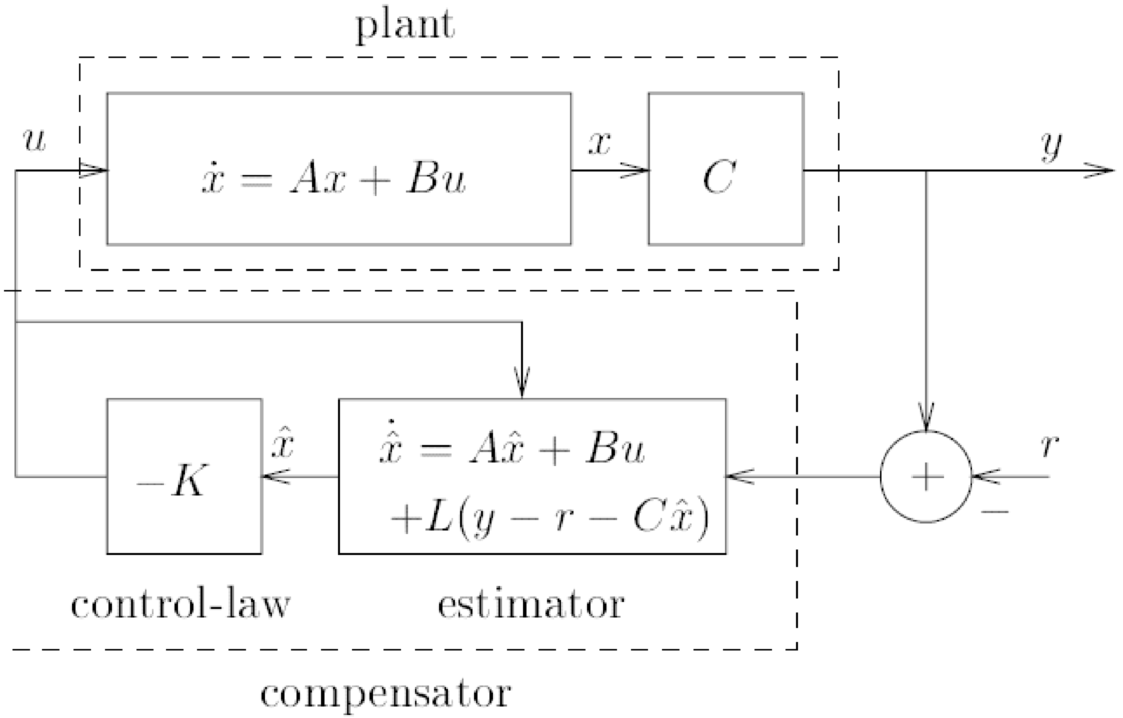
