Epistemic Exams

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Introduction

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An **exam** measures what you know.

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An **epistemic exam** measures what you know *and* what you know you know.

Example

Student 1 gets 6/10 correct answers on an exam

Result. C C C C C C W W W W

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and after the exam reports *"I'm very confident my first 6 answers are correct. Moreover, though these are my best guesses, I'm very confident my last 4 answers are wrong."*

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Student 2 gets 8/10 correct answers on an exam

Result. C C C C C C C C W W

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Student 2 gets 8/10 correct answers on an exam

Result. C C C C C C C C W W

and after the exam reports *"I'm very confident my last 8 answers are correct. Moreover, though these are my best guesses, I'm very confident my first 2 answers are wrong."*

Beliefs. W W C C C C C C C C

Example

Student 1	:	Result.	C C C C C C W W W W
		Beliefs.	C C C C C C W W W W
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		Beliefs.	W W C C C C C C C C

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- Student 2 **knows** more than Student 1.

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- Student 2 **knows** more than Student 1.
  - That is, Student 2 will get a higher fraction of her answers correct than Student 1, given similar questions
- Student 1 **knows what she knows** more than Student 2.
  - That is, Student 1 can perfectly identify which of her answers will be correct and which will be wrong, while Student 2 cannot

Example

How should we score each of these students?

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How should we score each of these students?

That's what this paper is about

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Food for Thought and Preview

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  - Other times it's better to have the student who knows what they know more, even if they know less
- The desired scoring rule will be a function of the characteristics of the situation you face

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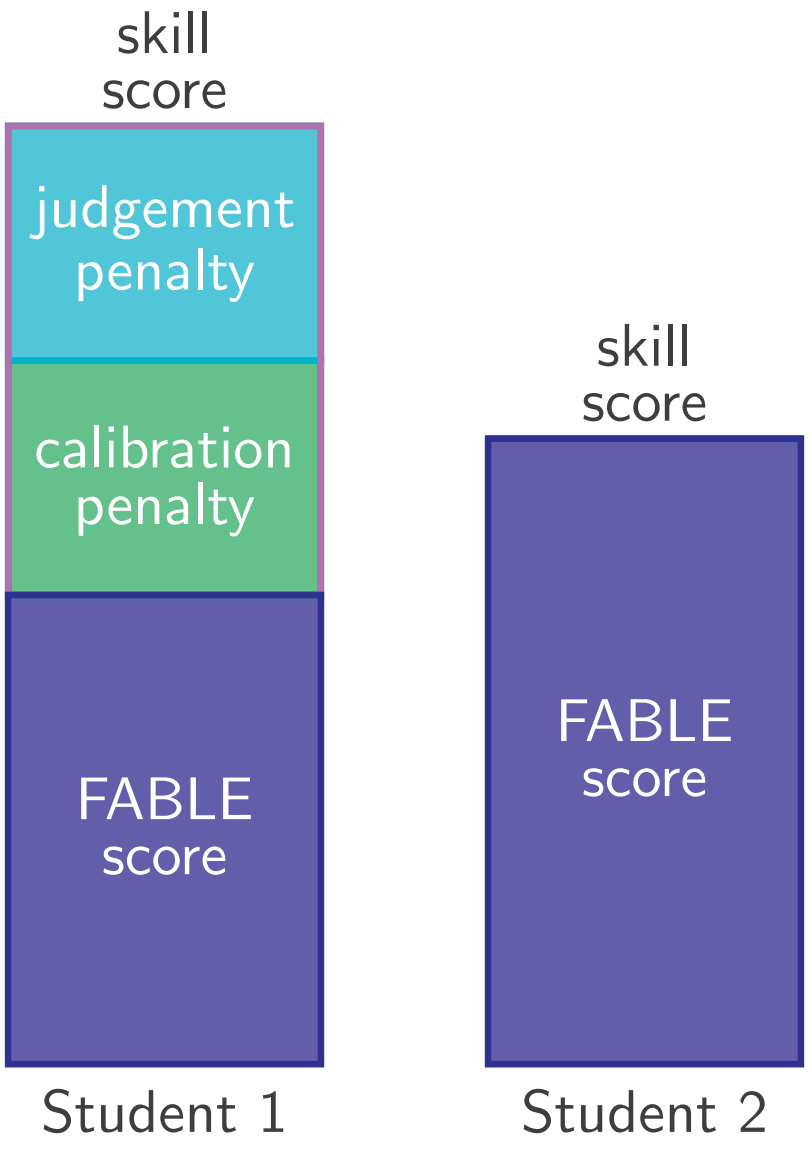
2. To help train AIs to behave closer to how we would like them to behave

In particular, current AIs know a lot, but don't always know what they know, and as a result are **unreliable**

Preview


skill
score
judgement
penalty
calibration
penalty
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score
Student 1
skill
score
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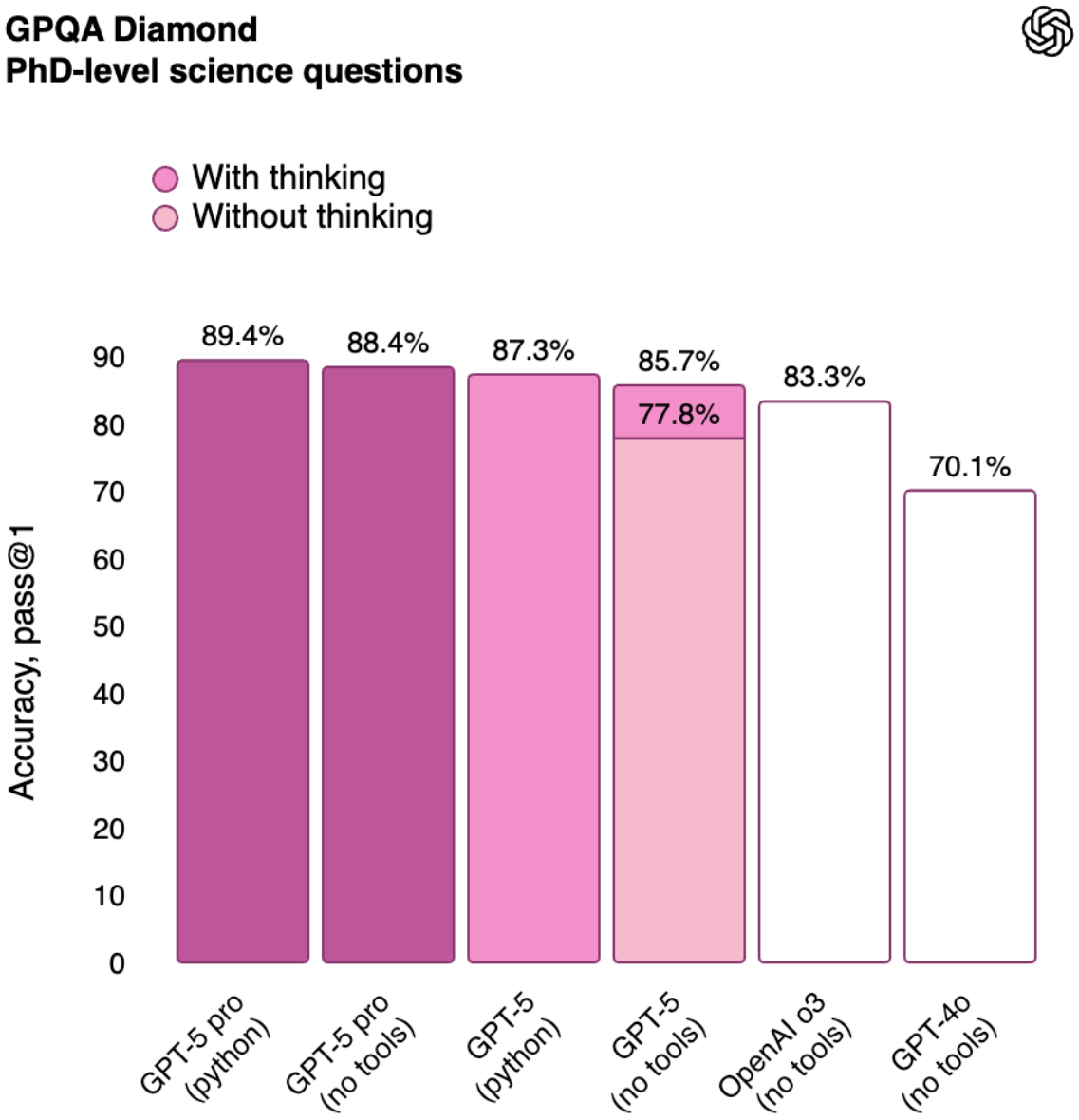
score
Student 2

Instead Of


GPQA Diamond
PhD-level science questions
With thinking
Without thinking
Accuracy, pass@1
0
10
20
30
40
50
60
70
80
90
89.4%
88.4%
87.3%
85.7%
77.8%
83.3%
70.1%
GPT-5 pro (python)
GPT-5 pro (no tools)
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GPT-5 (no tools)
OpenAI o3 (no tools)
GPT-4o (no tools)

Roadmap

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1. Scoring Rules

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2. Incentive-Compatible Scoring Rules

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3. Utilitarian Scoring Rules

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Scoring Rules

Forecasting

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We would like to incentivize the forecaster to report p by providing a score based on the realized outcome

Forecasting Environment

Definition. A *scoring rule* (S_0, S_1) is a pair of functions $S_0 : [0,1] \to [-\infty, \infty]$ and $S_1 : [0,1] \to [-\infty, \infty]$ where

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The agent submits a pair (x, p) where $x \in X$ and $p \in [0, 1]$

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Incentive-Compatible Scoring Rules

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Proper Scoring Rules

Definition. A scoring rule (S_0, S_1) is *(strictly) proper* if the forecaster (uniquely) maximizes her expected score by reporting her true belief.

