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The dual choice of geodetic horizontal reference systems for Ukraine

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ABSTRACT

Almost all European countries use at least two horizontal reference systems, which allow them to minimize all risks when changing the coordinate system. The first system is used for the needs of the national survey in general, and also for technical applications in engineering surveys. A brief description of this system can be written by the formula: Geocentric Ellipsoid (GRS80) + Projection (UTM). More than half of the European countries for the second national systems use the GRS80 geocentric ellipsoid together with a projection other than UTM, some of them continue to use the projection of the old coordinate systems. The rest of the countries, excluding Ukraine, adapted the existing national coordinate systems to modern requirements, removing deformations from them concerning the ETRS89. A small number of European countries have implemented the GRS80 geocentric ellipsoid together with the UTM projection for cadastral surveys and topographic maps with a scale of 1:5000 and larger. Only one country, Ukraine, chose a variant for the new coordinate system (UCS2000), which is briefly written by the formula: Old Not Geocentric Ellipsoid + New Projection. The analysis of modern coordinate systems of Europe made it possible to formulate two variants of the future coordinate system of Ukraine based on a geocentric ellipsoid. It is shown that the transition to a geocentric ellipsoid will change the coordinates and heights of the points, but practically will not change the distance between the points and the area of the parcels.

1. Introduction

The introduction of high-precision GPS-surveys in geodetic work required the creation of national high-precision geodetic reference coordinate systems. As a result, at the turn of the century, a series of high-precision global coordinate frames ITRF [1,2] was created, and on their basis, following the recommendations of IAG (EUREF), national reference coordinate systems. However, while in geodesy, cartography and GIS, the transition to a new coordinate system was smooth enough, in the cadastre the transition to a new system turned out to be problematic. It was found that all the numerical characteristics of land parcels: coordinates, heights, lengths of lines, and areas, change with the change in coordinate system. Replacing one coordinate system with another brings about an unavoidable evil - the change in the coordinates of points, which is technically solved by recalculating coordinates. However, changing the area of parcels is a serious problem since the area affects a normative valuation, rent price and parcel tax. As pointed out in Ref. [3], a change in the area of the land parcels may lead to claims and lawsuits of land users against the State as the guarantor of possession rights. That is why geodesists of many countries turned to the idea of dualism when choosing a national coordinate system.

2. The idea of dualism when choosing a national coordinate system

For the first time, the idea of dualism for coordinate systems was formulated by Ref. [4], who substantiated this idea with various

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requirements presented by national surveying on the one hand and cadastral surveying on the other.

According to this idea, for the needs of geodesy and national surveying in general, as well as for technical applications in engineering surveying, the first coordinate system based on the European, ETRS89 is being developed and used. Without considering the height system, a brief description of this system can be written in the form of a formula as follows:

$$\text{Geocentric Ellipsoid (GRS80)} + \text{Projection (UTM)} \quad (1)$$

However, for cadastral surveying, for topographic mapping with a scale of 1:5000 and larger, the second special system is selected, which minimizes all possible risks when changing the coordinate system occurs. As a result, in many European countries there are not one but two or more coordinate systems.

Only a few European countries have abandoned dualism in the choice of coordinate systems. In particular, in Spain, according to Ref. [5], the official geodetic system is the ETRS89, used, as indicated in Ref. [6], together with the UTM projection.

Geodesists of Serbia [7] and Montenegro [8] have adopted national systems for topographic and cadastral mapping based on the geocentric ellipsoid GRS80 and the UTM projection (see Eq. (1)) at the legislative level. The territories of both countries lie entirely within one zone of UTM 34 N, with a prime meridian of 21°E.

In Germany, the ETRS89 system with the UTM projection [9] has also been officially adopted for geodesy and cadastre. However, since the geodesy and real estate cadastre in Germany is organized at the federal state level, Saarland left the Gauss-Kruger (G-K) projection in use, which is different from the UTM that indicates only a partial rejection of dualism in Germany's coordinate systems. In Fig. 1, the territories of countries that have officially abandoned the dual choice of coordinate systems are colored pink. German territory is also colored pink, except for Saarland.

