Outside options and risk attitude

Gregorio Curello
Mannheim

Ludvig Sinander
Oxford

Mark Whitmeyer
Arizona State

19 February 2026

paper: arXiv.org/abs/2509.14732

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

Examples:

prospects
projects

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

Examples:

prospects
projects
health-insurance plans

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

Examples:

prospects
projects
health-insurance plans
occupations
spouses
locations

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

In most economic contexts, chosen prospect is not all there is:

prospects	outside option	
projects	job loss	(for manager)
health-insurance plans		
occupations spouses locations		

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

In most economic contexts, chosen prospect is not all there is:

prospects	outside option	
projects	job loss	(for manager)
health-insurance plans	public option	
occupations		
spouses		
locations		

Motivation

(Effective) risk attitude: how a decision-maker chooses among risky prospects (=lotteries).

A key determinant of many economic phenomena.

In most economic contexts, chosen prospect is not all there is:

prospects	outside option
projects	job loss (for manager)
health-insurance plans	public option
occupations	retraining
spouses	divorce
locations	bailout (e.g. FEMA)

Questions

Framework:

$$\text{effective risk att.} \quad = \quad \text{`true' risk att.} \quad + \quad \text{effect of o.o.}$$

Questions

Framework:

$$\underbrace{\text{effective risk att.}}_{\text{observable}} \quad = \quad \underbrace{\text{`true' risk att.}}_{\text{unobservable}} \quad + \quad \underbrace{\text{effect of o.o.}}_{\text{unobservable}}$$

Identification: what do choices reveal about 'true' risk attitude?

– important for policy / welfare analysis

Questions

Framework:

$$\underbrace{\text{effective risk att.}}_{\text{observable}} \quad = \quad \underbrace{\text{`true' risk att.}}_{\text{unobservable}} \quad + \quad \underbrace{\text{effect of o.o.}}_{\text{unobservable}}$$

Identification: what do choices reveal about ‘true’ risk attitude?

– important for policy / welfare analysis
– Thm 1(i): ‘true’ is <u>more risk-averse</u> than effective.

Questions

Framework:

$$\underbrace{\text{effective risk att.}}_{\text{observable}} \quad = \quad \underbrace{\text{`true' risk att.}}_{\text{unobservable}} \quad + \quad \underbrace{\text{effect of o.o.}}_{\text{unobservable}}$$

Identification: what do choices reveal about ‘true’ risk attitude?

– important for policy / welfare analysis
– Thm 1(i): ‘true’ is <u>more risk-averse</u> than effective.
– Thm 1(ii): nothing more can be learned.

Questions

Framework:

effective risk att. = 'true' risk att. + effect of o.o.

Identification: what do choices reveal about 'true' risk attitude?

– important for policy / welfare analysis
– Thm 1(i): 'true' is <u>more risk-averse</u> than effective.
– Thm 1(ii): nothing more can be learned.

Comparative statics: how does effective risk att. vary with 'true' risk att. & o.o.? (Th'm 2)

Questions

Framework:

effective risk att. = 'true' risk att. + effect of o.o.

Identification: what do choices reveal about 'true' risk attitude?

- important for policy / welfare analysis
- Thm 1(i): 'true' is <u>more risk-averse</u> than effective.
- Thm 1(ii): nothing more can be learned.

Comparative statics: how does effective risk att. vary
with 'true' risk att. & o.o.? (Th'm 2)

Specialness: which transform'ns of a decision problem
necessarily reduce risk-aversion?

- adding an o.o.: yes by Th'm 1(i)

Questions

Framework:

effective risk att. $=$ 'true' risk att. $+$ effect of o.o.

Identification: what do choices reveal about 'true' risk attitude?

- important for policy / welfare analysis
- Thm 1(i): 'true' is <u>more risk-averse</u> than effective.
- Thm 1(ii): nothing more can be learned.

Comparative statics: how does effective risk att. vary with 'true' risk att. & o.o.? (Th'm 2)

Specialness: which transform'ns of a decision problem necessarily reduce risk-aversion?

- adding an o.o.: yes by Th'm 1(i)
- <u>any</u> other transform'n: <u>no</u>! (Th'm 3)

Literature

Th'm 1(i) formalises idea that o.o. increases risk appetite.

An old idea. For example, Adam Smith (1776) argued that limited liability (o.o. = bankruptcy) increases risk-taking.

Details: slide 30. See also e.g. Jensen and Meckling (1976, section 4.1),
Golbe (1981, 1988),
Gollier, Koehl and Rochet (1997).

An important idea, e.g. for financial stability.

Literature

Th'm 1(i) formalises idea that o.o. increases risk appetite.

An old idea. For example, Adam Smith (1776) argued that limited liability (o.o. = bankruptcy) increases risk-taking.

Details: slide 30. See also e.g. Jensen and Meckling (1976, section 4.1),
Golbe (1981, 1988),
Gollier, Koehl and Rochet (1997).

An important idea, e.g. for financial stability.

Th'ms 1(ii), 2, 3: no close parallels that we know of.

Broader literature

Various literatures recognise that

effective risk att. $\quad = \quad$ ‘true’ risk att. $\quad + \quad$ economic forces

Broader literature

Various literatures recognise that

$$\text{effective risk att.} \quad = \quad \text{`true' risk att.} \quad + \quad \text{economic forces}$$

Some economic forces that have been studied:

- background risk . . . . papers: slide 33
- contracts
  - employment . . . . papers: slide 34
  - financing . . . . papers: slide 35
- having an audience
  - career concerns . . . . papers: slide 36
  - disclosure . . . . Ben-Porath, Dekel and Lipman, 2018
- (in)flexibility . . . . papers: slide 37
- competition . . . . papers: slides 38–41

Application: unemployment insurance

This paper grew out of an ongoing project on UI design.

Idea in policy debate: higher UI $\implies$ better matches.
(potential Pareto improvement)

Application: unemployment insurance

This paper grew out of an ongoing project on UI design.

Idea in policy debate: higher UI $\Longrightarrow$ better matches.
(potential Pareto improvement)

Some empirical lit: Nekoei & Weber (2017),
survey by Schmieder & von Wachter (2016).

Mechanism: UI shapes search direction (<u>which</u> jobs to apply to).
Applying to better matches = riskier.

Application: unemployment insurance

This paper grew out of an ongoing project on UI design.

Idea in policy debate: higher UI $\Longrightarrow$ better matches.
(potential Pareto improvement)

Some empirical lit: Nekoei & Weber (2017),
survey by Schmieder & von Wachter (2016).

Mechanism: UI shapes search direction (<u>which</u> jobs to apply to).
Applying to better matches = riskier.

prospects	(per-period) outside option
search directions	unemployment benefit

Application: unemployment insurance

This paper grew out of an ongoing project on UI design.

Idea in policy debate: higher UI $\Longrightarrow$ better matches.
(potential Pareto improvement)

Some empirical lit: Nekoei & Weber (2017),
survey by Schmieder & von Wachter (2016).

