

# Application of Kalman Filter in GPS Position Estimation

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**Abstract-** Over the past few years Kalman filter has gained attraction and significant position among the researchers, as this filtering technique can be applied to variety of applications. One of its applications that is presented in this paper is GPS based position estimation. Kalman filter is utilized for better position estimates over those provided by direct GPS measurements. GPS used for position estimates are prone to errors due to various sources. These errors accumulate and provide erroneous results. One of the ways to get accurate position estimate is to include more sensors like inertial measurement unit or some other devices, but all this would not be cost effective. Thus, by integrating Kalman filter as post processing filter, the error can be reduced.

**Keywords**—Kalman filter, Extended Kalman filter (EKF), Global Positioning System (GPS), Standard Position Services (SPS); Precise Position Services (PPS), Geometric dilution of precision (GDOP), Total electron content (TEC).

## I. INTRODUCTION

The Global Positioning System (GPS) was first introduced by US department of defense in 1973. It was space based radio navigation system owned by United States government[1]. It was a satellite navigation system used for providing geolocation and time information to a GPS receiver present anywhere on earth where there was direct path between GPS receiver and the satellite. GPS services are of two types 1) Standard Position Services (SPS). 2) Precise Position Services (PPS). The former is for civilians and the latter is for military purposes. The SPS is freely available to unlimited users all around the world. It has an accuracy of 100m in horizontal plane and 156m in vertical plane. The PPS is primarily intended for military and other government agency. It has an accuracy of at least 22m in horizontal and 27m in vertical[1].

GPS-enabled devices with GPS-enabled are growing rapidly nowadays. A large sector of this growth is due to the emergence of smartphones. Simple location-based applications require, rough position estimate while other applications such as navigation is directly connected with accuracy of GPS positioning. The standard positioning service of GPS technology is available 24 hours per day[2]. The accuracy of this service is thus not consistent and is varying with no. of satellite time of the day and place. Buildings can block or reflect the signals, atmosphere delays it, satellite clock and an

orbit error introduces some difference in time. Many times an application needs high accuracy of GPS positioning. Then this simple SPS services would not provide fruitful results. Thus, some different methodology has to be adopted as a means of improving GPS positioning.

## II. ERRORS IN GPS SIGNALS

The GPS satellite signal is subjected to various disturbances before reaching the receiver. These disturbances degrade the signal, decreasing the pseudorange accuracies and hence the overall estimated position. Some of the errors in GPS signals are described below[3].

### A. Geometric dilution of precision

The geometry of how the satellites are arranged in their orbits with respect to the receiver has an effect on position estimation. The perfect situation is when, one satellite on top and the remaining satellite dispersed evenly around the receiver near the surface. Satellite grouped together would yield nearly equal pseudorange estimates and will not provide sufficient information. The effect of how the satellites are placed in the orbit is called geometric dilution of precision (GDOP)[4].

### B. Ephemeris errors

The positions of satellite are effected by the solar radiation and the gravitational pull of the sun and the moon, thus disturbing their orbital positions. These changes can be observed by the control segment from the monitor station placed at known location. The predicted error on the satellite orbit position is of the range 1-6m. The average pseudorange error due to ephemeris prediction error is about 0.8 m[4][5].

### C. Satellite clock errors

The atomic clocks placed on the satellite are very accurate and precise, but small bias do occur over time. This changes leads to huge effect on the location estimate error margin if not eliminated. For example an error of 10ns in atomic clock results in 3m range error at the receiver. This deviation in atomic clock is closely monitored by observation facilities in the control segment. It is difficult to synchronize the time across all the satellite, thus a clock correction data is generated and sent along the GPS message so that the receiver can correct for the clock bias[4].

#### D. Atmospheric errors

Signal from satellite passes through different layers of the atmosphere that affects its speed. Factors such as refractive index of the medium through which the signal propagated, and the distance of air mass the signal must pass through before it reaches the receiver. Satellite present nearer to the horizon relative to receiver experience large amount of air mass compared to the satellite at larger angles. Given the rough position estimate and some metrological parameters this error can be modeled and largely compensated for [4].

#### E. Ionospheric errors

The ionosphere, ranging from 85km of the earth's surface up to 1000km, consisting of ionized gases by solar radiation. These ionized gases disperse the GPS signal. The error in the signal is proportional to amount of ionization suffer due to dispersion, or the total electron content (TEC). The TEC varies depending on latitude and the amount of solar radiation. The ionospheric error mainly decreases transmission speed of the signal, causing a delay. By comparing the different times of arrival of the  $L_1$  and  $L_2$  frequency, the ionospheric delay error can be estimated and compensated for higher accuracy[4][3].