3. The variants of coordinate systems for cadastre and large-scale mapping

Thus, most European countries implement the idea of dualism when choosing a coordinate system. The first is a system that satisfies condition of Eq. (1). The second system in each country is selected individually, taking into account various requirements. Therefore, each country has its unique geodetic coordinate system for the cadastral and large-scale surveying. However, all these systems can be grouped into the following four variants.

Variant No. 1 can be described by the following formula:

$$\text{New Geocentric Ellipsoid (GRS80)} + \text{New Projection (Not UTM)} \quad (2)$$

This is the most radical solution to the problem of changing the outdated coordinate system, in which not only the old non-geocentric ellipsoid is replaced, but also the projection associated with it. The main advantage of this variant is the use of a geocentric ellipsoid, which is part of the ETRS89 system, allows the coordinate systems to maintain high accuracy of measurements using GPS.

In Fig. 1, countries whose coordinate systems are described by Eq. (2) are shown in yellow. A classic example of coordinate systems of this variant is the systems LKS94/Lithuania TM [10], LKS92/Latvia TM [11] and EST97 [12] created in Lithuania, Latvia and Estonia.

The basis of the Cyprus Geodetic Reference System 1993 (CGRS93) is not the GRS80 ellipsoid, but the WGS84 ellipsoid [13]. Paradoxically, the Helmert parameters for the transformation of coordinates between CGRS93 and WGS84 systems are not equal to zero [14]. Therefore, theoretically, this system cannot be attributed to this variant of coordinate systems. However, it should be taken into account that the parameters of the GRS80 and WGS84 ellipsoids practically coincide [15], the distance between the center of the CGRS93 system and the center of the WGS84 system does not exceed 10 m, the angular parameters of the Helmert transformation