Mechanism: UI shapes search direction (which jobs to apply to).
Applying to better matches = riskier.

prospects	(per-period) outside option
search directions	unemployment benefit

Questions: How can UI shape search? What UI is optimal?

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \left\{ p : X \to [0,1] \ : \ |\mathrm{supp}(p)| < \infty \ \& \ \textstyle\sum_{x \in \mathrm{supp}(p)} p(x) = 1 \right\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \left\{ p : X \to [0,1] \ : \ |\text{supp}(p)| < \infty \ \& \ \textstyle\sum_{x \in \text{supp}(p)} p(x) = 1 \right\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

$\succeq$ is <u>EU</u> iff $\exists\, u : X \to \mathbf{R}$ s.t. $p \succeq q \iff \int u \mathrm{d}p \geq \int u \mathrm{d}q$.

$\hookrightarrow$ the function u is called <u>risk attitude</u>

$\hookrightarrow$ $\alpha u + \beta$ is identified off $\succeq$ $(\alpha > 0 \ \& \ \beta \in \mathbf{R}$ unknown$)$

Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \left\{ p : X \to [0,1] \ : \ |\mathrm{supp}(p)| < \infty \ \& \ \sum_{x \in \mathrm{supp}(p)} p(x) = 1 \right\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

$\succeq$ is <u>EU</u> iff $\exists\, u : X \to \mathbf{R}$ s.t. $p \succeq q \iff \int u \mathrm{d}p \geq \int u \mathrm{d}q$.

$\hookrightarrow$ the function u is called <u>risk attitude</u>

$\hookrightarrow$ $\alpha u + \beta$ is identified off $\succeq$ ($\alpha > 0$ & $\beta \in \mathbf{R}$ unknown)

$\succeq$ is <u>less risk-averse than</u> $\succeq'$ iff $\forall\, x \in X$ & $\forall\, p \in \Delta^0(X)$,

$$x \succeq (\succ)\, p \quad \Longrightarrow \quad x \succeq' (\succ')\, p.$$

Setup

Alternatives: $x, y \in X \neq \varnothing$.

Risky prospects / simple lotteries: $p, q \in \Delta^0(X)$

$$:= \left\{ p : X \to [0,1] \ : \ |\mathrm{supp}(p)| < \infty \ \& \ \textstyle\sum_{x \in \mathrm{supp}(p)} p(x) = 1 \right\}$$

Preferences $\succeq, \succeq'$: complete transitive relations on $\Delta^0(X)$.

$\succeq$ is <u>EU</u> iff $\exists\, u : X \to \mathbf{R}$ s.t. $p \succeq q \iff \int u \mathrm{d}p \geq \int u \mathrm{d}q$.

$\hookrightarrow$ the function u is called <u>risk attitude</u>

$\hookrightarrow$ $\alpha u + \beta$ is identified off $\succeq$ ($\alpha > 0$ & $\beta \in \mathbf{R}$ unknown)

u is <u>less risk-averse than</u> v iff $\forall\, x \in X$ & $\forall\, p \in \Delta^0(X)$,

$$u(x) \geq (>) \int u \mathrm{d}p \implies v(x) \geq (>) \int v \mathrm{d}p.$$

Pratt's theorem (part 1)

For $X \neq \varnothing$ and $u, v : X \to \mathbf{R}$, the following are equivalent:

(A) u is less risk-averse than v.

Pratt's theorem (part 1)

For $X \neq \varnothing$ and $u, v : X \to \mathbf{R}$, the following are equivalent:

(A) u is less risk-averse than v.

(B) $\exists$ increasing convex $\phi : \mathrm{co}(v(X)) \to \mathbf{R}$
that is strictly increasing on $v(X)$ & satisfies $u = \phi \circ v$.

Pratt's theorem (part 1)

For $X \neq \varnothing$ and $u, v : X \to \mathbf{R}$, the following are equivalent:

(A) u is less risk-averse than v.

(B) $\exists$ increasing convex $\phi : \mathrm{co}(v(X)) \to \mathbf{R}$
that is strictly increasing on $v(X)$ & satisfies $u = \phi \circ v$.

(C) The following two properties hold:

(I) $\forall\, x, y \in X,\ u(x) \geq (>)\, u(y) \implies v(x) \geq (>)\, v(y)$.

(II) $\forall\, x, y, z \in X$, if $u(x) < u(y) < u(z)$, then

$$\frac{u(z) - u(y)}{u(y) - u(x)} \geq \frac{v(z) - v(y)}{v(y) - v(x)}.$$

Pratt's theorem (part 2)

If in addition X is an open convex subset of $\mathbf{R}$

& u, v are C^2 with $u' > 0 < v'$, then

u is less risk-averse than v iff $u''/u' \geq v''/v'$.

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

The outside-option model

Decision-maker has

– EU risk attitude $v : X \to \mathbf{R}$

– outside option worth $\sim F$.

The outside-option model

Decision-maker has

- EU risk attitude $v : X \to \mathbf{R}$
- outside option worth $\sim F$.

  o.o. may be unavailable, i.e. 'worth $-\infty$'

  $\implies F : \mathbf{R} \to [0,1]$ is an extended CDF. (def'n: slide 31)

The outside-option model

Decision-maker has

- EU risk attitude $v : X \to \mathbf{R}$
- outside option worth $\sim F$.

  o.o. may be unavailable, i.e. 'worth $-\infty$'

  $\implies F : \mathbf{R} \to [0, 1]$ is an <u>extended</u> CDF. (def'n: slide 31)

Decision-maker's valuation of alternative $x \in X$ if o.o. worth k:

$$\max\{v(x), k\}$$

(Compare realised o.o. with x; if better, exercise.)

The outside-option model

Decision-maker has

– EU risk attitude $v : X \to \mathbf{R}$

– outside option worth $\sim F$.

o.o. may be unavailable, i.e. 'worth $-\infty$'

$\implies$ $F : \mathbf{R} \to [0, 1]$ is an <u>extended</u> CDF. (def'n: slide 31)

Decision-maker's valuation of alternative $x \in X$:

$$\int \max\{v(x), k\} F(\mathrm{d}k) \quad = \quad F(v(x))v(x) + \int_{(v(x), +\infty)} kF(\mathrm{d}k).$$

(Compare realised o.o. with x; if better, exercise.)

Where do outside options come from?

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

$$\implies \quad F(k) = \mu(\{y \in Y \cup \{\varnothing\} : w(y) \leq k\}) \qquad \forall\ k \in \mathbf{R}. \qquad (\star)$$

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

$$\implies F(k) = \mu(\{y \in Y \cup \{\varnothing\} : w(y) \leq k\}) \qquad \forall\, k \in \mathbf{R}. \qquad (\star)$$

Special case: X-valued o.o., i.e. $Y = X$ & $w = v$.

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

$$\implies \quad F(k) = \mu(\{y \in Y \cup \{\varnothing\} : w(y) \leq k\}) \qquad \forall\, k \in \mathbf{R}. \qquad (\star)$$

Special case: X-valued o.o., i.e. $Y = X$ & $w = v$.

