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# Position and Orientation Estimation Using Kalman Filtering and Particle Filtering with One IMU and One Position Sensor

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*Abstract*— **In this paper, a novel position and orientation estimation method that relies on Kalman filtering and particle filtering is proposed. The orientation calculation error by using gyros increases over time due to the integration of angular velocity measurement errors. This paper describes how to estimate the orientation and position with a high accuracy when one inertial measurement unit (IMU) and one position sensor are available. The proposed filter takes advantage of the particle filtering component to estimate the orientation, and the Kalman filtering component to estimate the position of each orientation particle. The simulation results of the orientation calculation with no filter, with a Kalman filter (KF), and with the proposed filter are compared and discussed. The proposed filter is proven to reduce the position error and the rotation matrix error significantly.**

## I. Introduction

The advent of a Micro Electro-Mechanical System (MEMS) IMU, which consists of three accelerometers and three gyros, has launched new applications in many fields such as manufacturing [1] and medical [2]. Although the position and the orientation of an object can be calculated by the numerical integration of the acceleration and angular velocity measurements from an IMU, the position and the orientation errors grow over time due to numerical integration errors. In addition, the integration of the non-linearity, noise, and bias error of the sensor outputs leads to increased errors in the state estimation. Two integration steps are required to calculate the position from the acceleration, whereas only one integration step is required to calculate the orientation from the angular velocity. Thus, the position error increases at a much faster rate than the orientation error. Therefore, when an IMU is used as a position sensor, the IMU is usually hybridized with a secondary position sensor to achieve a higher accuracy. In this case, the IMU can be used as an orientation sensor as well. Since the orientation calculation by an IMU also drifts over time due to integration, some researchers have used multiple position sensors to estimate the orientation [3]-[6]. A multi-antenna GPS receiver is utilized to find the orientation of an object for outdoor applications in [3]-[5]. Foxlin [6] has shown that the orientation of an object can be corrected by using multiple ultrasonic transmitters. However, these methods require some distance between the position sensors to maintain the orientation accuracy.

To integrate one position sensor with an IMU, a KF or an extended Kalman filter (EKF) is widely used [6]-[8]. Since both the KF and EKF are not computationally intensive, they are widely used in real-time applications such as inertial navigation systems. On the other hand, when the governing equation is highly non-linear or non-Gaussian, a particle filter provides better results. In order to achieve reliable results with a particle filter, the number of particles should be large enough to represent the true posterior probability density function (PDF). However, since most inertial navigation systems or motion sensing systems need to be in real-time, the number of particles is limited. Therefore, such navigation systems sometimes utilize the Rao-blackellized particle filter which uses Kalman filtering for the linear Gaussian part and uses particle filtering for the non-linear or non-Gaussian part [9], [10]. In [9], the Rao-Blackwellized particle filter demonstrates better accuracy than the particle filter alone, even with only one fifth of the number of particles used for the particle filter alone.

This paper proposes a filtering method that estimates the orientation with particle filtering and estimates the position of each orientation particle with Kalman filtering. Simulations are generated to analyze the effects of the proposed filtering technique. The resultant errors of the proposed filter are compared with the errors with no filter and with a KF.

## II. Kalman Filter

### A. Orientation Estimation Using Quaternions

The quaternion representation is used to determine the orientation of the body frame with respect to the fixed frame. This method uses one real number and three imaginary numbers ($i$, $j$, and, $k$) to represent the rotation. A unit quaternion $q$ can be written as

$$q = q_0 + q_1 i + q_2 j + q_3 k \quad (1)$$

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that satisfies

$$q_0^{\,2}+q_1^{\,2}+q_2^{\,2}+q_3^{\,2}=1\,. \tag{2}$$

To determine the orientation of a 3D vector, the vector should be multiplied by the quaternion $q$ and the conjugate of quaternion $q$, denoted as $q^*$, as follows:

