Fusion of GPS and Redundant IMU Data for Attitude Estimation

Conference Paper · August 2012

DOI: 10.2514/6.2012-5030

Matthew B Rhudy
Pennsylvania State University, Reading, PA

Jason N. Gross
West Virginia University

Yu Gu
University of Bristol

AIAA Guidance, Navigation, and Control Conference
13 - 16 August 2012, Minneapolis, Minnesota

AIAA 2012-5030

Fusion of GPS and Redundant IMU Data for Attitude Estimation

Matthew Rhudy[*], Jason Gross[†], Yu Gu[‡], and Marcello R. Napolitano[§]
Department of Mechanical and Aerospace Engineering, West Virginia University, Morgantown, WV, 26506

Attitude estimation using Global Positioning System/Inertial Navigation System (GPS/INS) was used as an example application to study three different methods of fusing redundant multi-sensor data used in the prediction stage of a nonlinear recursive filter. Experimental flight data were collected with an Unmanned Aerial Vehicle (UAV) containing GPS position and velocity calculations and four redundant Inertial Measurement Unit (IMU) sensors. Additionally, the aircraft roll and pitch angles were measured directly with a high-quality mechanical vertical gyroscope to be used as a 'truth' reference for evaluating attitude estimation performance. A simple formulation of GPS/INS sensor fusion using an Extended Kalman Filter (EKF) was used to calculate the results for this study. Each of the three presented fusion methods was shown to be effective in reducing the roll and pitch errors as compared to corresponding results using single IMU GPS/INS sensor fusion. Additionally, the fusion methods were shown to be effective in estimating roll and pitch angles without the aid of GPS (dead reckoning).

Nomenclature

$\mathbf{A}$	=	state prediction Jacobian matrix
a_x, a_y, a_z	=	acceleration in aircraft body frame (m/s^2)
$\mathbf{f}$	=	state prediction function
g	=	acceleration due to gravity (m/s^2)
$\mathbf{H}$	=	observation matrix
$\mathbf{I}$	=	identity matrix
$\mathbf{K}$	=	Kalman gain matrix
k	=	discrete time index
$\mathbf{P}$	=	state covariance matrix
p	=	roll rate (deg/s)
$\mathbf{Q}$	=	process noise covariance matrix
q	=	pitch rate (deg/s)
$\mathbf{R}$	=	measurement noise covariance matrix
r	=	yaw rate (deg/s)
T	=	averaging time (s)
T_s	=	sampling time (s)
$\mathbf{u}$	=	input vector
$\mathbf{v}$	=	measurement noise vector
V_x, V_y, V_z	=	local components of velocity (m/s)
W	=	random walk coefficient
$\mathbf{w}$	=	process noise vector
$\mathbf{x}$	=	state vector
$\mathbf{y}$	=	output vector
$\mathbf{z}$	=	measurement vector

[*] Ph.D. Candidate, Mechanical and Aerospace Engineering (MAE) Department, PO Box 6106, mrhudy1@mix.wvu.edu, AIAA Student Member.
[†] Ph.D., MAE Department, now at Jet Propulsion Lab.
[‡] Assistant Professor, MAE Department, PO Box 6106, AIAA Senior Member.
[§] Professor, MAE Department, PO Box 6106, AIAA Senior Member.

θ	=	pitch angle (deg)
σ_A	=	root Allan variance
ϕ	=	roll angle (deg)
ψ	=	yaw angle (deg)

I. Introduction

THE problem of multi-sensor fusion has become a popular research topic and has been considered by several researchers. Recent advancements in electronics have reduced the weight, cost, and size of various sensors, thus allowing for multiple potentially redundant sensors to be integrated into many existing systems for various applications. This increase in sensor data, however, leads to an entirely new area of research dealing with the challenges of how to best integrate or fuse this information. Different fusion architectures have been proposed conceptually by Bass for cyberspace intrusion detection[1] and by Hall and Llinas for military applications[2]. Additionally, more generalized approaches have been presented by Willner *et al*.[3], Xiong and Svensson[4], and Luo and Yih[5]. Other authors have presented decentralized approaches[6-8], including applications in image processing[9], simulated target tracking using sigma point information filtering[10], and land vehicle positioning[11]. Varshney presented a binary decision fusion approach with application to image fusion[12]. Sun and Deng derived an optimal information fusion approach and presented simulation results for radar tracking[13]. Drolet *et al*. used an underwater remotely operated vehicle to study positioning using acoustic positioning and accelerometers, including the combination of redundant measurements[14]. Some two-sensor fusion work has been performed by Bar-Shalom and Campo[15], Chang *et al*.[16], and Roecker and McGillem[17], considering the coupled effects between the two sensors in tracking applications.

Another active area of research related to multiple sensors is associated with voting schemes[18,19]. The focus of voting scheme research is to use sensor redundancy in order to improve reliability of the system. The improvement in reliability is achieved by developing fault-tolerant algorithms which feature different forms of logic to define a voting scheme, such as a fuzzy voting scheme[20], a dynamic voting scheme[21], or a secure and fault-tolerant voting scheme[22]. Additional work by Oosterom and Babuska is based on a virtual sensor technique that uses voting for fault detection and isolation[23]. Alternatively to reliability enhancement, redundant sensors can also be used in order to improve the performance of the system. The work herein focuses on this aspect of multi-sensor fusion, and allows for expansion of this work to include a synthesis of both reliability and performance enhancement through the use of multiple sensors.

Current Kalman filter based multi-sensor fusion work assumes redundancy only in the measurement update, which can be handled using the information filter[24], or its nonlinear variants such as the Extended Information Filter (EIF)[25] or Unscented Information Filter (UIF)[26]. This work, however, approaches the problem of multi-sensor fusion where the sensor information is used in the prediction stage of a nonlinear estimator. Since the redundancy is in the prediction, the current information filtering tools cannot directly be used, because these methods are formulated only for multiple measurements used in the measurement update stage of the filter. Using previous multi-sensor fusion work as inspiration, including that of Gan and Harris[27] and Drolet *et al*.[14], three different fusion methods were developed for handling redundant prediction information. Each of these developed methods utilize the Extended Kalman Filter (EKF)[24], with principles of information filtering used to fuse together redundant information. Here, the EKF was selected over the EIF since the two filters are mathematically equivalent for applications with a single measurement vector.

