Outside options and risk attitude

Gregorio Curello — Mannheim
Ludvig Sinander — Oxford
Mark Whitmeyer — Arizona State

19 February 2026

paper: arXiv.org/abs/2509.14732

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

Examples:

prospects
projects
health-insurance plans
occupations
spouses
locations

Motivation

<u>(Effective) risk attitude:</u> how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

In most <u>economic</u> contexts, chosen prospect is not all there is:

prospects	outside option
projects	job loss (for manager)
health-insurance plans	public option
occupations	retraining
spouses	divorce
locations	bailout (e.g. FEMA)

Questions

Framework:

$$\underbrace{\text{effective risk att.}}_{\text{observable}} \quad = \quad \underbrace{\text{`true' risk att.}}_{\text{unobservable}} \quad + \quad \underbrace{\text{effect of o.o.}}_{\text{unobservable}}$$

Identification: what do choices reveal about 'true' risk attitude?

- important for policy / welfare analysis
- Thm 1(i): 'true' is <u>more risk-averse</u> than effective.
- Thm 1(ii): nothing more can be learned.

Comparative statics: how does effective risk att. vary
with 'true' risk att. & o.o.? (Th'm 2)

Specialness: which transform'ns of a decision problem
necessarily reduce risk-aversion?

- adding an o.o.: yes by Th'm 1(i)
- <u>any</u> other transform'n: <u>no</u>! (Th'm 3)

Literature

Th'm 1(i) formalises idea that o.o. increases risk appetite.

An old idea. For example, Adam Smith (1776) argued that limited liability (o.o. = bankruptcy) increases risk-taking.

Details: slide 30. See also e.g. Jensen and Meckling (1976, section 4.1),
Golbe (1981, 1988),
Gollier, Koehl and Rochet (1997).

An important idea, e.g. for financial stability.

Th'ms 1(ii), 2, 3: no close parallels that we know of.

Broader literature

Various literatures recognise that

$$\text{effective risk att.} \quad = \quad \text{`true' risk att.} \quad + \quad \text{economic forces}$$

Some economic forces that have been studied:

- background risk .... papers: slide 33
- contracts
  - employment .... papers: slide 34
  - financing .... papers: slide 35
- having an audience
  - career concerns .... papers: slide 36
  - disclosure .... Ben-Porath, Dekel and Lipman, 2018
- (in)flexibility .... papers: slide 37
- competition .... papers: slides 38–41

Application: unemployment insurance

This paper grew out of an ongoing project on UI design.

Idea in policy debate: higher UI $\Longrightarrow$ better matches.
(potential Pareto improvement)

Some empirical lit: Nekoei & Weber (2017),
survey by Schmieder & von Wachter (2016).

Mechanism: UI shapes search direction (which jobs to apply to).
Applying to better matches = riskier.

prospects	(per-period) outside option
search directions	unemployment benefit

Questions: How can UI shape search? What UI is optimal?

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

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Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \left\{ p : X \to [0,1] \ : \ |\mathrm{supp}(p)| < \infty \ \& \ \textstyle\sum_{x \in \mathrm{supp}(p)} p(x) = 1 \right\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

$\succeq$ is <u>EU</u> iff $\exists\, u : X \to \mathbf{R}$ s.t. $p \succeq q \iff \int u \mathrm{d}p \geq \int u \mathrm{d}q$.

$\hookrightarrow$ the function u is called <u>risk attitude</u>

$\hookrightarrow$ $\alpha u + \beta$ is identified off $\succeq$ ($\alpha > 0$ & $\beta \in \mathbf{R}$ unknown)

$\succeq$ is <u>less risk-averse than</u> $\succeq'$ iff $\forall\, x \in X$ & $\forall\, p \in \Delta^0(X)$,

$$x \succeq (\succ)\ p \quad \Longrightarrow \quad x \succeq' (\succ')\ p.$$

Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \Big\{ p : X \to [0,1] \ : \ |\mathrm{supp}(p)| < \infty \ \& \ \textstyle\sum_{x \in \mathrm{supp}(p)} p(x) = 1 \Big\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

$\succeq$ is <u>EU</u> iff $\exists\, u : X \to \mathbf{R}$ s.t. $p \succeq q \iff \int u \mathrm{d}p \geq \int u \mathrm{d}q$.