#### F. Multipath effects

Multipath error is caused when the GPS signal received by the receiver is not only directly from the satellite, but also from reflected and diffracted from the local objects. This multipath error disturbs the pseudorange measurement, since the multipath signal take longer time to reach than the direct path, thus affecting the positioning accuracy. The amount of error not only depends on the time delay also on the power of multipath signal received compared to the direct signal. In order to reduce multipath error, an antenna that only records the incoming signal from where the signal is expected to arrive should be used[6][4][7].

### III. METHODOLOGY

The prime motivation behind this work is to utilize an algorithm that minimizes the errors and provide the most accurate position determination solution. The proposed algorithm is such, as to provide better navigation solution than the direct GPS navigation solution.

Steps involved in methodology are:

Step 1: GPS data is collected from hand held Garmin GPS 12 xl device for 500 sec.

Step 2: GPS data collected has file format .kml which is converted into .gpx format via gps visualizer converter, so that it can be easily imported into MATLAB for post processing.

Step 3: Then the data is given to Kalman filter block for the position, velocity estimates. The results provide more accurate positioning then the GPS system alone.

These steps can be performed through the following section:

#### A. KALMAN FILTER

A Kalman filter is a linear quadratic estimator given a dynamic state space model, a group of measurement, and knowledge of statistics surrounding the model and its measurements[8]. It has been applied and used for the last 60

years in applications ranging from space missions to robotics. It is one of the most used tools in state estimation and control due to its robust nature and applicability to many mechatronics systems including robotics, aircraft, automation, and much more. The Kalman filter is implemented via a program and mathematical description of the system so that the best estimate of the dynamic state may be achieved. In the field of autonomous vehicles, which generally uses many sensors to define the state of the system, they are a perfect platform to exercise Kalman filtering because every task depends on some interpretation of sensor data. It is a recursive data processing algorithm which takes a series of data observed over time, which is perturbed by noise and other inaccuracies for the estimation of unknown variables of interest with more accuracy. It was proposed by R.E Kalman in 1960[9]. Practically, it is one of the greatest discoveries in the history of estimation theory. It's most important application is control of complex dynamic systems such as manufacturing process, aircraft, spacecraft or ships. To control such complex system one must infer what it is doing. For such systems it's not always possible to determine every state variable you want to control, and the Kalman filter provides the means for getting the unknown data from the noisy measurements.

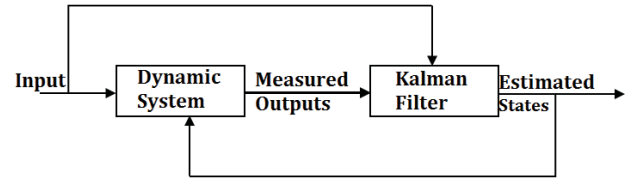


Figure 1: Basic Block Diagram of Kalman Filter

The purpose of the Kalman filter is to estimate the state of a dynamic system and is often referred to as an observer model or state estimator. It requires a model of the dynamic system of interest, however, this model is usually only an estimated model of the real world dynamics[10]. The general assumption in a Kalman filter is that there is some dynamic system which is measured by sensors, and a known control input is applied to the dynamic system. With the inputs, outputs, and an estimate of the system dynamics model, one can generate an estimate of the state variables and thus provide estimated full state feedback for the controller. In this paper desired estimate is that of position, velocity required to track the vehicle continuously.

This method revolves between two steps, prediction stage and the measurement update stage[11]. In the prediction stage the system states is predicted forward in time using a model based prediction, given the present system state as input. In the measurement update stage the forecasted stage is corrected using a weighted average of noisy sensory input based on the noise and estimated confidence for each sensor.

Let a nonlinear system represented by following state space equations

$$x_{t+1} = A_t x_t + B_t u_t + v_t \quad (1)$$

$$z_t = H_t x_t + w_t \quad (2)$$

Where the random variable  $v_t$  and  $w_t$  are process and measurement noise, respectively. Both the process noise and measurement noise are assumed to be white, Gaussian and independent[12]. The normal probability distribution of these noises is known to be

$$P(v) \sim N(0, Q) \quad (3)$$

$$P(w) \sim N(0, R) \quad (4)$$

The  $n \times n$  system matrix  $A$  in equation (1) relates the earlier state at instant  $t-1$  to the present state  $t$ , in the absence of either input function or process noise. The  $n \times 1$  matrix  $B$  relates the control input to the predicted state  $x$ . The  $H$  matrix in the measurement equation gives the relation between states and the corresponding measurement  $z_t$ .

