

# Comparative statics with adjustment costs and the Le Chatelier principle

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16 September 2024

paper: arXiv.org/abs/2206.00347

# Motivation

Comparative statics: under what circumstances  
does a parameter shift  
'increase' optima / equilibria?

Adjustment costs: key feature of many economic models, e.g.

- capital investment  $\left( \begin{array}{l} \text{e.g. Jorgenson, 1963; Hayashi, 1982;} \\ \text{Cooper \& Haltiwanger, 2006} \end{array} \right)$
- sticky prices  $\left( \begin{array}{l} \text{e.g. Mankiw, 1985; Caplin \& Spulber, 1987;} \\ \text{Golosov \& Lucas, 2007; Midrigan, 2011} \end{array} \right)$
- trade in illiquid assets  $\left( \begin{array}{l} \text{e.g. Kyle, 1985; Back, 1992} \end{array} \right)$
- wealthy hand-to-mouth  $\left( \begin{array}{l} \text{e.g. Kaplan \& Violante, 2014; Berger \&} \\ \text{Vavra, 2015; Chetty \& Szeidl, 2016} \end{array} \right)$
- labour supply  $\left( \begin{array}{l} \text{e.g. Chetty, Friedman, Olsen \&} \\ \text{Pistaferri, 2011; Chetty, 2012} \end{array} \right)$
- labour demand  $\left( \begin{array}{l} \text{e.g. Hamermesh, 1988;} \\ \text{Bentolila \& Bertola, 1990} \end{array} \right)$

# Motivation

Comparative statics: under what circumstances  
does a parameter shift  
'increase' optima / equilibria?

Adjustment costs: key feature of many economic models.

This paper: comparative statics for adjustment-cost models.

# Example: sticky-price models

Central plank of new Keynesian macro models: sticky prices.

Most important micro-foundation: adjustment ('menu') costs.  
(e.g. Mankiw, 1985; Golosov & Lucas, 2007; Midrigan, 2011)

Simplest model: monopolist with constant marg. cost  $c \geq 0$ ,  
decr. demand curve  $D(\cdot, \eta)$ ,  
parameter  $\eta$  shifts |elasticity|.   
Adjusting price by  $\epsilon$  costs  $C(\epsilon) \geq 0$ .

Under what ass'ns on demand  $D(\cdot, \eta)$  & adj. cost  $C(\cdot)$  do

supply shocks ( $c \nearrow$ )  $\implies$  inflation?

demand-elasticity shocks ( $\eta \searrow$ )  $\implies$  inflation?

# Overview

Basic setting: one-off parameter shift,  
agent adjusts subject to cost.

Basic insight (Th'm 1): need only very weak  
assumptions on cost.

Consequence (Th'm 2): new, greatly generalised  
Le Chatelier principle.

Consequence (Th'ms 3–6): results extend to  
infinite-horizon model.

Applications: factor demand, pricing, investment,  
labour supply, saving.

# Setting

Action  $x \in L$ ,  $L \subseteq \mathbf{R}^n$  ( $L$  a sublattice)

Objective  $F(x, \theta)$  depends on parameter  $\theta$   
( $\in \Theta$ , a partially ordered set)

At initial parameter  $\underline{\theta}$ , agent chose  $\underline{x} \in \arg \max_{x \in L} F(x, \underline{\theta})$

Parameter  $\nearrow$  to  $\bar{\theta}$ , agent adjusts  $x \in L$

Adjusting by  $\epsilon = x - \underline{x}$  costs  $C(\epsilon) \geq 0$

Agent maximises  $G(x, \bar{\theta}) := F(x, \bar{\theta}) - C(x - \underline{x})$ .

# Cost assumptions

Cost  $C : \Delta L \rightarrow [0, \infty]$  where  $\Delta L := \{x - y : x, y \in L\} \subseteq \mathbf{R}^n$ .

Assume little about  $C$ :  $C(0) < \infty$  and

- for first result :  $C$  minimally monotone
- for other results:  $C$  monotone

Allows

- non-convex costs (even non-quasiconvex)
- some adjustments infeasible:  $C(\epsilon) = \infty$
- non-separability between dimensions (as in Midrigan (2011))

# Minimal monotonicity

$C$  is minimally monotone iff  $C(\epsilon \wedge 0) \leq C(\epsilon) \geq C(\epsilon \vee 0)$   $\forall \epsilon$ .

