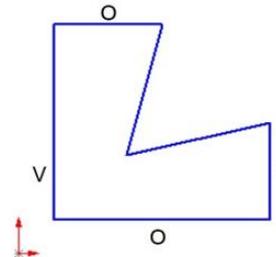
Initial construction of a free form and addition of geometric and dimensional constraints



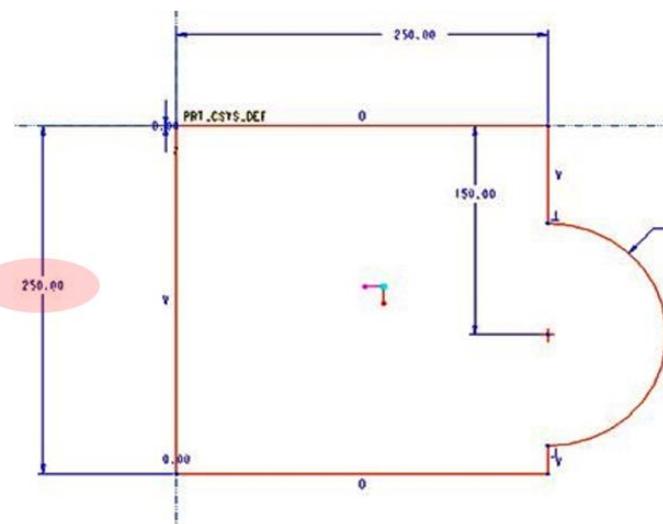
# Parametric vs Variational approaches



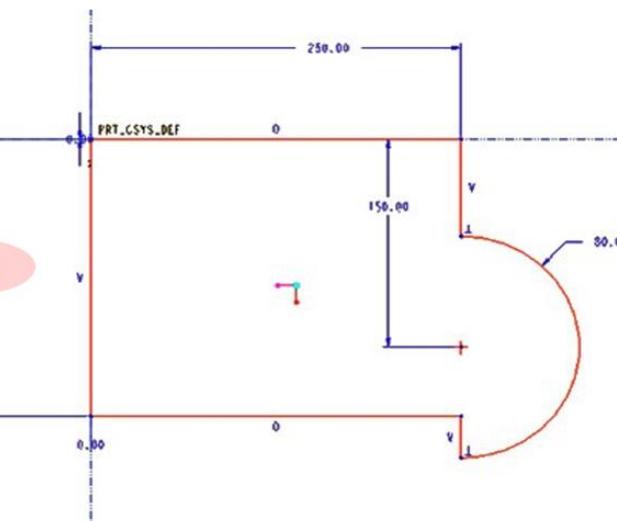
Under constrained sketch



Drawbacks in updating parameters



*Even a fully-constrained sketch  
may not retain the designer's  
intent during modification*



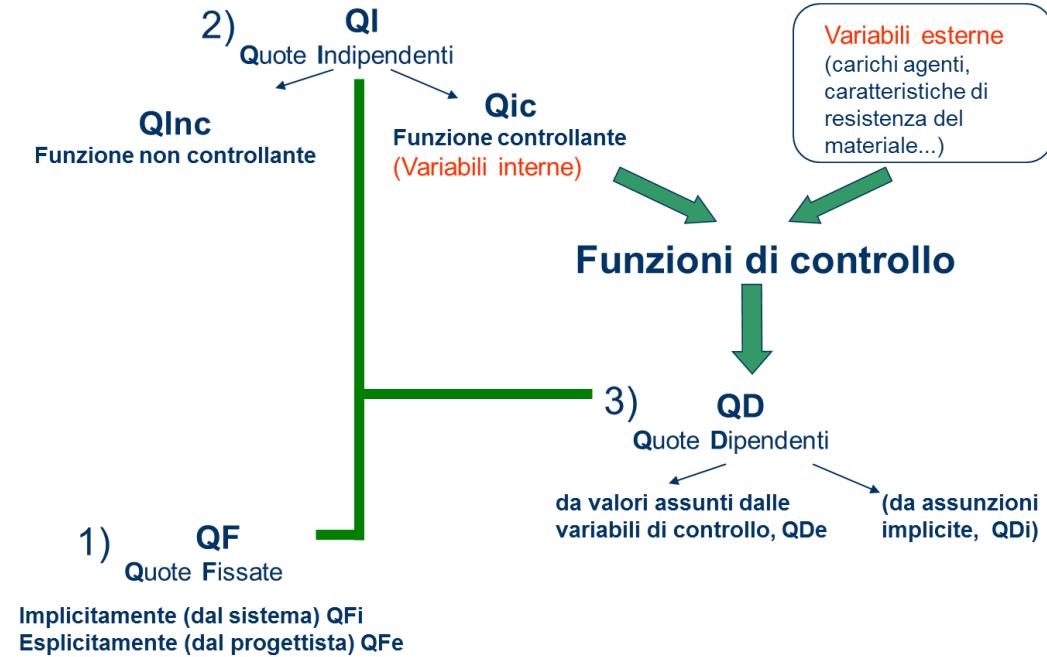
*It is necessary to define  
a parametric-variational  
control system*



# PARAMETRIC-VARIATIONAL CONTROL SYSTEM

The **QF** are quotas "fixed" by morphological-geometric design choice; they are the dimensions that constrain the modeled object to maintain a defined geometric shape. Among them we can distinguish those fixed

- "implicitly" **QFi**, (assumptions made by the modeler during the sketching phase) - for example angles of 90 ° between elements assumed perpendicular to each other;
- "explicitly" **QFe**, those fixed directly by the designer (for example the dimensions of the internal angles of an equilateral triangle).