Kalman filter equation is given in Table 1:

Table: 1

Prediction Step	Measurement update/Correction step
State vector is given by $\hat{x}_{t(-)} = \Phi \hat{x}_{t-1(+)}$	Kalman gain is given by, $K_t = P_{t(-)} H_t^T (H_t P_{t(-)} H_t^T + R_t)^{-1}$
Covariance matrix is given by $P_{t(-)} = \Phi_t P_{t-1(+)} \Phi_t^T + Q_{t-1}$	State estimation is given by, $\hat{x}_{t(+)} = \hat{x}_{t(-)} + K_t (z_t - H_t \hat{x}_{t(-)})$
	Corrected covariance matrix is given by, $P_{t(+)} = P_{t(-)} - K_t H_t P_{t(-)}$

$P_{t(-)}$  and  $\hat{x}_{t(-)}$  are the estimated/initial estimate of covariance matrix and state vector, where  $t$  represents the time index.  $H_t$  represents the measurement sensitivity matrix and  $K_t$  is the Kalman gain,  $\Phi_t$  is the state transition matrix,  $Q_{t-1}$  represents the covariance of noise uncertainties and  $R_t$  is the measurement uncertainties,  $P_{t(+)}$  and  $\hat{x}_{t(+)}$  are the corrected estimate of covariance and state vector[13].

### B. VEHICLE MODEL DESCRIPTION

In this paper, we take a problem to estimate position and velocity of a vehicle using noisy position measurements of GPS sensors.

The vehicle can be represented by a simple point-mass model Where, the vehicle states are (from fig 2):

$x_e(t)$  = east position (m).

$x_n(t)$  = north position (m)

$s(t)$  = speed (m/s).

$\theta(t)$  = orientation from east (deg).

$$\frac{d}{dt} \begin{bmatrix} x_e(t) \\ x_n(t) \\ s(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} s(t) \cos(\theta(t)) \\ s(t) \sin(\theta(t)) \\ P \frac{u_r}{s(t)} - A C_d s(t)^2 / m \\ s(t) \tan(u_\phi(t)) / L \end{bmatrix} \quad (5)$$

The vehicle position can be represented in two dimensional planes as

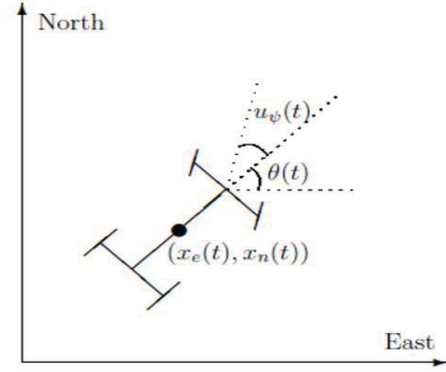


Figure 2: Vehicle position in two dimensions

Where  $x_n(t)$  and  $x_e(t)$  is the vehicle's north and east position from the origin,  $\theta(t)$  is orientation from east and  $u_\phi(t)$  is the steering angle of the vehicle.  $t$  is the continuous-time variable.

Vehicle parameters:

Table: 2

P	10000	Peak engine power(w)
A	1	Frontal Area(m <sup>2</sup> )
C <sub>d</sub>	0.3	Drag coefficient unit less
M	1250	Vehicle mass(Kg)
L	2.5	Wheelbase length(m)

### C. MATHEMATICAL DESCRIPTION OF KALMAN FILTER

In this paper Kalman filter is applied on a linear model to estimate the state of the unknown variable. The linear model describes how the estimated variable gets updated over time in response to model initial condition as well as known and unknown inputs.

$$\hat{x}[n] = \begin{bmatrix} \hat{x}_e[n] \\ \hat{x}_n[n] \\ \hat{x}_s[n] \\ \hat{x}_\theta[n] \end{bmatrix} \quad (6)$$

Where,

- $\hat{x}_e[n]$  East position estimate (m).
- $\hat{x}_n[n]$  North position estimate (m).
- $\hat{\dot{x}}_e[n]$  East velocity estimate (m/s).
- $\hat{\dot{x}}_n[n]$  North velocity estimate (m/s).

The term  $\dot{x}$  represents velocities in east and north direction. N is the discrete-time index.

