Inferencia de Tipos

Paradigmas (de Lenguajes) de Programación

Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires

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Introducción

¿Tiene tipo? ¿Cuál es el tipo? ¿Es el más general? ¿Qué necesitamos saber del contexto?

- $(\lambda x.\ isZero(x))\ true$
- $\lambda x.\ succ(x)$
- $\lambda x.\ succ(y)$
- $\emptyset \rhd \lambda x : Nat.\ x : Nat \to Nat$
- $\emptyset \rhd \lambda x : X_1.\ x : X_1 \to X_1$

Introducción

Generalidad

¿Qué significa ser el juicio *más general*? Que todos los juicios derivables para $\lambda x.\ x$ son instancias de $\emptyset \rhd \lambda x : X_1.\ x : X_1 \to X_1$. Por ejemplo:

- $\emptyset \rhd \lambda x : Nat.\ x : Nat \to Nat$
- $\emptyset \rhd \lambda x : Bool.\ x : Bool \to Bool$
- $\{y : Bool\} \rhd \lambda x : X_2 \to Nat.\ x : (X_2 \to Nat) \to X_2 \to Nat$
- ...

Ejemplos a ojo

Inferir el juicio de tipado de las siguientes expresiones:

1. $\lambda x.\ y$
2. $f\ true$
3. $\text{iszero}(x)$

Algoritmo de Martelli Montanari

Determinar el resultado de aplicar el algoritmo MGU sobre las siguientes ecuaciones:

1. $\texttt{MGU}\{X_2 \to X_1 \to \mathsf{Bool} \stackrel{?}{=} X_2 \to X_3\}$
2. $\texttt{MGU}\{(X_2 \to X_1) \to \mathsf{Nat} \stackrel{?}{=} X_2 \to X_3\}$
3. $\texttt{MGU}\{X_1 \to \mathsf{Bool} \stackrel{?}{=} \mathsf{Nat} \to \mathsf{Bool}, X_2 \stackrel{?}{=} X_1 \to X_1\}$
4. $\texttt{MGU}\{X_1 \to X_2 \stackrel{?}{=} X_3 \to X_4, X_3 \stackrel{?}{=} X_2 \to X_1\}$

Algoritmo de inferencia

¿Qué tipo tienen las siguientes expresiones?

1. $\mathbb{W}(\lambda f.\, \lambda x.\, f(f\ x))$
2. $\mathbb{W}(x\ (\lambda x.\ \mathsf{succ}(x)))$
3. $\mathbb{W}(\lambda x.\, x\ y\ x)$

Algoritmo de inferencia

Ejercicio

Dada la siguiente extensión al conjunto de términos para el cálculo λ con listas:

$$M ::= \ldots | map_{\sigma,\tau} | foldr_{\sigma,\tau}$$

La modificación al sistema de tipos es la introducción de dos axiomas de tipado para $map_{\sigma,\tau}$ y $foldr_{\sigma,\tau}$:

$$\mathbb{W}(map) \stackrel{\text{def}}{=} \emptyset \triangleright map_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$\mathbb{W}(foldr) \stackrel{\text{def}}{=} \emptyset \triangleright foldr_{X_1,X_2} : (X_1 \to X_2 \to X_2) \to X_2 \to [X_1] \to X_2$$

siendo X_1 y X_2 variables de tipo frescas. Se asumen dadas las extensiones correspondientes para `Erase` y `mgu`. Usar el algoritmo $\mathbb{W}()$ con esta nueva extensión para tipar la siguiente expresión:

$$foldr\ map$$

Algoritmo de inferencia

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$S = MGU\{???\}$

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$\mathbb{W}(foldr) = \emptyset \triangleright foldr_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$
$\mathbb{W}(map) = \emptyset \triangleright map_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$

$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$

$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$

$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$

$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 / X_3\}$

$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 / X_3\}$

$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

$\longmapsto^4 \{[X_2] \to [X_2 \to X_2] \to [X_2] \stackrel{?}{=} X_5\} \mid \{X_2 / X_1\} \circ \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

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$\longmapsto^3 \{X_5 \stackrel{?}{=} [X_2] \to [X_2 \to X_2] \to [X_2]\} \mid \{X_2 / X_1\} \circ \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

Algoritmo de inferencia

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Algoritmo de inferencia

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$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

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$\longmapsto^4 \{\} \mid \{[X_2] \to [X_2 \to X_2] \to [X_2] / X_5\} \circ \{X_2 / X_1\} \circ \{[X_1] / X_4\} \circ \{X_1 \to X_2 / X_3\}$

Algoritmo de inferencia

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$\mathbb{W}(\mathit{map}) = \emptyset \rhd \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$

$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$
$= \{X_1 \to X_2 / X_3,\ [X_1] / X_4,\ X_2 / X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2] / X_5\}$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = S\emptyset \cup S\emptyset \rhd S\ (\mathit{foldr}_{X_3,X_4}\ \mathit{map}_{X_1,X_2}) : Se$$

$\mathbb{W}(\mathit{foldr}) = \emptyset \rhd \mathit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$
$\mathbb{W}(\mathit{map}) = \emptyset \rhd \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$

$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$
$= \{X_2 \to X_2 / X_3,\ [X_2] / X_4,\ X_2 / X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2] / X_5\}$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr\ map}) = \emptyset \rhd \mathit{foldr}_{X_2 \to X_2, [X_2]}\ \mathit{map}_{X_2, X_2} : [X_2] \to [X_2 \to X_2] \to [X_2]$$

$$\mathbb{W}(\mathit{foldr}) = \emptyset \rhd \mathit{foldr}_{X_3, X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$$
$$\mathbb{W}(\mathit{map}) = \emptyset \rhd \mathit{map}_{X_1, X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$
$$= \{X_2 \to X_2 \,/ X_3,\ [X_2] \,/ X_4,\ X_2 \,/ X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2] \,/ X_5\}$$