To evaluate the developed fusion methods, the problem of attitude estimation using Global Positioning System/Inertial Navigation System (GPS/INS) sensor fusion is used as a case study, taking advantage of experimentally collected Unmanned Aerial Vehicle (UAV) flight data containing redundant Inertial Measurement Unit (IMU) measurements. Attitude estimation is an important problem in aerospace applications such as remote sensing with UAVs[28-31]. Low-cost approaches such as GPS/INS integration[32,33] are often used instead of high quality military grade sensors[34,35] due to cost and weight restrictions[35,36]. Previous GPS/INS sensor fusion work has relied on combining information from a single IMU and a single GPS receiver[37-42]. However, due to the relatively low cost and weight of available IMU sensors, adding additional IMUs to the aircraft payload is a viable option. While it is easy to add more sensors to an aircraft payload, the most effective utilization of this additional information becomes a more difficult problem. Three fusion methods are presented and compared for the combination of redundant IMU data in this application.

The paper is organized as follows. Section II describes the formulation of the considered GPS/INS sensor fusion algorithm and an outline of the three information fusion methods. Section III discusses the experimental setup including modeling of the sensor errors. Section IV presents the empirical results. Finally Section V provides a summary and analysis of the results.

II. Problem Formulation

A. GPS/INS Sensor Fusion Formulation

The attitude estimation problem using GPS/INS sensor fusion has been studied by different research groups using different formulations of the problem[37-42]. This work considers a relatively simple formulation of GPS/INS sensor fusion that does not consider any complex on-line noise modeling[42] or joint parameter estimation, such as bias tracking on sensor measurements[38,39,41]. This simple formulation is used in order to demonstrate the usefulness of redundant sensor fusion, even with simple methods. The considered formulation is a 6-state formulation of GPS/INS sensor fusion with state vector, **x**, consisting of the three local Cartesian components of velocity, V_x, V_y, V_z, and the aircraft Euler angles, ϕ, θ, ψ. IMU measurements of three axis accelerations, a_x, a_y, a_z, and angular rates, p, q, r, are used as an input vector, **u**, while the GPS velocity calculations are used as the measurement vector, **z**. The output vector, **y**, is given by the three components of velocity. In summary, the following definitions are used:

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} V_x & V_y & V_z & \phi & \theta & \psi \end{bmatrix}^T & \qquad \mathbf{y} &= \begin{bmatrix} V_x & V_y & V_z \end{bmatrix}^T \\ \mathbf{u} &= \begin{bmatrix} a_x & a_y & a_z & p & q & r \end{bmatrix}^T & \qquad \mathbf{z} &= \begin{bmatrix} V_x^{GPS} & V_y^{GPS} & V_z^{GPS} \end{bmatrix}^T \end{aligned} \tag{1}$$

Using the above definitions, the state equations are determined from coordinate transformations[43,44] between the aircraft body frame as measured by the IMU and the Earth-fixed local frame, and also considering a first order discretization[45]:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + T_s \left(\begin{bmatrix} 0 \\ 0 \\ g \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c\phi c\psi & -c\phi s\psi + s\phi s\theta c\psi & s\phi s\psi + c\phi s\theta c\psi & 0 & 0 & 0 \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & -s\phi c\psi + c\phi s\theta s\psi & 0 & 0 & 0 \\ -s\theta & s\phi c\theta & c\phi c\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & s\phi t\theta & c\phi t\theta \\ 0 & 0 & 0 & 0 & c\phi & -s\phi \\ 0 & 0 & 0 & 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix} (\mathbf{u}_k + \mathbf{w}_k) \right) \tag{2}$$

where 'c', 's', and 't' denote the cosine, sine, and tangent functions respectively. The Euler angles ϕ, θ, ψ, are evaluated at discrete time k-1, T_s is the sampling time, and **w** is the process noise vector, which is assumed to be zero-mean, additive to the input vector, and have covariance matrix **Q**. The output equations are determined by extracting the measured states of the system, as in:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k = \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{0}_{3x3} \end{bmatrix} \mathbf{x}_k + \mathbf{v}_k \tag{3}$$

where **H** is the observation matrix, **I** is an identity matrix of given dimensions, **0** is a matrix of zeros of given dimensions, and **v** is the measurement noise vector, which is assumed to be zero-mean with covariance matrix **R**.

B. Methods of Fusing Redundant IMU Information

This section outlines three different methods of fusing redundant IMU information. Each of the considered methods features the Extended Kalman Filter (EKF) as a nonlinear estimator[24]. The basic principles of the information filter[24] are also used to fuse redundant information sources. In particular, for a set of vectors, $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$, each with corresponding covariance matrices, $\mathbf{P}_1$, $\mathbf{P}_2$, … $\mathbf{P}_n$, the information of these vectors can be combined using:

$$\mathbf{P}_{eq}^{-1} = \sum_{i=1}^{n} \mathbf{P}_i^{-1}, \quad \mathbf{x}_{eq} = \mathbf{P}_{eq} \sum_{i=1}^{n} \mathbf{P}_i^{-1} \mathbf{x}_i \tag{4}$$

where $\mathbf{P}_{eq}$ and $\mathbf{x}_{eq}$ denote the equivalent or combined covariance and vector estimates.

1. Method #1: Fusion of IMU Information Prior to Filtering

This method combines the information from each of the redundant IMUs to obtain one combined equivalent input vector of IMU information. This fusion is done using (4) where the considered vector is the input vector, $\mathbf{u}$, whose covariance is given by $\mathbf{Q}$. This combined input vector is then used in the attitude estimation EKF to obtain the state estimates. This method is illustrated in Fig. 1. Note that the (-) superscripts on $\mathbf{x}$ and $\mathbf{P}$ denote the predicted values, of the state vector and covariance matrix respectively.

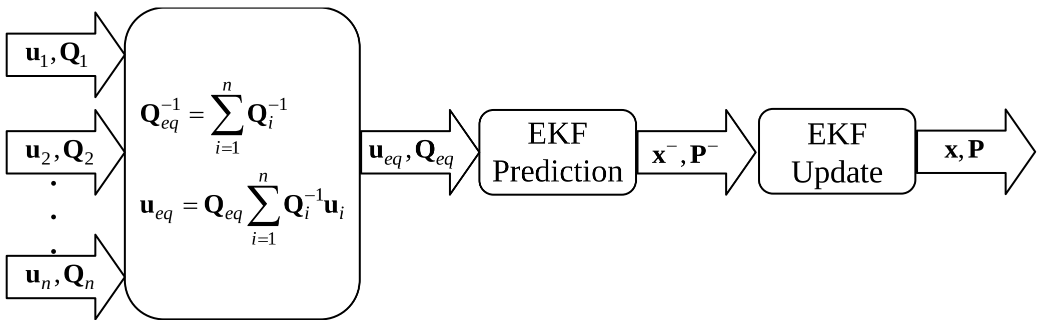

Fig. 1. Fusion Method #1 Diagram

2. Method #2: Fusion of EKF Prediction Information Prior to Updating

For this method, the input vector from each IMU is used to obtain the predicted state and covariance. Then the predicted information is fused using (4). The fused prediction information is passed into a single measurement update to obtain the state and covariance. Note for this method that only a single state vector and covariance matrix are propagated in time, therefore each parallel prediction uses the same *a priori* information. Fig. 2 shows a diagram detailing this method.