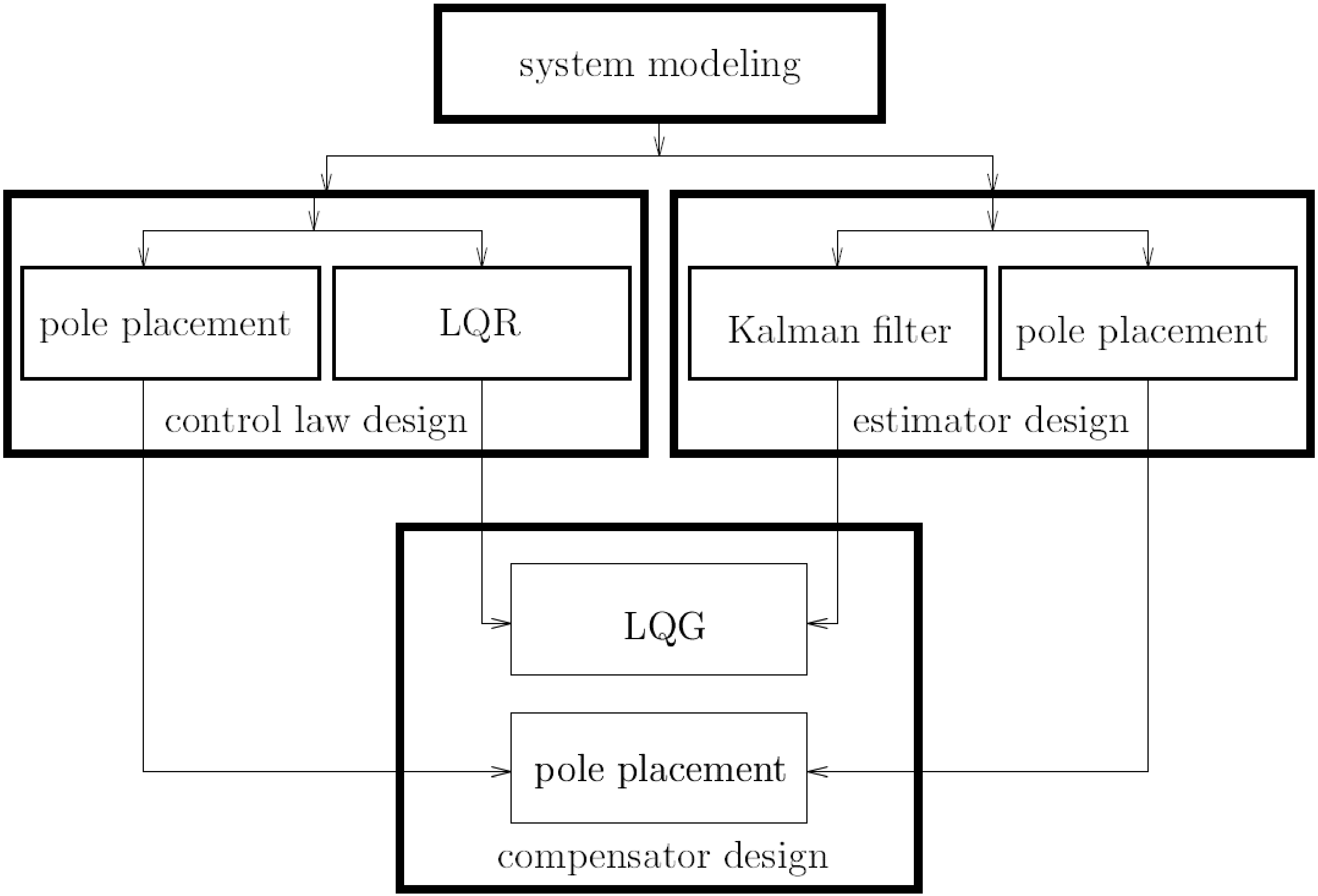
SESSION 2: Overview problem solving