$$v' = q\cdot v\cdot q^* \tag{3}$$

where vector $v'$ is the rotated 3D vector, and $q^*$ is defined as

$$q^* = q_0 - q_1 i - q_2 j - q_3 k\ . \tag{4}$$

Then, the rotation matrix from the body frame to the fixed frame by using quaternions is represented as

$$C_b^f = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}. \tag{5}$$

To find the quaternion terms from the angular velocities, the derivative of the quaternions should be determined from

$$\begin{bmatrix} \dot q_0 \\ \dot q_1 \\ \dot q_2 \\ \dot q_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}\cdot\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{6}$$

where $\omega_x$, $\omega_y$, and $\omega_z$ are the angular velocity of each axis.

The KF is widely used to estimate the orientation with the quaternion orientation representation [11] because (6) has state a space model, and the measurement noise PDF of a gyro is assumed Gaussian. However, when only gyros are used to estimate the orientation, there are no redundant measurements to correct the quaternion terms.

### B. Position Estimation

An IMU measures the acceleration and angular velocity with respect to the body frame. For a short-distance navigation application, the fixed frame is attached to a stationary point which rotates with the earth. In order to find the acceleration of the IMU with respect to the fixed frame, the simplified equation of motion is

$$\dot v^f = C_b^f f^b + g^f \tag{7}$$

where $\dot v^f$ is the acceleration (derivative of the velocity) of the IMU with respect to the fixed frame, $f^b$ is the specific force acceleration which the IMU is subjected to, and $g^f$ is the local gravity force with respect to the fixed frame. The rotation matrix is determined from three gyros by computing (5) and (6), the specific force acceleration is obtained from the accelerometers, and the local gravity force is predetermined. The position equation is

$$\dot p^f = v^f\ , \tag{8}$$

where $\dot p^f$ is the derivation of the position with respect to the fixed frame. Since (7) and (8) are in a linear state space model, and the sensor noise can be assumed to be zero-mean Gaussian, a KF can be used. The acceleration measurements from the accelerometers are used to predict the velocity and the position states, and the position sensor measurements are used to update the position states.

## III. Orientation Correction Method Using Particle Filtering

The particle filter is a suboptimal observer that can estimate the states of a nonlinear non-Gaussian system. Equation (7) shows that the acceleration equation is coupled with the orientation equation. When the rotation matrix of (7) is incorrect, there is a misalignment between the true body frame and the calculated body frame which leads to errors in the body frame acceleration. Since the double integration of the acceleration error results in a position error, the higher the position error is, the higher the probability is for the orientation errors to be higher.

The proposed method uses particle filtering to estimate the orientation. Since a quaternion has four components, each particle set consists of four particles, and an $N$ number of particle sets is used to estimate the orientation. For each particle set, the position is estimated by using the IMU measurements only. In parallel, the KF is employed to estimate the position and the velocity of the IMU for each particle set. For each time step, the estimated position states for each quaternion particle set are evaluated against the KF position outputs by

$$PE_j^i = ((P_{Px}^i - P_{Kx}^i)^2 + (P_{Py}^i - P_{Ky}^i)^2 + (P_{Pz}^i - P_{Kz}^i)^2)^{0.5}\,, \tag{9}$$

where $PE_j^i$ represents the position error of the $i^{\text{th}}$ particle at the $j^{\text{th}}$ sampling time, $P_{P(axis)}^i$ represents the position difference from the $j$-1$^{\text{th}}$ sampling time to the $j^{\text{th}}$ sampling time in each axis of the i$^{\text{th}}$ particle set by using the IMU measurement only, and $P_{K(axis)}^i$ is the position difference from the $j$-1$^{\text{th}}$ sampling time to the $j^{\text{th}}$ sampling time in each axis of the $i^{\text{th}}$ particle set by using the KF. The weights of each particle set are calculated with the summation of $PE_j^i$ for a period of time (ΔT) as follows:

$$w^i = \sum_{j=1}^{K} PE_j^i\ , \tag{10}$$

where $w^i$ is the weight of the particle set $i$, $K$ is the number of samples before the weights are calculated ( $K = \Delta T / t$ ), and $t$

is the sampling time. The higher the weight is, the higher the probability is for the particle set to be correct. After the weights are evaluated, $j$ is initialized to 1 to calculate the new $PE_j^i$. By using a sampling importance resampling (SIR) particle filter [12], the higher weights of the quaternion particle sets survive and are selected more for the next estimation.