- Implies F concentrated on $v(X) \cup \{-\infty\}$.

Where do outside options come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\varnothing\})$, ‘$\varnothing$’ means ‘unavailable’

payoff f’n $w : Y \to \mathbf{R}$ convention: $w(\varnothing) := -\infty$

$$\implies F(k) = \mu(\{y \in Y \cup \{\varnothing\} : w(y) \leq k\}) \qquad \forall\, k \in \mathbf{R}. \qquad (\star)$$

Special case: X-valued o.o., i.e. $Y = X$ & $w = v$.

- Implies F concentrated on $v(X) \cup \{-\infty\}$.
- Conversely, any ECDF F concentrated on $v(X) \cup \{-\infty\}$ arises via $(\star)$ from some X-valued o.o. $\mu \in \Delta(X \cup \{-\infty\})$.
- (Detail: above makes sense provided $v(X)$ is Borel.)

(skip to slide 21)

Behavioural implications

Say that (v, F) is o.o. representation of $\succeq$ iff

$$p \succeq q \quad \iff \quad \begin{cases} \displaystyle\int \left(\int \max\{v(x), k\} F(\mathrm{d}k) \right) p(\mathrm{d}x) \\ \\ \displaystyle\geq \int \left(\int \max\{v(x), k\} F(\mathrm{d}k) \right) q(\mathrm{d}x). \end{cases}$$

Behavioural implications

Say that (v, F) is o.o. representation of $\succeq$ iff

$$
p \succeq q \quad \iff \quad \begin{cases} \displaystyle\int \underbrace{\left(\int \max\{v(x), k\} F(\mathrm{d}k) \right)}_{=\alpha u(x)+\beta} p(\mathrm{d}x) \\ \displaystyle\geq \int \overbrace{\left(\int \max\{v(x), k\} F(\mathrm{d}k) \right)} q(\mathrm{d}x). \end{cases}
$$

Note: (v, F) o.o. rep'n of $\succeq$

$\implies$ $\succeq$ is EU with risk att. u where $\exists\, \alpha > 0$ & $\beta \in \mathbf{R}$ s.t.

$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \forall x \in X.$

$\hookrightarrow$ u is decision-maker's effective risk attitude.

Behavioural implications

Say that (v, F) is <u>o.o. representation</u> of $\succeq$ iff

$$p \succeq q \quad \iff \quad \begin{cases} \displaystyle\int \underbrace{\left(\int \max\{v(x), k\} F(\mathrm{d}k) \right)}_{=\alpha u(x)+\beta} p(\mathrm{d}x) \\ \displaystyle\geq \int \overbrace{\left(\int \max\{v(x), k\} F(\mathrm{d}k) \right)} q(\mathrm{d}x). \end{cases}$$

Note: (v, F) o.o. rep'n of $\succeq$

$\implies$ $\succeq$ is EU with risk att. u where $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \qquad \forall x \in X.$$

$\hookrightarrow$ u is decision-maker's <u>effective</u> risk attitude.

Corollary: $\succeq$ admits an o.o. representation iff $\succeq$ is EU.

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

How an o.o. shapes risk attitude

Question: how does shape of o.o. dist'n F shape wedge between effective risk att. u from 'true' risk att. v ?

How an o.o. shapes risk attitude

Proposition 1. For $X \neq \varnothing$,
$u, v : X \to \mathbf{R}$,
& ECDF F,

the following are equivalent:

(i) $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x),\ \ k\ \ \} F(\mathrm{d}k) \qquad \forall\ x \in X.$$

(ii) $\int_{(0,+\infty)} k F(\mathrm{d}k) < +\infty$, and

$\exists\ \phi : \mathrm{co}(v(X)) \to \mathbf{R}$ s.t. $u = \phi \circ v$,

ϕ is abs. cont's & $\phi' \propto F$.

How an o.o. shapes risk attitude: smoothish case

Proposition 1. For $X \neq \varnothing$,
$u, v : X \to \mathbf{R}$,
& ECDF F that is C^1 with $F > 0$ on $(\inf v(X), +\infty)$,

the following are equivalent:

(i) $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x),\ \ k\ \ \}F(\mathrm{d}k) \qquad \forall\ x \in X.$$

(ii′) $\int_{(0,+\infty)} kF(\mathrm{d}k) < +\infty$, and

$\exists\ \phi : \mathrm{co}(v(X)) \to \mathbf{R}$ s.t. $u = \phi \circ v$,

ϕ is C^2 & $\phi''/\phi' = F'/F$.

How an o.o. shapes risk attitude: monetary case

Proposition 1. For $X \neq \varnothing$ open convex subset of $\mathbf{R}$,
$u, v : X \to \mathbf{R}$ that are C^2 with $u' > 0 < v'$,
& ECDF G that is C^1 with $G > 0$ on $(\inf X, +\infty)$
& concentrated on $X \cup \{-\infty\}$,
the following are equivalent:

(i) $\exists\, \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), v(y)\} G(\mathrm{d}y) \qquad \forall\, x \in X.$$

(ii″) $\int_{(0,+\infty)} v \mathrm{d}G < +\infty$, and

$$\frac{u''}{u'} = \frac{v''}{v'} + \frac{G'}{G}.$$

$G \equiv F \circ v$ is dist'n of o.o. denominated in <u>dollars</u> (not utils).

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

Revealed risk attitude

Question: what can be learned about v from observing choice among lotteries?

Revealed risk attitude

Question: what can be learned about v from $\alpha u + \beta$?

Revealed risk attitude

Theorem 1. For $X \neq \varnothing$ and bounded-above $u, v : X \to \mathbf{R}$ with u Lipschitz w.r.t. v,*
the following are equivalent:

(a) u is less risk-averse than v.

Revealed risk attitude

Theorem 1. For $X \neq \varnothing$ and bounded-above $u, v : X \to \mathbf{R}$ with u Lipschitz w.r.t. v,* the following are equivalent:

(a) u is less risk-averse than v.

(b) $\exists\, \alpha > 0$, $\beta \in \mathbf{R}$ & an ECDF F such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \text{for every } x \in X$$

& $F > 0$ on $(\underline{\inf}\, v(X), +\infty)$.†

Revealed risk attitude

Theorem 1. For $X \neq \varnothing$ and bounded-above $u, v : X \to \mathbf{R}$ with u Lipschitz w.r.t. v,[*] the following are equivalent:

(a) u is less risk-averse than v.

(b) $\exists\ \alpha > 0$, $\beta \in \mathbf{R}$ & an ECDF F such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \text{for every } x \in X$$

& $F > 0$ on $(\underline{\inf}\, v(X), +\infty)$.[†]

[*] $\exists\ L \geq 0 \quad \text{s.t.} \quad \forall\ x, y \in X, \quad |u(x) - u(y)| \leq L|v(x) - v(y)|.$

Revealed risk attitude

Theorem 1. For $X \neq \varnothing$ and bounded-above $u, v : X \to \mathbf{R}$ with u Lipschitz w.r.t. v,* the following are equivalent:

(a) u is less risk-averse than v.