(‘ $\wedge$ ’ is entry-wise min; ‘ $\vee$ ’ is entry-wise max.)

Interpretation: cancelling all upward adjustments lowers cost;  
likewise for downward adjustments.

For 1D action  $L \subseteq \mathbf{R}$ ,  $C$  minimally monotone  
 $\iff C$  minimised at 0.

For additively separable  $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$ ,  
 $C$  minimally monotone  
 $\iff$  each  $C_i$  minimised at 0.

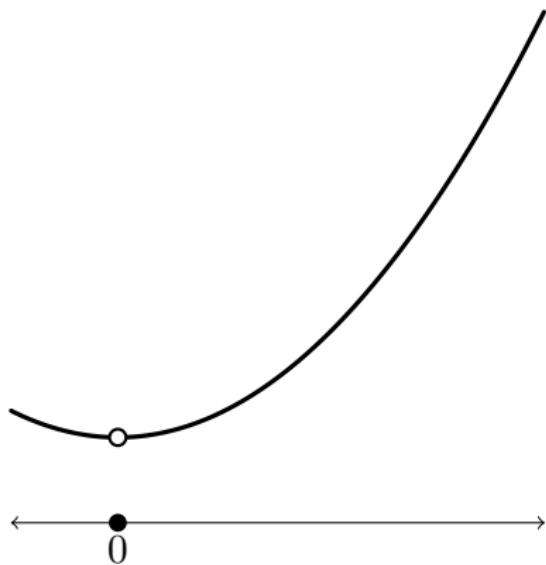
# Minimal monotonicity: example

1D action  $L \subseteq \mathbf{R}$

fixed cost  $k > 0$

variable cost  $a\epsilon^2$  ( $a > 0$ )

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } \epsilon \neq 0 \end{cases}$$



# Minimal monotonicity: example

1D action  $L \subseteq \mathbf{R}$

$\infty$  —————

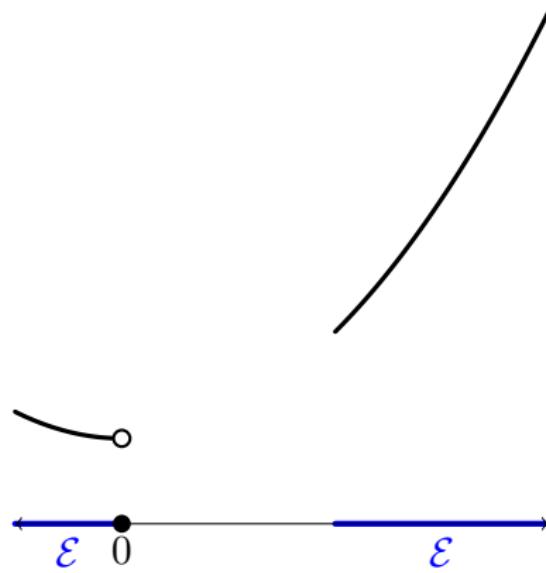
fixed cost  $k > 0$

variable cost  $a\epsilon^2$  ( $a > 0$ )

constraint set  $\mathcal{E}$  ( $\ni 0$ )

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } 0 \neq \epsilon \in \mathcal{E} \\ \infty & \text{for } \epsilon \notin \mathcal{E} \end{cases}$$

As in Field–Pande–Papp–Rigol 2013,  
Bari–Malik–Meki–Quinn 2021



# Review: costless adjustment

**Basic result** (see Milgrom & Shannon, 1994): if  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) complementarity btw. action dimensions  $x = (x_1, \dots, x_n)$

then  $\bar{x} \geq \underline{x}$  for some  $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$  provided  $\arg \max$  is not empty.

