Task 1.1: Multiplying $\tilde{\boldsymbol{l}}$ by a non-zero scalar represents the same line as the points that satisfy the line equation for $\tilde{\boldsymbol{l}}$ still satisfy the line equation for $s\tilde{\boldsymbol{l}}, s \neq 0$.

To normalize an arbitrary homogeneous line we divide by $\sqrt{a^2 + b^2}$:

$$(\cos\theta, \sin\theta, -\rho) = \frac{1}{\sqrt{a^2 + b^2}}(a, b, c). \tag{1}$$

If we normalize, then the line equation $\tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{l}}$ is equal to the perpendicular distance in the image from the point to the line, assuming that $\tilde{\boldsymbol{x}} = (\tilde{x}, \tilde{y}, 1)$. This is zero if the point lies on the line, but otherwise we can use this in Part 4 to compute the epipolar distances.

Task 1.2: Inserting either point into the line equation, i.e.

$$\tilde{\boldsymbol{x}}_a^T\tilde{\boldsymbol{l}} = \tilde{\boldsymbol{x}}_a^T(\tilde{\boldsymbol{x}}_a \times \tilde{\boldsymbol{x}}_b), \tag{2}$$
$$\tilde{\boldsymbol{x}}_b^T\tilde{\boldsymbol{l}} = \tilde{\boldsymbol{x}}_b^T(\tilde{\boldsymbol{x}}_a \times \tilde{\boldsymbol{x}}_b), \tag{3}$$

gives the value 0, because the cross product $\tilde{\boldsymbol{x}}_a \times \tilde{\boldsymbol{x}}_b$ produces a vector that is perpendicular to both $\tilde{\boldsymbol{x}}_a$ and $\tilde{\boldsymbol{x}}_b$.

Task 1.3: When $\lambda \to 0$, the left term with the rotation matrix vanishes and $\tilde{\boldsymbol{x}}_2 \to \mathbf{t}$.

When $\lambda \to \infty$, the left term with the rotation matrix dominates and $\tilde{\boldsymbol{x}}_2 \to \lambda\mathbf{R}\tilde{\boldsymbol{x}}_1$ $\mathbf{R}\tilde{\boldsymbol{x}}_1$. We can drop the λ because this quantity is homogeneous, and any scalar multiple represents the same 2D point.

When these are dehomogenized, we find that

- $\tilde{\boldsymbol{x}}_2 \to \mathbf{t}$ is the projection of the first camera's center into the second image, i.e. the epipole.
- $\tilde{\boldsymbol{x}}_2 \to \mathbf{R}\tilde{\boldsymbol{x}}_1$ is the viewing ray rotated into the second image. Intuitively, when the point is very far away, the translation between the cameras no longer affects its position in the image (zero parallax). You can observe this when going out at night and looking at the stars; they are sufficiently far away that only your orientation affects their position in your field of view.

Task 1.4: From Task 1.2 we know that we can use the cross product to obtain the line between any pair of points. We found two image points in the above task, which correspond to the projection of the beginning and end of the epipolar line. Because lines in the world remain lines in the image, we can simply connect the two image points to find the epipolar line. Thus,

$$\tilde{\boldsymbol{l}}_2 = \mathbf{t} \times (\mathbf{R}\tilde{\boldsymbol{x}}_1) = [\mathbf{t}]_\times\mathbf{R}\tilde{\boldsymbol{x}}_1, \tag{4}$$

where $[\mathbf{t}]_\times$ is the matrix form of the cross product operator. This is related to the essential matrix, which is simply the constant part of this expression written as a 3×3 matrix: $\mathbf{E} = [\mathbf{t}]_\times\mathbf{R}$.

Task 1.5: There can be more than one essential matrix.

Task 3.1: It necessarily goes toward zero, because the SVD solution imposes the constraint $||\tilde{\mathbf{X}}|| = 1$.

Task 4.3: These are incorrect correspondences that happen to satisfy the epipolar constraint. It is not possible to detect them from a single pair of images, unless assumptions are made about the scene (e.g. it's unlikely to have isolated floating points). With more images they would be revealed to violate the epipolar constraint between a different image pair.

Task 4.4: I expect it to find the best solution less than 99% of the time, because the probability of getting an all-inlier sample is not equal to the probability of getting a good all-inlier sample.