$\hookrightarrow$ the function u is called <u>risk attitude</u>

$\hookrightarrow$ $\alpha u + \beta$ is identified off $\succeq$ ($\alpha > 0$ & $\beta \in \mathbf{R}$ unknown)

u is <u>less risk-averse than</u> v iff $\forall\, x \in X$ & $\forall\, p \in \Delta^0(X)$,

$$u(x) \geq (>) \int u \mathrm{d}p \implies v(x) \geq (>) \int v \mathrm{d}p.$$

Pratt's theorem (part 1)

For $X \neq \varnothing$ and $u, v : X \to \mathbf{R}$, the following are equivalent:

(A) u is less risk-averse than v.

(B) $\exists$ increasing convex $\phi : \mathrm{co}(v(X)) \to \mathbf{R}$
that is strictly increasing on $v(X)$ & satisfies $u = \phi \circ v$.

(C) The following two properties hold:

(I) $\forall\, x, y \in X,\ u(x) \geq (>)\, u(y) \implies v(x) \geq (>)\, v(y).$

(II) $\forall\, x, y, z \in X,$ if $u(x) < u(y) < u(z)$, then

$$\frac{u(z) - u(y)}{u(y) - u(x)} \geq \frac{v(z) - v(y)}{v(y) - v(x)}.$$

Pratt's theorem (part 2)

If in addition X is an open convex subset of $\mathbf{R}$

& u, v are C^2 with $u' > 0 < v'$, then

u is less risk-averse than v iff $u''/u' \geq v''/v'$.

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The outside-option model

Decision-maker has

- EU risk attitude $v : X \to \mathbf{R}$
- outside option worth $\sim F$.

  o.o. may be unavailable, i.e. 'worth $-\infty$'

  $\implies$ $F : \mathbf{R} \to [0,1]$ is an <u>extended</u> CDF. (def'n: slide 31)

Decision-maker's valuation of alternative $x \in X$:

$$\int \max\{v(x), k\} F(\mathrm{d}k) \quad = \quad F(v(x))v(x) + \int_{(v(x),+\infty)} kF(\mathrm{d}k).$$

(Compare realised o.o. with x; if better, exercise.)

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

$$\implies \quad F(k) = \mu(\{y \in Y \cup \{\varnothing\} : w(y) \leq k\}) \qquad \forall\, k \in \mathbf{R}. \qquad (\star)$$

Special case: X-valued o.o., i.e. $Y = X$ & $w = v$.

- Implies F concentrated on $v(X) \cup \{-\infty\}$.
- Conversely, any ECDF F concentrated on $v(X) \cup \{-\infty\}$ arises via $(\star)$ from some X-valued o.o. $\mu \in \Delta(X \cup \{-\infty\})$.
- (Detail: above makes sense provided $v(X)$ is Borel.)

(skip to slide 21)

Behavioural implications

Say that (v, F) is o.o. representation of $\succeq$ iff

$$p \succeq q \quad \iff \quad \begin{cases} \displaystyle\int \left(\int \max\{v(x), k\} F(\mathrm{d}k) \right) p(\mathrm{d}x) \\ \\ \displaystyle\geq \int \left(\int \max\{v(x), k\} F(\mathrm{d}k) \right) q(\mathrm{d}x). \end{cases}$$

Note: (v, F) o.o. rep'n of $\succeq$

$\Longrightarrow$ $\succeq$ is EU with risk att. u where $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \forall x \in X.$$

$\hookrightarrow$ u is decision-maker's effective risk attitude.

Corollary: $\succeq$ admits an o.o. representation iff $\succeq$ is EU.

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How an o.o. shapes risk attitude

Question: how does shape of o.o. dist'n F shape wedge between effective risk att. u from 'true' risk att. v ?

