Computergestuurde Regeltechniek
exercise session 2

Pole placement and LQG control design

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In this exercise, we learn about pole placement and LQG control design. First, download the corresponding mat file from the toledo website. Then, in MATLAB type at the prompt:

```
load fileN
```

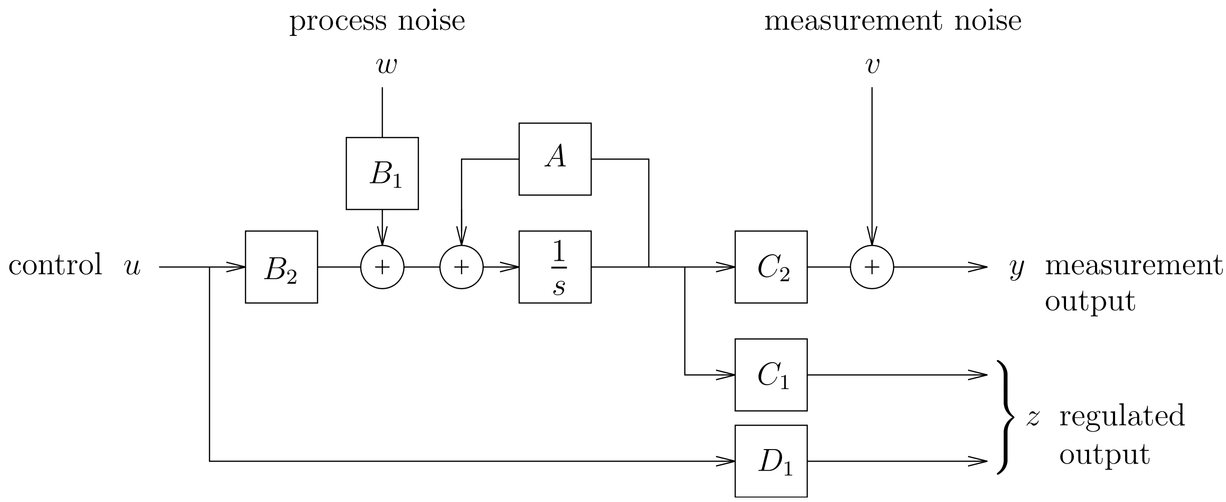
where `fileN` is the name of the .mat-file you just downloaded. This will load your personal MIMO system

$$\left[\begin{array}{c|ccc} A & B_1 & 0 & B_2 \\ \hline C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_1 \\ C_2 & 0 & I & 0 \end{array}\right]$$

into the MATLAB environment. Your state–space system $G(s)$ is described as :

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \\ z &= \begin{bmatrix} C_1 x \\ D_1 u \end{bmatrix}, \\ y &= C_2 x + v, \end{aligned}$$

where x is the state vector, z is the regulated output vector, y is the measurement output vector, u is the control input vector and w and v are white noises. The system is also described by the following figure (standard plant formulation, see also course notes page 191 & 192) :



As you can see, a standard plant has 2 types of inputs : the actuator inputs u and the exogenous or noise inputs w and v, which are called process noise and measurement noise respectively. Keep in mind that only the actuator inputs u are accessible by the controller. There are 2 types of outputs : the measurement or sensor outputs y and the regulated outputs z. The regulated outputs are not measured, but are used in the LQR optimisation criterion and serve as a kind of reference to compare different design results. Matrices C_1 and D_1 can be chosen by you as a designer in order to have your desired closed–loop response. For the moment leave C_1 and D_1 unchanged. You will be allowed to change C_1 and D_1 later on.

Questions

1. Control Law Design : Pole Placement (course notes page 99 – 100, 108 – 110)

   a. Using the Sylvester equation approach, find a state feedback matrix K_1 for the system $G(s)$ such that the poles are placed at $-3 \pm j$ and -1. Find another state feedback matrix K_2 that places the poles at $-10 \pm 2.5j$ and -12.1. Check if the poles of the closed–loop system are indeed in the right position.

      Use **lyap** to obtain the feedback matrices.

   b. Consider the system presented on the previous page. Add K_1 or K_2 to the scheme and close the loop. Now make a step input to the first entry of w. Keep v and the other entries of w equal to 0. Consider the effect on y, z and u. Compare K_1 with K_2. In order to solve question 1b you first have to derive an appropriate state–space model for the closed–loop system.

2. Control Law Design : LQR (course notes page 145 – 146)

   a. Find an optimal LQR control law K for the system $G(s)$, based on the C_1 and D_1 you loaded into the MATLAB workspace. The cost function to be minimised

is $\int_0^\infty (z^T z)dt$.
Use **are** to obtain the feedback matrix.

b. Obtain the same K, now using the **lqr** command.

c. Change the objective function by playing around with the weighting matrices Q and R as defined on page 145.[1] Plug K into the scheme and close the loop. Make an impulse input to the first entry of w. Keep v and the other entries of w equal to 0. What is the influence of Q and R on the impulse response ?

3. Observer design : Pole Placement (course notes page 162 – 168)

a. For your system $G(s)$, make a state observer L_1 whose dynamics are slightly faster than the closed loop poles obtained from K in 2a. Design another observer L_2 whose dynamics are about an order of magnitude faster than those of L_1. Use **place** to obtain the estimator gains.

b. Consider the error system which describes the estimator dynamics (cf. course notes page 167) :

$$\begin{aligned} \dot{\tilde{x}} &= (A - LC_2)\tilde{x} + \begin{bmatrix} B_1 & -L \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}, \\ z_{\tilde{x}} &= I\tilde{x}, \end{aligned}$$

where L is the observer feedback gain, and $\tilde{x}$ is the difference between the actual states x and the estimated states $\hat{x}$. The equations describe the time evolution of the estimation error. Apparently, the evolution of the estimation error is only determined by the measurement and the process noise. Make sure that you understand where the estimator equations come from.
Plot an initial condition response, with $x(0) = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$ and with w and v white noises. Adjust the noise power and choose an appropriate time scaling in order to see the effect of both the initial condition and the noise. Compare the estimation error for both observers (L_1 and L_2) by simulating the error system. Try to obtain a plot similar to the one you find in the course notes on page 168. Discuss.
Use MATLAB command **lsim** for the simulation and **randn** to obtain w and v.

4. Observer design : Kalman Filter (course notes page 183 – 184)

a. Let w and v be white noises with unit covariance[2]. Find an optimal state observer or Kalman filter L such that the objective function $\int_0^\infty (x-\hat{x})^T(x-\hat{x})dt$ is minimized.
Use MATLAB function **lqe** to find the optimal observer feedback gain.

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[1] If you don't know what Q and R are : type **help lqr**.
[2] i.e. $Q = R = I$. If you don't know what Q and R are : type **help lqe**.

b. Play around with the noise covariance matrices Q and R. Assume for instance that there is more process noise than measurement noise (or vice versa) entering the system. Now design two Kalman filters, the first one based on the correct information, i.e. with Q (large) and R (small) being the correct estimates for the process and measurement noise covariances, and a second observer which is optimally tuned for a different setting, e.g. Q small and R large. Check the noise robustness of both estimators.

5. Compensator Design : LQG

   a. Now find the closed–loop system combining the state feedback K with the estimator L. Derive a state–space model which relates the inputs w and v to the outputs z.

   b. What are the poles of this closed–loop system ?