(b) $\exists\ \alpha > 0,\ \beta \in \mathbf{R}$ & an ECDF F such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \text{for every } x \in X$$

& $F > 0$ on $(\underline{\inf}\, v(X), +\infty)$.†

*$\exists\ L \geq 0$ s.t. $\forall\ x, y \in X$, $|u(x) - u(y)| \leq L|v(x) - v(y)|$.
†Ordinarily $\underline{\inf}\, A \coloneqq \inf A$. The exception: $\underline{\inf}\, A \coloneqq \inf(A \setminus \{\inf A\})$ if $\inf A < \inf(A \setminus \{\inf A\}) \notin A$.

(Non-)identification of the o.o. dist'n

Question: what can be learned about F from observing choice among lotteries?

(Non-)identification of the o.o. dist'n

Question: what can be learned about F from $\alpha u + \beta$?

(Non-)identification of the o.o. dist'n

Question: what can be learned about F from $\alpha u + \beta$?

Answer: essentially nothing. (details: slide 32)

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

Comparative statics

Question: how does effective risk att. u vary with (v, F)?

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$u, v : X \to \mathbf{R}$,
ECDF F with $F > 0$ on $(\underline{\inf}\, v(X), +\infty)$.
Suppose $\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), \quad k \quad\} F(\mathrm{d}k) \qquad \forall x \in X.$$

Assume ($v(X)$ Borel and) F concentrated on $v(X) \cup \{-\infty\}$.

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \quad k \quad\}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume ($\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \quad k \quad\}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume ($\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \quad k \quad\}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume $(\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff $\widehat{F}$ is better than F in the RHRO on $v(X) \setminus \{\sup v(X)\}$.

$\hookrightarrow$ RHRO (reverse hazard rate order) on $K \subseteq \mathbf{R}$:
$F(\ell)\widehat{F}(k) \leq F(k)\widehat{F}(\ell)$ for all $k < \ell$ in K.

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x),\ \ k\ \ \}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume ($\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{F}$ is better than F in the RHRO on $v(X) \setminus \{\sup v(X)\}$.

(b) Sps $F \circ v = \widehat{F} \circ \widehat{v}$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{v}$ is less risk-averse than v.

$\hookrightarrow$ $F \circ v = \widehat{F} \circ \widehat{v}$ means utility units change $(v \rightsquigarrow \widehat{v})$,
physical o.o. dist'n $\mu \in \Delta(X \cup \{\varnothing\})$ held fixed. $\binom{\text{recall}}{\text{sl. 13}}$

Comparative statics

Theorem 2. Fix $X \neq \varnothing$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$,
ECDF $\widehat{F}$ with $\widehat{F} > 0$ on $(\underline{\inf}\, \widehat{v}(X), +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \ k\ \}\widehat{F}(\mathrm{d}k) \qquad \forall x \in X.$$

Assume ($\widehat{v}(X)$ Borel and) $\widehat{F}$ concentrated on $\widehat{v}(X) \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{F}$ is better than F in the RHRO on $v(X) \setminus \{\sup v(X)\}$.

(b) Sps $F \circ v = \widehat{F} \circ \widehat{v}$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{v}$ is less risk-averse than v.

Comparative statics: monetary case

Theorem 2. Fix $X \neq \varnothing$ open convex subset of $\mathbf{R}$,
$\widehat{u}, \widehat{v} : X \to \mathbf{R}$ that are str. incr.,
ECDF $\widehat{G}$ with $\widehat{G} > 0$ on $(\inf X, +\infty)$.
Suppose $\exists\ \widehat{\alpha} > 0$ & $\widehat{\beta} \in \mathbf{R}$ such that

$$\widehat{\alpha}\widehat{u}(x) + \widehat{\beta} = \int \max\{\widehat{v}(x), \widehat{v}(y)\}\widehat{G}(\mathrm{d}y) \qquad \forall x \in X.$$

Assume $\widehat{G}$ concentrated on $X \cup \{-\infty\}$.

(a) Sps $\widehat{v} = v$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{G}$ is better than G in the RHRO on $X \setminus \{\sup X\}$.

$\hookrightarrow$ RHRO (reverse hazard rate order) on $Y \subseteq \mathbf{R}$:
$G(y)\widehat{G}(x) \leq G(x)\widehat{G}(y)$ for all $x < y$ in Y.

(b) Sps $G = \widehat{G}$. Then $\widehat{u}$ is less risk-averse than u iff
$\widehat{v}$ is less risk-averse than v.

Application to unemployment insurance: model

Consumption $x \in X \subseteq \mathbf{R}_+$, risk attitude $v : X \to \mathbf{R}$,

discount factor $\beta \in [0, 1)$. Worker unemployed since $t = 0$.

Application to unemployment insurance: model

Consumption $x \in X \subseteq \mathbf{R}_+$, risk attitude $v : X \to \mathbf{R}$,
discount factor $\beta \in [0, 1)$. Worker unemployed since $t = 0$.

Start of period t: worker chooses search direction
$\iff$ which wage-offer dist'n $p \in \mathcal{P} \subseteq \Delta^0(X)$ to sample from.

Extension: also choose search <u>effort</u>.

Application to unemployment insurance: model

Consumption $x \in X \subseteq \mathbf{R}_+$, risk attitude $v : X \to \mathbf{R}$,
discount factor $\beta \in [0,1)$. Worker unemployed since $t = 0$.

Start of period t: worker chooses search direction
$\iff$ which wage-offer dist'n $p \in \mathcal{P} \subseteq \Delta^0(X)$ to sample from.

Extension: also choose search effort.

End of period t: wage offer x drawn from p, accept/reject.

– if accept: payoff $v(x)$ today, tomorrow, day after…

Extensions: saving (hard), jobs not permanent.

Application to unemployment insurance: model

Consumption $x \in X \subseteq \mathbf{R}_+$, risk attitude $v : X \to \mathbf{R}$, discount factor $\beta \in [0, 1)$. Worker unemployed since $t = 0$.

Start of period t: worker chooses search direction

$\iff$ which wage-offer dist'n $p \in \mathcal{P} \subseteq \Delta^0(X)$ to sample from.

Extension: also choose search effort.

End of period t: wage offer x drawn from p, accept/reject.

- if accept: payoff $v(x)$ today, tomorrow, day after...
- if reject: payoff $v(b_t) + y_t$ today, search again tomorrow.
    - b_t = unemployment benefit
    - $y_t \overset{\text{iid}}{\sim} H$ captures leisure, home production, stigma...

Extensions: saving (hard), jobs not permanent.