How an o.o. shapes risk attitude

Proposition 1. For $X \neq \varnothing$,
$u, v : X \to \mathbf{R}$,
& ECDF F,

the following are equivalent:

(i) $\exists\ \alpha > 0\ \ \&\ \ \beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x),\ \ k\ \ \} F(\mathrm{d}k) \qquad \forall\ x \in X.$$

(ii) $\int_{(0,+\infty)} kF(\mathrm{d}k) < +\infty$, and

$\exists\ \phi : \mathrm{co}(v(X)) \to \mathbf{R}$ s.t. $u = \phi \circ v$,

ϕ is abs. cont's & $\phi' \propto F$.

How an o.o. shapes risk attitude: smoothish case

Proposition 1. For $X \neq \varnothing$,
$u, v : X \to \mathbf{R}$,
& ECDF F that is C^1 with $F > 0$ on $(\inf v(X), +\infty)$,

the following are equivalent:

(i) $\exists\, \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), \quad k \quad\} F(\mathrm{d}k) \qquad \forall\, x \in X.$$

(ii′) $\int_{(0,+\infty)} k F(\mathrm{d}k) < +\infty$, and

$\exists\, \phi : \mathrm{co}(v(X)) \to \mathbf{R}$ s.t. $u = \phi \circ v$,

ϕ is C^2 & $\phi''/\phi' = F'/F$.

How an o.o. shapes risk attitude: monetary case

Proposition 1. For $X \neq \varnothing$ open convex subset of $\mathbf{R}$,
$u, v : X \to \mathbf{R}$ that are C^2 with $u' > 0 < v'$,
& ECDF G that is C^1 with $G > 0$ on $(\inf X \quad , +\infty)$
& concentrated on $X \cup \{-\infty\}$,
the following are equivalent:

(i) $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), v(y)\} G(\mathrm{d}y) \qquad \forall\ x \in X.$$

(ii$''$) $\int_{(0,+\infty)} v \mathrm{d}G < +\infty$, and

$$\frac{u''}{u'} = \frac{v''}{v'} + \frac{G'}{G}.$$

$G \equiv F \circ v$ is dist'n of o.o. denominated in dollars (not utils).

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Revealed risk attitude

Question: what can be learned about v from observing choice among lotteries?

Revealed risk attitude

Question: what can be learned about v from $\alpha u + \beta$?

Revealed risk attitude

Theorem 1. For $X \neq \varnothing$ and bounded-above $u, v : X \to \mathbf{R}$ with u Lipschitz w.r.t. v,* the following are equivalent:

(a) u is less risk-averse than v.

(b) $\exists\ \alpha > 0,\ \beta \in \mathbf{R}$ & an ECDF F such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \text{for every } x \in X$$

& $F > 0$ on $(\underline{\inf}\, v(X), +\infty)$.†

---

*$\exists\ L \geq 0$ s.t. $\forall\ x, y \in X$, $|u(x) - u(y)| \leq L|v(x) - v(y)|$.

†Ordinarily $\underline{\inf}\, A \coloneqq \inf A$. The exception: $\underline{\inf}\, A \coloneqq \inf(A \setminus \{\inf A\})$ if $\inf A < \inf(A \setminus \{\inf A\}) \notin A$.

(Non-)identification of the o.o. dist'n

Question: what can be learned about F from observing choice among lotteries?

(Non-)identification of the o.o. dist'n

Question: what can be learned about F from $\alpha u + \beta$?

Answer: essentially nothing. (details: slide 32)

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Comparative statics

Question: how does effective risk att. u vary with (v, F)?

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \ k\ \}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume $(\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{F}$ is better than F in the RHRO on $v(X) \setminus \{\sup v(X)\}$.

$\hookrightarrow$ RHRO (reverse hazard rate order) on $K \subseteq \mathbf{R}$:
$F(\ell)\widehat{F}(k) \leq F(k)\widehat{F}(\ell)$ for all $k < \ell$ in K.