The **QD** Quotas are "controlled". Among them we can distinguish those:

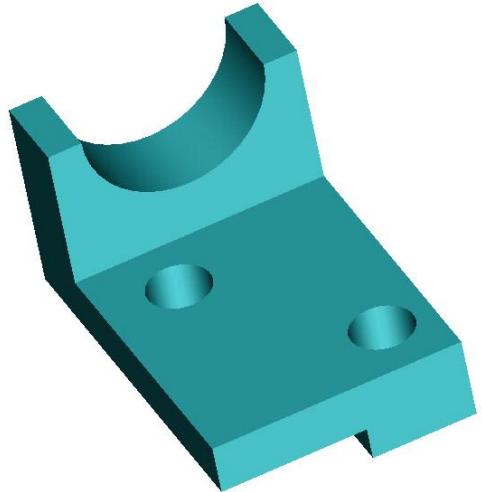
- **QDi** "implicitly assumed"; (assumptions made by the modeler in the sketch phase) - for example, dimensions of elements assumed by the modeler of the same length compared to an element dimensioned by the designer;
- **QDe** "controlled" by values assumed by control factors;

The **QI** are Quotas remained "independent". Among them we can distinguish those:

- **Qic** "parent companies" other quotas; they are quotas whose value determines that of "controlled quotas" **QD**. Control can be exercised through control functions in which the controlling quotas are assumed to control variables of an "internal" type.
- **QInc** "non-controlling" other quotas; they are "isolated" dimensions in the sense that any modification of the value does not involve changes in any of the other dimensions of the model.

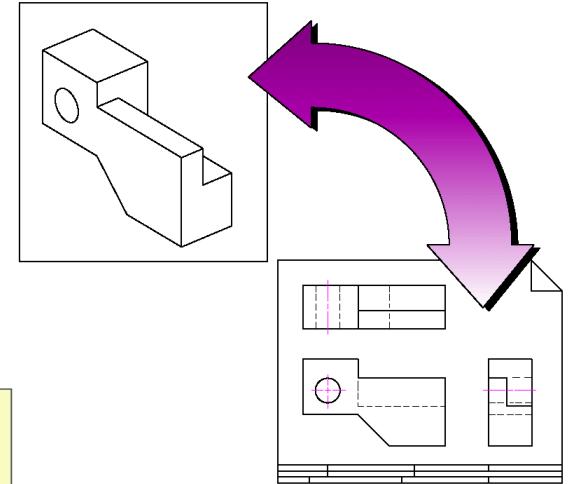
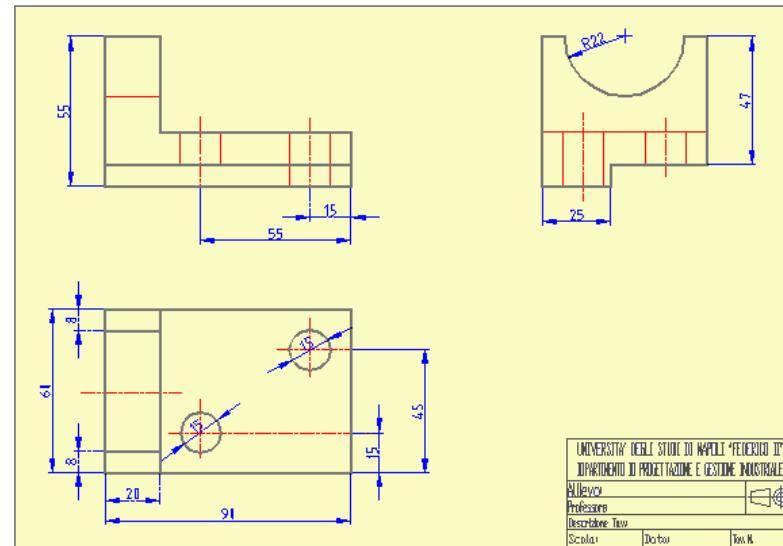