Here

- (1) means single-crossing differences in  $(x, \theta)$ : for any  $x' > x$ ,  
$$F(x', \theta) \geq (>) F(x, \theta) \implies F(x', \theta') \geq (>) F(x, \theta') \quad \text{if } \theta < \theta'$$
- (2) means quasi-supermodularity in  $x$ : for any  $x, x' \in L$ ,  
$$F(x, \theta) \geq (>) F(x \wedge x', \theta) \implies F(x \vee x', \theta) \geq (>) F(x', \theta) \quad \forall \theta$$

# Review: ordinal vs. cardinal complementarity

SCD & QSM are

- ordinal: preserved by  $\nearrow$  transformations
- not inherited by sums: sum of SCD (QSM) functions  
generally not SCD (QSM)

Cardinal sufficient conditions

that are (not ordinal, but) inherited by sums:

incr. differences in  $(x, \theta)$ : marginal return  $F(x', \theta) - F(x, \theta)$   
 $\nearrow$  in  $\theta$  (for  $x' > x$ )

supermodularity in  $x$ : marginal return

$$F((x'_i, \textcolor{blue}{x_j}, x_{-ij}), \theta) - F((x_i, \textcolor{blue}{x_j}, x_{-ij}), \theta)$$
$$\nearrow \text{in } \textcolor{blue}{x_j} \quad (\text{for } x'_i > x_i, i \neq j)$$

# Comparative statics with costly adjustment

Recall: agent maximises  $G(x, \bar{\theta}) := F(x, \bar{\theta}) - C(x - \underline{x})$ .

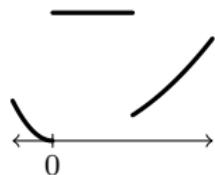
**Theorem 1:** if  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) complementarity btw. action dimensions  $x = (x_1, \dots, x_n)$

and cost  $C$  is minimally monotone,

then  $\hat{x} \geq \underline{x}$  for some  $\hat{x} \in \arg \max_{x \in L} G(x, \bar{\theta})$  provided  $\arg \max$  is not empty.

Costs need not even be single-dipped! E.g.  $C =$



Only ordinal complementarity on  $F$ . (Not inherited by  $G$ !)

# Proof of Theorem 1

Fix  $x' \in \arg \max_{x \in L} G(x, \bar{\theta})$ .

Clearly  $\underline{x} \vee x' \geq \underline{x}$ . Will show  $\underline{x} \vee x' \in \arg \max_{x \in L} G(x, \bar{\theta})$ .

Standard steps:

$$\begin{aligned} F(\underline{x}, \underline{\theta}) &\geq F(\underline{x} \wedge x', \underline{\theta}) && \text{by def'n of } \underline{x} \\ \implies F(\underline{x} \vee x', \underline{\theta}) &\geq F(x', \underline{\theta}) && \text{by QSM} \\ \implies F(\underline{x} \vee x', \bar{\theta}) &\geq F(x', \bar{\theta}) && \text{by SCD.} \end{aligned}$$

New step:

$$\begin{aligned} C(\underline{x} \vee x' - \underline{x}) \\ = C((x' - \underline{x}) \vee 0) \\ \leq C(x' - \underline{x}) && \text{by minimal monotonicity.} \end{aligned}$$

So

$$\begin{aligned} G(\underline{x} \vee x', \bar{\theta}) &= F(\underline{x} \vee x', \bar{\theta}) - C(\underline{x} \vee x' - \underline{x}) \\ &\geq F(x', \bar{\theta}) - C(x' - \underline{x}) = G(x', \bar{\theta}). \end{aligned} \quad \text{QED}$$

# Monotonicity

Cost  $C$  is monotone iff for each adj. vector  $\epsilon$  & each  $i$ ,

$$C(\epsilon_1, \dots, \epsilon_{i-1}, \epsilon'_i, \epsilon_{i+1}, \dots, \epsilon_n) \leq C(\epsilon) \text{ whenever } 0 \leq \epsilon'_i \leq \epsilon_i \\ \text{or } 0 \geq \epsilon'_i \geq \epsilon_i.$$

Interpretation: adjusting less lowers cost.

$$\begin{aligned} \text{For } L \subseteq \mathbf{R}, \quad & C \text{ monotone} \\ \iff & C \text{ single-dipped \& minimised at 0.} \\ & (\text{i.e. } \searrow \text{ on } (-\infty, 0], \nearrow \text{ on } [0, \infty)) \end{aligned}$$

For additively separable  $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$ ,

$$\begin{aligned} & C \text{ monotone} \\ \iff & \text{each } C_i \text{ single-dipped \& minimised at 0.} \end{aligned}$$