(b) Sps $F \circ v = \widehat{F} \circ \widehat{v}$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{v}$ is less risk-averse than v.

$\hookrightarrow$ $F \circ v = \widehat{F} \circ \widehat{v}$ means utility units change $(v \rightsquigarrow \widehat{v})$,
physical o.o. dist'n $\mu \in \Delta(X \cup \{\varnothing\})$ held fixed. $\binom{\text{recall}}{\text{sl. 13}}$

Comparative statics: monetary case

Theorem 2. Fix $X \neq \varnothing$ open convex subset of $\mathbf{R}$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$ that are str. incr.,
ECDF $\widehat{G}$ with $\widehat{G} > 0$ on $(\inf X, +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \widehat{v}(y)\}\widehat{G}(\mathrm{d}y) \qquad \forall x \in X.$$

Assume $\widehat{G}$ concentrated on $X \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{G}$ is better than G in the RHRO on $X \setminus \{\sup X\}$.

$\hookrightarrow$ RHRO (reverse hazard rate order) on $Y \subseteq \mathbf{R}$:
$G(y)\widehat{G}(x) \leq G(x)\widehat{G}(y)$ for all $x < y$ in Y.

(b) Sps $G = \widehat{G}$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{v}$ is less risk-averse than v.

Application to unemployment insurance: model

Consumption $x \in X \subseteq \mathbf{R}_+$, risk attitude $v : X \to \mathbf{R}$,
discount factor $\beta \in [0, 1)$. Worker unemployed since $t = 0$.

Start of period t: worker chooses search direction
$\iff$ which wage-offer dist'n $p \in \mathcal{P} \subseteq \Delta^0(X)$ to sample from.

Extension: also choose search <u>effort</u>.

End of period t: wage offer x drawn from p, accept/reject.

- if accept: payoff $v(x)$ today, tomorrow, day after...
- if reject: payoff $v(b_t) + y_t$ today, search again tomorrow.
  - b_t = unemployment benefit
  - $y_t \overset{\text{iid}}{\sim} H$ captures leisure, home production, stigma...

Extensions: saving (hard), jobs not permanent.

Application to unemployment insurance: analysis

Bellman eq'n for value function V_t:

$$V_t = \max_{p \in \mathcal{P}} \int \int \max\left\{ \frac{v(x)}{1-\beta}, \quad [v(b_t) + y] + \beta V_{t+1} \right\} H(\mathrm{d}y) \; p(\mathrm{d}x)$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t =$$

$$\max_{p \in \mathcal{P}} \int \int \max\left\{ v(x) \, , (1-\beta)[v(b_t) + y] + \beta W_{t+1} \right\} H(\mathrm{d}y) \; p(\mathrm{d}x)$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$\hookrightarrow$ F_β^ℓ is CDF of r.v. $(1-\beta)Y + \ell$ when r.v. $Y \sim H$.

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p\in\mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^\ell(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

Assume H log-concave. (Holds for nearly all standard dist'ns.)

$\implies$ as $\ell \uparrow$, $\quad F_\beta^\ell \uparrow$ in RHRO.

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\Big\{ v(x)\,, \qquad\qquad k \qquad\qquad \Big\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
b_t \searrow \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & H \text{ log-concave} \arrow[r, Rightarrow] & F_\beta^{\ell_t} \searrow \text{ in RHRO} \arrow[d, Rightarrow, "\text{Th'm 2}"] \\
W_t \searrow \arrow[r, Rightarrow] & \ell_t \searrow \arrow[ur, Rightarrow] & \text{risk-aversion of } u_\beta^{\ell_t} \nearrow
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
b_t \uparrow\uparrow \forall t \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & H \text{ log-concave} \arrow[r, Rightarrow] & \begin{array}{c} F_\beta^{\ell_t} \uparrow\uparrow \forall t \\ \text{in RHRO} \end{array} \arrow[d, Rightarrow, "\text{Th'm 2}"] \\
W_t \uparrow\uparrow \forall t \arrow[r, Rightarrow] & \ell_t \uparrow\uparrow \forall t \arrow[ur, Rightarrow] & \begin{array}{c} \text{risk-aversion} \\ \text{of } u_\beta^{\ell_t} \downarrow\downarrow \forall t \end{array}
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=: \, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