Application to unemployment insurance: analysis

Bellman eq'n for value function V_t:

$$V_t = \max_{p\in\mathcal{P}} \int \max\left\{\frac{v(x)}{1-\beta}, \quad v(b_t) \quad + \beta V_{t+1} \right\} \quad p(\mathrm{d}x)$$

Application to unemployment insurance: analysis

Bellman eq'n for value function V_t:

$$V_t = \max_{p \in \mathcal{P}} \int \int \max\left\{ \frac{v(x)}{1-\beta}, \quad [v(b_t) + y] + \beta V_{t+1} \right\} H(\mathrm{d}y) \; p(\mathrm{d}x)$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p\in\mathcal{P}} \int \int \max\left\{ v(x)\, , (1-\beta)[v(b_t)+y] + \beta W_{t+1} \right\} H(\mathrm{d}y)\ p(\mathrm{d}x)$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \int \max\Big\{ v(x)\,, \qquad k \qquad \Big\} F_\beta^{\ell_t}(\mathrm{d}k)\, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

$\hookrightarrow$ F_β^{ℓ} is CDF of r.v. $(1-\beta)Y + \ell$ when r.v. $Y \sim H$.

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)}\, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

$\hookrightarrow$ F_β^ℓ is CDF of r.v. $(1-\beta)Y + \ell$ when r.v. $Y \sim H$.

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\Big\{ v(x)\,, \qquad\qquad k \qquad\qquad \Big\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)}\, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^\ell(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

Assume H log-concave. (Holds for nearly all standard dist'ns.)

$\Longrightarrow \quad$ as $\ell \uparrow$, $\quad F_\beta^\ell \uparrow$ in RHRO.

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad\qquad k \qquad\qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$\boxed{b_t \searrow}$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x) \, , \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$$\boxed{b_t \searrow} \Downarrow \boxed{W_t \searrow}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
b_t \searrow \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\
W_t \searrow \arrow[r, Rightarrow] & \ell_t \searrow
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t := (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t := (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) := H\left(\dfrac{k-\ell}{1-\beta}\right)$.

$$\begin{tikzcd}
b_t \searrow \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & H \text{ log-concave} \\
W_t \searrow \arrow[r, Rightarrow] & \ell_t \searrow
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\Big\{ v(x)\,, \qquad k \qquad \Big\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\dfrac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
b_t \searrow \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & H \text{ log-concave} \arrow[r, Rightarrow] & \begin{array}{c} F_\beta^{\ell_t} \searrow \\ \text{in RHRO} \end{array} \\
W_t \searrow \arrow[r, Rightarrow] & \ell_t \searrow \arrow[ur, Rightarrow] &
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p\in\mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)}\, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
\boxed{b_t \searrow} \arrow[d, Rightarrow] \arrow[drr, Rightarrow] & & \boxed{H \text{ log-concave}} \arrow[rr, Rightarrow] & & \boxed{F_\beta^{\ell_t} \searrow \text{ in RHRO}} \arrow[d, Rightarrow, "\text{Th'm 2}"] \\
\boxed{W_t \searrow} \arrow[rr, Rightarrow] & & \boxed{\ell_t \searrow} \arrow[urr, Rightarrow] & & \boxed{\text{risk-aversion of } u_\beta^{\ell_t} \nearrow}
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$$\begin{tikzcd}
\boxed{b_t \uparrow\uparrow \forall t} \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \boxed{H \text{ log-concave}} \arrow[r, Rightarrow] & \boxed{\begin{array}{c} F_\beta^{\ell_t} \uparrow\uparrow \forall t \\ \text{in RHRO} \end{array}} \arrow[dd, Rightarrow, "\text{Th'm 2}"] \\
\boxed{W_t \uparrow\uparrow \forall t} \arrow[r, Rightarrow] & \boxed{\ell_t \uparrow\uparrow \forall t} \arrow[ur, Rightarrow] & \\
& & \boxed{\begin{array}{c} \text{risk-aversion} \\ \text{of } u_\beta^{\ell_t} \downarrow\downarrow \forall t \end{array}}
\end{tikzcd}$$

Application to unemployment insurance: analysis

Bellman eq'n for normalised value function $W_t \coloneqq (1-\beta)V_t$:

$$W_t = \max_{p \in \mathcal{P}} \int \underbrace{\int \max\left\{ v(x)\,, \qquad k \qquad \right\} F_\beta^{\ell_t}(\mathrm{d}k)}_{=:\, u_\beta^{\ell_t}(x)} \, p(\mathrm{d}x)$$

where $\quad \ell_t \coloneqq (1-\beta)v(b_t) + \beta W_{t+1} \quad \& \quad F_\beta^{\ell}(k) \coloneqq H\left(\frac{k-\ell}{1-\beta}\right).$

$$\text{So} \quad \begin{cases} \underbrace{\text{declining UI}}_{b_t \searrow} \quad \Longrightarrow \quad \underbrace{\text{declining search ambition}}_{\text{risk-aversion of } u_\beta^{\ell_t} \nearrow} \\ \underbrace{\text{raise UI}}_{b_t \uparrow\uparrow \forall t} \quad \Longrightarrow \quad \underbrace{\text{raise search ambition}}_{\text{risk-aversion of } u_\beta^{\ell_t} \downarrow\downarrow \forall t} \end{cases}$$

Plan

Setup and background

The outside-option model

How an o.o. shapes risk attitude (Proposition 1)

Identification (Theorem 1)

Comparative statics (Theorem 2)

What is special about o.o.? (Theorem 3)

Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Idea

Adding an o.o. to a decision problem transforms it by replacing each $x \in X$ with an x-contingent lottery. (Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Question: which transformations decrease risk-aversion?

Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Question: which transformations decrease risk-aversion?

- adding o.o.: yes, by Th'm 1.
- adding background risk: no, not in general.

Idea

Adding an o.o. to a decision problem transforms it
by replacing each $x \in X$ with an x-contingent lottery.
(Viz. lottery which returns x or the o.o., whichever is better.)

Another example of such transform'n: adding background risk.

Question: which transformations decrease risk-aversion?

Answer: <u>only</u> transform'ns that amount to adding an o.o.

The question, formally

Transform'n: replace each x with draw from measure μ_x on X.

'True' risk att. $v : X \to \mathbf{R} \quad \rightsquigarrow \quad$ effective risk att. $x \mapsto \int v \mathrm{d}\mu_x$.

The question, formally

Transform'n: replace each x with draw from measure μ_x on X.

'True' risk att. $v : X \to \mathbf{R} \quad \rightsquigarrow \quad$ effective risk att. $x \mapsto \int v \mathrm{d}\mu_x$.

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes,
which families $(\mu_x)_{x \in X}$ have the property that
$\forall\ v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

The question, reformulated

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes, which families $(\mu_x)_{x\in X}$ have the property that $\forall\ v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

Since u less risk-averse than $v \implies u, v$ ordinally equivalent, restrict to $\mathcal{V}$s whose members are mutually ordinally equivalent.

The question, reformulated

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes, which families $(\mu_x)_{x \in X}$ have the property that $\forall\, v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

Since u less risk-averse than $v \implies u, v$ ordinally equivalent, restrict to $\mathcal{V}$s whose members are mutually ordinally equivalent.

No other restrictions $\implies \mathcal{V}$ is an ordinal equivalence class.

The question, reformulated

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes, which families $(\mu_x)_{x \in X}$ have the property that $\forall\ v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

Since u less risk-averse than $v \implies u, v$ ordinally equivalent, restrict to $\mathcal{V}$s whose members are mutually ordinally equivalent.