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Definition. A scoring rule (S_0, S_1) is *(strictly) proper* if the forecaster (uniquely) maximizes her expected score by reporting her true belief. That is, for any $p^* \in [0, 1]$ and $p \neq p^*$,

$$p^* S_1(p^*) + (1 - p^*) S_0(p^*) \geq (>)\, p^* S_1(p) + (1 - p^*) S_0(p).$$

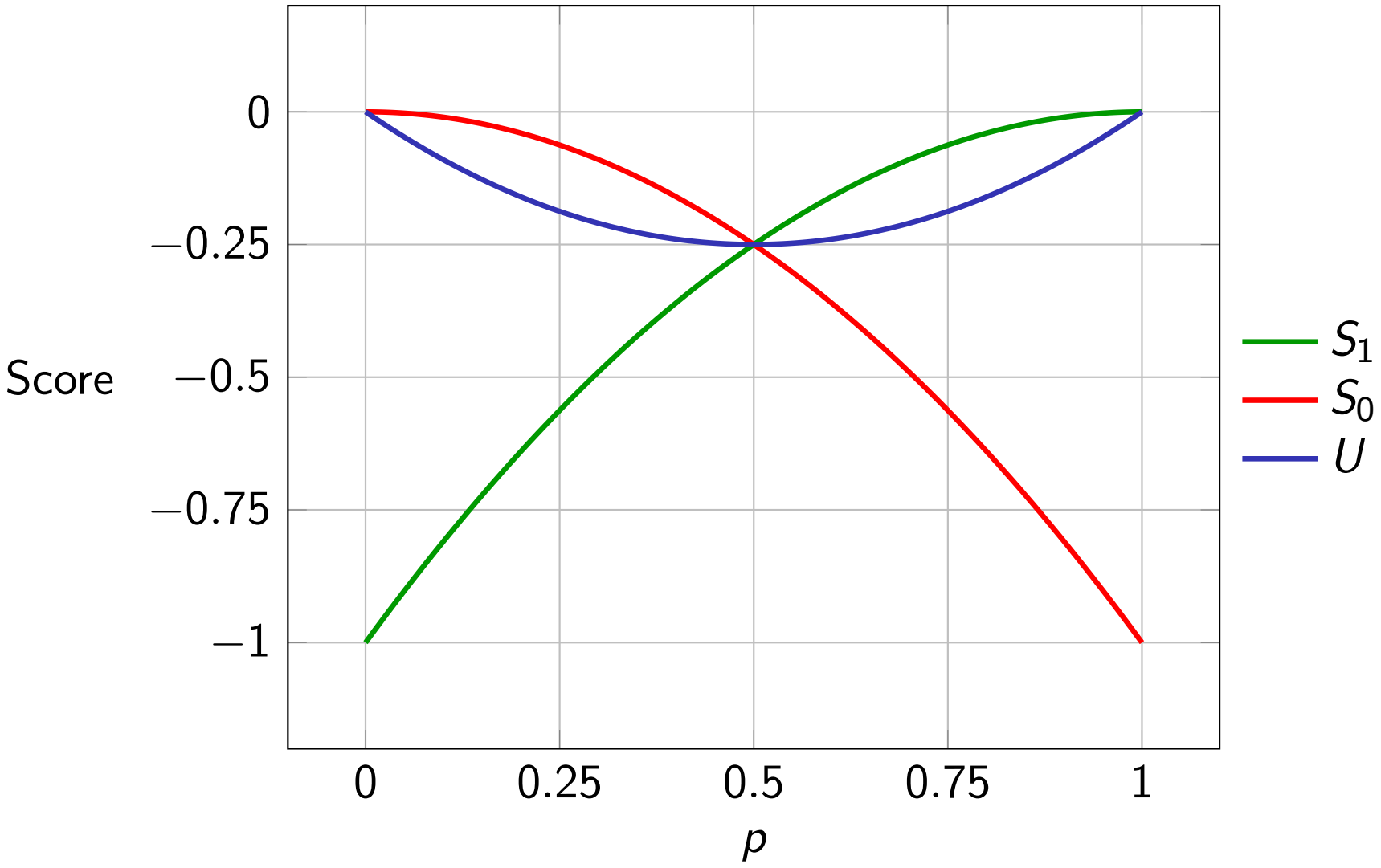
Quadratic Scoring Rule

For example, the *quadratic scoring rule* is defined by

$$S_0(p) = -p^2 \quad \text{and} \quad S_1(p) = -(1-p)^2$$

and is a strictly proper scoring rule.

Quadratic Scoring Rule



Characterization of Proper Scoring Rules

Theorem 1 (McCarthy, Savage). A scoring rule S is (strictly) proper if and only if

$$S_0(p) = U(p) - pU'(p) \quad \text{and} \quad S_1(p) = U(p) + (1-p)U'(p)$$

where $U : [0,1] \to \mathbb{R}$ is a (strictly) convex function.

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where $U : [0,1] \to \mathbb{R}$ is a (strictly) convex function.

Moreover, U is precisely the expected score function, i.e.,

$$U(p) = pS_1(p) + (1-p)S_0(p).$$

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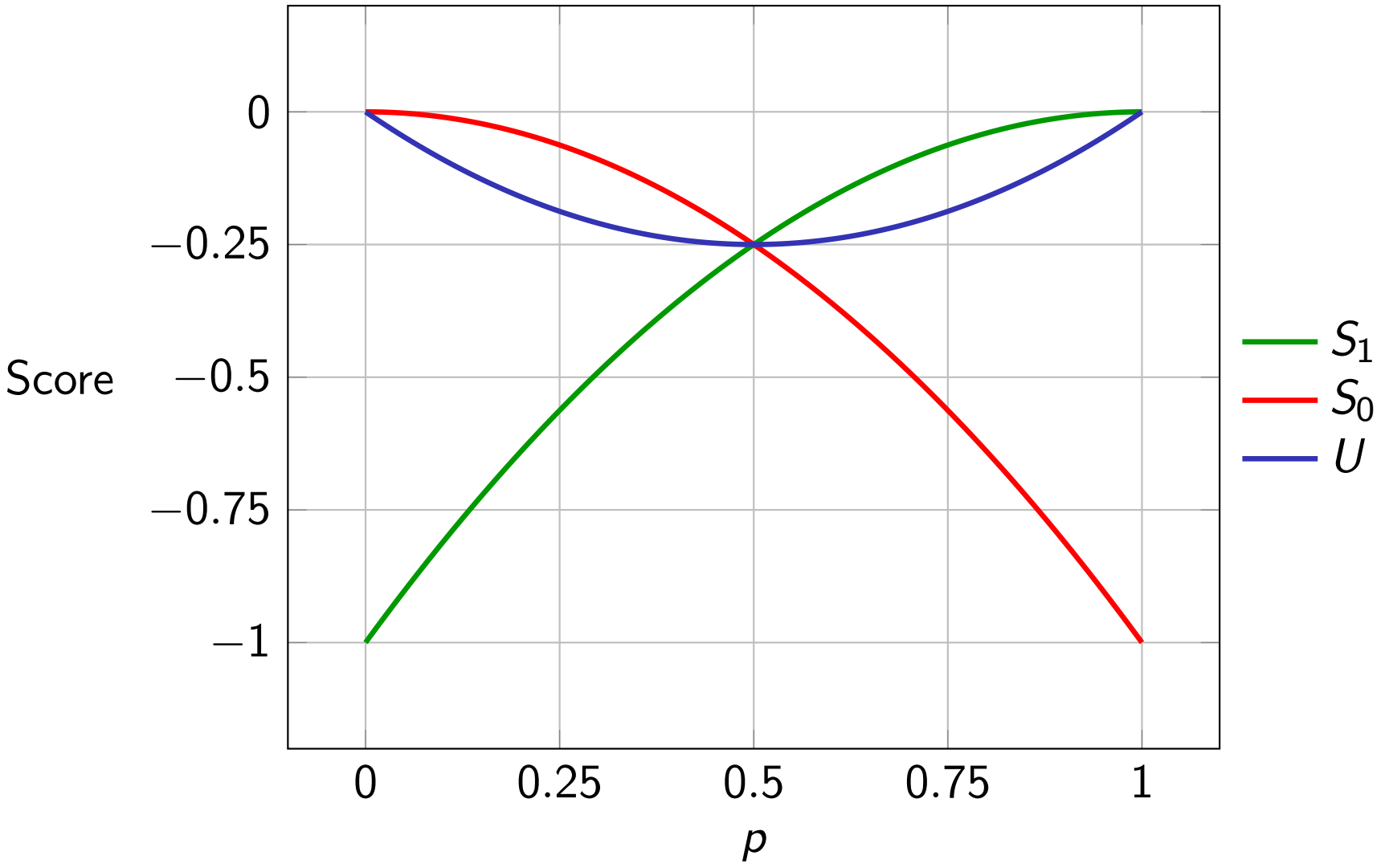

Test-Taking

Proper Epistemic Exam Scoring Rules

Definition. A scoring rule (S_0, S_1) is a *(strictly) proper epistemic exam scoring rule* if the test-taker (uniquely) maximizes her expected score by submitting her best answer x^* along with her true belief that it is correct $p(x^*)$, no matter the feasible set of answers X.

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That is, for any X, if $x \neq x^*$ or $\pi \neq p(x^*)$, then

$$p(x^*)S_1(p(x^*)) + (1 - p(x^*))S_0(p(x^*)) \geq (>)\, p(x)S_1(\pi) + (1 - p(x))S_0(\pi),$$

where $x^* \in \arg\max_{x \in X} p(x)$.

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Alternatively, (2) can be replaced by

3. the score for correct answers is everywhere above the score for incorrect answers, $S_1(p) \geq S_0(p)$ for all p.

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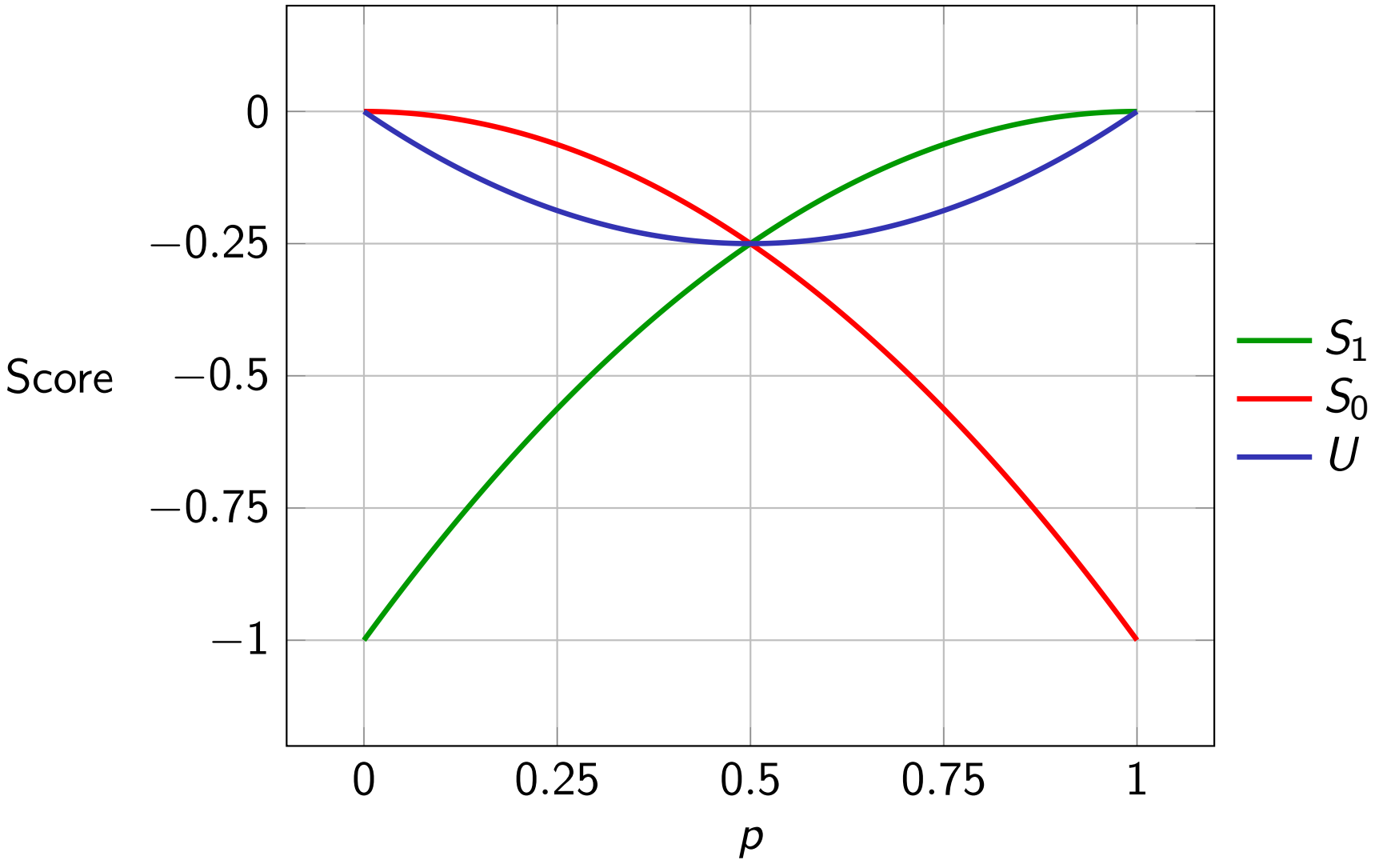