So
$$\begin{cases} \underbrace{\text{declining UI}}_{b_t \searrow} \quad \Longrightarrow \quad \underbrace{\text{declining search ambition}}_{\text{risk-aversion of } u_\beta^{\ell_t} \nearrow} \\ \underbrace{\text{raise UI}}_{b_t \uparrow\uparrow \forall t} \quad \Longrightarrow \quad \underbrace{\text{raise search ambition}}_{\text{risk-aversion of } u_\beta^{\ell_t} \downarrow\downarrow \forall t} \end{cases}$$

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Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Question: which transformations decrease risk-aversion?

- adding o.o.: yes, by Th'm 1.
- adding background risk: no, not in general.

Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Question: which transformations decrease risk-aversion?

Answer: <u>only</u> transform'ns that amount to adding an o.o.

The question, formally

Transform'n: replace each x with draw from measure μ_x on X.

'True' risk att. $v : X \to \mathbf{R} \quad \rightsquigarrow \quad$ effective risk att. $x \mapsto \int v \mathrm{d}\mu_x$.

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes,
which families $(\mu_x)_{x \in X}$ have the property that
$\forall\, v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

The question, reformulated

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes, which families $(\mu_x)_{x \in X}$ have the property that $\forall\ v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

Since u less risk-averse than $v \implies u, v$ ordinally equivalent, restrict to $\mathcal{V}$s whose members are mutually ordinally equivalent.

No other restrictions $\implies \mathcal{V}$ is an ordinal equivalence class.

Given such $\mathcal{V}$, fix $v_0 \in \mathcal{V}$ & embed X in $\mathbf{R}$ via $x \mapsto v_0(x)$.
Then $X \subseteq \mathbf{R}$ & $\mathcal{V}$ is the set of all str. incr. f'ns $X \to \mathbf{R}$.

Question′: which families $(G_x)_{x \in X}$ of CDFs concentr'd on X have the property that $\forall$ str. incr. $v : X \to \mathbf{R}$, $x \mapsto \int v \mathrm{d}G_x$ is less risk-averse than v?

The answer

Theorem 3. For non-empty Borel $X \subseteq \mathbf{R}$ with $\sup X \notin X$ & family $(G_x)_{x \in X}$ of CDFs concentrated on X, the following are equivalent:

(a) $\forall$ bounded str. incr. $v : X \to \mathbf{R}$, $x \mapsto \int v \mathrm{d} G_x$ is less risk-averse than v.

(b) $\exists$ ECDF G $\begin{Bmatrix} \text{concentrated on } X \cup \{-\infty\}, \\ \text{with } G > 0 \text{ on } (\inf X, +\infty) \end{Bmatrix}$ such that

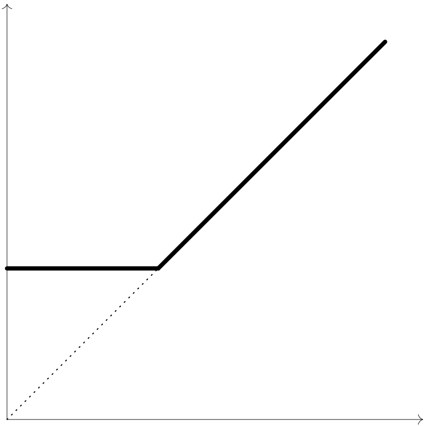
$\forall$ bounded str. incr. $v : X \to \mathbf{R}$,

$\exists\, \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha \int v \mathrm{d} G_x + \beta = \int \max\{v(x), v(y)\} G(\mathrm{d}y) \quad \text{for every } x \in X,$$

where by convention $v(-\infty) := -\infty$.

Thanks!