# Concept of ASSOCIATIVITY



3D model

Automatic drawing  
associativity



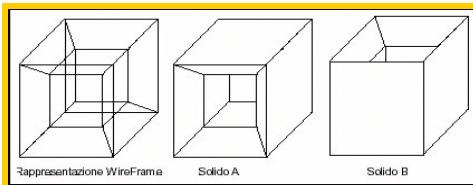
# Modern 3D CAD systems

## Modeling techniques

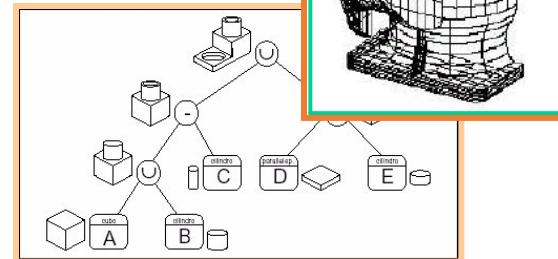
**Computer Aided Drafting**

**Computer Aided Design**

**Wireframe**



**Surfaces**



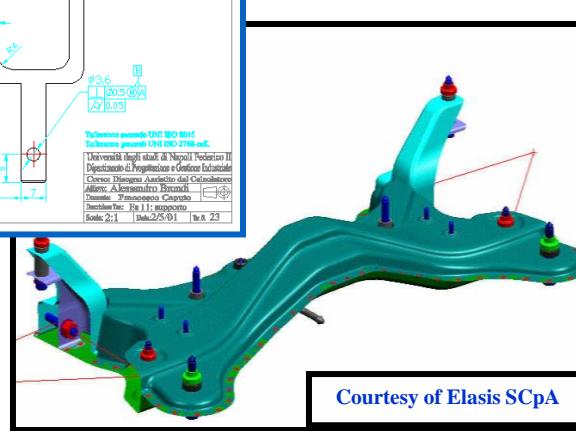
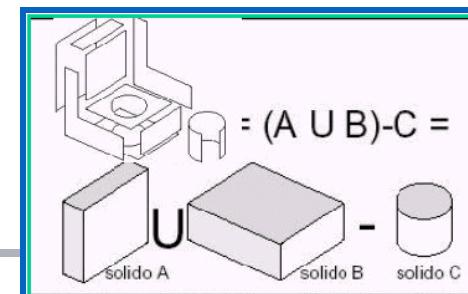
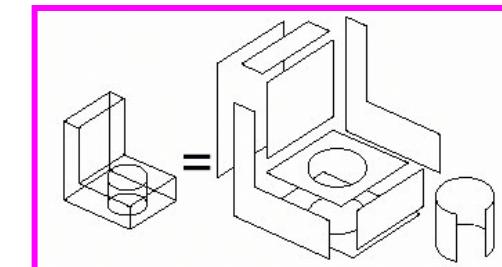
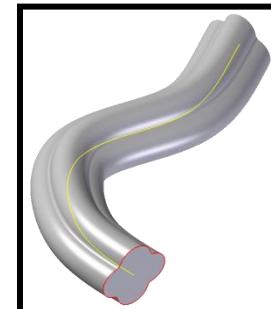
**Solids**

**CSG**

**B-Rep**

**Sweep**

**Hybrid**

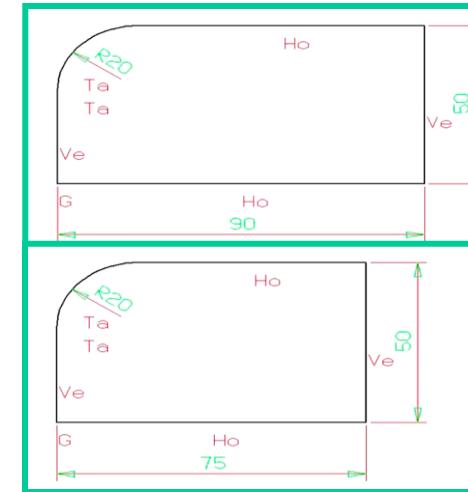
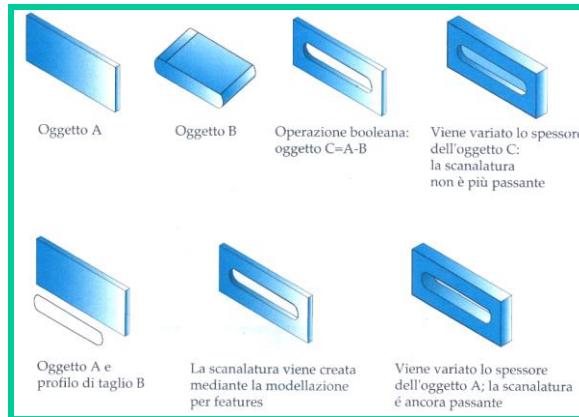


# Modern 3D CAD systems

## Characteristics

**Explicit**  
**Variable-driven**  
**Feature-based**

**Parametric**  
**Variational**  
**Param.\Variaz.**



## Kernel

**Licenced**

**Proprietary**

**ACIS**  
**Parasolid**  
**Open Cascade**  
**SMlib**

CAD application	Kernel		
	ACIS	Parasolid	Proprietary
AutoCAD	•		
CADKEY	•		
CATIA			•
I-DEAS			•
IronCAD	•		
IX Design	•		
Mechanical Desktop	•		
MicroStation		•	
Pro/ENGINEER			•
SolidEdge		•	
SolidWorks		•	
ThinkDesign			•
Unigraphics		•	
VX CAD/CAM		•	