The linear model used in Kalman filter is of form:

$$\hat{x}[n+1] = A\hat{x}[n] + Gw[n] \quad (7)$$

$$y[n] = C\hat{x}[n] + v[n] \quad (8)$$

Where,

$\hat{x}$  is the state vector,  $y$  is the measurement,  $w$  is the process noise, and  $v$  is the measurement noise. Kalman filter assumes that  $w$  and  $v$  are zero-mean, independent random variable with known variance  $E[ww^T] = Q$ ,  $E[vv^T] = R$  and  $E[wv^T] = N$ . and A,G and C matrix are:

$$A = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} T_s/2 & 0 \\ 0 & T_s/2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Where  $T_s = 1s$ .

#### IV. SIMULATION RESULTS AND DISCUSSION

The overall Simulink model containing the vehicle model, plant model, Kalman filter, noise model is shown below:

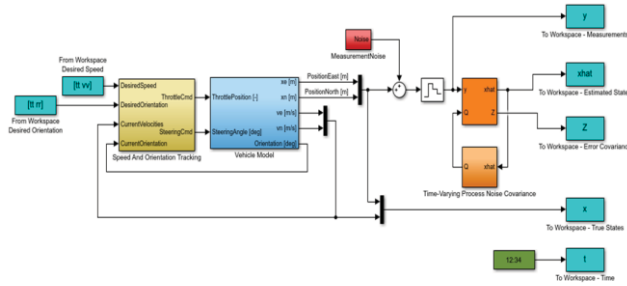


Figure 3: Simulink Model of overall System

The vehicle is allowed to move in known direction in Simulink environment. After simulation, the actual, measured and Kalman estimates are obtained and shown in Fig 4:

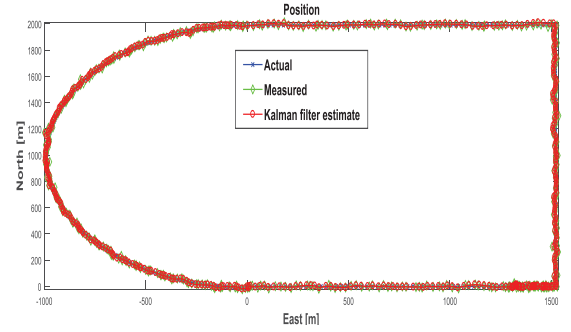


Figure 4: Actual, Measured, Kalman estimate of vehicle position

Fig 4 indicates the actual movement of the vehicle in north-east plane. It also shows how the GPS measured and the Kalman filter tracks the actual movement of the vehicle.

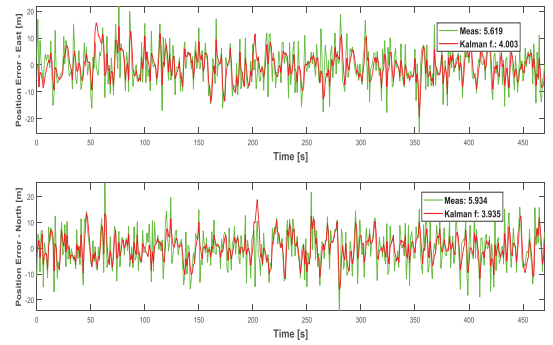


Figure 5: Position error b/w measured and actual and error b/w Kalman filter and actual

Fig 5 represents the position error graph in east and north direction the error for measured model in east and north came out to be 5.609m and 5.934m, the error for Kalman filter model in east and north came out to be 4.003m and 3.935m.

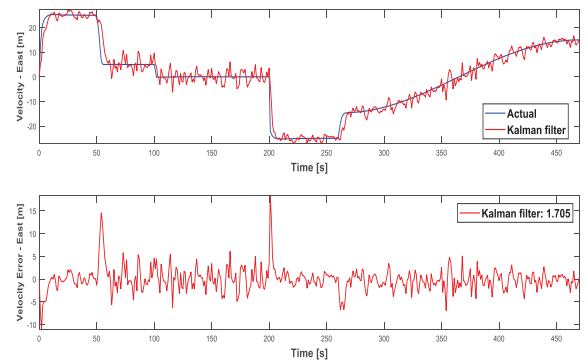


Figure 6: Velocity in east direction and its estimate And Velocity error in east direction

Fig 6 represents the velocity tracking for Kalman filter in east direction and the velocity error in 1.705 in east direction.

## V. CONCLUSION

This paper gives an overview about the sources of error in GPS signal, and how some of the errors can be reduced with Kalman filter. In this paper MATLAB Simulink environment has been used. A problem is defined to estimate the states (position and velocity) of ground vehicle based on noisy position measurements. Here, Kalman filter is used to estimate the position and velocity of a vehicle. The vehicle is required to follow a directed path. Simulation results show that the least error between position vs. time comes out to be that of Kalman filter than the direct GPS measurement. Further each of these error sources can be dealt individually for further improving of GPS results.

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