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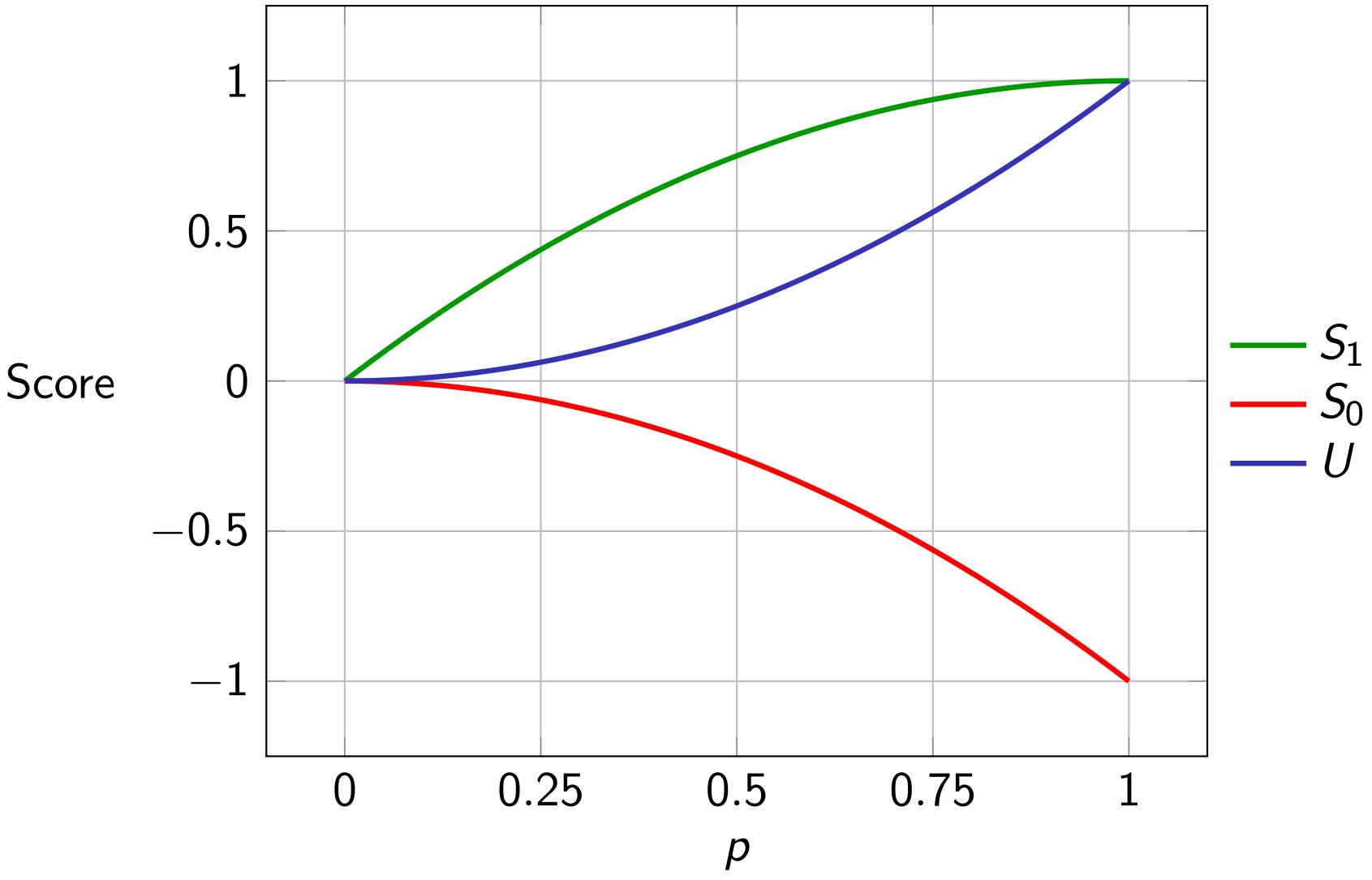

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For any desired expected payoff for a truthful agent U, then there are infinitely many scoring rules (S_0, S_1) that generate it (e.g., $S_1 = S_0 = U$)

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For any desired expected payoff for a truthful agent U, then there are infinitely many scoring rules (S_0, S_1) that generate it (e.g., $S_1 = S_0 = U$)

If U is **not convex**, then none of these scoring rules are proper scoring rules

If U is **convex**, then exactly one of these scoring rules is a proper scoring rule

If U is **convex and increasing**, then exactly one of these scoring rules is a proper epistemic exam scoring rule

Utilitarian Scoring Rules

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- the end-user of the forecast in the forecasting environment, or
- the end-user of the answer in the test-taking environment

Forecasting

Forecasting Environment with User

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Definition. A *utilitarian scoring rule* is a scoring rule that reflects the expected utility to society of employing that agent, i.e., for all $p \in [0, 1]$,

$$S_0(p) = U_0(a^*(p)) \quad \text{and} \quad S_1(p) = U_1(a^*(p)).$$

Proposition 1

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Intuitively, the forecaster and the user are perfectly aligned—and the user makes optimal decisions given the forecast—so the forecaster wants to give the user the true forecast

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I'd be curious afterwards if you have any references or thoughts on this

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Utilitarian Scoring Rule

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Definition. A *utilitarian scoring rule* is a scoring rule that reflects the expected utility to society of employing that agent, i.e., for all $p \in [0, 1]$,

$$S_0(p) = U_0(a^*(p, x)) \quad \text{and} \quad S_1(p) = U_1(a^*(p, x)).$$

Proposition 2

Proposition 2. In a test-taking environment, every utilitarian scoring rule is a proper epistemic exam scoring rule.

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This follows from Proposition 1 and the fact that U is weakly increasing

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Proposition 2 shows that if we want to measure an AI's usefulness to society, then that scoring rule will also be incentive-compatible.

FABLE

Deriving FABLE

We are now going to derive FABLE

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First, we will derive a simpler FABE scoring rule, and then we will derive the full FABLE scoring rule

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We would like to design a utilitarian scoring rule that reflects the payoffs to society of employing the AI that we are testing

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We will consider three actions the user can take:

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1. **Disregard** : Don't implement

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3. **Implement** : Do implement

The User's (Society's) Payoffs

Let's define the user's payoffs

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First, the two normalized payoffs

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From here, there will be 5 parameters

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  - $E = \frac{1}{n}$ where n is the number of answers at which the user would be just indifferent between checking n answers to get one correct answer and not bothering

The 5 Parameters (continued)

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- L : luck
  - the confidence level associated with random guessing (1 / the number of multiple choice answers)

The 5 Parameters

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- $F = 9$ (failure is 9x worse than success)
- $\alpha = 0.05$
- $\beta = 0.2$
- $L = 0.25$ (4 multiple choice answers)
- $E = 0.33$: (evaluation costs 0.33 of the benefit of a success)

Deriving FABLE

Let's now write the payoffs/scores assuming the user always disregards, evaluates, and implements, respectively

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Then we will combine them and take the max to find the overall scoring rule

Disregard

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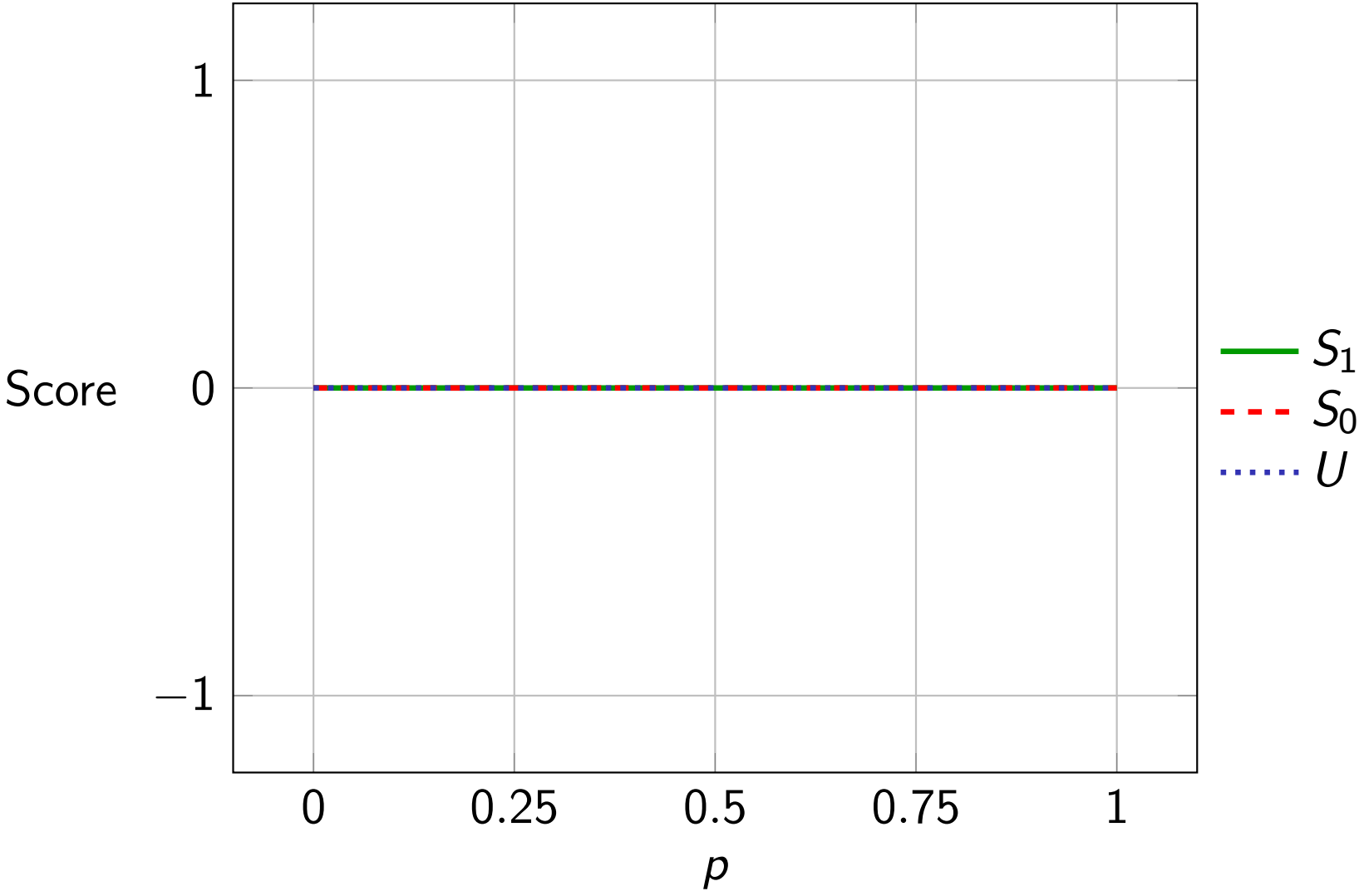
$$S_1(p) = 0$$

