**Supplementary Information of Beyond Mobility: Socioeconomic Context Shapes the Dynamics and Hidden States of COVID-19 Transmission.**

**Supplementary Figures**

**Supplementary Figure N°1.** Viterbi-decoded sequence by commune using a four-state Non-Homogeneous Hidden Markov Model (nHMM).

Gráfico

El contenido generado por IA puede ser incorrecto.

This figure displays the Viterbi sequence for each commune based on a Non Homogeneous Hidden Markov Model (nHMM) with four latent states. The model converged with a log-likelihood of -20448.6 (df=443), and yielded AIC=41783.21 and BIC=44836.52. Compared to the three-state model, the four-state model improves the differentiation of transitional epidemic phases, especially during the mid-period of the timeline. Visually, it introduces a higher leve lof detail while preserving the interpretability and temporal coherence across comunes. This model offers a favorable trade-off between statistical performance and visual clarity, avoiding the excessive fragmentation observed in more complex specifications.

**Supplementary Figure N°2.** Viterbi-decoded sequence by commune using a five-state Non-Homogeneous Hidden Markov Model (nHMM).

Gráfico

El contenido generado por IA puede ser incorrecto.

This figure shows the Viterbi-decoded hidden states under a five-state Non Homogeneous Hidden Markov Model. The model converged with a log-likelihood of -18959.39 (df=734), achieving the lowest AIC (39386.77) and BIC (44445.75) among all candidate models. Despite this statistical improvement, visual inspection reveals signs of overfitting. Several comunes exhibit erratic transitions and abrupt state changes lacking epidemiological plausibility. These patterns suggest the model may be capturing noise rather tan meaningful shifts in epidemic dynamics. Consequently, although the five-state model scores better in AIC/BIC, its interpretability is compromised, limiting its utility for comparative and policy-oriented analyses.

**Supplementary Tables**

**Supplementary Table N°1.**Variance decomposition and model fit for each transition and model (A, B, C and D).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Transition** | **Model** | **Inter-Variance** | **Intra-Variance** | **Inter-Proportion (%)** | **Marginal R2** | **Conditional R2** |
| 1->2 | A | 0.115302 | 0.010097 | 91.95 | 0.25162 | 0.94227 |
|  | B | 0.21896 | 0.010088 | 95.60 | 0.28712 | 0.93447 |
|  | C | 0.142007 | 0.010087 | 93.37 | 0.37958 | 0.90113 |
|  | D | 0.151177 | 0.010089 | 93.74 | 0.33605 | 0.90461 |
| 2->2 | A | 0.142058 | 0.014912 | 90.50 | 0.12229 | 0.93388 |
|  | B | 0.259290 | 0.014910 | 94.43 | 0.17148 | 0.92680 |
|  | C | 0.208853 | 0.014898 | 93.34 | 0.23314 | 0.84995 |
|  | D | 0.226930 | 0.014888 | 93.84 | 0.23479 | 0.87174 |
| 2->3 | A | 0.28065 | 0.032680 | 89.57 | 0.25926 | 0.93150 |
|  | B | 1.070577 | 0.032642 | 97.04 | 0.45430 | 0.93631 |
|  | C | 0.333904 | 0.032629 | 91.10 | 0.44836 | 0.89619 |
|  | D | 0.727473 | 0.032587 | 95.71 | 0.47456 | 0.92325 |
| 3->3 | A | 0.108558 | 0.019726 | 84.62 | 0.45791 | 0.94035 |
|  | B | 0.988542 | 0.019704 | 98.05 | 0.53850 | 0.92417 |
|  | C | 0.188177 | 0.019707 | 90.52 | 0.58939 | 0.92820 |
|  | D | 0.417336 | 0.019668 | 95.50 | 0.49034 | 0.94446 |
| 3->2 | A | 0.007030 | 0.001219 | 85.22 | 0.23742 | 0.90800 |
|  | B | 0.033655 | 0.001220 | 96.50 | 0.17990 | 0.90558 |
|  | C | 0.019729 | 0.001218 | 94.19 | 0.33561 | 0.85860 |
|  | D | 0.021130 | 0.001217 | 94.55 | 0.28616 | 0.87227 |
| 2->1 | A | 0.003408 | 0.000145 | 95.93 | 0.32698 | 0.98755 |
|  | B | 0.076598 | 0.000145 | 99.81 | 0.32689 | 0.99552 |
|  | C | 0.013163 | 0.000145 | 98.89 | 0.36370 | 0.98049 |
|  | D | 0.015267 | 0.000145 | 99.06 | 0.21412 | 0.97595 |

Supplementary Table N°1 presents the results of the linear mixed models (LMMs) applied to the transition probabilites between epidemic latent states, estimated using an nHMM model. Each transition was modeled independently under four specifications: model A included exclusively mobility indicators (internal and external mobility indexes of each comunne); model B incorporated sociodemographic covariates derived from the 2017 Census; model C considered the six dimensions of the 2019 Urban Quality of Life Index (ICVU); and model D combined variables from both structural domains. The selection of covariates for models B, C and D was performed by Elastic Net regularization (Supplementary Table N°2) which allowed us to identify an optimal subset of predictors by simultaneously penalizing the magnitude and redundancy of the coefficients.