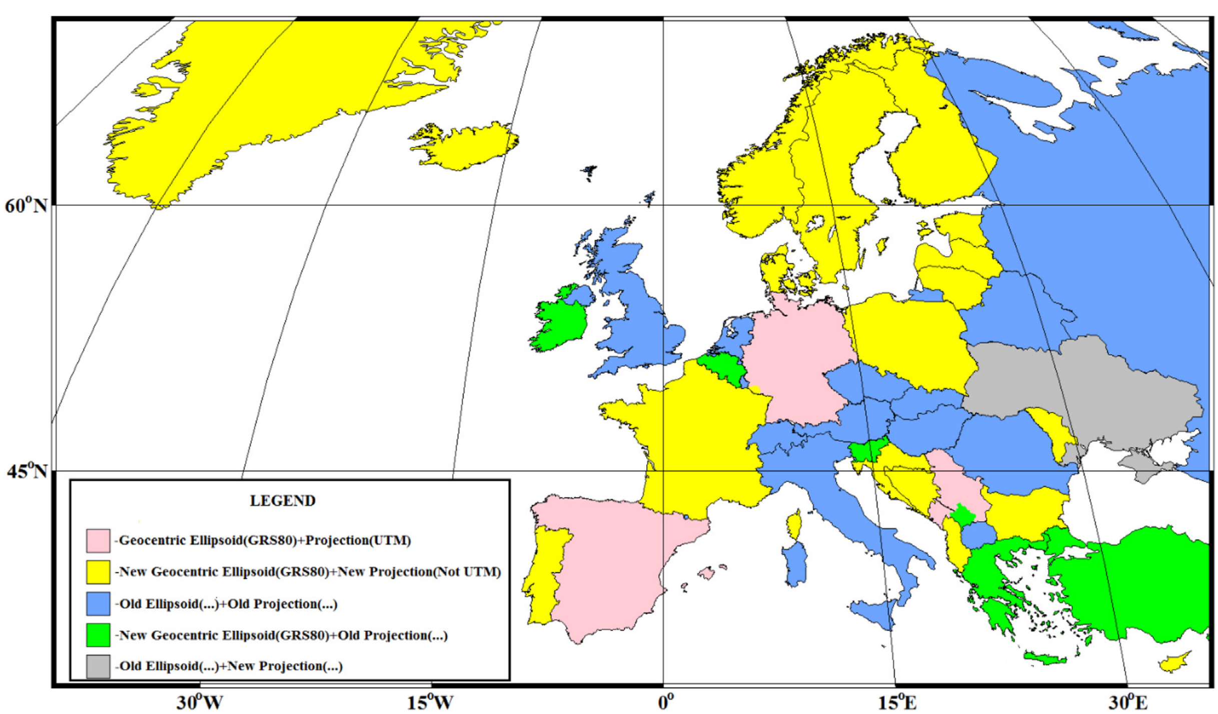

Fig. 1. Variants of the dual choice of geodetic reference horizontal coordinate systems in Europe for large-scale topographic mapping and cadastre.

do not exceed 0.2 arc seconds and the scale factor is less than 1 ppm [14]. Therefore, in practice, the CGRS93 system can be described by Eq. (2), as the territory of Cyprus is shown in yellow in Fig. 1.

The difference between Eqs. (1) and (2) is associated with the choice of the new projection for the coordinate system.

In this work, an old projection refers to a projection that surveyors previously used in a particular country as part of a previous national coordinate system. A new projection is a projection that changes the scale of the lengths in points during the transition from the old projection, which means that even if the name of the projection remains the same, for example, the Lambert projection or the Gauss-Kruger projection, if the projection parameters that change the length scale are changed, the projection is called new. For the Lambert projection, these parameters are the latitudes of the standard parallels; for the Gauss-Kruger projection, these parameters are the longitude of the prime meridian and the scale factor on this meridian. Changing projection parameters such as false northing, false easting does not change the internal properties of projection. In this case, the projection is considered old.

When choosing a new projection, geodesists take into account many different factors, the main of which are:

1. Maximum coverage of the country territory by each projection zone, minimizing the number of zones;
2. Minimum distortion of line lengths (areas) inside each projection zone.

These requirements are antagonistic. The larger the area of the zone, the greater the distortion. Therefore, geodesists from different countries individually select the optimal balance from their point of view between the first and second requirements.

For example, geodesists of Bulgaria [16], having a negative experience in the use of four latitudinal zones of the Lambert projection (CS70), which were unsuccessfully selected for its territory in the 1970s. So, they chose the one latitudinal zone of Lambert projection for the entire territory of the country (BGS2005), taking into account the first requirement, as the basis for choosing a new projection [17].

On the contrary, geodesists of France, to reduce distortions of lengths and areas, replaced four latitudinal zones of the Lambert projection (NTF) with nine zones of Lambert projection (RGF93) spaced evenly at 1° in latitude [18].

Similarly, geodesists of Finland plan to replace six 3° longitudinal zones of the Gauss-Kruger projection (KKJ) with 13 longitudinal zones of the same projection (ETRS-GKNFIN), with prime meridians chosen at 1° intervals [19,20].

An even more complex solution for dividing the country's territory into longitudinal zones in the Gauss-Kruger projection was implemented by Sweden in the new coordinate system SWEREF 99. The country's territory is divided into southern and northern. Each part of the territory has its system of longitudinal zones. The southern part contains six zones; the northern one contains seven zones. The zones are drawn at intervals of 1°30', expect for the zone with the prime meridian 18°45', which belongs to both the northern and southern parts of the country [21].

Thus, when the geodesists of France, Finland and Sweden chose a new projection for the coordinate systems, they first took into account the requirement to minimize length distortions within each zone, choosing zones with intervals from 1° to 1°30'.