## IV. Simulations

To analyze the effects of the proposed method on the position and the orientation estimation (6DOF), simulations, which mimic a random hand motion, are generated. Then, the resultant errors of the proposed filter are compared with the errors where no filter is applied and the errors where a KF is used. In order to mimic the sensor outputs, a zero-mean Gaussian noise with a variance of 0.01m, 0.03 m/s$^2$, and 0.5°/s are added to the simulated position, acceleration, and gyro signal, respectively. The number of particle sets should be large enough to obtain good results, but increasing the number of particle sets increases the computational cost. A faster sampling rate results in a better accuracy because the position and orientation calculations with an IMU require integration steps, but this also increases the computational cost. Therefore, the number of particle sets and the sampling rate should be optimized according to the required accuracy and the computational power. In the simulation, the proposed filter uses 20 particle sets, and a sampling rate of 200 Hz (t = 0.005 second) is chosen for both the position sensor and the IMU. The newly developed filter is applied to evaluate the weights of each particle set at every 0.5 second intervals ($\Delta T$ = 0.5 second). The particle set with the highest weight is chosen as the outputs for both orientation and position until the next weight evaluation. Since the bias of a gyro cannot be perfectly estimated, small biases are added to the gyros in the simulation. For the first test, a bias of 0.001 rad/sec is added to each output of the gyros. The simulation is repeated with the bias 10 and 100 times higher than that of the first simulation to analyze the effects on the higher orientation error. Additional errors such as random walk and gain factor error lead to further errors but are not included.

For the first simulation, the bias of each axis is set to 0.001 rad/sec. The orientation errors of the three different orientation calculation methods are compared in Fig. 1. The discontinuities exist (the error suddenly approaches 0˚) in the figures because the range of the roll, pitch, and yaw angles are limited within the range of -90° to 90° when the quaternion is transformed to an Euler angle. For example, 91° becomes 89° due to the angle limit. Fig. 1 illustrates that the errors in all three angles are significantly reduced when the proposed filter is used.

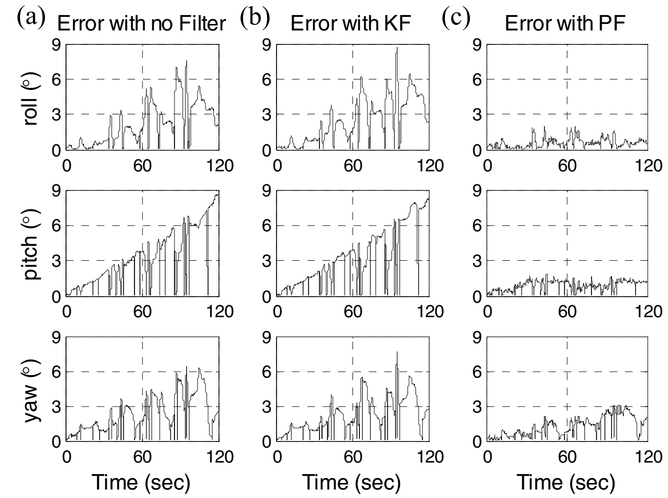


Fig. 1: Orientation errors using (a) only gyro integration, (b) the KF, and (c) the proposed filter (PF), when the bias of each axis is 0.001 rad/sec.