No other restrictions $\implies \mathcal{V}$ is an ordinal equivalence class.

Given such $\mathcal{V}$, fix $v_0 \in \mathcal{V}$ & embed X in $\mathbf{R}$ via $x \mapsto v_0(x)$. Then $X \subseteq \mathbf{R}$ & $\mathcal{V}$ is the set of all str. incr. f'ns $X \to \mathbf{R}$.

The question, reformulated

Question: given rich class $\mathcal{V}$ of possible 'true' risk attitudes, which families $(\mu_x)_{x \in X}$ have the property that $\forall\, v \in \mathcal{V}, \quad x \mapsto \int v \mathrm{d}\mu_x$ is less risk-averse than v?

Since u less risk-averse than $v \implies u, v$ ordinally equivalent, restrict to $\mathcal{V}$s whose members are mutually ordinally equivalent.

No other restrictions $\implies \mathcal{V}$ is an ordinal equivalence class.

Given such $\mathcal{V}$, fix $v_0 \in \mathcal{V}$ & embed X in $\mathbf{R}$ via $x \mapsto v_0(x)$. Then $X \subseteq \mathbf{R}$ & $\mathcal{V}$ is the set of all str. incr. f'ns $X \to \mathbf{R}$.

Question′: which families $(G_x)_{x \in X}$ of CDFs concentr'd on X have the property that $\forall$ str. incr. $v : X \to \mathbf{R}$, $x \mapsto \int v \mathrm{d}G_x$ is less risk-averse than v?

The answer

Theorem 3. For non-empty Borel $X \subseteq \mathbf{R}$ with $\sup X \notin X$ & family $(G_x)_{x \in X}$ of CDFs concentrated on X, the following are equivalent:

(a) $\forall$ bounded str. incr. $v : X \to \mathbf{R}$,
$x \mapsto \int v \mathrm{d} G_x$ is less risk-averse than v.

The answer

Theorem 3. For non-empty Borel $X \subseteq \mathbf{R}$ with $\sup X \notin X$ & family $(G_x)_{x \in X}$ of CDFs concentrated on X, the following are equivalent:

(a) $\forall$ bounded str. incr. $v : X \to \mathbf{R}$, $x \mapsto \int v \mathrm{d} G_x$ is less risk-averse than v.

(b) $\exists$ ECDF G $\begin{cases} \text{concentrated on } X \cup \{-\infty\}, \\ \text{with } G > 0 \text{ on } (\inf X, +\infty) \end{cases}$ such that

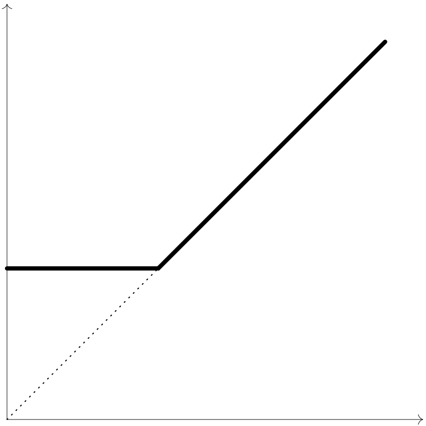
$\forall$ bounded str. incr. $v : X \to \mathbf{R}$,

$\exists\ \alpha > 0$ & $\beta \in \mathbf{R}$ such that

$$\alpha \int v \mathrm{d} G_x + \beta = \int \max\{v(x), v(y)\} G(\mathrm{d}y) \quad \text{for every } x \in X,$$

where by convention $v(-\infty) := -\infty$.

Thanks!



Adam Smith on limited liability and risk-taking

From the *Wealth of Nations*:[3]

> *In a private copartnery, each partner is bound for the debts contracted by the company to the whole extent of his fortune. In a joint stock company, on the contrary, each partner is bound only to the extent of his share. This total exemption from trouble and from risk, beyond a limited sum, encourages many people to become adventurers in joint stock companies, who would, upon no account, hazard their fortunes in any private copartnery.*

(back to slide 4)

[3]Pages. 740–1 in the Glasgow edition (Smith, 1776/1976).

Extended CDFs

Extended CDF: $k \mapsto \mathbf{P}(K \leq k)$ for a $[-\infty, +\infty)$-valued random variable K.

Observation: $F : \mathbf{R} \to [0,1]$ is an ECDF iff increasing, right-continuous, $\lim_{k \nearrow +\infty} F(k) = 1$.

Expectation of meas'ble $g : [-\infty, +\infty) \to \mathbf{R}$ w.r.t. ECDF F:

$$\int g \mathrm{d}F := \left(\lim_{k \searrow -\infty} F(k) \right) g(-\infty) + \int_{\mathbf{R}} g \mathrm{d}F.$$

(back to slide 12)

(Non-)identification of the o.o. dist'n: details

Proposition 2. For $X \neq \varnothing$, $u : X \to \mathbf{R}$ & ECDF F, the following are equivalent:

(I) $\int_{(0,+\infty)} kF(\mathrm{d}k) < +\infty$, and

if $\int_{(-\infty,0]} kF(\mathrm{d}k) > -\infty$ then u is bounded below.

(Non-)identification of the o.o. dist'n: details

Proposition 2. For $X \neq \varnothing$, $u : X \to \mathbf{R}$ & ECDF F, the following are equivalent:

(I) $\int_{(0,+\infty)} kF(\mathrm{d}k) < +\infty$, and

if $\int_{(-\infty,0]} kF(\mathrm{d}k) > -\infty$ then u is bounded below.

(II) $\exists\ \alpha > 0$, $\beta \in \mathbf{R}$, & $v : X \to \mathbf{R}$ such that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(\mathrm{d}k) \quad \forall\ x \in X.$$

(back to slide 19)

Forces shaping risk att.: background risk

E.g. Ross (1981), Kihlstrom, Romer and Williams (1981), Pratt and Zeckhauser (1987), Kimball (1993), Eeckhoudt, Gollier and Schlesinger (1996), Gollier and Pratt (1996), Pomatto, Strack and Tamuz (2020) and Mu, Pomatto, Strack and Tamuz (2024).

(back to slide 5)

Forces shaping risk att.: employment contracts

E.g. Wilson (1969), Ross (1974), Amihud and Lev (1981), Lambert (1986), Hirshleifer and Suh (1992), Diamond (1998), Garicano and Rayo (2016) and Barron, Georgiadis and Swinkels (2020).

(back to slide 5)

Forces shaping risk att.: financing contracts

E.g. Galai and Masulis (1976), Jensen and Meckling (1976), Stiglitz and Weiss (1981), Green (1984) and Hébert (2018).

(back to slide 5)

Forces shaping risk att.: career concerns

E.g. Holmström (1982/1999), Hirshleifer and Thakor (1992), Hermalin (1993) and Chen (2015).