When creating the national system KRJSH-2010 based on ETRS89 [22], Albanian geodesists chose such a prime meridian of the longitude zone that it divides the country into two equal parts. In this case, one zone was selected for the country due to the successful choice of the prime meridian with a longitude of 20°. Similarly, the Bosnia and Herzegovina coordinate system, BH_ETRS89/TM, contains a Gauss-Kruger projection with a single zone slightly larger than 4° with the prime meridian of 18° passing almost through the center of the country [23]. Croatia's coordinate system (HTRS96/TM) also contains one 6° zone with a prime meridian of 16°30' [24,25]. However, the most unusual prime meridian, not a multiple of even whole seconds, equal to 08°07'59.19"W, is used in the Portuguese coordinate system ETRS89/Portugal TM06 for the Gauss-Kruger projection [26,27].

Intelligent choice of the prime meridian of the Gauss-Kruger projection allowed geodesists in Albania, Croatia, Portugal, Bosnia and Herzegovina to find a balance between the two requirements for the projection: to minimize distortions of the projection and to cover the entire country within one zone.

The numerical characteristics of land parcels change in this variant for the following reasons:

1. Transition from the surface of an old non-geocentric ellipsoid to the surface of a new geocentric one;
2. Transition from an old projection to a new one;
3. Deformations of an old coordinate system concerning a new one.

Problems associated with a change in land parcels that arise for all of the above reasons are discussed in Ref. [28]. In Bulgaria, the procedure recalculation of the area and other numerical characteristics of land parcels changed during the transition from the CS70 system to the BGS2005 system is legal according to Ref. [29].

Variant No. 2 is described using the formula:

$$\text{Old Ellipsoid}\ (\ldots) + \text{Old Projection}\ (\ldots) \tag{3}$$

Eq. (3) is the most conservative solution to the problem of creating a new coordinate system. The new system is based on an old but a ubiquitous system. Systematic deformations in relation to the ETRS89 system are removed from the old system. The new system has the same non-geocentric ellipsoid and the same projection as the old system. The coordinates of the new system only in places of deformations differ greatly from the coordinates of the old system. The lengths of lines and areas of land parcels also change locally.

In Fig. 1, countries choose Eq. (3) are shown in blue.

The disadvantage of Eq. (3) is that, as indicated in Ref. [21], the accuracy of the points coordinates obtained by the GPS method will reduce when the transition from WGS84 to the national system takes place, because transformation formulae are not free from errors. The advantage of this variant lies in the numerical characteristics of land parcels are almost completely preserved, which makes it possible not to change or correct most of the information in the databases and documentation.

In favor of Eq. (3), Czech experts claim in Ref. [30] that drastic changes in the coordinate system will negatively reflect on many technical areas in practice, and the actual implementation of the transition will entail costs that will be very difficult to justify.

As indicated in Ref. [31], due to the large scale, the creation of cadastral maps is costly. Even updating the cadastre requires significant funds from State budgets. Therefore, the coordinate system for cadastral works rarely is changed.

According to Ref. [4], the transition from a coordinate system based on the old national system CH1903+ to a system based on ETRS89 (CHTRF95) will change the coordinates of the points of the Swiss Geodetic Network up to 1.6 m. These changes are not significant for users of the maps with a scale of 1: 25,000 or less. For users of maps with a scale of 1: 5000 and larger, these changes will require recalculation and transformation of a large amount of geospatial information, which is the reason for choosing a coordinate system for the cadastre in Switzerland based on the old system with the non-geocentric ellipsoid Bessel 1851 and the Rosenmund Swiss map projection. As pointed out in Ref. [4], there is no alternative to CH1903 for surveying in Switzerland.

This group of national coordinate systems includes systems containing projections that Clifford J. Mugnier calls unique [32,33], and Hans van der Marel calls exotic [34]. These are such systems as S-JTSK+ used in conjunction with the Krovak projection in the Czech Republic [35] and the Slovak Republic [36], the HD72 system with the EOV projection in Hungary [37,38], the RD2000 system with the Schreiber projection [34] for the Netherlands, and the aforementioned coordinate system CH1903+ used in Switzerland and Liechtenstein with the Rosenmund projection [32,33].