To analyze the total orientation error, the RMS errors of the rotation matrix are compared in Fig. 2. All three graphs show that the orientation errors tend to increase over time. Fig. 2 shows that the orientation estimation using KF does not show significant improvement because there is no redundant measurement to correct the quaternion terms. However, when the proposed filter was used, the error of the rotation matrix is significantly reduced.

The position error comparison is denoted in Fig. 3. The figures show that the position error is reduced to almost one third by using the KF and the proposed filter. Fig. 3 also illustrates that there is no significant difference between the position error of the proposed filter and that of the KF when the orientation error is small.

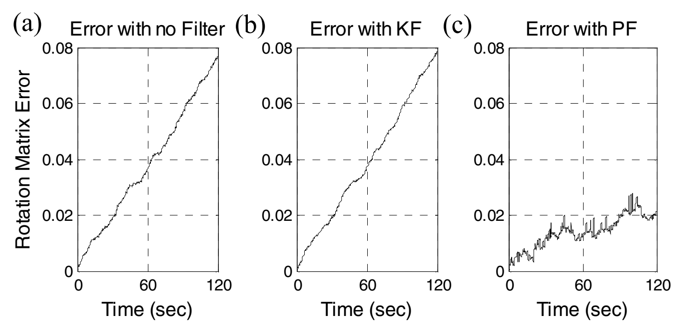


Fig. 2: RMS errors of the orientation matrices using (a) only gyro integration, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.001 rad/sec.

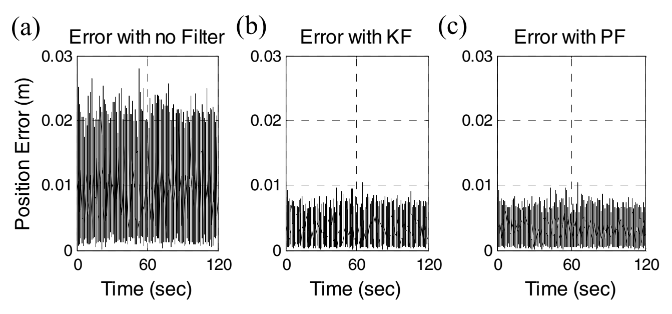


Fig. 3: Position errors using (a) only position sensor, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.001 rad/sec.

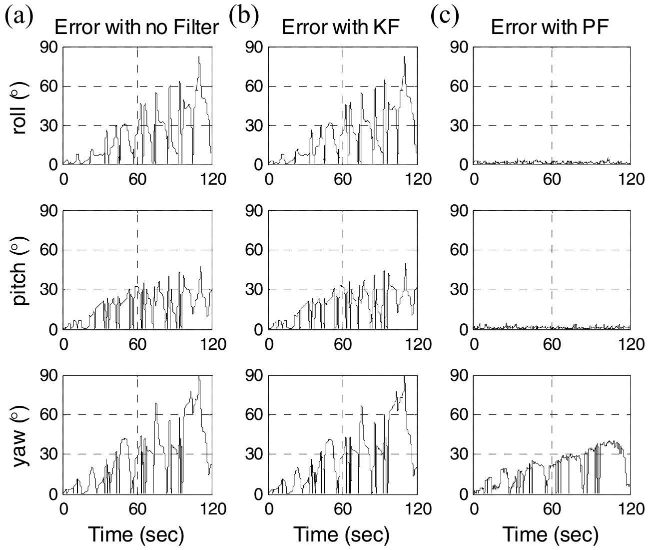


Fig. 4: Orientation errors using (a) only gyro integration, (b) the KF, and (c) the proposed filter (PF), when the bias of each axis is 0.01 rad/sec.