(back to slide 5)

Forces shaping risk att.: flexibility

E.g. Drèze and Modigliani (1966/1972), Mossin (1969), Spence and Zeckhauser (1972), Machina (1982, 1984), Bodie, Merton and Samuelson (1992), Gollier (2005), Chetty and Szeidl (2007) and Postlewaite, Samuelson and Silverman (2008).

(back to slide 5)

Forces shaping risk att.: R&D races

E.g. Dasgupta and Stiglitz (1980), Klette and de Meza (1986), Dasgupta and Maskin (1987), Cabral (2003) and Anderson and Cabral (2007).

(back to slide 5)

Forces shaping risk att.: competition for status

E.g. Robson (1992), Rosen (1997), Becker, Murphy and Werning (2005), Ray and Robson (2012) and Hopkins (2018).

(back to slide 5)

Forces shaping risk att.: competition generally

E.g. Hvide (2002), Hvide and Kristiansen (2003), Taylor (2003), Kräkel and Sliwka (2004), Seel and Strack (2013, 2016), Nutz and Zhang (2022) and Fang et al. (2025).

(back to slide 5)

Forces shaping risk att.: competition in banking

Surveys: Beck (2008), Carletti (2008), Vives (2016) and Berger, Klapper and Turk-Ariss (2017).

(back to slide 5)

References I

Amihud, Y., & Lev, B. (1981). Risk reduction as a managerial motive for conglomerate mergers. *Bell Journal of Economics*, *12*(2), 605–617. https://doi.org/10.2307/3003575

Anderson, A., & Cabral, L. M. B. (2007). Go for broke or play it safe? Dynamic competition with choice of variance. *RAND Journal of Economics*, *38*(3), 593–609. https://doi.org/10.1111/j.0741-6261.2007.00102.x

Barron, D., Georgiadis, G., & Swinkels, J. (2020). Optimal contracts with a risk-taking agent. *Theoretical Economics*, *15*(2), 715–761. https://doi.org/10.3982/TE3660

Beck, T. (2008). *Bank competition and financial stability: Friends or foes?* [working paper, Jun 2008].

References II

Becker, G. S., Murphy, K. M., & Werning, I. (2005). The equilibrium distribution of income and the market for status. *Journal of Political Economy*, *113*(2), 282–310. https://doi.org/10.1086/427463

Ben-Porath, E., Dekel, E., & Lipman, B. L. (2018). Disclosure and choice. *Review of Economic Studies*, *85*(3), 1471–1501. https://doi.org/10.1093/restud/rdx064

Berger, A. N., Klapper, L. F., & Turk-Ariss, R. (2017). Bank competition and financial stability. In J. A. Bikker & L. Spierdijk (Eds.), *Handbook of competition in banking and finance* (pp. 185–204). Elgar. https://doi.org/10.4337/9781785363306.00018

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life cycle model. *Journal of Economic Dynamics and Control*, *16*(3–4), 427–449. https://doi.org/10.1016/0165-1889(92)90044-F

References III

Cabral, L. M. B. (2003). R&D competition when firms choose variance. *Journal of Economics & Management Strategy*, *12*(1), 139–150. https://doi.org/10.1111/j.1430-9134.2003.00139.x

Carletti, E. (2008). Competition and regulation in banking. In A. V. Thakor & A. W. A. Boot (Eds.), *Handbook of financial intermediation and banking* (pp. 449–482). Elsevier. https://doi.org/10.1016/B978-044451558-2.50022-2

Chen, Y. (2015). Career concerns and excessive risk taking. *Journal of Economics & Management Strategy*, *24*(1), 110–130. https://doi.org/10.1111%2Fjems.12085

Chetty, R., & Szeidl, A. (2007). Consumption commitments and risk preferences. *Quarterly Journal of Economics*, *122*(2), 831–877. https://doi.org/10.1162/qjec.122.2.831

References IV

Dasgupta, P., & Maskin, E. (1987). The simple economics of research portfolios. *The Economic Journal*, *97*(387), 581–595. https://doi.org/10.2307/2232925

Dasgupta, P., & Stiglitz, J. (1980). Uncertainty, industrial structure, and the speed of r&d. *11*(1), 1–28. https://doi.org/10.2307/3003398

Diamond, P. (1998). Managerial incentives: On the near linearity of optimal compensation. *Journal of Political Economy*, *106*(5), 931–957. https://doi.org/10.1086/250036

Drèze, J. H., & Modigliani, F. (1966). Épargne et consommation en avenir aléatoire. *Cahiers du Séminaire d'Économétrie*, *9*, 7–33.

Drèze, J. H., & Modigliani, F. (1972). Consumption decisions under uncertainty. *Journal of Economic Theory*, *5*(3), 308–335. https://doi.org/10.1016/0022-0531(72)90044-0

References V

Eeckhoudt, L., Gollier, C., & Schlesinger, H. (1996). Changes in background risk and risk taking behavior. *Econometrica*, *64*(3), 683–689. https://doi.org/10.2307/2171866

Fang, D., Ke, C., Kubitz, G., Noe, T. H., Page, L., & Liu, Y. (2025). *Winning ways: How rank-based incentives shape risk-taking decisions* [working paper, 5 Feb 2025]. https://doi.org/10.2139/ssrn.4856341

Galai, D., & Masulis, R. W. (1976). The option pricing model and the risk factor of stock. *Journal of Financial Economics*, *3*(1–2), 53–81. https://doi.org/10.1016/0304-405X(76)90020-9

Garicano, L., & Rayo, L. (2016). Why organizations fail: Models and cases. *Journal of Economic Literature*, *54*(1), 137–192. https://doi.org/10.1257/jel.54.1.137

References VI

Golbe, D. L. (1981). The effects of imminent bankruptcy on stockholder risk preferences and behavior. *Bell Journal of Economics*, *12*(1), 321–328. https://doi.org/10.2307/3003528

Golbe, D. L. (1988). Risk-taking by firms near bankruptcy. *Economics Letters*, *28*(1), 75–79. https://doi.org/10.1016/0165-1765(88)90075-4

Gollier, C. (2005). *Does flexibility enhance risk tolerance?* [working paper, 8 Jun 2025].

Gollier, C., Koehl, P.-F., & Rochet, J.-C. (1997). Risk-taking behavior with limited liability and risk aversion. *Journal of Risk and Insurance*, *64*(2), 347–370. https://doi.org/10.2307/253734

Gollier, C., & Pratt, J. W. (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica*, *64*(5), 1109–1123. https://doi.org/10.2307/2171958

References VII

Green, R. C. (1984). Investment incentives, debt, and warrants. *Journal of Financial Economics*, *13*(1), 115–136. https://doi.org/10.1016/0304-405X(84)90034-5

Hébert, B. (2018). Moral hazard and the optimality of debt. *Review of Economic Studies*, *85*(4), 2214–2252. https://doi.org/10.1093/restud/rdx080

Hermalin, B. E. (1993). Managerial preferences concerning risky projects. *Journal of Law, Economics, & Organization*, *9*(1), 127–135. https://doi.org/10.1093/oxfordjournals.jleo.a037049

Hirshleifer, D., & Suh, Y. (1992). Risk, managerial effort, and project choice. *Journal of Financial Intermediation*, *2*(3), 308–345. https://doi.org/10.1016/1042-9573(92)90004-W

Hirshleifer, D., & Thakor, A. V. (1992). Managerial conservatism, project choice, and debt. *Review of Financial Studies*, *5*(3), 437–470. https://doi.org/10.1093/rfs/5.3.437

References VIII

Holmström, B. (1982). Managerial incentive problems: A dynamic perspective [reprinted in Holmström (1999)]. In *Vetenskap och företagsledning: Studier i ekonomi och ledarskap tillägnade Lars Wahlbeck* (pp. 209–230). Swedish School of Economics and Business Administration, Helsinki.