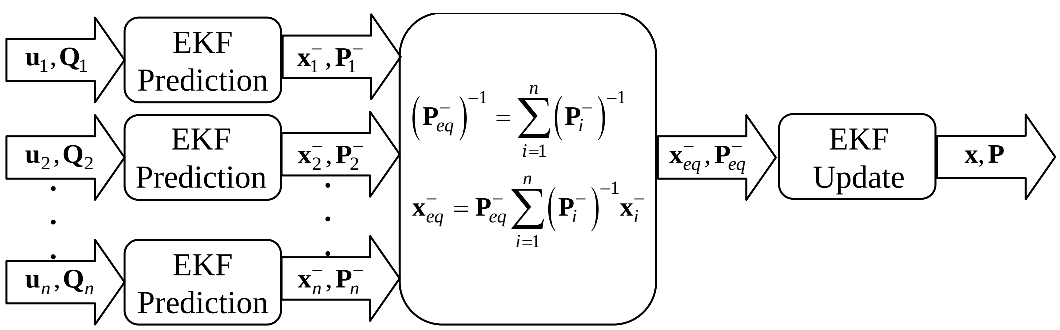

Fig. 2. Fusion Method #2 Diagram

3. Method #3: Fusion of Parallel EKF Estimation Results

Each IMU input vector is used independently in this method to obtain estimates of the state and covariance. Basically, n parallel attitude estimation algorithms are executed, then the results of these parallel filters are fused with (4), as shown in Fig. 3. For this method, each of the individual filters are decoupled, and do not communicate any information between them, therefore this is a decentralized[46] method. Unlike *Method #2*, each prediction for this method uses different *a prioi* information for each input vector. A limitation of this method is that it requires the repeated use of the same GPS information, which is undesirable since each information source ideally should only be used once.

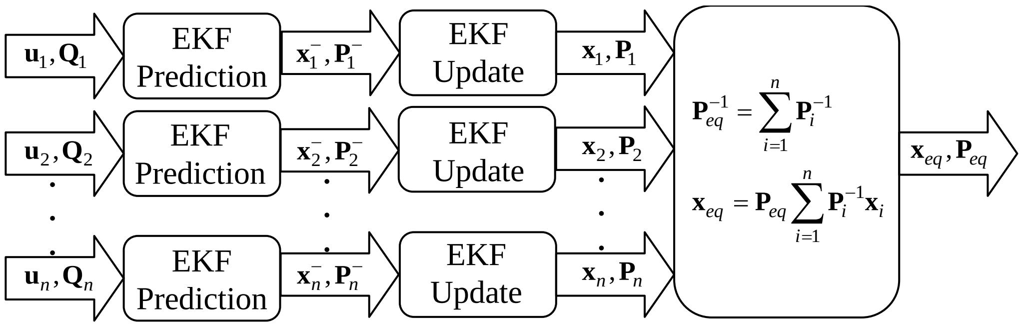

Fig. 3. Fusion Method #3 Diagram

III. Experimental Platform

A. Aircraft Testbed

The flight data used for this study were collected with a Propulsion Assisted Control (PAC) Small Unmanned Aerial Vehicle (SUAV) which was designed, manufactured, and instrumented by researchers at the Flight Control Systems Lab (FCSL) at West Virginia University (WVU). The avionic payload includes a custom designed printed circuit board (PCB) featuring four Analog Devices® Inertial Measurement Units (IMUs) and a Novatel OEM-V1 GPS receiver, as shown in Fig. 4.

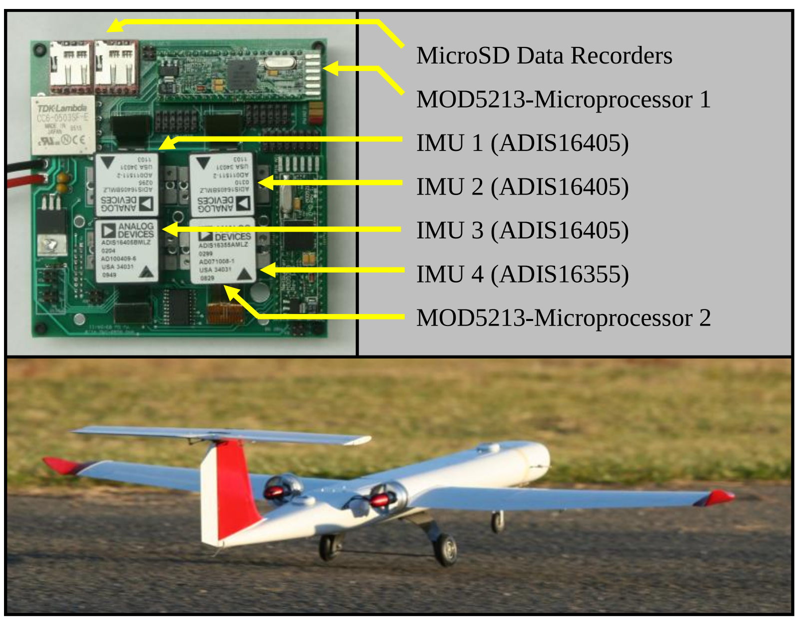

Fig. 4. WVU PAC SUAV with Multi-IMU PCB

The PCB shown in Fig. 4 records measurements using two MicroSD data loggers which are interfaced with the various measurement systems using two MOD-5213 microprocessors. Of the four IMUs implemented in this system, three are ADIS16405, and one is an ADIS16355. Some reported sensor specifications for these two IMUs are provided in Table 1. Although each IMU has an actual resolution of 14-bit, the resolution is improved by oversampling the signals at 200 Hz, then averaging down to 50 Hz, thus achieving 18-bit resolution. A Pulse Per Second (PPS) signal from the GPS receiver is recorded with the IMU data using an Analog to Digital (A/D) port on the MOD-5213 microprocessor. This PPS signal is utilized for precision time alignment between the IMU and GPS data. The GPS receiver uses satellite information to calculate Cartesian position and velocity. In addition to IMU and GPS data, a high-quality Goodrich® mechanical vertical gyroscope was used to obtain direct measurements of the roll and pitch of the aircraft, which are used as 'truth' measurements for this study. These measurements are recorded using a 3.3 V A/D at 16-bit resolution. All data were recorded at 50 Hz.