Disregard

$$S_1(p) = 0$$

$$S_0(p) = 0$$

Disregard



Evaluate

Evaluate

$$
\begin{aligned}
S_1(p) &= -E + (1-\beta) \times 1 + \beta \times 0 \\
&= 1 - \beta - E
\end{aligned}
$$

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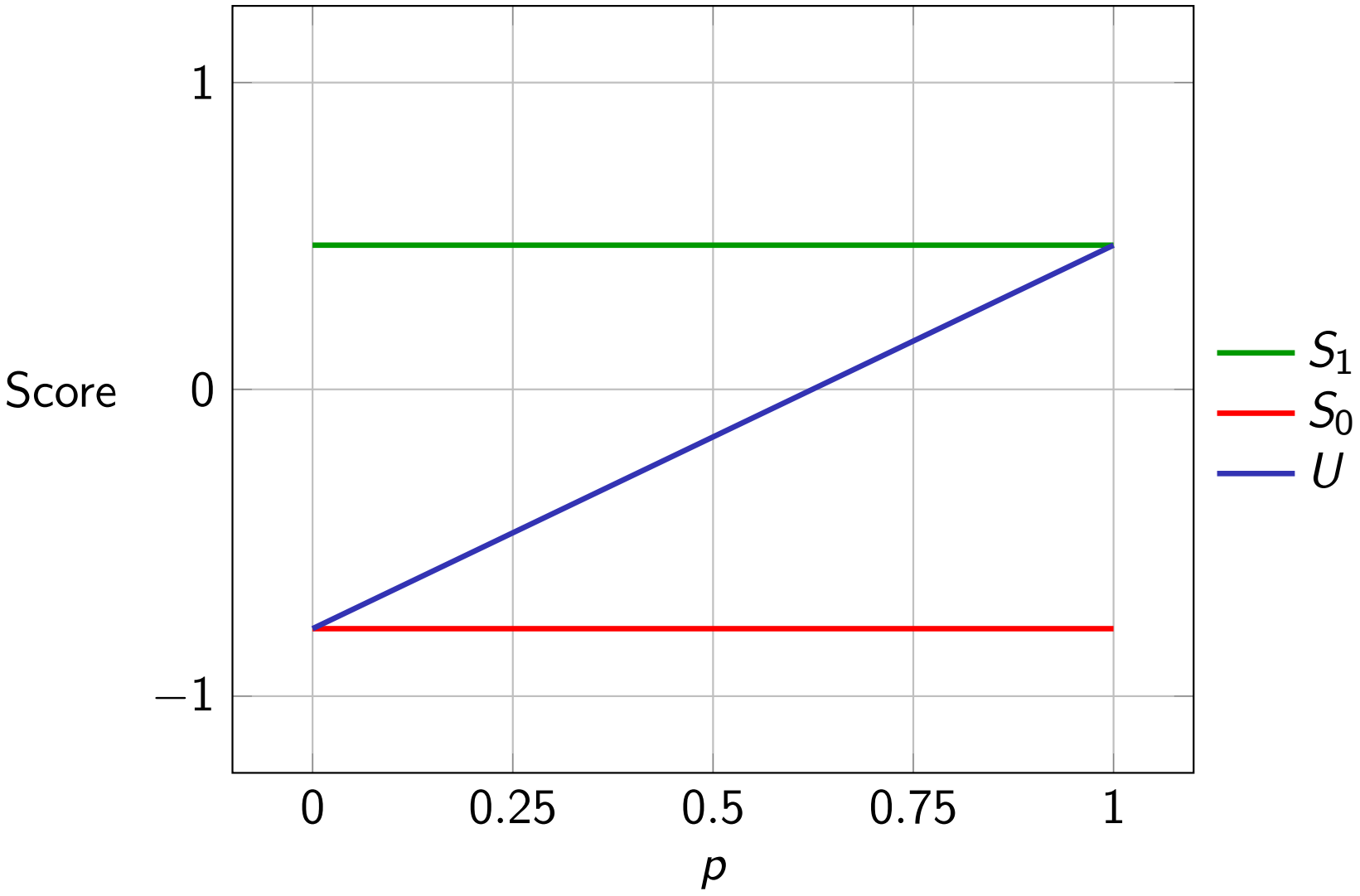
$$

$$
\begin{aligned}
S_0(p) &= -E + (1-\alpha) \times 0 + \alpha \times (-F) \\
&= -\alpha F - E
\end{aligned}
$$