Extensión del algoritmo de inferencia

Listas

$$\sigma ::= \dots \mid [\sigma]$$

$$M, N, O ::= \dots \mid [\,]_\sigma \mid M :: N \mid \mathit{Case}\ M\ \mathit{of}\ [\,] \rightsquigarrow N \,;\, h :: t \rightsquigarrow O$$

$$\frac{}{\Gamma \rhd [\,]_\sigma\ : [\sigma]} \qquad \frac{\Gamma \rhd M : \sigma \qquad \Gamma \rhd N : [\sigma]}{\Gamma \rhd M :: N : [\sigma]}$$

$$\frac{\Gamma \rhd M : [\sigma] \qquad \Gamma \rhd N : \tau \qquad \Gamma \cup \{h : \sigma, t : [\sigma]\} \rhd O : \tau}{\Gamma \rhd \mathit{Case}\ M\ \mathit{of}\ [\,] \rightsquigarrow N \,;\, h :: t \rightsquigarrow O : \tau}$$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \rhd [\,]_X : [X]$ con X variable fresca

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \rhd S(M :: N)$:

- $\mathbb{W}(U) = \Gamma_1 \rhd M : \sigma$
- $\mathbb{W}(V) = \Gamma_2 \rhd N : [\sigma]$
- $S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \rhd [\,]_X : [X]$ con X variable fresca

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \rhd S(M :: N)$:

- $\mathbb{W}(U) = \Gamma_1 \rhd M : X_1$
- $\mathbb{W}(V) = \Gamma_2 \rhd N : X_2$
- $S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \triangleright [\,]_X : [X]$ con X variable fresca

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \triangleright S(M :: N) : S\tau$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma$

$\mathbb{W}(V) = \Gamma_2 \triangleright N : \tau$

$S = MGU\{\tau \stackrel{?}{=} [\sigma]\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

$\mathbb{W}(\mathit{Case}\ U\ \mathit{of}\ [\,] \leadsto V\ ; h :: t \leadsto W) \stackrel{\text{def}}{=}$
$S\Gamma_1 \cup S\Gamma_2 \cup S\Gamma_{3'} \ \triangleright S\,(\mathit{Case}\ M\ \mathit{of}\ [\,] \leadsto N\ ; h :: t \leadsto O) : S\tau$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma \qquad \mathbb{W}(V) = \Gamma_2 \triangleright N : \tau \qquad \mathbb{W}(W) = \Gamma_3 \triangleright O : \rho$

$\tau_h = \begin{cases} \alpha \text{ si } h : \alpha \in \Gamma_3, \\ \text{var fresca si no} \end{cases} \qquad \tau_t = \begin{cases} \beta \text{ si } t : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$

$\Gamma_{3'} = \Gamma_3 \ominus \{h, t\}$

$S = MGU(\{\sigma \stackrel{?}{=} [\tau_h],\ \rho \stackrel{?}{=} \tau,\ \tau_t \stackrel{?}{=} \sigma\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_i,\ x : \sigma_2 \in \Gamma_j,\ i,j \in \{1,2,3'\}\})$

Dar el tipo de: $\mathit{Case}\ \text{succ}(0) :: x\ \mathit{of}\ [\,] \leadsto x\ ; x :: y \leadsto \text{succ}(x) :: [\,]$

Extensión del algoritmo de inferencia

Listas por Comprensión

$\mathbb{W}([\ U \mid x \leftarrow V, W\ ]) \stackrel{\text{def}}{=} S\Gamma_{1'} \cup S\Gamma_2 \cup S\Gamma_{3'} \ \triangleright S\,([\ M \mid X \leftarrow N, O\ ]) : S[\sigma_1]$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma_1$
$\mathbb{W}(V) = \Gamma_2 \triangleright N : \sigma_2$
$\mathbb{W}(W) = \Gamma_3 \triangleright O : \sigma_3$

$\tau_{x1} = \begin{cases} \alpha \text{ si } x : \alpha \in \Gamma_1, \\ \text{var fresca si no} \end{cases} \qquad \tau_{x2} = \begin{cases} \beta \text{ si } x : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$

$\Gamma_{1'} = \Gamma_1 \ominus \{x\} \qquad \Gamma_{3'} = \Gamma_3 \ominus \{x\}$

$S = MGU(\{\tau_{x1} \stackrel{?}{=} \tau_{x2},\ \sigma_2 \stackrel{?}{=} [\tau_{x1}],\ \sigma_3 \stackrel{?}{=} \text{Bool}\}$
$\quad \cup \{\rho_1 \stackrel{?}{=} \rho_2 \mid y : \rho_1 \in \Gamma_i,\ y : \rho_2 \in \Gamma_j,\ i,j \in \{1',2,3'\}\})$

Dar el tipo de: $[\ \text{if } x \text{ then } \underline{0} \text{ else } \underline{1} \mid x \leftarrow \mathit{false} :: \text{iszero}(x) :: [\,], \mathit{true}\ ]$

Extensión del algoritmo de inferencia

Listas por comprensión

$$M ::= \ldots \mid [\ M \mid x \leftarrow M, M\ ]$$

Consideremos el Cálculo Lambda extendido con las listas por comprensión vistas en la práctica 4.

La regla de tipado es la siguiente:

$$\frac{\Gamma \cup \{x : \sigma\} \ \triangleright M\colon \tau \quad \Gamma \triangleright N\colon [\sigma] \quad \Gamma \cup \{x : \sigma\} \ \triangleright \ O : \text{Bool}}{\Gamma \triangleright [\ M \mid x \leftarrow N, O\ ]\colon\ [\tau]}$$