This procedure effectively reduced dimensionality without sacrificing explanatory capacity, with validation RMSE errors closet o 22.7 for the structural models.

In the transition from moderate to severe states (1🡪2), all models captured a high proportion of intercommunity variance, above 91%, which confirms the existence of relevant structural differences between comunes. Model C, base don ICVU dimensions, achieved the highest marginal R2 (0.37958), while model D achieved a comparable performance (0.33605), integrating information from both census conditions and the urban environment.

As for the severe state persistence transitions (2🡪2), they also presented high levels of intercommunal variance. Although the marginal fit of the models was more modest compared to other transitions, the models that incorporated urban structures (C and D) show a better explanatory capacity than those that included only mobility or census data, suggesting that permanence in adverse epidemic situations depends on structural determinants that are difficult to modify in the short term.

In the case of the transition from severe to critical (2🡪3), the greatest structural dispersion between communes was observed. Model D, with a hybrid architecture, achieved both a high proportion of intercommune variance (0.727473) and the highest marginal R2 (0.47456), surpassing the partial versions. The covariates selected by Elastic Net in this model reflect both conditions of social vulnerability and aspects of the urban environment related to employment, health and connectivity, therefore, it is clear that the progression towards critical epidemic scenarios is a complex interaction of the structural and social domain.

In the critical state persistence transition (3🡪3), model C again demonstrated a remarkable explanatory capacity, reaching the highest marginal R2 (0.58939), reinforcing the hypothesis that factors related to the urban environment play a central role in the spread of severe epidemic conditions, especially in territories with structural deficits in housing, services and health.

Finally, in the regressive transitions (3🡪2, 2🡪1), the proportions of intercommunal variance were particularly high, exceeding 99% in some cases. In 2🡪1, where a substantial improvement in the epidemiological situation is observed, model B achieved the highest conditional R2 (0.99552), reflecting the direct influence of sociodemographic characteristics such as per capita income, education, housing precariousness index, among others. Although model C obtained the best marginal performance (0.36370), model D maintained a balance between fit and explanatory capacity, confirming its overall robustness.

**Supplementary Table N°2.** Elastic Net Coefficients and RMSE for Models B, C, and D including Socio-demographic and Urban Quality variables.

|  |  |  |  |
| --- | --- | --- | --- |
| **Model** | **RMSE** | **Variable** | **Elastic Net Coef** |
| B | 22,92972 | Internal Mobility Index | -6,44389 |
|  |  | External Mobility Index | -5,03525 |
|  |  | Per Capita Income (USD) | 2,61930 |
|  |  | Percentage Overcrowding | 0,09401 |
|  |  | Average Persons Households | 0,18306 |
|  |  | Housing Precariousness Index | 1,93892 |
|  |  | Percentage Higher Education Completed | -1,21381 |
|  |  | Percentage Households with Immigrants | 0,42076 |
| C | 22,77792 | Internal Mobility Index | -4,21061 |
|  |  | External Mobility Index | -6,35017 |
|  |  | Employment Conditions | -4,22224 |
|  |  | Business Climate | 2,28145 |
|  |  | Socio-cultural Factors | 1,64871 |
|  |  | Connectivity and Mobility | -0,58358 |
|  |  | Health and Ecology | 2,12862 |
|  |  | Housing and Urban Space | -1,65299 |
| D | 22,69835 | Internal Mobility Index | -4,77879 |
|  |  | External Mobility Index | -6,55418 |
|  |  | Per Capita Income (USD) | 1,63088 |
|  |  | Percentage Overcrowding | 0,00000 |
|  |  | Average Persons Households | 0,00000 |
|  |  | Housing Precariousness Index | 0,00000 |
|  |  | Percentage Higher Education Completed | -0,87650 |
|  |  | Percentage Households with Immigrants | 1,65439 |
|  |  | Employment Conditions | -4,53999 |
|  |  | Business Climate | 1,87196 |
|  |  | Socio-cultural Factors | 1,55538 |
|  |  | Connectivity and Mobility | -0,92635 |
|  |  | Health and Ecology | 2,29027 |
|  |  | Housing and Urban Space | -1,66662 |