Evaluate



Implement

Implement

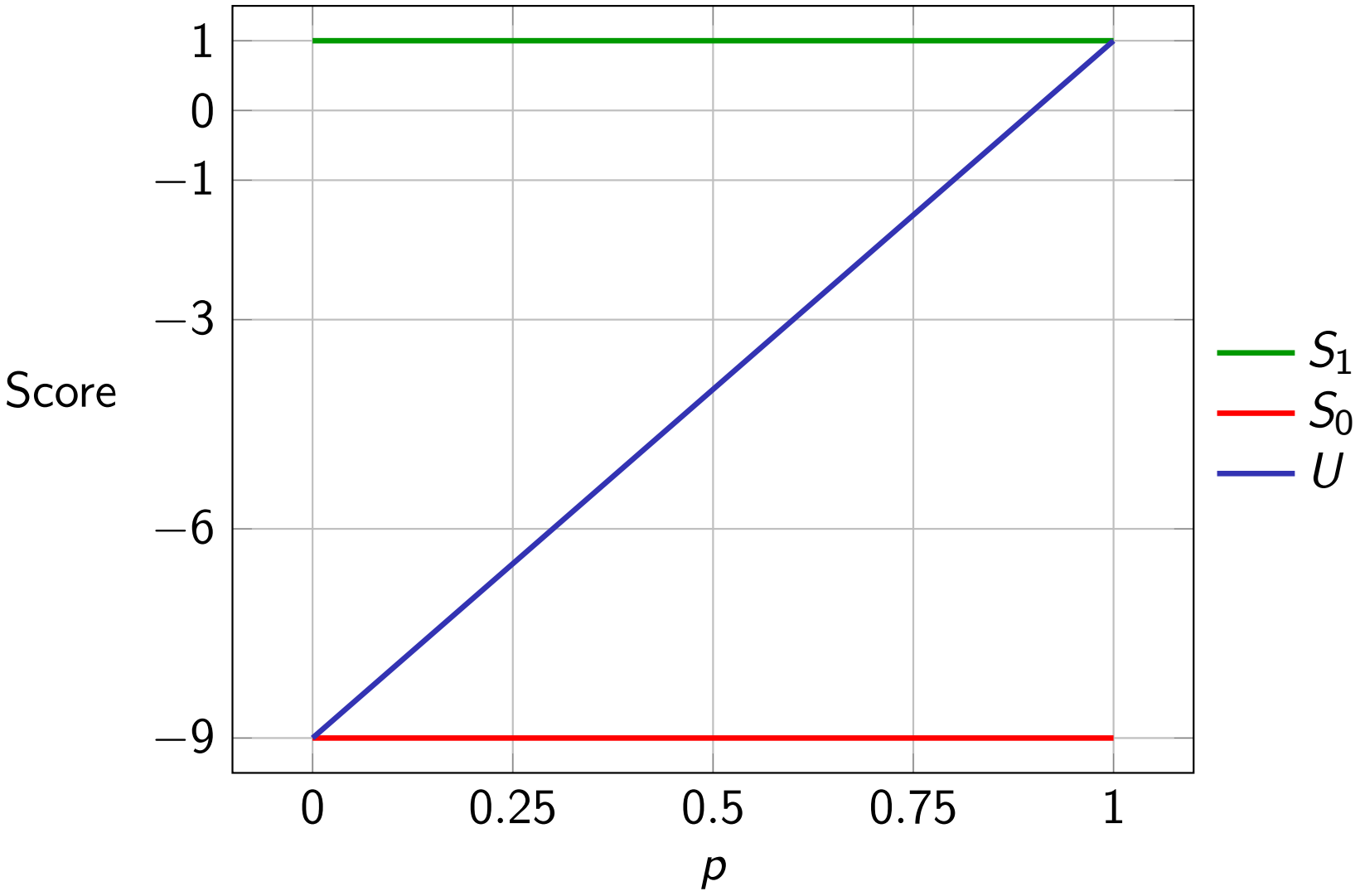
$$S_1 = 1$$

Implement

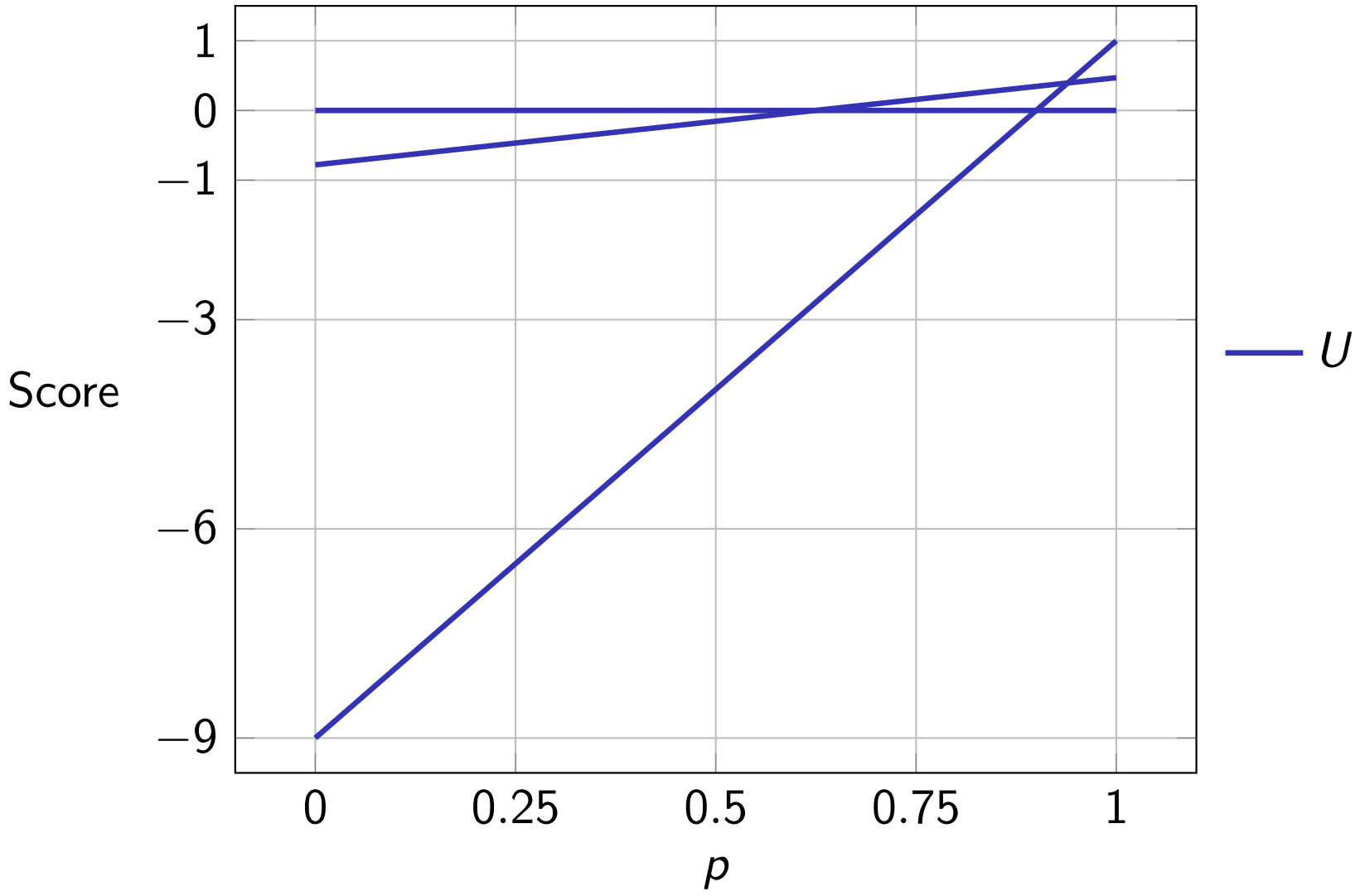
$$S_1 = 1$$

$$S_0 = -F$$

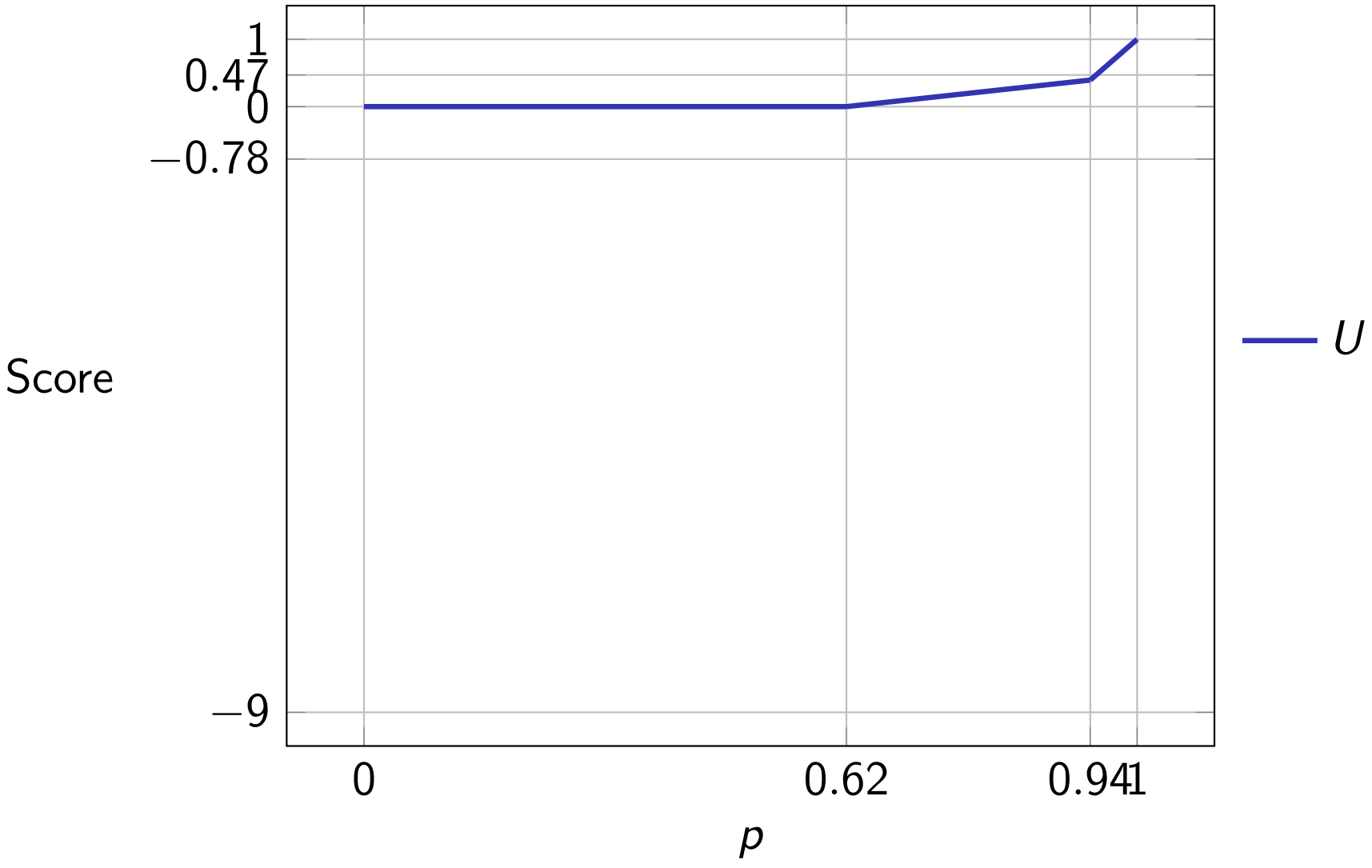
Implement



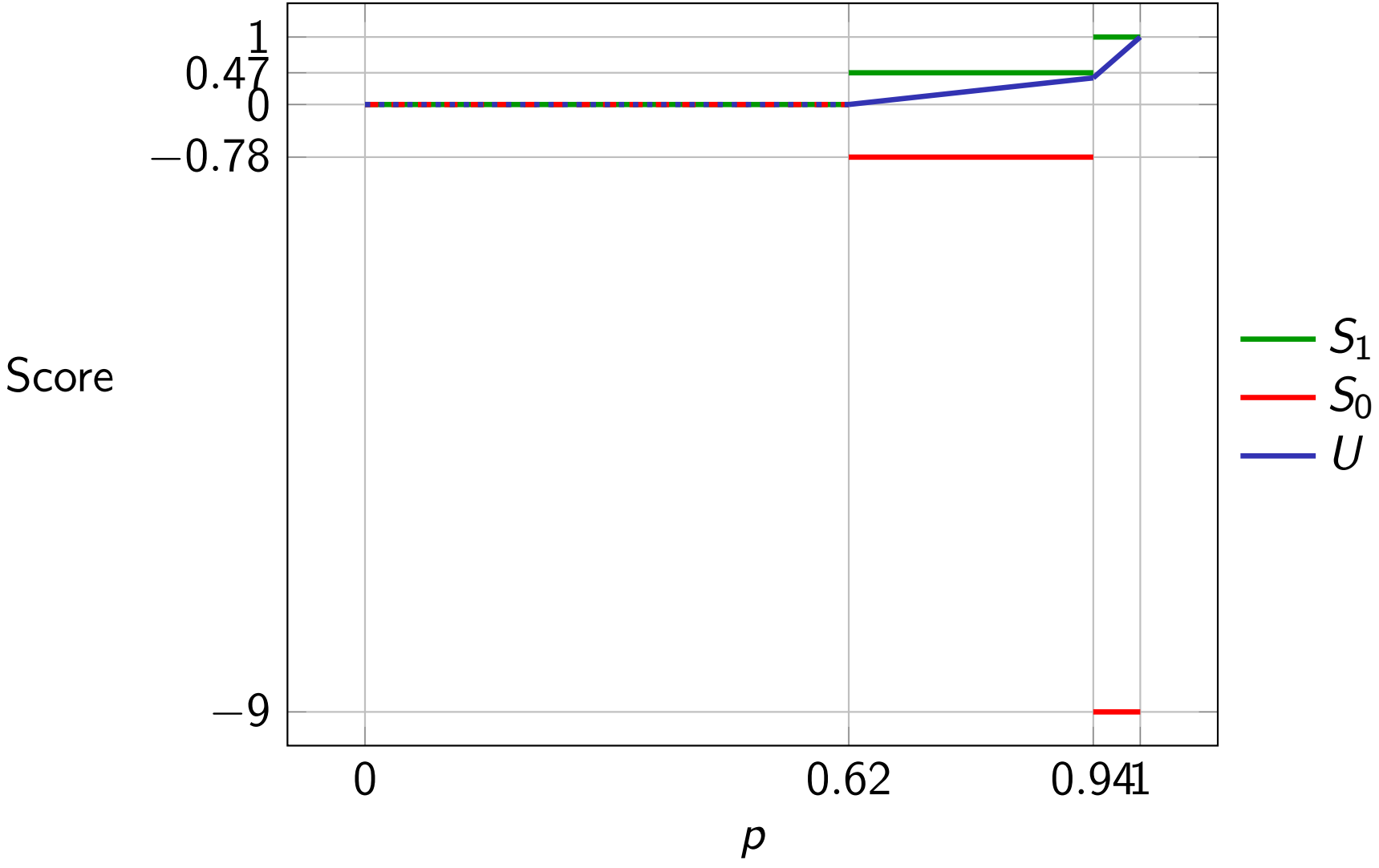
All Payoffs Together



Taking the Max



Adding S_0 and S_1 Back In



Incentives

Notice that U is weakly convex and weakly increasing

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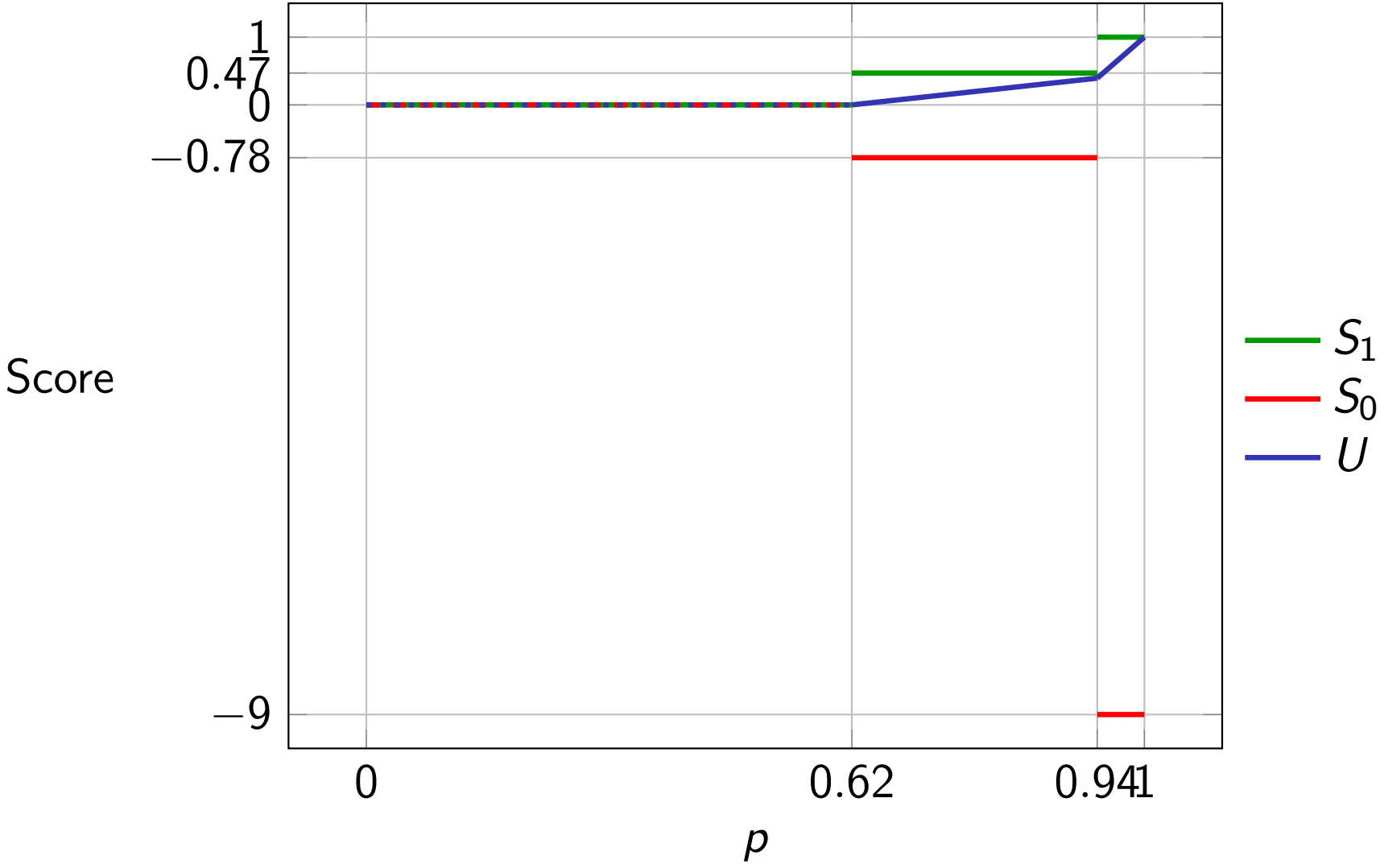
Since we didn't use the L parameter for this, I will call this the FABE scoring rule