- **Control system design by state feedback**
  1. **Control-law design: state feedback**
  2. **Estimator design: estimation of the state vectors**
  3. **Compensator design: combination of control law and estimator**

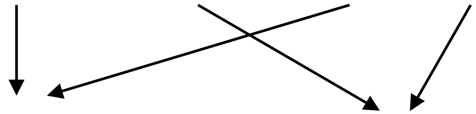


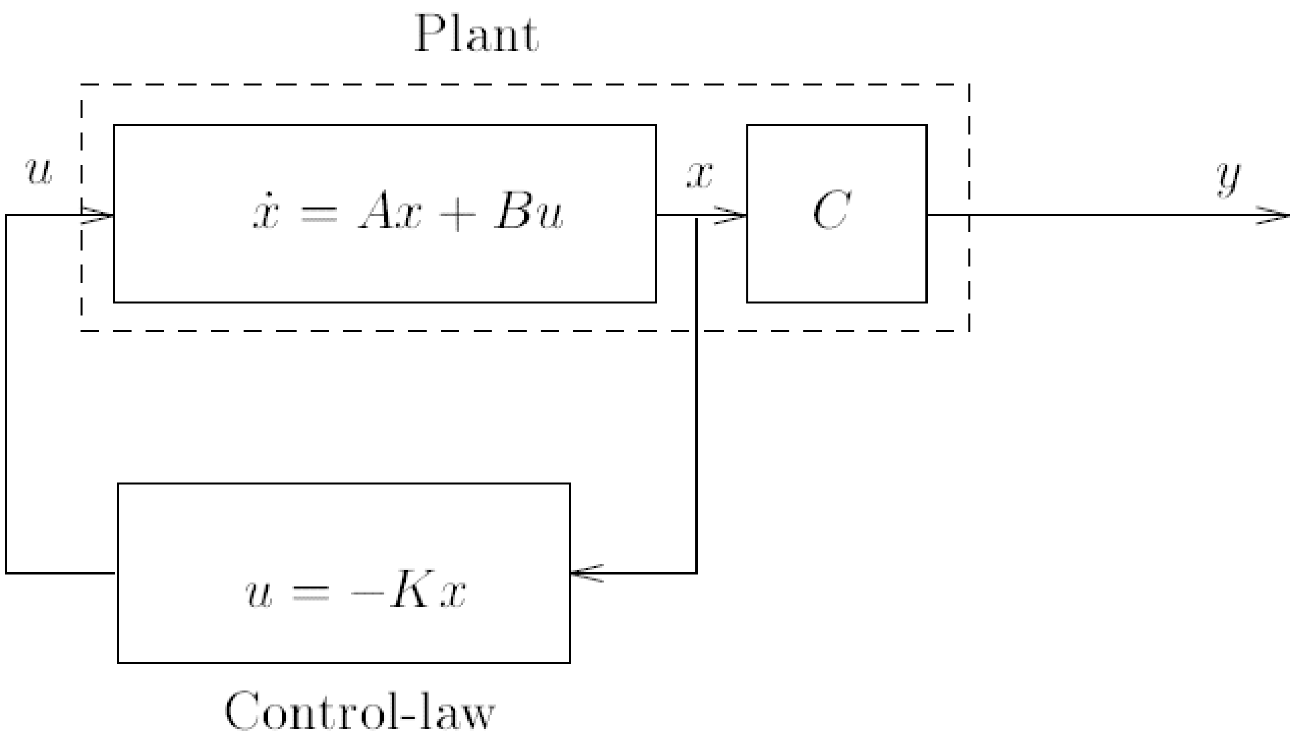
SESSION 2: Overview problem solving



Control law

- **u = - K x** ⇨ (**A-BK**, B, **C-DK**, D)





Control law – pole placement

- Closed-loop system: $dx/dt = (A - BK)\,x$
- Poles = roots of $\det(sI - (A - BK))$
- 3 ways to choose K such that poles are in specified positions

1. **Direct** pole placement:

   $\det(sI - A + BK) = (s - s_1)(s - s_2)\ldots(s - s_n)$

2. **Ackermann**: control canonical form
3. **Sylvester**: similarity transformation $X^{-1}(A-BK)X = S$

Course notes p. 60 + 102 - 110

Control law – pole placement

- **Ackermann pole placement**
  - transformation to control canonical form
  - $\det(sI - A_c + B_cK_c) = (s - s_1)(s - s_2)\ldots(s - s_n)$
  - system (A,B) must be controllable
  - controllability not affected by nonsingular transformations
  - For MI, work with derived controllable SI system:

    $(A - BK_r\ ,\ Bv) \Rightarrow A - BK_r - (Bv)K_s$

    $\Rightarrow A - B(K_r + vK_s) = A - BK$
  - disadvantage: lacks numerical accuracy for large number of states

Control law – pole placement

- **Sylvester equation:**
  - **eig(S) are wanted closed-loop poles**
  - **X^{-1} (A-BK) X = S**
  - **AX-XS = BKX = BG with G auxiliary matrix**
  - **solve AX-XS = BG for known A, S, B,**

    solve KX = G to find K
- **Use *lyap.m* in Matlab, or *place.m***
- **For SI, K is unique, independent of G**