Exotic projections have been used in national systems for many decades. As a result, a stable tradition to use these projections has developed in national cartography. These projections are computationally unique since they are double, that is, the surface of the ellipsoid is projected onto a sphere initially, and then the surface of the sphere is mapped onto a plane. As shown in Ref. [4], using the Rosemund projection as an example, the transition from exotic projections to any other projection will cause large changes in the coordinates of points on the earth's surface. Therefore, it is quite difficult for the countries using exotic projections to abandon their use.

As stated in Ref. [39], since 2008, the reference system ETRS89 has been used in Romania in parallel with the national reference system S42 (Krassovsky ellipsoid). The old system S42 temporarily continues to be used due to the enormous cadastral information that needs to be converted into a new reference system.

Variant No. 3 is represented by the formula:

$$\text{New Geocentric Ellipsoid (GRS80)} + \text{Old Projection } (\ldots) \qquad (4)$$

Eq. (4) is a compromise between the two previous variants. It is interesting because it has the main advantage of the first variant (Eq. (2)): a geocentric ellipsoid and does not have its disadvantages associated with the transition to a new projection. This group includes countries (shown in Fig. 1 in green) whose geodetic authorities are cautious and balanced in their approach to the transition to a new coordinate system.

The change in the numerical characteristics of land parcels takes place in this variant only for two reasons: when the ellipsoid changes and because of the deformation of the old system in relation to the new one.

Geodesists of Slovenia [40] use new coordinate systems based on the GRS80 geocentric ellipsoid. However, the projection of these systems is the Gauss-Kruger projection, which was used on the territory of these countries at the beginning of the 20th century as part of the system MGI 1901.

In Greece, both the ΕΓΣΑ87 system and the HTRS07 system are based on the GRS80 ellipsoid. Nevertheless, the ΕΓΣΑ87 system, unlike HTRS07, is not geocentric. Both systems use the same Gauss-Kruger projection [41].

In Belgium, several versions of a new system based on a geocentric ellipsoid (ETRS89/Belgian Lambert 2005, ETRS89/ Belgian Lambert 20008) have already been created. However, according to Ref. [42], the official system is still the old non-geocentric system Belge 1972/Belgian Lambert 72, based on the ellipsoid International 1924. The above systems use the Lambert projection with the same standard parallels, as Philippe Lambot notes in Ref. [42], this selection of projection allows users to take full advantages of the new coordinate system.

Variant No. 4 is described by the following formula:

$$\text{Old Ellipsoid } (\ldots) + \text{New Projection } (\ldots) \qquad (5)$$

Eq. (5) is the most unfortunate of all four variants for choosing a new coordinate system. It does not have the advantages of the first and third variants (Eq. (2) and (4)) associated with the choice of the GRS80 geocentric ellipsoid. At the same time, it has all the disadvantages of the first variant associated with the transition to a new projection. In Eq. (5), the main advantage of the second variant is also missing - the preservation of the numerical characteristics of land parcels. Only one country, Ukraine, has chosen this variant for the new coordinate system UCS2000.

One of the advantages of choosing Eq. (5), the creators of the UCS2000 system, called the possibility of using paper maps of the Soviet period [43]. But this advantage has lost relevance over the thirty years of the country's existence. Apparently, this is the reason that, despite the official proclamation of UCS2000 as the only State coordinate system for Ukraine, the old CS63 system continues to be used in the cadastre to calculate the areas of land parcels [28].

The territory of the country (Ukraine) whose coordinate system satisfies Eq. (5) is shown in Fig. 1 in gray.

4. Geocentric ellipsoid GRS80 for the future coordinate system of Ukraine

According to the Decision State Geologic and Subsoil Survey of Ukraine 11, 2020 No. 2/1 [44], the land parcels for subsoil use are issued with WGS84 with the involvement of the UTM projection. The official refusal to use the state Ukrainian system UCS2000 in geology indicates its serious shortcomings. The main disadvantages are non-geocentricity and the absence of a modern high-precision national elevation system. Therefore, Ukrainian geodesists are tasked with creating a new high-precision geodetic reference coordinate system without the disadvantages inherent in the UCS2000.