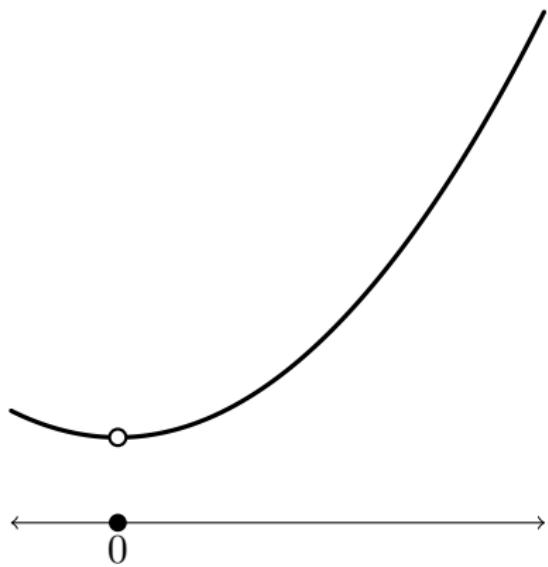
# Example of monotonicity

1D action  $L \subseteq \mathbf{R}$

fixed cost  $k > 0$

variable cost  $a\epsilon^2$  ( $a > 0$ )

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } \epsilon \neq 0 \end{cases}$$



# Example of monotonicity violation

1D action  $L \subseteq \mathbf{R}$

$\infty$  —————

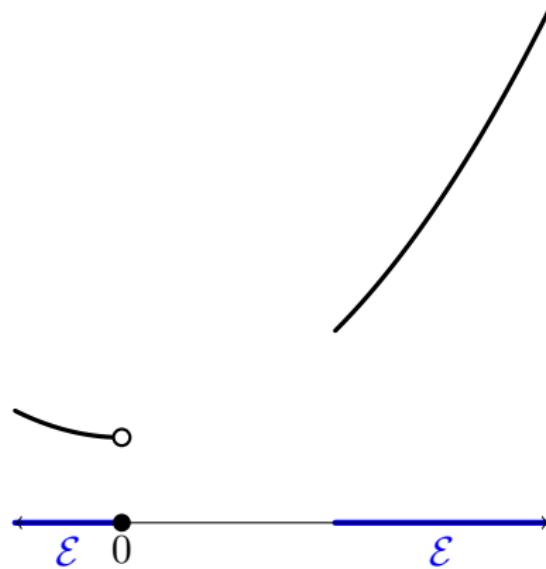
fixed cost  $k > 0$

variable cost  $a\epsilon^2$  ( $a > 0$ )

constraint set  $\mathcal{E}$  ( $\ni 0$ )

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } 0 \neq \epsilon \in \mathcal{E} \\ \infty & \text{for } \epsilon \notin \mathcal{E} \end{cases}$$

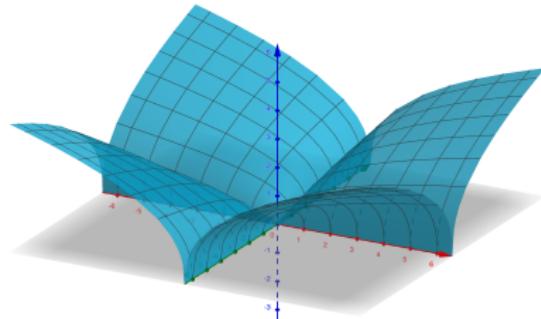
As in Field–Pande–Papp–Rigol 2013,  
Bari–Malik–Meki–Quinn 2021



# Examples of monotonicity

- Additively separable:  $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$ ,  
each  $C_i$  single-dipped & minimised at 0.
- Euclidean:  $C(\epsilon) = \sqrt{\sum_{i=1}^n \epsilon_i^2}$ .
- Cobb–Douglas:  $C(\epsilon) = \prod_{i=1}^n |\epsilon_i|^{a_i}$   
where  $a_1, \dots, a_n > 0$ .

(not quasiconvex)



# The Le Chatelier principle

Le Chatelier principle:  $|LR \text{ elasticity}| \geq |SR \text{ elasticity}|.$

Usual story: only some dimensions of  $x$  adjustable in SR.

**Formalisation** (Milgrom & Roberts, 1996): suppose  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) complementarity btw. action dimensions  $x = (x_1, \dots, x_n)$ .