For the second simulation, the bias of each axis is changed from 0.001 rad/sec to 0.01 rad/sec, and the process noise variance for the particle filter is multiplied by ten. Fig. 4 displays the orientation errors in the roll, pitch, and yaw angles, and Fig. 5 shows the RMS errors of the rotation matrix. Both Fig. 4 and Fig. 5 indicate that there is almost no difference in the orientation error between the KF and the gyro integration method. However, the orientation error is significantly reduced when the proposed filter is used. The roll and pitch angle errors of the proposed filter do not increase over time because the roll and pitch angles are related to the gravity vector. When the errors in the roll and pitch angles increase, the acceleration error increases as well due to the misalignment between the calculated gravity vector and the true gravity vector. The higher the acceleration error is, the higher the position error is. Since the particle sets with higher position errors have lower probability to survive and be selected for the next estimation, the particle sets with lower roll and pitch angle errors are selected more. Thus, the proposed filtering method has low roll and pitch angle errors. However, since the gravity vector has no effect on the yaw angle, the yaw angle error tends to increase over time.

Fig. 6 reflects the position error comparison. The position error with the KF increases over time because the roll and pitch angle errors increase over time. This leads to a higher acceleration error due to the gravity vector misalignment and results in a higher position error. However, the position error of the proposed filter with the bias of 0.01 rad/sec is almost the same as the position error with the bias of 0.001 rad/sec because the errors of the roll and pitch angles are kept at a low level.

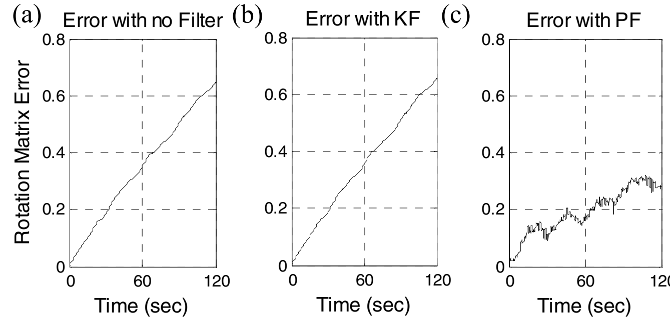


Fig. 5: RMS errors of the orientation matrices using (a) only gyro integration, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.01 rad/sec.

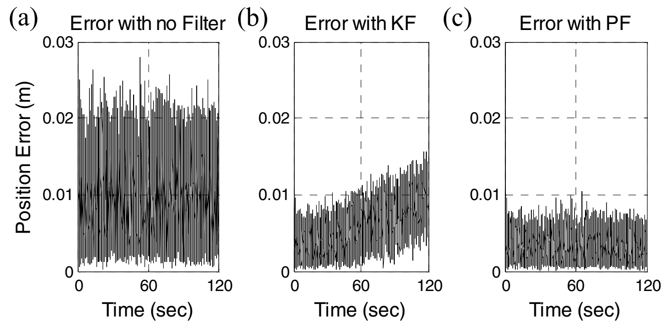


Fig. 6: Position errors using (a) only position sensor, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.01 rad/sec.

For the third simulation, the bias of each axis are changed from 0.001 rad/sec to 0.1 rad/sec, and the process noise variance for the particle filter is multiplied by hundred. The orientation errors are represented in Fig. 7 and Fig. 8. Fig. 7 reveals that the orientation errors increase more than those in Fig. 4 due to a higher bias. Although the roll and pitch angle errors are increased, the errors remain at low level compared to those of the other two methods. However, the pitch angle error increases as much as the errors of the other two methods. Fig. 8 indicates that the proposed filter has a very high rotation matrix error due to the high yaw angle error.