Table 1. IMU Sensor Specifications

Characteristic	ADIS16405	ADIS16355
Resolution	14-bit	14-bit
Accelerometer Range	±18 g	±10 g
Accelerometer Random Walk	0.2 $m/s/\sqrt{hr}$	2.0 $m/s/\sqrt{hr}$
Rate Gyroscope Range	±150 deg/s	±150 deg/s
Rate Gyroscope Random Walk	2.0 deg/$\sqrt{hr}$	4.2 deg/$\sqrt{hr}$

B. Sensor Error Modeling

Allan variance analysis has been shown to be a useful tool in determining the error characteristics of inertial sensors[42,47,48]. To obtain experimental results for Allan variance, a four hour static test was conducted with 50 Hz sampling rate using the avionics system shown in Fig. 4. The data from this test were then averaged over bins of specified averaging times, *T*, as in[49]:

$$\bar{\Omega}_k(T) = \frac{1}{T}\int_{t_k}^{t_k+T} \Omega_k(t)dt \tag{5}$$

The Allan variance is then calculated as the variance among the bins:

$$\sigma_A^2(T) = \frac{1}{2T^2(N-2n)}\sum_{j=1}^{N-2i}\left[\bar{\Omega}_{j+n}(T) - \bar{\Omega}_j(T)\right]^2 \tag{6}$$

where N is the total number of data points, and n is the number of data points in an averaging bin. This process is then repeated for a range of averaging times, and the root Allan variance curve is constructed by plotting the range of averaging times against the root Allan variance on a logarithmic scale.

A root Allan variance curve can be used to identify different noise sources in the signal by analyzing the different slope components of the curve[49,50]. Although more complex noise models have been considered for inertial sensors[42,47,48], this formulation considers a simple noise model for the IMU sensor measurements, as in (2). Within this study, for simplicity, only the variance of the uncorrelated (wide band) noise was determined. To obtain the variance of this wide band noise, the following Allan variance approach is utilized. A pure white noise model appears in a root Allan variance curve as a line with -1/2 slope. To determine the variance of the wide band noise, the portion of the root Allan variance curve with -1/2 slope is extended to where the averaging time is equal to the sampling time, T_s (0.02 s)[42,47,48]. An example figure demonstrating this process is shown in Fig. 5.

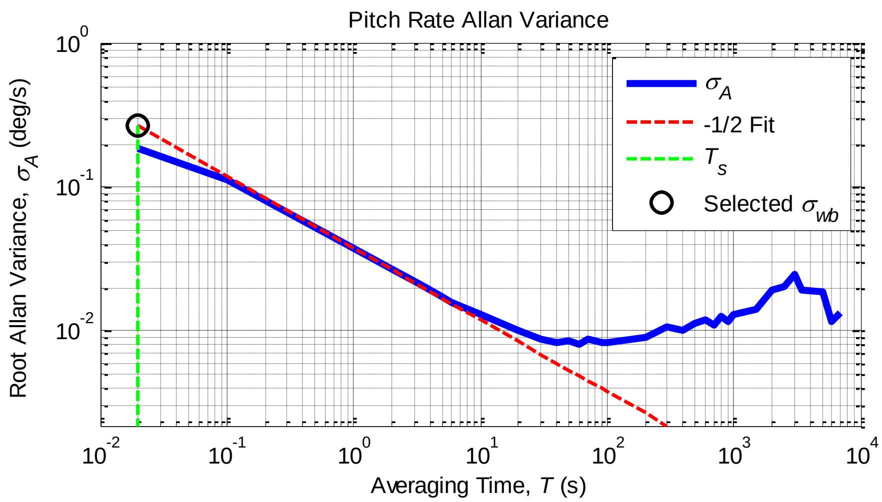

Fig. 5. Example Wide Band Noise Approximation Using Allan Variance

This process was repeated for each measurement of each of the four IMUs, and the results are given in Table 2.

Table 2. Wide Band Noise Standard Deviation Results

Measurement	IMU 1 (ADIS16405)	IMU 2 (ADIS16405)	IMU 3 (ADIS16405)	IMU 4 (ADIS16355)
a_x (g)	0.002889	0.002770	0.002740	0.003534
a_y (g)	0.002899	0.003144	0.002808	0.004713
a_z (g)	0.002471	0.002692	0.002831	0.005488
p (deg/s)	0.2157	0.2225	0.2204	0.2851
q (deg/s)	0.2681	0.2516	0.2140	0.2916
r (deg/s)	0.2365	0.2222	0.2305	0.3380

For comparison, the corresponding wide band noise was calculated based on the reported error characteristics from the manufacturer, as in Table 1. The Allan variance for pure random walk noise satisfies[49]:

$$\sigma_A^2\left(T\right) = \frac{W^2}{T} \tag{7}$$

where W is the random walk coefficient. As with the previous Allan variance analysis, to determine the wide band noise, the Allan variance must be evaluated where the averaging time is equal to the sampling time, T_s. Using the random walk coefficients from Table 1 and the sampling time of 0.02 s, the wide band noise was calculated for each IMU model, and the results are provided in Table 3.

Table 3. Manufacturer Reported Wide Band Noise Standard Deviation

Sensor Type	ADIS16405	ADIS16355
Accelerometer σ_{wb} (g)	0.002403	0.02403
Rate Gyroscope σ_{wb}(deg/s)	0.2357	0.4950

Although the wide band noise estimates for the ADIS16405 from Table 2 are reasonably close to the manufacturer reported values, these values on average are slightly higher than the reported values. Much larger differences are observed for the ADIS16355. Since the Allan variance analysis results from Table 2 represent realistic performance characteristics within the actual studied system, these values are used over the reported values in the noise assumptions for filtering. The process noise covariance matrix, **Q**, for each of the IMUs is constructed by taking the squares of the entries of Table 2 to form a diagonal matrix of variances.