Considering the experience of the European countries discussed in section 2 and the IAG recommendations [45], the new system should be based on the geocentric ellipsoid GRS80. As indicated by Ref. [45], the geocentric coordinate system will preserve the accuracy of data obtained from GPS measurements and use global high-precision geoids such as EGM2008.

The transition to the new system involves a variety of risks. Some of these risks can be pre-analyzed and the necessary documentation to neutralize the negative consequences of the country's transition to a new coordinate system. It is easiest to consider the changes in quantitative characteristics, such as coordinates of points, lengths of lines, and areas of land parcels.

Further, these changes are considered to transition from the Krassovsky ellipsoid of the UCS2000 non-geocentric system to the GRS80 geocentric ellipsoid.

4.1. Changing the coordinates and heights of points

There are many methods for calculating the change in the coordinates of points during the transition from one ellipsoid to another. The two most famous can be found in Ref. [46]:

1. The exact method using Helmert's formula;
2. The approximate method using Molodensky's formula.

This work used Helmert's formulae to analyze changes in all quantitative characteristics of land parcels. Molodensky's approximate formulae were used to control.

Regardless of the method, the differences in geodetic coordinates are determined:

$$\Delta\varphi = \varphi_2 - \varphi_1, \Delta\lambda = \lambda_2 - \lambda_1 \tag{6}$$

where φ_1, λ_1 are the geodetic latitude and longitude of the point on the Krassovsky ellipsoid of UCS2000; φ_2, λ_2 are the geodetic latitude and longitude of the point on the GRS80 geocentric ellipsoid.

The displacement vector of the points on the earth's surface during the transition from one ellipsoid to another, its size ΔS and azimuth α are directly determined by the Equation in Ref. [47]:

$$\Delta S = \sqrt{M^2\Delta\varphi^2 + r^2\Delta\lambda^2}, \alpha = \arctan\frac{\Delta\lambda}{\Delta\varphi} \tag{7}$$

In Eq. (7) M, r are the radius of curvature of the meridian and the radius of the parallel, respectively.

Since the points with φ_1, λ_1 and φ_2, λ_2 coordinates are located on different surfaces, the first is on the surface of the Krassovsky ellipsoid, and the second is on the surface of the GRS80 ellipsoid, M, r are calculated as the arithmetic averages of the corresponding values of both ellipsoids.

Fig. 2 shows the vectors of change calculated by Eq. (7) in the coordinates of points in meters. The coordinates of points on the surface of the Krassovsky ellipsoid of UCS2000, φ_1, λ_1 were determined using the regular 15' × 15' arcminute grid. The geodetic coordinates of the same points on the surface of the GRS80 ellipsoid, φ_2, λ_2 were determined by the Helmert method [46] (see Eqs. (A1) and (A3) of Appendix A).

When the transition from the surface of one ellipsoid to another, not only does the geodetic latitude and longitude change, but also the geodetic height. To estimate these changes, the heights of the points of the original ellipsoid were equated to zero, i.e.

$$H_1 = 0 \tag{8}$$

In this case, the change in height can be estimated by the Equation:

$$\Delta H = H_2 - H_1 = H_2 \tag{9}$$

Change in height, ΔH, was also calculated on a 15' × 15' arcminute grid in both latitude and longitude using Eqs. (A1) and (A3) of Appendix A.

Fig. 3 shows the contour plot for changes in the geodetic height of points during the transition from the ellipsoid of the coordinate system UCS2000 to the ellipsoid GRS80. Thus, Figs. 2–3 are built using the parameters of the Helmert transformation obtained in Ref. [48].

Table 1 shows extreme changes in the coordinates and heights of points on the earth's surface on the territory of Ukraine during the transition from the Krassovsky ellipsoid (UCS2000) to the GRS80 ellipsoid.

4.2. Changing line lengths and areas of parcels

It is easiest to estimate changes in the lengths of lines between points and areas of parcels using the formulae of mathematical cartography, determining the scale factors of lengths and areas of projections. In this case, it is necessary to consider the transition process from one ellipsoid to another as the process of creating a projection of one surface, the surface of the Krassovsky ellipsoid, onto another surface, the surface of the GRS80 ellipsoid.