Adam Smith on limited liability and risk-taking

From the *Wealth of Nations:*[3]

> *In a private copartnery, each partner is bound for the debts contracted by the company to the whole extent of his fortune. In a joint stock company, on the contrary, each partner is bound only to the extent of his share. This total exemption from trouble and from risk, beyond a limited sum, encourages many people to become adventurers in joint stock companies, who would, upon no account, hazard their fortunes in any private copartnery.*

(back to slide 4)

[3] Pages. 740–1 in the Glasgow edition (Smith, 1776/1976).

Extended CDFs

Extended CDF: $k \mapsto \mathbf{P}(K \le k)$ for a $[-\infty, +\infty)$-valued random variable K.

Observation: $F : \mathbf{R} \to [0,1]$ is an ECDF iff increasing, right-continuous, $\lim_{k \nearrow +\infty} F(k) = 1$.

Expectation of meas'ble $g : [-\infty, +\infty) \to \mathbf{R}$ w.r.t. ECDF F:

$$\int g \mathrm{d}F := \Big(\lim_{k \searrow -\infty} F(k)\Big) g(-\infty) + \int_{\mathbf{R}} g \mathrm{d}F.$$

(back to slide 12)

(Non-)identification of the o.o. dist'n: details

Proposition 2. For $X \neq \varnothing$, $u : X \to \mathbf{R}$ & ECDF F, the following are equivalent:

(I) $\int_{(0,+\infty)} kF(\mathrm{d}k) < +\infty$, and

if $\int_{(-\infty,0]} kF(\mathrm{d}k) > -\infty$ then u is bounded below.

(II) $\exists\ \alpha > 0$, $\beta \in \mathbf{R}$, & $v : X \to \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \forall\ x \in X.$$

(back to slide 19)

Forces shaping risk att.: background risk

E.g. Ross (1981), Kihlstrom, Romer and Williams (1981), Pratt and Zeckhauser (1987), Kimball (1993), Eeckhoudt, Gollier and Schlesinger (1996), Gollier and Pratt (1996), Pomatto, Strack and Tamuz (2020) and Mu, Pomatto, Strack and Tamuz (2024).

(back to slide 5)

Forces shaping risk att.: employment contracts

E.g. Wilson (1969), Ross (1974), Amihud and Lev (1981), Lambert (1986), Hirshleifer and Suh (1992), Diamond (1998), Garicano and Rayo (2016) and Barron, Georgiadis and Swinkels (2020).

(back to slide 5)

Forces shaping risk att.: financing contracts

E.g. Galai and Masulis (1976), Jensen and Meckling (1976), Stiglitz and Weiss (1981), Green (1984) and Hébert (2018).

(back to slide 5)

Forces shaping risk att.: career concerns

E.g. Holmström (1982/1999), Hirshleifer and Thakor (1992), Hermalin (1993) and Chen (2015).

(back to slide 5)

Forces shaping risk att.: flexibility

E.g. Drèze and Modigliani (1966/1972), Mossin (1969), Spence and Zeckhauser (1972), Machina (1982, 1984), Bodie, Merton and Samuelson (1992), Gollier (2005), Chetty and Szeidl (2007) and Postlewaite, Samuelson and Silverman (2008).

(back to slide 5)

Forces shaping risk att.: R&D races

E.g. Dasgupta and Stiglitz (1980), Klette and de Meza (1986), Dasgupta and Maskin (1987), Cabral (2003) and Anderson and Cabral (2007).

(back to slide 5)

Forces shaping risk att.: competition for status

E.g. Robson (1992), Rosen (1997), Becker, Murphy and Werning (2005), Ray and Robson (2012) and Hopkins (2018).

(back to slide 5)

Forces shaping risk att.: competition generally

E.g. Hvide (2002), Hvide and Kristiansen (2003), Taylor (2003), Kräkel and Sliwka (2004), Seel and Strack (2013, 2016), Nutz and Zhang (2022) and Fang et al. (2025).

(back to slide 5)

Forces shaping risk att.: competition in banking

Surveys: Beck (2008), Carletti (2008), Vives (2016) and Berger, Klapper and Turk-Ariss (2017).

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