For the GPS measurements, the studied receiver reports the standard deviation of the corresponding reported position and velocity navigation solution in the Earth Centered Earth Fixed (ECEF) coordinate frame. These reported standard deviations can be used to construct a measurement noise covariance matrix:

$$\mathbf{R}_k^{ECEF} = diag\left(\left[\sigma^2(V_{x,k}^{ECEF}), \quad \sigma^2(V_{y,k}^{ECEF}), \quad \sigma^2(V_{z,k}^{ECEF})\right]\right) \tag{8}$$

Since this covariance matrix is represented in the ECEF frame, it must be converted to the local frame using a linear transformation[51], **T**, to obtain the measurement noise covariance matrix, **R**, to be used in the filter:

$$\mathbf{R}_k = \mathbf{T} \cdot \mathbf{R}_k^{ECEF} \cdot \mathbf{T}^T \tag{9}$$

Note that for this matrix, unlike the considered **Q**, can contain off-diagonal terms and is time-varying.

IV. Results

A single set of flight data was collected with the WVU PAC SUAV for use in this study. This flight data segment is approximately 5 minutes in duration including takeoff and landing of the aircraft. This flight included four lateral and four longitudinal pilot-injected doublet maneuvers. A top-down view of the GPS reported flight path is provided in Fig. 6, where the darkening color is used to indicate progression in time, with black corresponding to the end of the flight just after landing.

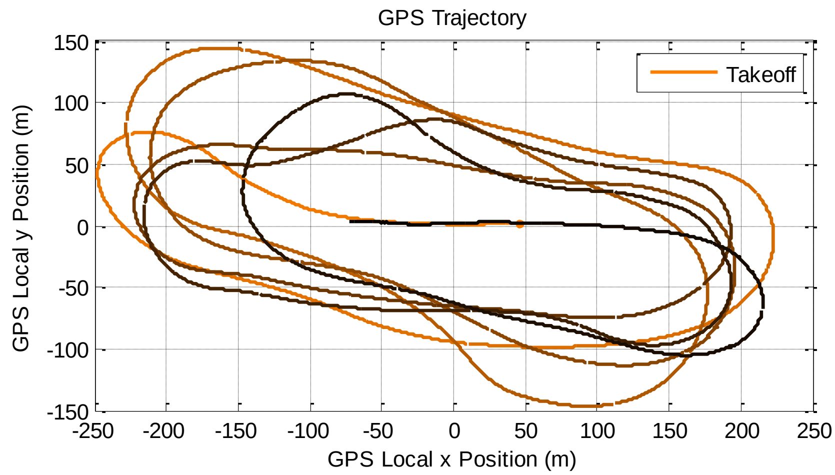

Fig. 6. GPS Flight Path

Using this set of experimentally collected flight data, the multi-sensor fusion results are presented using each of the three considered fusion methods, followed by some prediction only, or dead reckoning results.

A. Multi-Sensor Fusion Performance Evaluation

Using the presented GPS/INS formulation, attitude estimation results were generated for each individual IMU. Various combinations of redundant IMU information were processed using each of the three presented fusion methods. Roll and pitch measurements from the onboard mechanical vertical gyroscope are used as a 'truth' reference in order to calculate the error of each of the considered methods. Performance is evaluated in terms of the standard deviation of the roll and pitch error. Additionally, the average of these two standard deviations, denoted by σ_m, is calculated to provide a single overall value to gauge performance. First, the performance of single IMU GPS/INS sensor fusion is presented using each individual IMU in Table 4.

Table 4. Individual IMU Sensor Fusion Performance

IMU Used	Roll Error Std. Dev., $\sigma(\Delta\phi)$ (deg)	Pitch Error Std. Dev., $\sigma(\Delta\theta)$ (deg)	$\sigma_m = 0.5\ \sigma(\Delta\phi) + 0.5\ \sigma(\Delta\theta)$ (deg)
IMU 1 (ADIS16405)	1.75	1.57	1.66
IMU 2 (ADIS16405)	1.58	1.41	1.49
IMU 3 (ADIS16405)	3.02	3.06	3.06
IMU 4 (ADIS16355)	2.96	1.70	2.33

It is interesting to note that although IMUs 1, 2, and 3 are the same model (ADIS16405), the results are significantly different. Next, performance results are presented for all combinations of the different IMUs using each of the three presented fusion methods in Table 5.

Table 5. Multiple IMU Sensor Fusion Performance (deg)

IMUs Used				Method #1			Method #2			Method #3		
				$\sigma(\Delta\phi)$	$\sigma(\Delta\theta)$	σ_m	$\sigma(\Delta\phi)$	$\sigma(\Delta\theta)$	σ_m	$\sigma(\Delta\phi)$	$\sigma(\Delta\theta)$	σ_m
1	2			1.14	0.92	1.03	1.31	0.99	1.15	1.36	1.02	1.19
1		3		1.44	1.19	1.32	1.49	1.23	1.39	1.56	1.25	1.41
1			4	2.25	1.52	1.88	2.26	1.60	1.93	2.25	1.65	1.95
	2	3		2.34	2.12	2.23	2.20	2.17	2.18	2.22	2.18	2.20
	2		4	1.47	0.90	1.19	1.73	1.00	1.37	1.71	0.99	1.35
		3	4	1.73	1.75	1.74	1.49	1.20	1.35	1.56	1.49	1.53
1	2	3		1.41	1.19	1.30	1.49	1.30	1.40	1.55	1.26	1.41
1	2		4	1.47	0.94	1.21	1.67	1.10	1.39	1.66	1.10	1.38
1		3	4	1.08	0.95	1.02	1.38	0.95	1.17	1.40	1.05	1.23
	2	3	4	1.55	1.51	1.53	1.46	1.23	1.35	1.54	1.44	1.49
1	2	3	4	1.06	0.98	1.02	1.35	0.99	1.17	1.40	1.07	1.24

Each of the considered methods in Table 5 were shown to be effective, in most cases, at reducing the errors in comparison to individual IMU GPS/INS sensor fusion. The roll and pitch error for each individual case as well as the three fusion methods using all four IMUs is shown in Fig. 7.