Let  $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$  satisfy  $\bar{x} \geq \underline{x}$ .<sup>1</sup> (LR optimum.)

Then  $\bar{x} \geq \hat{x} \geq \underline{x}$  for some SR-optimal  $\hat{x}$ .

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<sup>1</sup>Such  $\bar{x}$  exists by the ‘basic result’ (sl. 9), provided  $\arg\max$  nonempty.

# General Le Chatelier principle

Different story: SR adjustment is costly.

Nests usual story as (very) special case:  $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$ ,

- some dimensions have  $C_i \equiv 0$
- other dimensions have  $C_i(\epsilon_i) = \infty$  for every  $\epsilon_i \neq 0$ .

**Theorem 2:** suppose  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) complementarity btw. action dimensions  $x = (x_1, \dots, x_n)$

and cost  $C$  is monotone.

Let  $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$  satisfy  $\bar{x} \geq \underline{x}$ .<sup>2</sup> (LR optimum.)

Then  $\bar{x} \geq \hat{x} \geq \underline{x}$  for some  $\hat{x} \in \arg \max_{x \in L} G(x, \bar{\theta})$  provided  $\arg \max$  is not empty.

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<sup>2</sup>Such  $\bar{x}$  exists by the ‘basic result’ (sl. 9), provided  $\arg \max$  nonempty.

## Proof of Theorem 2

By Theorem 1, may choose  $x' \geq \underline{x}$  in  $\arg \max_{x \in L} G(x, \bar{\theta})$ .

Clearly  $\bar{x} \geq \bar{x} \wedge x' \geq \underline{x}$ . We show  $\bar{x} \wedge x' \in \arg \max_{x \in L} G(x, \bar{\theta})$ .

Standard step:

$$\begin{aligned} F(\bar{x} \vee x', \bar{\theta}) &\leq F(\bar{x}, \bar{\theta}) && \text{by def'n of } \bar{x} \\ \implies F(x', \bar{\theta}) &\leq F(\bar{x} \wedge x', \bar{\theta}) && \text{by QSM.} \end{aligned}$$

New step:

$$\begin{aligned} x' \geq \bar{x} \wedge x' \geq \underline{x} &\implies (x' - \underline{x}) \geq (\bar{x} \wedge x' - \underline{x}) \geq 0 \\ \implies C(x' - \underline{x}) &\geq C(\bar{x} \wedge x' - \underline{x}) && \text{by monotonicity.} \end{aligned}$$

So

$$\begin{aligned} G(x', \bar{\theta}) &= F(x', \bar{\theta}) - C(x' - \underline{x}) \\ &\leq F(\bar{x} \wedge x', \bar{\theta}) - C(\bar{x} \wedge x' - \underline{x}) \\ &= G(\bar{x} \wedge x', \bar{\theta}). \end{aligned}$$

**QED**

# Why not minimal monotonicity?

Minimal monotonicity is not enough for Le Chatelier. Example:

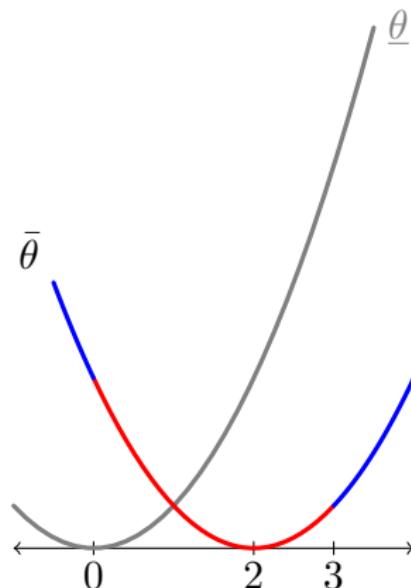
$$L = \mathbf{R} \quad F(x, \underline{\theta}) = -x^2 \quad F(x, \bar{\theta}) = -(x - 2)^2 \quad C(\epsilon) = \begin{cases} \infty & \text{if } 0 < \epsilon < 3 \\ 0 & \text{otherwise.} \end{cases}$$

$C$  is minimally monotone,  
not monotone.