Holmström, B. (1999). Managerial incentive problems: A dynamic perspective [reprinted from Holmström (1982)]. *Review of Economic Studies*, *66*(1), 169–182. https://doi.org/10.1111/1467-937X.00083

Hopkins, E. (2018). Inequality and risk-taking behaviour. *Games and Economic Behavior*, *107*, 316–328. https://doi.org/10.1016/j.geb.2017.11.007

Hvide, H. K. (2002). Tournament rewards and risk taking. *Journal of Labor Economics*, *20*(4), 709–978. https://doi.org/10.1086/342041

References IX

Hvide, H. K., & Kristiansen, E. G. (2003). Risk taking in selection contests. *Games and Economic Behavior*, *42*(1), 172–179. https://doi.org/10.1016/S0899-8256(02)00538-9

Jensen, M. C., & Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, *3*(4), 305–360. https://doi.org/10.1016/0304-405X(76)90026-X

Kihlstrom, R. E., Romer, D., & Williams, S. (1981). Risk aversion with random initial wealth. *Econometrica*, *49*(4), 911–920. https://doi.org/10.2307/1912510

Kimball, M. S. (1993). Standard risk aversion. *Econometrica*, *61*(3), 589–611. https://doi.org/10.2307/2951719

Klette, T., & de Meza, D. (1986). Is the market biased against risky R&D? *RAND Journal of Economics*, *17*(1), 133–139.

References X

Kräkel, M., & Sliwka, D. (2004). Risk taking in asymmetric tournaments. *German Economic Review*, *5*(1), 103–116. https://doi.org/10.1111/j.1465-6485.2004.00096.x

Lambert, R. A. (1986). Executive effort and selection of risky projects. *RAND Journal of Economics*, *17*(1), 77–88.

Machina, M. J. (1982). Flexibility and the demand for risky assets. *Economics Letters*, *10*(1–2), 71–76. https://doi.org/10.1016/0165-1765(82)90118-5

Machina, M. J. (1984). Temporal risk and the nature of induced preferences. *Journal of Economic Theory*, *33*(2), 199–231. https://doi.org/10.1016/0022-0531(84)90087-5

Mossin, J. (1969). A note on uncertainty and preferences in a temporal context. *American Economic Review*, *59*(1), 172–174.

References XI

Mu, X., Pomatto, L., Strack, P., & Tamuz, O. (2024). Background risk and small-stakes risk aversion. *American Economic Review: Insights*, *6*(2), 262–276. https://doi.org/10.1257/aeri.20220480

Nekoei, A., & Weber, A. (2017). Does extending unemployment benefits improve job quality? *American Economic Review*, *107*(2), 527–61. https://doi.org/10.1257/aer.20150528

Nutz, M., & Zhang, Y. (2022). Reward design in risk-taking contests. *SIAM Journal on Financial Mathematics*, *13*(1), 129–146. https://doi.org/10.1137/21M1397386

Pomatto, L., Strack, P., & Tamuz, O. (2020). Stochastic dominance under independent noise. *Journal of Political Economy*, *128*(5), 1877–1900. https://doi.org/10.1086/705555

References XII

Postlewaite, A., Samuelson, L., & Silverman, D. (2008). Consumption commitments and employment contracts. *Review of Economic Studies*, *75*(2), 559–578. https://doi.org/10.1111/j.1467-937X.2008.00472.x

Pratt, J. W., & Zeckhauser, R. J. (1987). Proper risk aversion. *Econometrica*, *55*(1), 143–154. https://doi.org/10.2307/1911160

Ray, D., & Robson, A. J. (2012). Status, intertemporal choice, and risk-taking. *Econometrica*, *80*(4), 1505–1531. https://doi.org/10.3982/ECTA9042

Robson, A. J. (1992). Status, the distribution of wealth, private and social attitudes to risk. *Econometrica*, *60*(4), 837–857. https://doi.org/10.2307/2951568

Rosen, S. (1997). Manufactured inequality. *Journal of Labor Economics*, *15*(2), 189–196. https://doi.org/10.1086/209829

References XIII

Ross, S. A. (1974). On the economic theory of agency and the principle of similarity. In M. Balch, D. McFadden, & S.-Y. Wu (Eds.), *Essays on economic behavior under uncertainty* (pp. 215–237). North-Holland.

Ross, S. A. (1981). Some stronger measures of risk aversion in the small and the large with applications. *Econometrica*, *49*(3), 621–638. https://doi.org/10.2307/1911515

Schmieder, J. F., & von Wachter, T. (2016). The effects of unemployment insurance benefits: New evidence and interpretation. *Annual Review of Economics*, *8*, 547–581. https://doi.org/10.1146/annurev-economics-080614-115758

Seel, C., & Strack, P. (2013). Gambling in contests. *Journal of Economic Theory*, *148*(5), 2033–2048. https://doi.org/10.1016/j.jet.2013.07.005

References XIV

Seel, C., & Strack, P. (2016). Continuous time contests with private information. *Mathematics of Operations Research*, *41*(3), 745–1160. https://doi.org/10.1287/moor.2015.0769

Smith, A. (1776). *An inquiry into the nature and causes of the wealth of nations*. W. Strahan; T. Cadell.

Smith, A. (1776/1976). *An inquiry into the nature and causes of the wealth of nations* (W. B. Todd, Ed.; Vol. 2). Oxford University Press. https://doi.org/10.1093/actrade/9780199269570.book.1

Spence, M., & Zeckhauser, R. (1972). The effect of the timing of consumption decisions and the resolution of lotteries on the choice of lotteries. *Econometrica*, *40*(2), 401–403. https://doi.org/10.2307/1909418

Stiglitz, J. E., & Weiss, A. (1981). Credit rationing in markets with imperfect information. *American Economic Review*, *71*(3), 393–410.

References XV

Taylor, J. (2003). Risk-taking behavior in mutual fund tournaments. *Journal of Economic Behavior & Organization*, *50*(3), 373–383. https://doi.org/10.1016/S0167-2681(02)00028-8

Vives, X. (2016). *Competition and stability in banking: The role of regulation and competition policy*. Princeton University Press. https://doi.org/10.23943/princeton/9780691171791.001.0001

Wilson, R. (1969). The structure of incentives for decentralization under uncertainty. In *La décision: Agrégation et dynamique des ordres de préférence* (pp. 287–307). Éditions du CNRS.