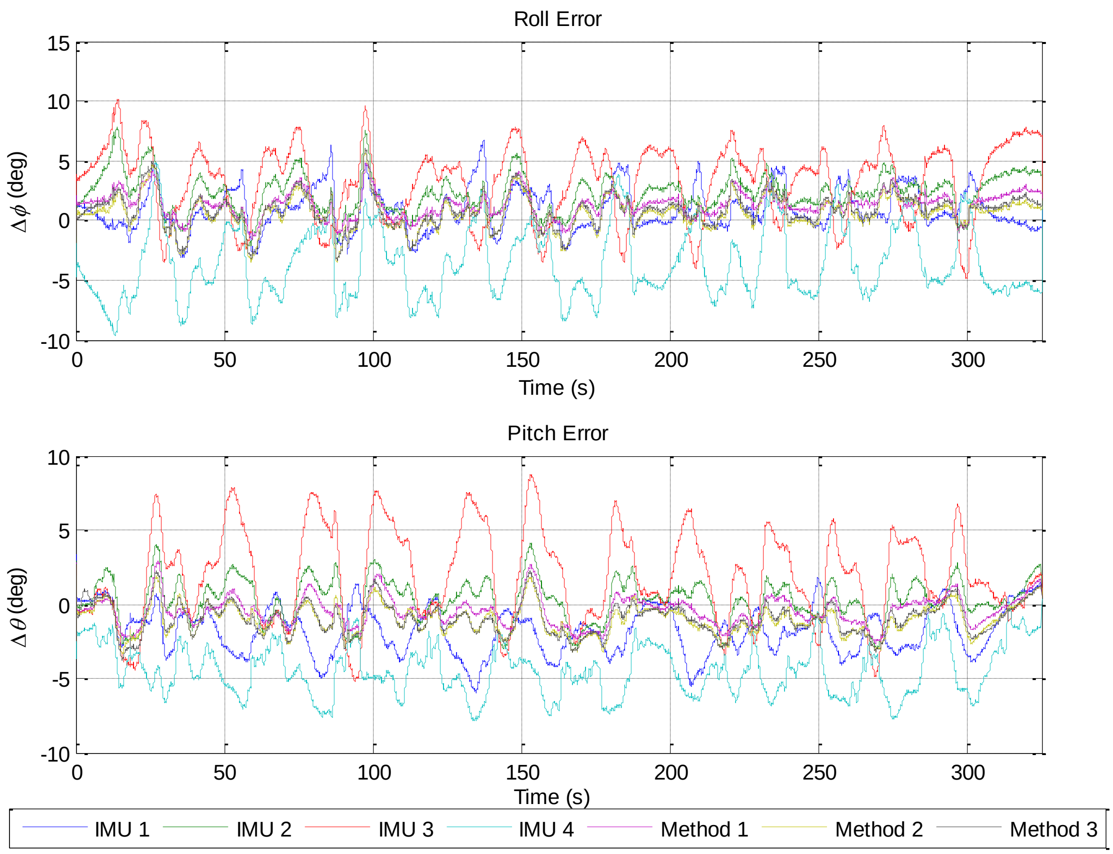

Fig. 7. Comparison of Roll and Pitch Error

Fig. 7 shows that each of the three fusion methods provides a similar estimation of the roll and pitch angles, while also demonstrating a clear reduction in the error relative to the individual IMU performance.

B. Multi-Sensor Dead Reckoning Performance Evaluation

To demonstrate the effectiveness of these multi-sensor fusion methods in the presence of GPS outages, prediction only or dead reckoning results were calculated for each individual IMU. Fusion method #1 can also be applied directly. With no measurement update, fusion methods #2 and #3 reduce to the same method consisting of the fusion of the predicted states from each individual prediction. For this analysis, only the combination of all four IMUs is presented for the different fusion methods. As in the previous section, the estimation algorithm was executed over the entire length of flight including takeoff and landing of the aircraft. The results of this analysis are shown in Table 6 and Fig. 8.

Table 6. Dead Reckoning Performance Results

Estimation Method	$\sigma(\Delta\phi)$ (deg)	$\sigma(\Delta\theta)$ (deg)	σ_m (deg)
IMU 1 only	6.84	8.66	7.75
IMU 2 only	12.6	10.1	11.4
IMU 3 only	13.7	13.0	13.4
IMU 4 only	22.3	17.5	19.9
Fusion Method #1	1.78	1.58	1.68
Fusion Method #2 & #3	7.14	4.53	5.83

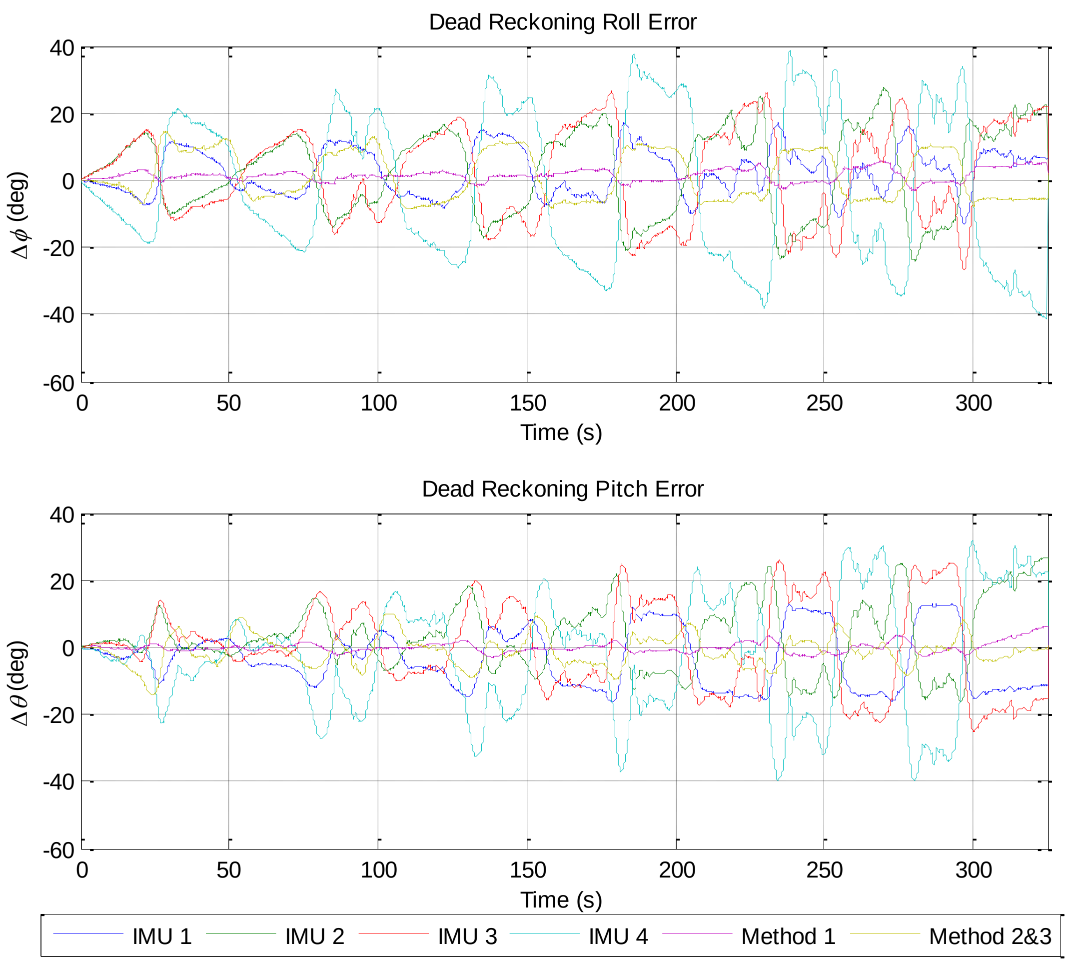

Fig. 8. Dead Reckoning Roll and Pitch Error

The dead reckoning roll and pitch error for each of the individual IMUs tend to drift with time as shown in Fig. 8, however, the fusion methods are capable of keeping the roll and pitch estimates closer to zero. Fusion method #1 yields more accurate dead reckoning estimation than the other methods.