Initial:  $\underline{x} = 0 \in \arg \min_{x \in \mathbf{R}} x^2$

SR:  $\hat{x} = 3 \in \arg \min_{x \in \mathbf{R} \setminus (0,3)} (x - 2)^2$

LR:  $\bar{x} = 2 \in \arg \min_{x \in \mathbf{R}} (x - 2)^2$



# Application to factor demand

Inputs  $(k, \ell)$ , real input prices  $(r, w)$ , production function  $f$

Monotone adjustment cost

Profit  $F(k, \ell, -w) = f(k, \ell) - rk - w\ell$

- supermodular in  $(k, \ell)$ : if  $f$  supermodular (complements)
- incr. diff. in  $((k, \ell), -w)$ :  $\nabla_{(k, \ell)} F = \begin{pmatrix} f_k - r \\ f_\ell - w \end{pmatrix} \nearrow$  in  $-w$

As  $w \searrow$ , both factor demands  $\nearrow$  (Theorem 1)

In LR, both factor demands further  $\nearrow$  (Theorem 2)

If instead  $f$  submodular (substitutes),  $\ell$  demand  $\nearrow$  but  
 $k$  demand  $\searrow$ .

# Application to pricing

Menu-cost model: monopolist with constant marg. cost  $c \geq 0$ ,  
decr. demand curve  $D(\cdot, \eta)$ ,  
parameter  $\eta$  shifts |elasticity|.   
Adjusting price by  $\epsilon$  costs  $C(\epsilon) \geq 0$ .

Profit  $F(p, (c, -\eta)) = (p - c)D(p, \eta)$

- $F$  QSM in  $p$ : automatic since  $p$  one-dimensional ( $\in \mathbf{R}$ )
- $\ln F$  has incr. differences in  $(p, (c, -\eta))$ :

$$\frac{d}{dp} \ln F = \frac{1}{p - c} + \frac{D'(p, \eta)}{D(p, \eta)} = \frac{1}{p - c} - \frac{|\text{elasticity}(p, \eta)|}{p}$$

$\nearrow$  in  $c$  &  $-\eta$

# Dynamic adjustment

Agent chooses  $x_t \in L$  in each period  $t \in \mathbf{N}$

Period- $t$  payoff:  $F(x_t, \theta_t) - C_t(x_t - x_{t-1})$

Taken as given:

- initial choice  $x_0 = \underline{x} \in \arg \max_{x \in L} F(x, \underline{\theta})$
- parameter path  $(\theta_t)_{t=1}^{\infty}$
- cost path  $(C_t)_{t=1}^{\infty}$

Agent forward-looking, discount rate  $\delta \in (0, 1)$

Chooses path  $(x_t)_{t=1}^{\infty}$  to max  $\sum_{t=1}^{\infty} \delta^t [F(x_t, \theta_t) - C_t(x_t - x_{t-1})]$ .

# Dynamic Le Chatelier principle

**Theorem 3:** suppose  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) complementarity btw. action dimensions  $x = (x_1, \dots, x_n)$ ,  
and that each cost  $C_t$  is monotone.

Fix  $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$  such that  $\bar{x} \geq \underline{x}$ .<sup>3</sup>

If  $\underline{\theta} \leq \theta_t \leq \bar{\theta} \quad \forall t,$  then  $\underline{x} \leq x_t \leq \bar{x} \quad \forall t$

for some solution  $(x_t)_{t=1}^{\infty}$ , provided a solution exists.

Proof: straightforward extension of Theorem 1+2 logic.

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<sup>3</sup>Such  $\bar{x}$  exists by the ‘basic result’ (sl. 9), provided  $\arg\max$  nonempty.

## Strong dynamic Le Chatelier principle

**Theorem 4:** suppose  $F(x, \theta)$  exhibits

- (1) complementarity btw. action  $x$  & parameter  $\theta$
- (2) cardinal complementarity btw. dimensions  $x = (x_1, \dots, x_n)$
- (3) boundedness in  $x$  on each compact set  $\subseteq L$ , for each  $\theta$ ,

and that  $C_t = C \ \forall t$  for  $C$  monotone & additively separable.

Fix  $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$  such that  $\bar{x} \geq \underline{x}$ .<sup>4</sup>

If  $\theta_t = \bar{\theta} \ \forall t$ , then  $\underline{x} \leq x_t \leq x_{t+1} \leq \bar{x} \ \forall t$

for some solution  $(x_t)_{t=1}^{\infty}$ , provided a solution exists.