Course notes p. 60 + 102 - 110

Control law – LQR

- **Can we use state feedback to place poles optimally, according to a cost function?**
- **LQR, linear (feedback) quadratic (cost function) regulator**
- **Solution via Lagrange multipliers leads to ricatti (difference) equation and** time-dependent **K (but independent of initial state..)**
- **For horizon near infinity, K ~ constant** $\Rightarrow$ **LQR!**

$$\min \int_0^{\infty} (z^T z)dt = \int_0^{\infty} (x^T \underbrace{C_1^T C_1}_{\mathbf{Q}} x + u^T \underbrace{D_1^T D_1}_{\mathbf{R}} u)dt$$

s.t. model equations

- **Variations on this theme, such as MPC (time-dependent)**

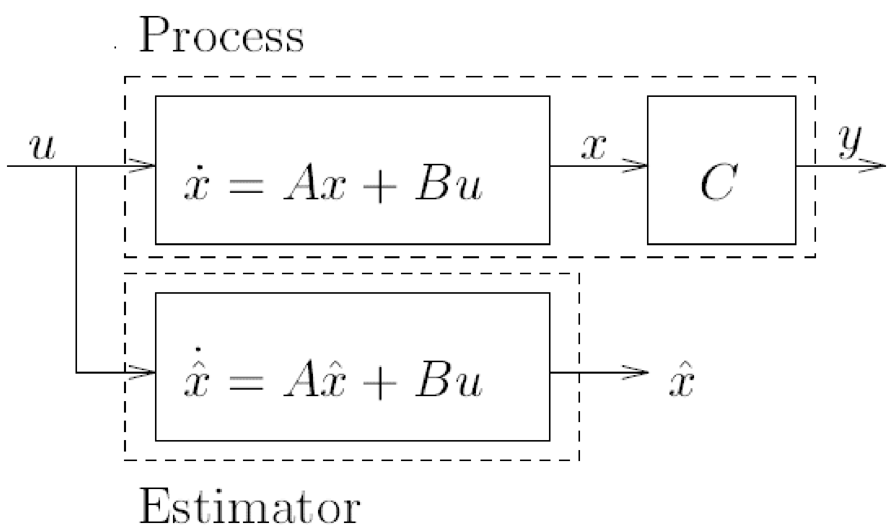
Observer

- **Measuring states not always possible**
- **Estimating states ⇒ observer**
  - **Simply by using open loop estimator:**

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\,\hat{\mathbf{x}} + \mathbf{B}\,\mathbf{u}$$

Error dynamics given by eig(A)

No convergence of error to 0 when A is unstable or when model dynamics are different from plant dynamics



Course notes p. 167-168

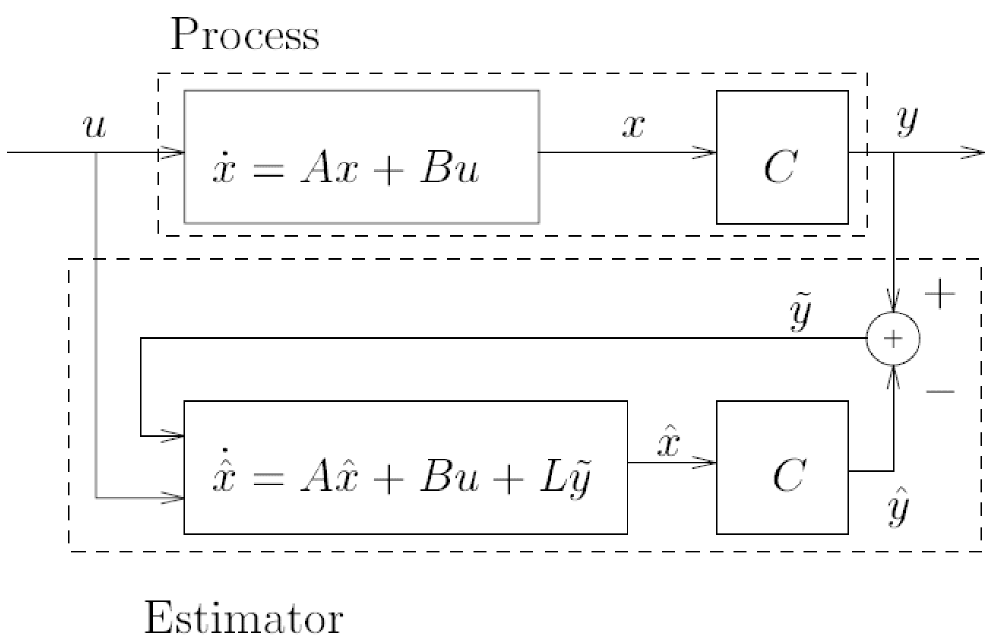
Observer

- **Estimating states ⟹ observer**
  - **Better, with correction for mismatch (D=0):**