V. Conclusion

This work presented three different methods of fusing together redundant IMU information for the application of attitude estimation using GPS/INS sensor fusion. A set of experimental flight data was used to evaluate and compare the different methods for all combinations of four redundant IMU sensors. Reduction in roll and pitch errors was shown for each of the considered fusion methods. For the GPS/INS sensor fusion problem, each of the three presented fusion methods showed similar results. For the case of dead reckoning, fusion method #1 appeared to be more accurate than the other fusion methods.

The described effort focused only on the utilization of redundant sensor data for performance enhancement. A natural extension of this work is to incorporate existing technology of reliability enhancement, *e.g.* through the use of voting schemes. Multiple redundant sensor data can effectively be synthesized in such a way that the reliability and the performance of the system can both be improved. This can be achieved, *e.g.*, by identifying and isolating faulty or poor quality measurements, then fusing the remaining 'good' sensors in order to obtain a higher accuracy fused solution. Additionally, a more extensive modeling of sensor errors could be used to enhance this study, such as by including bias states in the system.

Acknowledgments

This research was partially supported by NASA grant # NNX10AI14G.

References

[1]Bass, T., "Intrusion Detection Systems & Multisensor Data Fusion: Creating Cyberspace Situational Awareness," *Communications of the ACM*, 1999, pp. 100-105.

[2]Hall, D. L., and Llinas, J., "An Introduction to Multisensor Data Fusion," *Proc. IEEE*, Vol. 85, 1997, pp. 6-23.

[3]Willner, D., Chang, C. B., and Dunn, K. P., "Kalman Filter Algorithms for a Multi-Sensor System," *Proc. of IEEE Conference on Decision and Control*, Clearwater, FL, 1976, pp. 570-574.

[4]Xiong, N., and Svensson, P., "Multi-sensor management for information fusion: issues and approaches," *Information Fusion*, Vol. 3, 2002, pp. 163-186.

[5]Luo, R. C., Yih, C., "Multisensor Fusion and Integration: Approaches, Applications and Future Research Directions," *IEEE Sensors J.*, Vol. 2, No. 2, 2002, pp. 107-119.

[6]Carlson, N. A., "Federated Filter for Fault-Tolerant Integrated Navigation Systems," *Proc. IEEE Position Location and Navigation Symposium*, 1988, pp. 110-119.

[7]Durrant-Whyte, H. F., Rao, B. Y. S., and Hu, H., "Toward a Fully Decentralized Architecture for Multi-Sensor Data Fusion," *Proc. IEEE Int. Conf. Robotics and Automation*, 1990.

[8]Hashemipour, H. R., Roy, S., and Laub, A. J., "Decentralized Structures for Parallel Kalman Filtering," *IEEE Trans. on Automatic Control*, Vol. 33, No. 1, 1988, pp. 88-94.

[9]Rao, B. S., and Durrant-Whyte, H. F., "Fully decentralized algorithm for multisensor Kalman filtering," *IEEE Proceedings-D*, Vol. 138, No. 5, 1991, pp. 413-420.

[10]Vercauteren, T., and Wang, X., "Decentralized Sigma-Point Information Filters for Target Tracking in Collaborative Sensor Networks," *IEEE Trans. on Signal Processing*, Vol. 53, No. 8, 2005, pp. 2997-3009.

[11]Wei, M., and Schwarz, K. P., "Testing a Decentralized Filter for GPS/INS Integration," *IEEE Proc. Position Location and Navigation Symposium*, Las Vegas, NV, 1990, pp. 429-435.

[12]Varshney, P. K., "Multisensor data fusion," *Electronics & Communication Engineering Journal*, Vol. 9, No. 6, 1997, pp. 245-253.

[13]Sun, S., and Deng, Z., "Multi-sensor optimal information fusion Kalman filter," *Automatica*, Vol. 40, 2004, pp. 2017-1023.

[14]Drolet, L., Michaud, F., and Cote, J., "Adaptable Sensor Fusion Using Multiple Kalman Filters," *Proc. International Conference Intelligent Robots and Systems (IROS)*, Vol. 2, Takamatsu, Japan, 2000, pp. 1434-1439.

[15]Bar-Shalom, Y., and Campo, L., "The Effect of the Common Process Noise on the Two-Sensor Fused-Track Covariance," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 22, No. 6, 1986, pp. 803-805.

[16]Chang, K. C., Saha, R. K., and Bar-Shalom, Y., "On Optimal Track-to-Track Fusion," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 33, No. 4, 1997, pp. 1271-1276.

[17]Roecker, J. A., and McGillem, C. D., "Comparison of Two-Sensor Tracking Methods Based on State Vector Fusion and Measurement Fusion," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 24, No. 4, 1988, pp. 447-449.

[18]Parhami, B., "Voting Algorithms," *IEEE Trans. on Reliability*, Vol. 43, No. 4, Dec., 1994, pp. 617-629.

[19]Parhami, B., "A taxonomy of voting schemes for data fusion and dependable computation," Relibability Engineering and System Safety, Vol. 52, 1996, pp. 139-151.

[20]Latif-Shabgahi, G., and Hirst, A. J., "A fuzzy voting scheme for hardware and software fault tolerant systems," *Fuzzy Sets and Systems*, Vol. 150, 2005, pp. 579-598.

[21]Milliord, C. J., Sharma, C. A., and DeMara, R. F., "Dynamic Voting Schemes to Enhance Evolutionary Repair In Reconfigurable Logic Devices," *The International Conference on Reconfigurable Computing and FPGAs*, Puebla City, Mexico, Sept., 2005.

[22]Hardekopf, B., Kwiat, K., and Upadhyaya, S., "Secure and Fault-Tolerant Voting in Distributed Systems," *IEEE Proc. Aerospace Conference*, Vol. 3, 2001, pp. 1117-1126.