FABE $(9, 0.05, 0.2, 0.33)$

Confidence	**Outcome**	**Score**
None $[0, 0.62)$	$\times$	0
	$\checkmark$	0
Intermediate $[0.62, 0.94]$	$\times$	-0.78
	$\checkmark$	0.47
High $(0.94, 1]$	$\times$	-9
	$\checkmark$	1

FABE $(9, 0.05, 0.2, 0.33)$



From FABE to FABLE

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Moreover, I argue that this will be a closer approximation to the user's actual payoff than FABE

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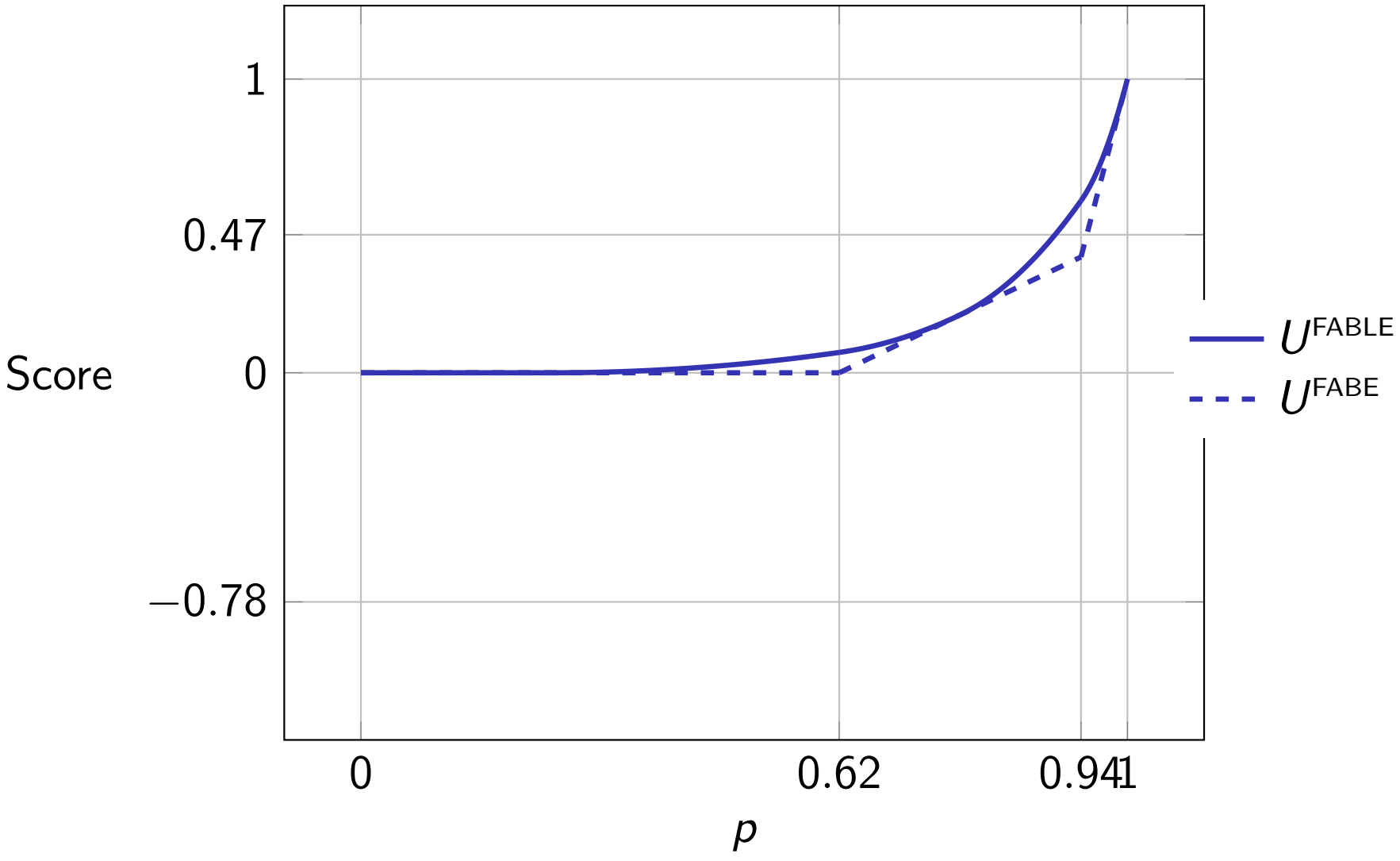
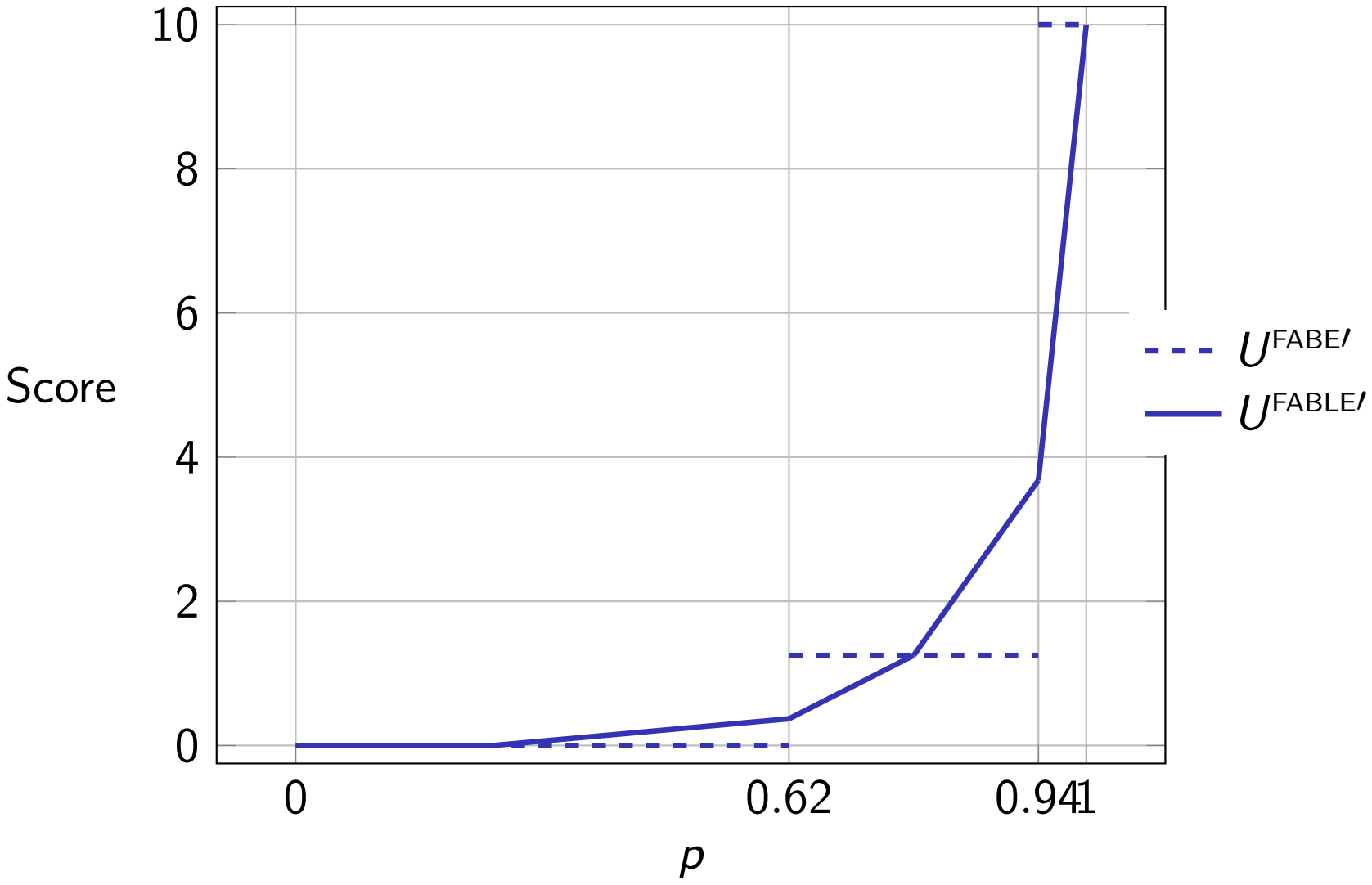
It turns out this pins down U^{FABLE} uniquely

U^{FABE} and U^{FABLE}



$U^{\text{FABE}\prime}$ and $U^{\text{FABLE}\prime}$



Why Do This?