As is known, the Helmert transformation gives a conformal projection of the three-dimensional space [34]. However, considering it only as the process of projecting one ellipsoid's surface onto another ellipsoid's surface, a non-conformal projection will be obtained. Therefore, to estimate the change in line lengths, the maximum and minimum distortions of the lengths were used, equal to the value in Ref. [47]:

$$Va = a - 1, Vb = b - 1 \tag{10}$$

Similarly, distortions of the area calculated from the next equality are used to estimate the change in the area of the parcels [47]:

$$Vp = p - 1 = ab - 1 \tag{11}$$

To assess the non-conformity of the projection, the distortions of the angles were calculated using the Equation of Ref. [47]:

$$\omega = 2\arcsin\left(\frac{a-b}{a+b}\right) \tag{12}$$

In Eqs. (10–12) a, b are the maximum and minimum scale factors, respectively; p is the area scale factor, and ω are the maximum angle distortion.

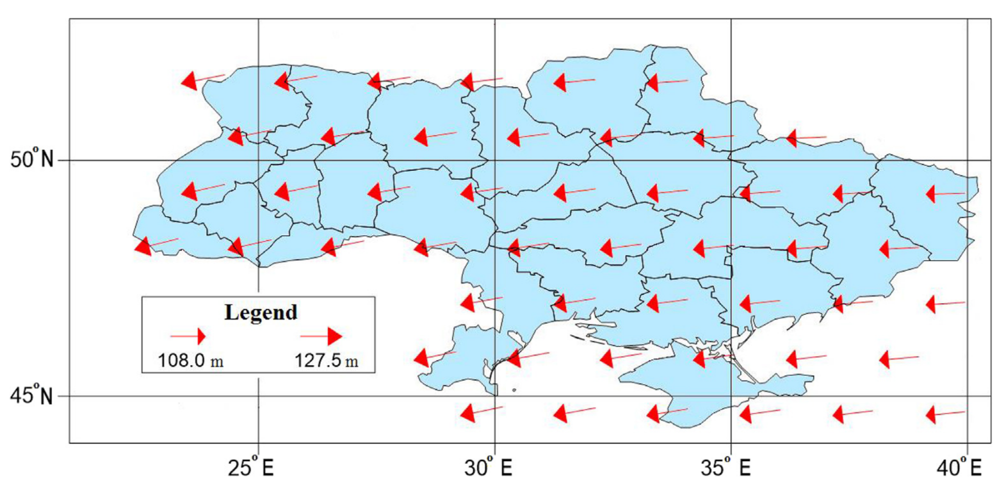

Fig. 2. The vectors of changes in the coordinates of points during the transition from the Krassovsky ellipsoid of UCS2000 to the geocentric ellipsoid GRS80. To visualize the scale of the change in the length of the radius vector, the legend shows the maximum and minimum length vectors.