[23]Oosterom, M., and Babuska, R., "Virtual Sensor for Fault Detection and Isolation in Flight Control Systems – Fuzzy Modeling Approach," Proc. of the 39th IEEE Conference on Decision and Control, Sydney, Australia, Dec., 2000, pp. 2645-2650.

[24]Simon, D., *Optimal State Estimation*, Wiley, New York, 2006.

[25]Mutambara, A. G. O., and Durrant-Whyte, H. F., "Estimation and Control for a Modular Wheeled Mobile Robot," *IEEE Trans. on Control Systems Technology*, Vol. 8, No. 1, January, 2000.

[26]Lee, D., "Nonlinear Estimation and Multiple Sensor Fusion Using Unscented Information Filtering," *IEEE Signal Processing Letters*, 2008.

[27]Gan, Q., and Harris, C. J., "Comparison of Two Measurement Fusion Methods for Kalman-Filter-Based Multisensor Data Fusion," IEEE Trans. on Aerospace and Electronic Systems, Vol. 37, No. 1, 2001, pp. 273-280.

[28]Changchun, L., Li, S., Hai-bo, W., Tianjie, L., "The Research on Unmanned Aerial Vehicle Remote Sensing and Its Applications," *Proc. of the IEEE 2010 International Conference on Advanced Computer Control (ICACC)*, Shenyang, 2010, pp. 644-647.

[29]Jensen, A., Baumann, M., and Chen, Y., "Low-Cost Multispectral Aerial Imaging using Autonomous Runway-Free Small Flying Wing Vehicles,". *of the IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, Boston, 2008, pp.506-509.

[30]Nagai, M., Chen, T., Shibasaki, R., Kumagai, H., and Ahmed, A., "UAV-Borne 3-D Mapping System by Multisensor Integration," *IEEE Trans. on Geoscience and Remote Sensing*, March 2009, Issue 3, Vol. 47, pp. 701-708.

[31]Suzuki, T., Amano, Y., and Hashizym, T., "Vision Based Localization of a Small UAV for Generating a Large Mosaic Image," *Proc. of 2010 SICE Annual Conference*, Taipei, 2010, pp. 2960-2964.

[32]Grewal, M. S., Weill, L. R., and Andrew, A. P., *Global Positioning, Inertial Navigation & Integration*, 2nd Ed, Wiley, NY, 2007.

[33]Kaplan, E., and Heagarty, C., *Understanding GPS Principles and Applications*, 2nd Ed., Arttech House, Norwood, MA, 2006.

[34]Everaerts, J., "The Use of Unmanned Aerial Vehicles (UAVs) for Remote Sensing and Mapping," *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, Beijing, 2008, Issue B1, Vol. XXXVI, pp. 1187-1192.

[35]Zhou, G., and Zang, D., "Civil UAV System for Earth Observation," *Proc. of the IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, Barcelona, 2007.

[36]Dascalu, S., "Remote Sensing Using Autonomous UAVs Suitable for Less Developed Countries," *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, Enschede, Netherlands, May 2006, Issue XXX, Vol. 34.

[37]Rhudy, M., Gu, Y., Gross, J., and Napolitano, M. R., "Sensitivity Analysis of EKF and UKF in GPS/INS Sensor Fusion," *AIAA Guidance, Navigation and Control Conference*, Portland, 2011.

[38]Rhudy, M., Gu, Y., Gross, J., Gururajan, S., and Napolitano, M. R., "Sensitivity and Robustness Analysis of EKF and UKF Design Parameters for GPS/INS Sensor Fusion," accepted to *AIAA Journal of Aerospace Computing, Information, and Communication*, July 2012.

[39]Gross, J., Gu, Y., Rhudy, M., Gururajan, S., and Napolitano, M., "Flight Test Evaluation of GPS/INS Sensor Fusion Algorithms for Attitude Estimation," *IEEE Trans. on Aerospace Electronic Systems*, Vol. 48, No. 3, July 2012, pp. 2128-2139.

[40]Crassidis, J. L., "Sigma-Point Kalman Filtering for Integrated GPS and Inertial Navigation," *AIAA Guidance, Navigation and Control Conference and Exhibit*, San Francisco, CA, 2005.

[41]Fiorenzani, T., Manes, C., Oriolo, G., and Peliti, P., "Comparative Study of Unscented Kalman Filter and Extended Kalman Filter for Position/Attitude Estimation in Unmanned Aerial Vehicles," *IASI-CNR*, R. 08-08, 2008.

[42]Gross, J. N., Gu, Y., Rhudy, M., Barchesky, F. J., and Napolitano, M. R., "On-line Modeling and Calibration of Low-Cost Navigation Sensors," *AIAA Modeling and Simulation Technologies Conference*, Portland, 2011.

[43]Roskam, J., *Airplane Flight Dynamics and Automatic Flight Controls*, DARcorporation, Lawrence, KS, 2003, Chap. 1.

[44]Stevens, B. L. and Lewis, F. L., Aicraft Control and Simulation. 2nd ed., Wiley & Sons, New Jersey, 2003, Chap. 1.

[45]Lewis, F. L., and Syrmos, V. L., *Optimal Control*. 2nd ed., Wiley & Sons, New York, 1995, Chap. 6.

[46]Berg, T. M, and Durrant-Whyte, H. F., "General Decentralized Kalman Filters," Proc. of the American Control Conference, Baltimore, MD, June 1994, pp. 2273-2274.

[47]Xing, Z. Gebre-Egziabher, D., "Modeling and Bounding Low Cost Inertial Sensor Errors," *IEEE/ION Position, Location and Navigation Symposium*, Monterey, CA, 2008, pp. 1122-1132.

[48]Xing, Z., "Over-bounding Integrated INS/GNSS Output Errors," PhD Dissertation, The University of Minnesota, Minneapolis, 2010.

[49]El-Sheimy, N., Hou, H. and Niu, X., "Analysis and Modeling of Inertial Sensors Using Allan Variance." *IEEE Trans. on Instrumentation and Measurement*, Issue 1, Vol. 57, Jan. 2008.

[50]Allan, D. W., "Statistics of atomic frequency standards." *Proc. IEEE*, Issue 2, Vol. 54, Feb. 1966.
[51]Rogers, Robert M., *Applied Mathematics in Integrated Navigation Systems*. [ed.] Joseph A. Schetz. 2nd Ed. American Institute of Aeronautics and Astronautics, Inc., Reston, VA, 2003.