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Of course, in reality, the user has many more actions, including

- evaluating the answer with different levels of effort
- using an incorrect answer as helpful inspiration

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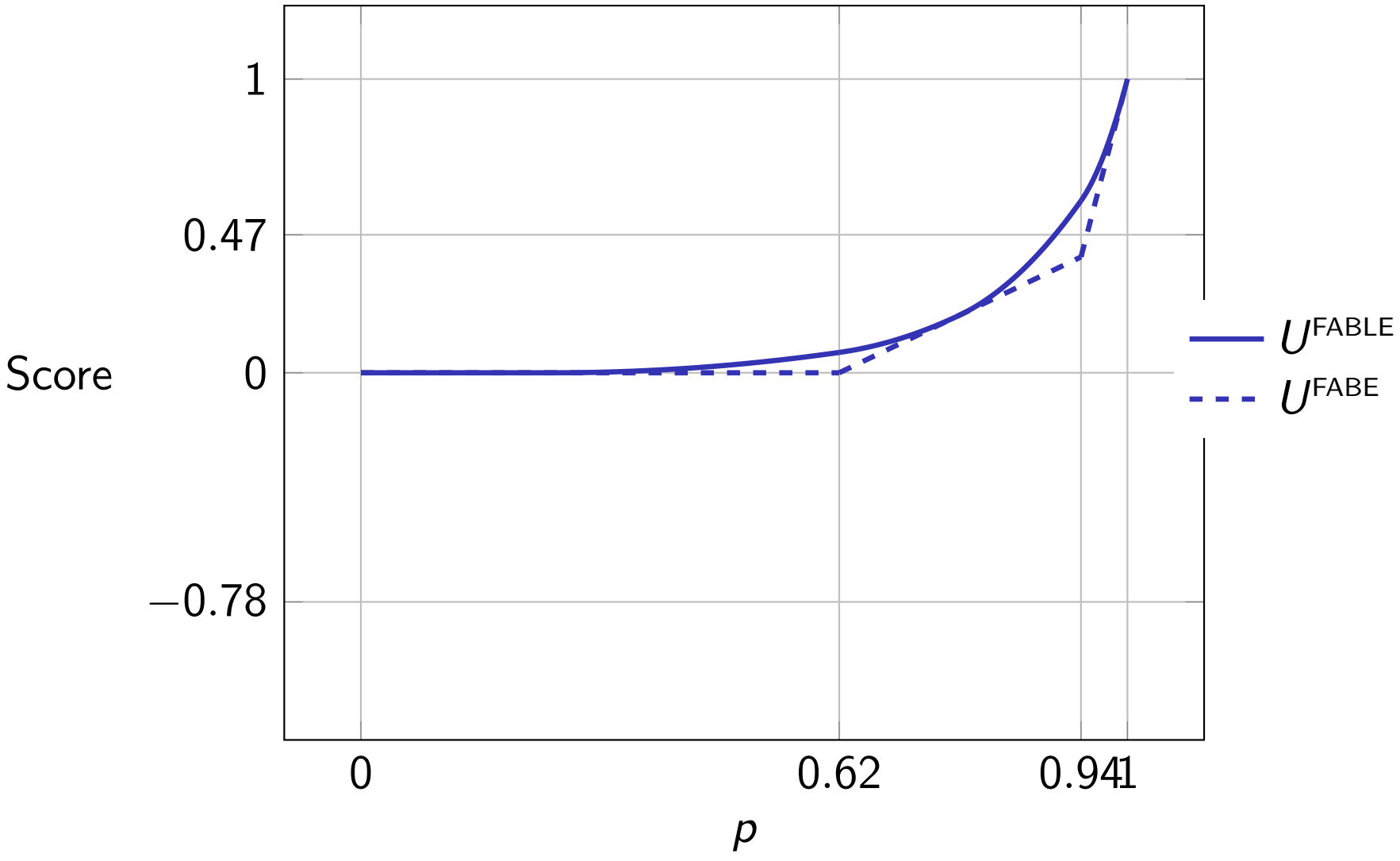
FABLE adds the "meat" to FABE's "bones," smoothing out U in between these points to capture the other actions an individual might take that are unnecessary to model explicitly

U^{FABE} and U^{FABLE}



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We've pinned down U^{FABLE}, but haven't yet pinned down (S_0, S_1)

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By Proposition 2, the scoring rule must be incentive compatible if it is to reflect the payoffs of society

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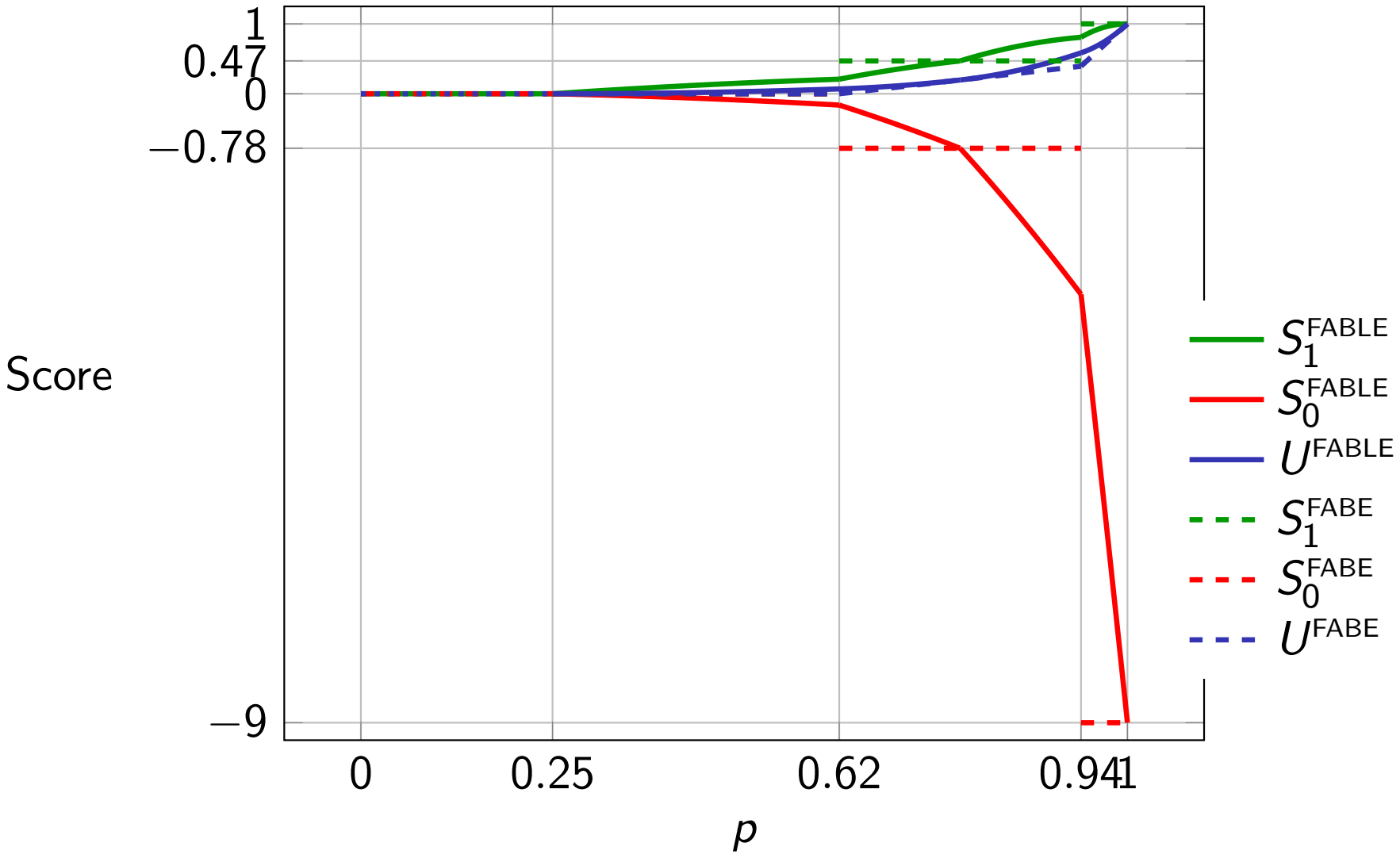
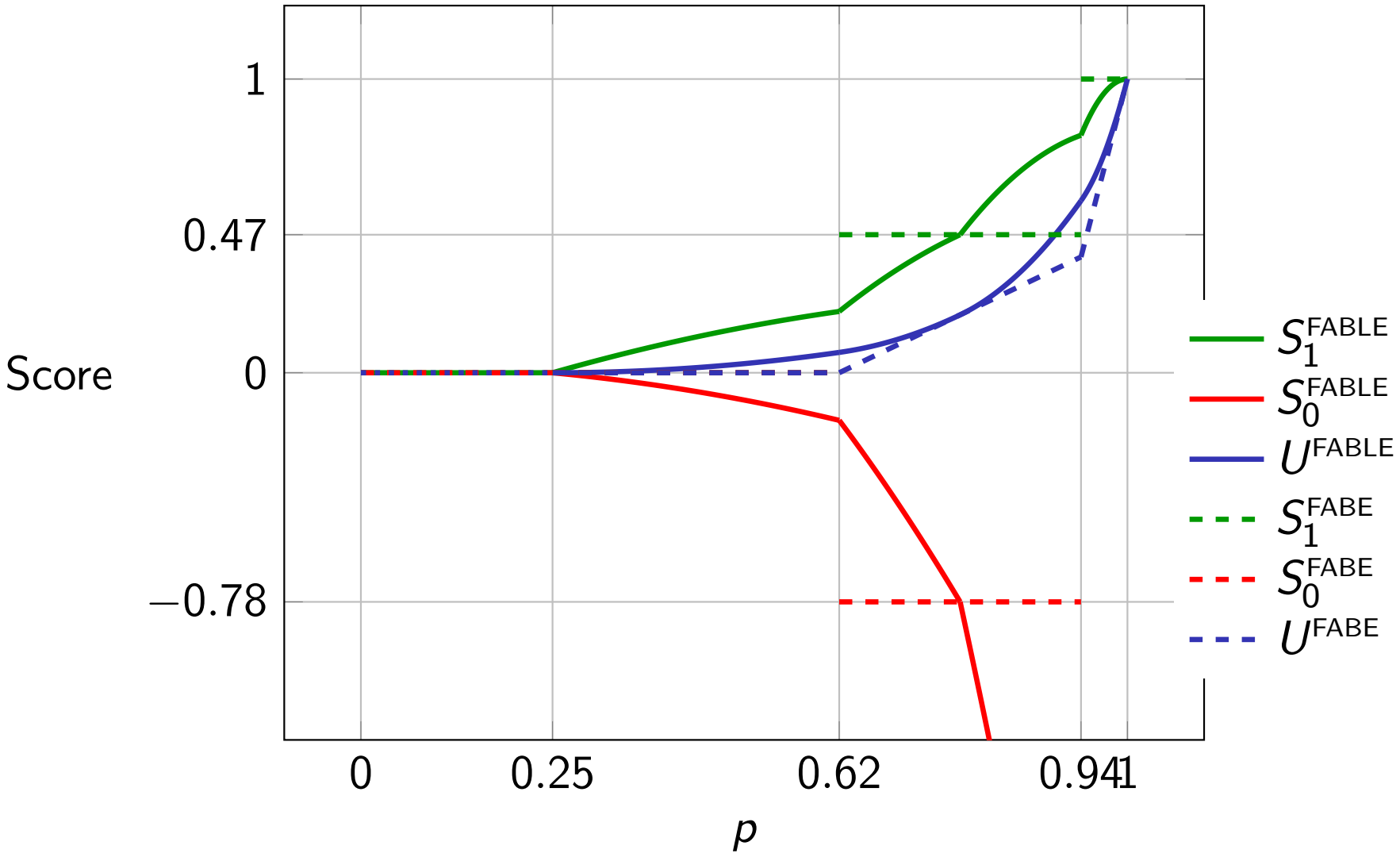
Adding the unique incentive-compatible (S_0, S_1) which generates U^{FABLE} gives us FABLE, and it looks like this

FABE and FABLE



FABE and FABLE



Online Demo

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Skill, Judgement, and Calibration

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the expected payoff to society of employing the agent if they had perfect judgement and calibration (knowing what they know exactly)