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\,\hat{\mathbf{x}} + \mathbf{B}\,\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\,\hat{\mathbf{x}})$$

Error dynamics given by eig(A - LC)

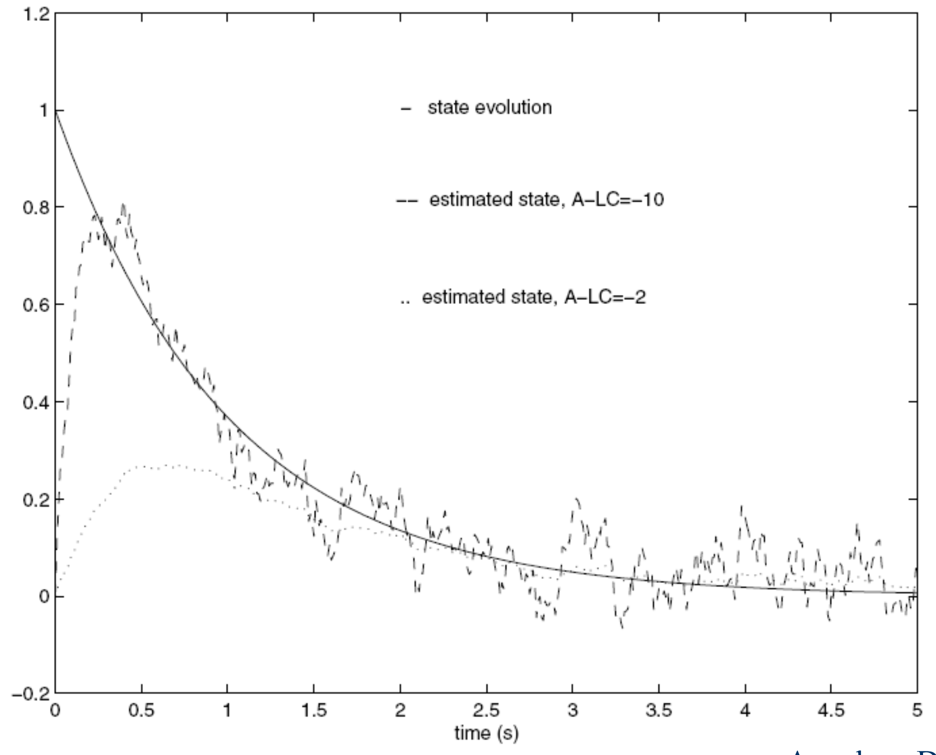
L represents trade-off between fast dynamic response and sensor noise reduction



Course notes p. 167-168

Observer – pole placement

- Observer: dual to pole placement for control law
  - Large L: fast dynamic response and reduction of process noise, but sensitive to sensor noise
  - Small L: Sensor noise reduction, but slow convergence of the error to 0