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<sup>4</sup>Such  $\bar{x}$  exists by the ‘basic result’ (sl. 9), provided  $\arg \max$  nonempty.

## Sketch proof of Theorem 4

Can ‘monotonise’ any  $(x_t)_{t=1}^{\infty}$  by replacing  $t^{\text{th}}$  entry  
with cumulative max  $x_1 \vee x_2 \vee \cdots \vee x_{t-1} \vee x_t$ .

Claim: monotonisation preserves optimality.

(Suffices since can then monotonise solution from Theorem 3.)

By boundedness & limit argument, suffices to show that  
if  $(x_t)_{t=1}^{\infty}$  optimal &  $x_1 \leq x_2 \leq \cdots \leq x_{k-1} \leq x_k$ ,  
remains optimal if replace  $t^{\text{th}}$  entry by  $x_{t-1} \vee x_t \quad \forall t \geq k+1$ .

Proved using supermodularity of  $F(\cdot, \bar{\theta})$ ,  
+ monotonicity & additive separability of  $C$ .  
(argument: slide 28)

# Application to pricing, continued

Assume cost  $C$  time-invariant,  
demand  $D(\cdot, \eta)$  upper semi-continuous.

Profit  $F(p, (c, -\eta)) = (p - c)D(p, \eta)$

- $F(p, (c, -\eta))$  SM in  $p$  &  $C$  additively separable:  
automatic since  $p$  one-dimensional ( $\in \mathbf{R}$ )
- $F(p, (c, -\eta))$  bounded in  $p$  on each compact set  $\subseteq \mathbf{R}_+$

Theorem 4:

- supply shock  $(c \nearrow)$   $\implies$  inflation at every horizon
- demand shock s.t.  $\eta \searrow$   $\implies$  inflation at every horizon

Thanks!



## Sketch proof of Theorem 4: main step

$(x_t)_{t=1}^{\infty}$  optimal  $\implies$  better than  $(x_t \wedge x_{t+1})_{t=1}^{\infty}$ :

$$\begin{aligned} & \sum_{t=k}^{\infty} \delta^{t-k} \left[ F(x_t, \bar{\theta}) - F(\textcolor{red}{x_t \wedge x_{t+1}}, \bar{\theta}) \right] \\ & - \sum_{t=k}^{\infty} \delta^{t-k} [C(x_t - x_{t-1}) - C(\textcolor{red}{x_t \wedge x_{t+1}} - x_{t-1} \wedge \textcolor{red}{x_t})] \geq 0. \end{aligned}$$

$$\begin{aligned} F(\cdot, \bar{\theta}) \text{ supermodular: } & F(\textcolor{blue}{x_t \vee x_{t+1}}, \bar{\theta}) - F(x_{t+1}, \bar{\theta}) \\ & \geq F(x_t, \bar{\theta}) - F(\textcolor{red}{x_t \wedge x_{t+1}}, \bar{\theta}) \quad \forall t \geq k \end{aligned}$$

$C$  monotone & additively separable: (argument omitted)

$$\begin{aligned} & C(\textcolor{blue}{x_t \vee x_{t+1} - x_{t-1} \vee x_t}) - C(x_{t+1} - x_t) \\ & \leq C(x_t - x_{t-1}) - C(\textcolor{red}{x_t \wedge x_{t+1} - x_{t-1} \wedge x_t}) \quad \forall t \geq k \end{aligned}$$

So (changing variables,)  $(x_{t-1} \vee x_t)_{t=1}^{\infty}$  better than  $(x_t)_{t=1}^{\infty}$ :

$$\begin{aligned} & \sum_{t=k+1}^{\infty} \delta^{t-(k+1)} \left[ F(\textcolor{blue}{x_{t-1} \vee x_t}, \bar{\theta}) - F(x_t, \bar{\theta}) \right] \\ & - \sum_{t=k+1}^{\infty} \delta^{t-(k+1)} [C(\textcolor{blue}{x_{t-1} \vee x_t - x_{t-2} \vee x_{t-1}}) - C(x_t - x_{t-1})] \geq 0. \end{aligned}$$

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