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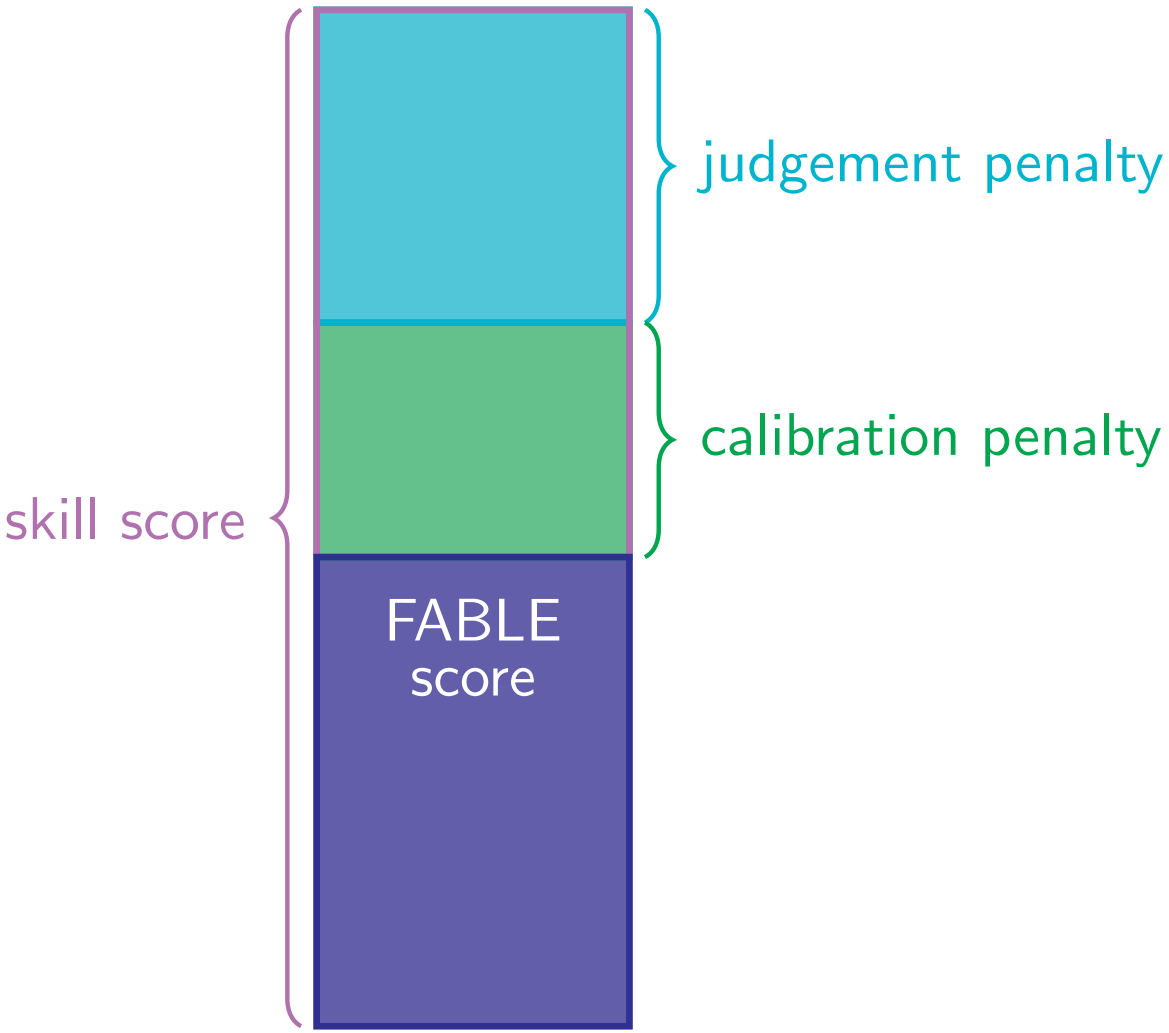
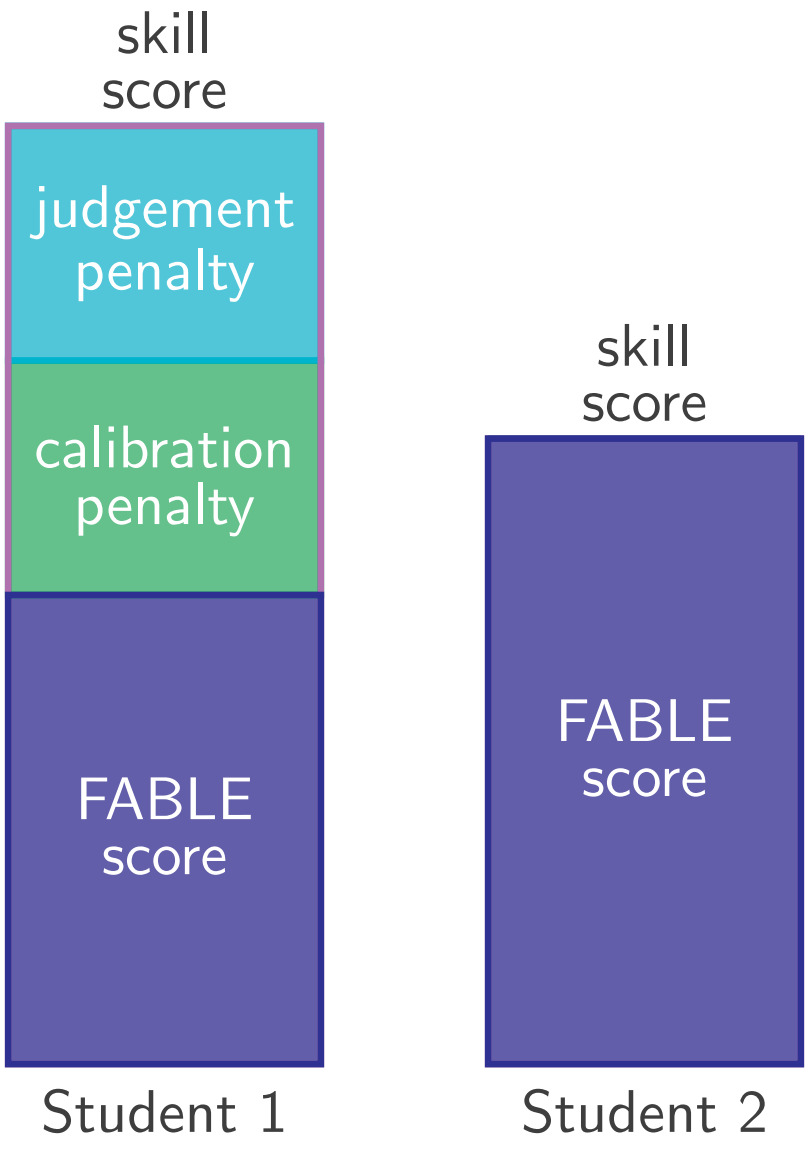
Skill Score + Judgement Penalty + Calibration Penalty = FABLE
the expected payoff to society of employing the agent

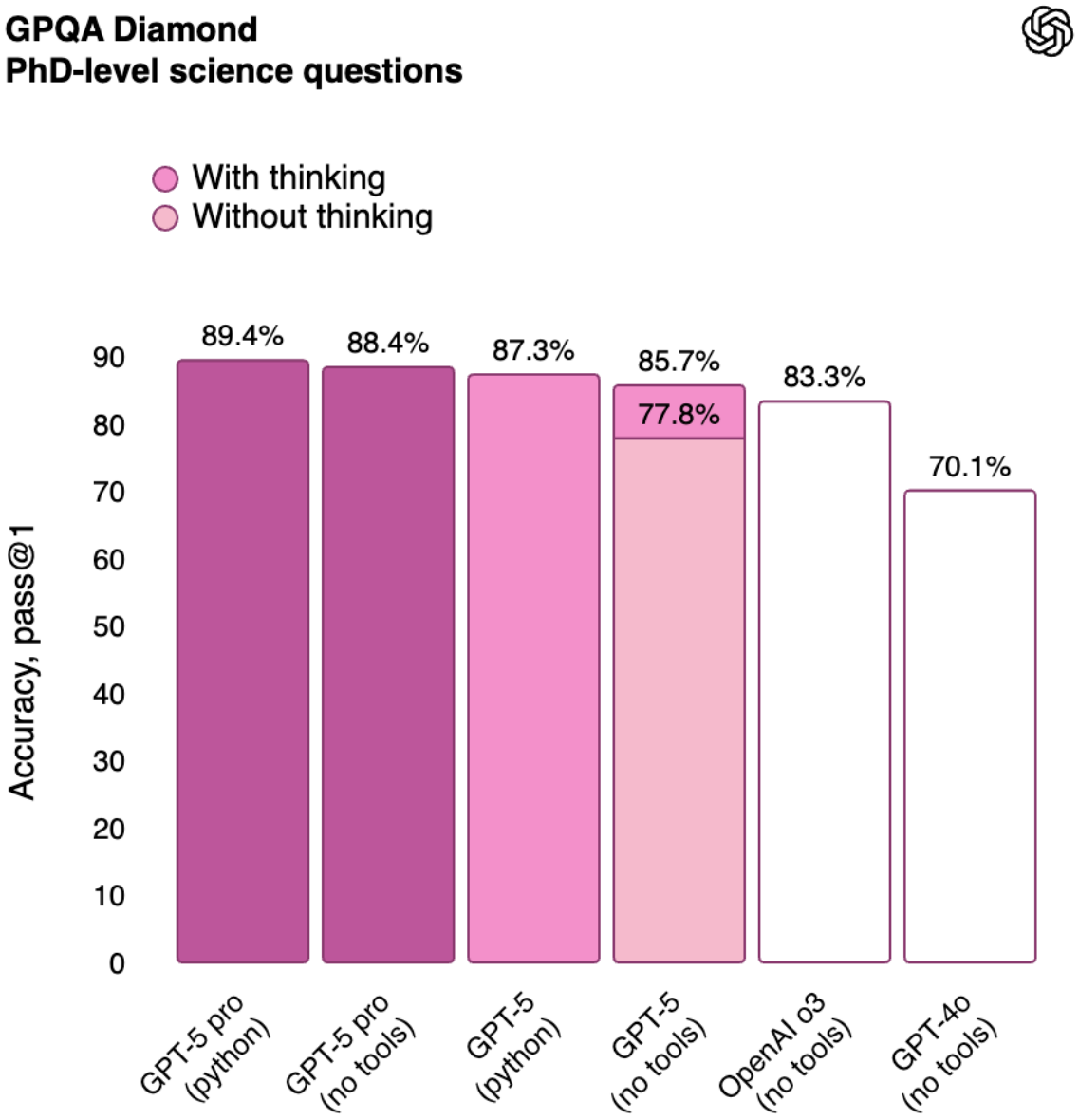
Skill, Judgement, and Calibration



Example


skill
score
judgement
penalty
calibration
penalty
FABLE
score
Student 1
skill
score
FABLE
score
Student 2

Instead Of


GPQA Diamond
PhD-level science questions
With thinking
Without thinking
Accuracy, pass@1
0
10
20
30
40
50
60
70
80
90
89.4%
88.4%
87.3%
85.7%
77.8%
83.3%
70.1%
GPT-5 pro (python)
GPT-5 pro (no tools)
GPT-5 (python)
GPT-5 (no tools)
OpenAI o3 (no tools)
GPT-4o (no tools)

Thank you for listening!

Questions, comments, or concerns?