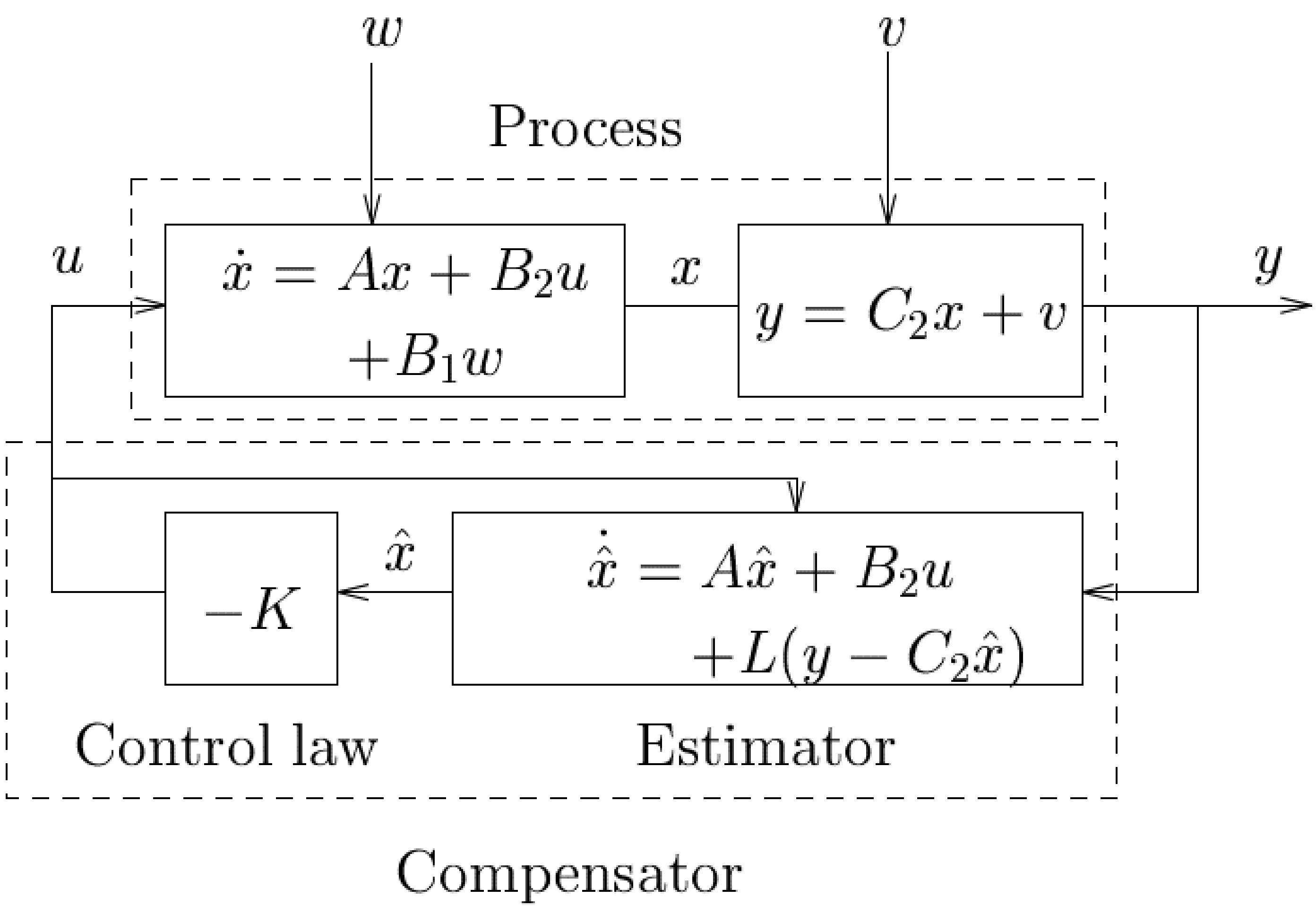
Course notes p. 167-168

Observer – Kalman filter

- Optimal observer w.r.t. gaussian noise?
  - When characteristics of the noise sources (process and measurement noise) are known
  - Kalman filter!
  - Ricatti equations again..
    - Ricatti equation to update the estimation error covariance matrix
    - This estimation error covariance determines optimal L
    - In principle, time dependent BUT for infinite horizon L is constant (cfr. LQR!)
  - Q and R = covariance matrices of process noise w and measurement noise v

Compensator

- **Compensator = Control law + Estimator**



Compensator – separation principle

- **Plant + compensator (= control law + estimator):**

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -B_2K \\ LC_2 & A - B_2K - LC_2 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix},$$
$$z = \begin{bmatrix} C_1 & 0 \\ 0 & -D_1K \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}.$$

- **The system with states x and** $\tilde{x} = x - \hat{x}$

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A - B_2K & B_2K \\ 0 & A - LC_2 \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ B_1 & -L \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix},$$
$$z = \begin{bmatrix} C_1 & 0 \\ -D_1K & D_1K \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}.$$

so that poles are $\mathrm{eig}(A - B_2K) \cup \mathrm{eig}(A - LC_2)$.

- **Linear Quadratic Gaussian Controller (LQG)**

Compensator – state feedback

- Compensator can be unstable.. even if
  - A – BK is stable
  - A – LC is stable
- zeros do not change through state feedback

Course notes p. 116 and 201