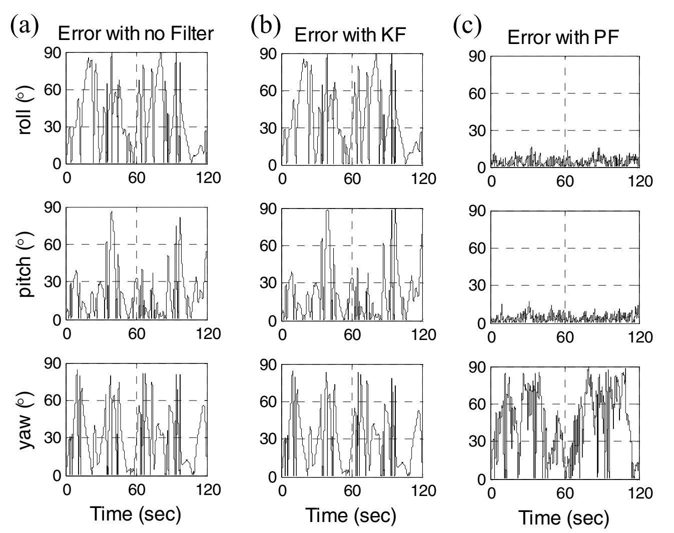


Fig. 7: Orientation errors using (a) only gyro integration, (b) the KF, and (c) the proposed filter (PF), when the bias of each axis is 0.1 rad/sec.

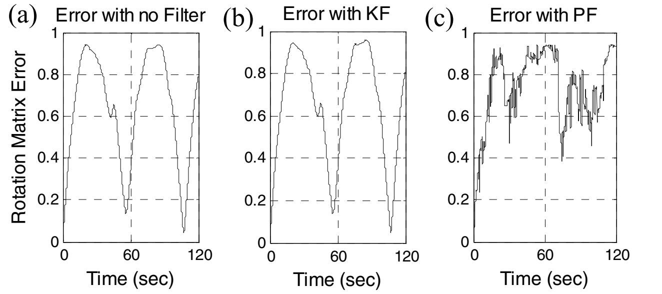


Fig. 8: RMS errors of the orientation matrices using (a) only gyro integration, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.1 rad/sec.

Fig. 9 shows the position error when the biases of each axes are 0.1 rad/sec. When the KF is employed, the position error increases as the orientation error increases. However, the position error of the proposed filter does not differ significantly from the position error in Fig. 3 although the rotation matrix error is as high as that of the other methods. This is due to the fact that the roll and pitch angle errors are kept at a low level.

## V. Conclusions and Future Work

This paper presents a novel position and orientation estimation method that utilizes the particle filtering technique to estimate the orientation and the Kalman filtering technique to estimate the position. The proposed filter is examined by conducting three simulations with different biases to investigate the effects on the different orientation errors.

There is almost no noticeable improvement in orientation when the KF is used. However, the proposed filtering technique provides significant improvement in the orientation estimation. The roll and pitch angle errors remain at a low level with at each different bias level. However, the yaw angle error increases over time, but much slower rate than the other two methods when the bias is low. When the bias is very high, the yaw angle fluctuates as much as in the other two methods.

The position estimations of both the KF and the proposed filter demonstrate improved results than the position sensor outputs. The position error is reduced to one third when the orientation error is small. However, when the orientation error increases, the position error of the KF increases. The position error with the proposed filtering technique does not show any noticeable change under the same condition. In fact, the position error with the proposed filtering technique has a better accuracy than the position estimation with the KF when the orientation error is high due to a higher orientation accuracy.

In the real world, both accelerometers and gyros exhibit many sources of errors such as the gain factor error and moving bias. Thus, experimental tests will be conducted in the future.

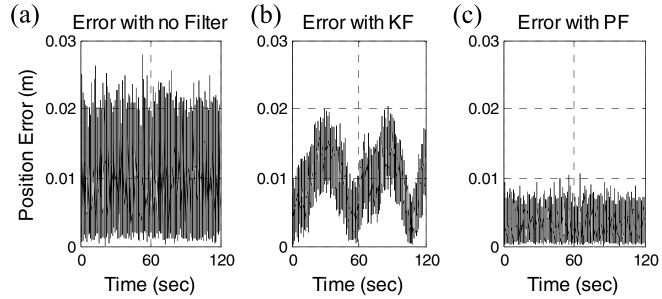


Fig. 9: Position errors using (a) only position sensor, (b) the KF, and (c) the proposed filter, when the bias of each axis is 0.1 rad/sec.

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