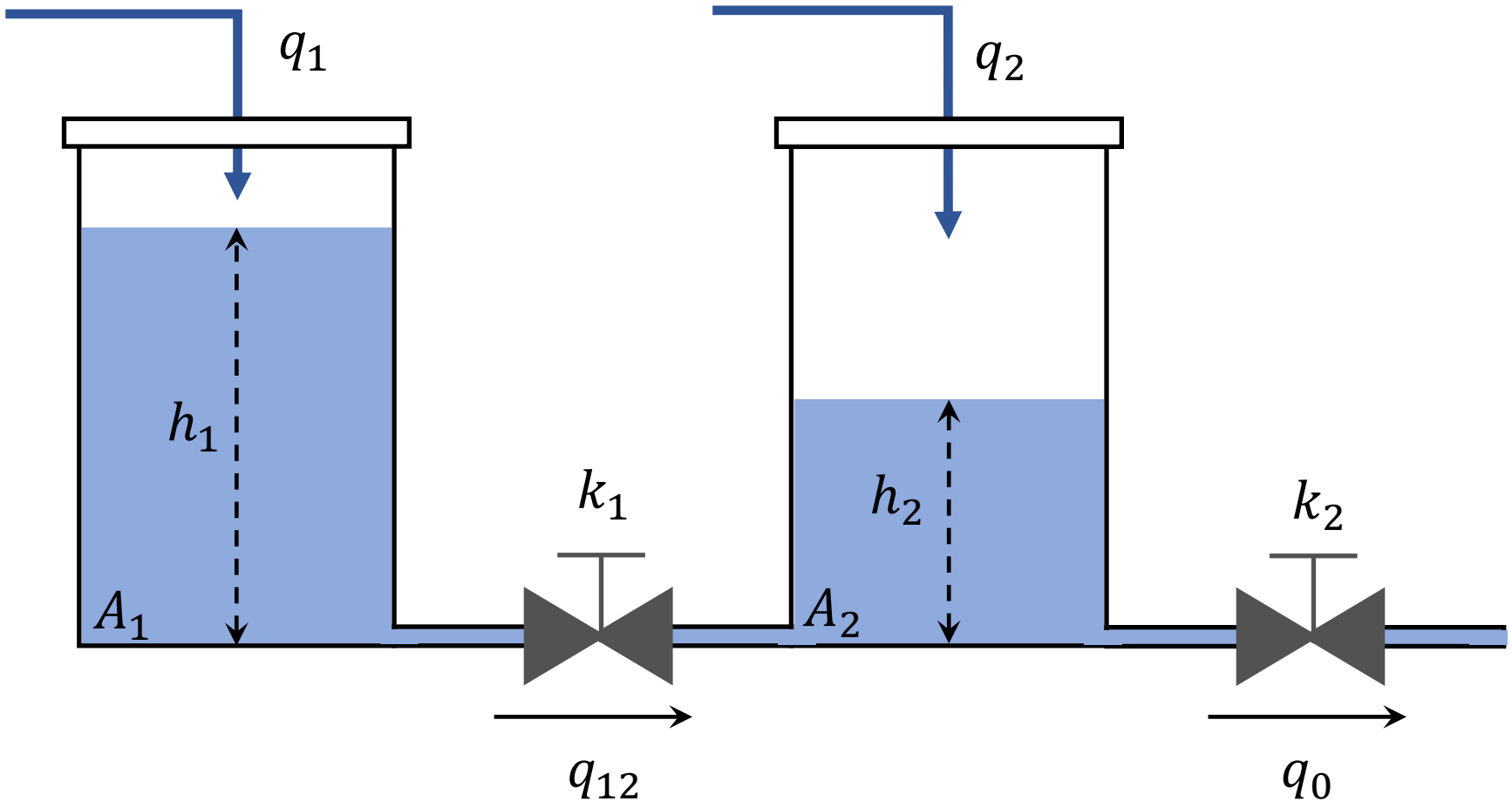
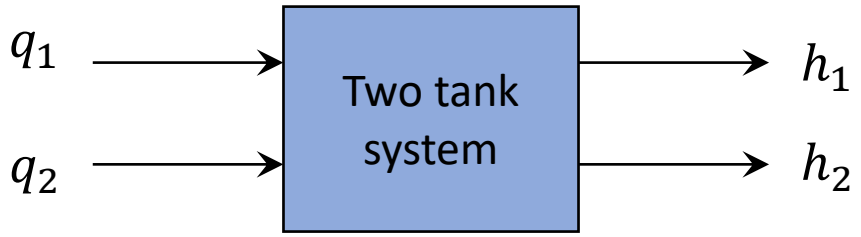
Example

Let’s design a tracking controller for the following two tank system:



Nonlinear model:



$$\dot{h_1} = \frac{1}{A_1}\left(q_1 - k_1 \cdot \text{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|}\right)$$

$$\dot{h_2} = \frac{1}{A_2}\left(q_2 + k_1 \cdot \text{sign}(h_1 - h_2)\sqrt{|h_1 - h_2|} - k_2\sqrt{h_2}\right)$$

Parameter		Value
A_1	Cross sectional area of Tank 1	$0.071\ \text{m}^2$
A_2	Cross sectional area of Tank 2	$0.071\ \text{m}^2$
k_1	Valve coefficient	$0.008\ \text{m}^{2.5}/\text{s}$
k_2	Valve coefficient	$0.018\ \text{m}^{2.5}/\text{s}$

Operating point (steady state, $\dot{h}_1 = \dot{h}_2 = 0$):

h_1^*	h_2^*	q_1^*	q_2^*
0.8 m	0.4 m	$0.0050596\ \mathrm{m}^3/\mathrm{s}$	$0.0063246\ \mathrm{m}^3/\mathrm{s}$

Linearized model around the operating point:

$$\begin{bmatrix}\Delta\dot{h}_1\\ \Delta\dot{h}_2\end{bmatrix} = \begin{bmatrix}-0.0891 & 0.0891\\ 0.0891 & -0.2895\end{bmatrix}\begin{bmatrix}\Delta h_1\\ \Delta h_2\end{bmatrix} + \begin{bmatrix}14.0845 & 0\\ 0 & 14.0845\end{bmatrix}\begin{bmatrix}\Delta q_1\\ \Delta q_2\end{bmatrix}$$

$$\begin{bmatrix}\Delta y_1\\ \Delta y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\Delta h_1\\ \Delta h_2\end{bmatrix}$$

Deviation variables: $\Delta h_1 = h_1 - h_1^*, \quad \Delta h_2 = h_2 - h_2^*, \quad \Delta q_1 = q_1 - q_1^*, \quad \Delta q_2 = q_2 - q_2^*$

Control goal: track step references for the water levels h_1 and h_2 by manipulating the inflows q_1 and q_2