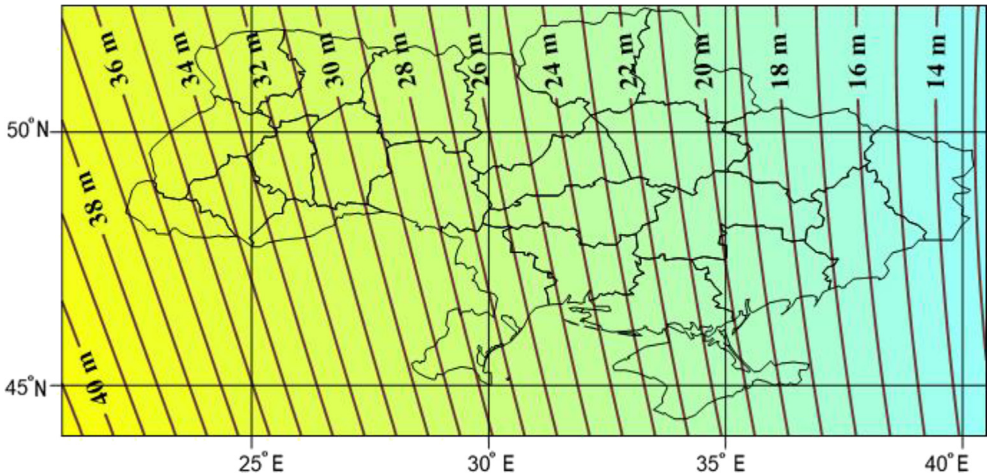

Fig. 3. Contour plot of changes in the geodetic height of points on the earth's surface on the territory of Ukraine during the transition from the Krassovsky ellipsoid (UCS2000) to the GRS80 ellipsoid.

Table 1
Changes in the quantitative characteristics of points, lines and areas of land parcels caused by the transition from the UCS2000 ellipsoid to the GRS80 ellipsoid.

Parameter	Value	Parameter	Value
$\Delta\varphi_{max}$ (arcsec)	0.166	Va_{max} (%)	2.8×10^{-6}
$\Delta\varphi_{min}$ (arcsec)	−1.212	Va_{min} (%)	2.0×10^{-6}
$\Delta\lambda_{max}$ (arcsec)	−4.851	Vb_{max} (%)	-2.6×10^{-6}
$\Delta\lambda_{min}$ (arcsec)	−6.543	Vb_{min} (%)	-4.7×10^{-6}
ΔS_{max} (m)	125.145	Vp_{max} (%)	-0.1×10^{-6}
ΔS_{min} (m)	108.368	Vp_{min} (%)	-2.3×10^{-6}
ΔH_{max} (m)	37.095	ω_{max} (arcsec)	0.015
ΔH_{min} (m)	12.992	ω_{min} (arcsec)	0.010

Formulae for calculating extreme length scale factors are provided in Appendix A.

4.3. Changing line lengths and areas of parcels in the Gauss-Kruger projection, caused by the offset of the prime meridian on the earth's ellipsoid

Since the earth's ellipsoid changes, the geodetic coordinates of all points on the earth's surface, the positions of the axial meridians will also be changed, which in turn, will cause changes in the lengths of the lines between the points and the areas of the parcels. The changes in areas are quite easy to estimate using the Equation presented in Ref. [28], namely:

$$\Delta P \cong 2\Delta\lambda(\lambda - \lambda_0)\cos^2\varphi\left(1 + e'^2\cos^2\varphi\right)P \tag{13}$$

Where $\Delta\lambda$ is the offset of the prime meridian caused by the transition from one ellipsoid to another; λ_0 is the longitude of the prime meridian, defined in radians; P is the area of the parcel on the original ellipsoid (Krassovsky); φ, λ are geodetic latitude and longitude of the central point of the land parcel on the original ellipsoid; e'^2 is square of the second eccentricity of the original ellipsoid.

Since the length distortions in conformal projections are two times less than the area distortions [47], the changes in the lengths of the lines can be determined by the Equation:

$$\Delta L \cong \Delta\lambda(\lambda - \lambda_0)\cos^2\varphi\left(1 + e'^2\cos^2\varphi\right)L \tag{14}$$

where L is the line length on the original ellipsoid.

$$\mathrm{abs}(\Delta\lambda) \leq 6.543\ (\mathrm{arcsec})\ .$$

The difference between the longitudes $(\lambda - \lambda_0)$ of the points of the 6° zone does not exceed:

$$\mathrm{abs}(\lambda - \lambda_0) \leq 3^\circ$$

Thus, changes in the area of parcels and line length in the Gauss-Kruger projection, caused by a change in the position of the prime meridian during the transition from one ellipsoid (Krassovsky) to another (GRS80), can be estimated by the following formula:

$$\mathrm{abs}(\Delta P) \leq 3.3 \times 10^{-6} \cdot P, \mathrm{abs}(\Delta L) \leq 1.7 \times 10^{-6} \cdot L \tag{15}$$

5. Results and discussion

Therefore, according to the left part of Table 1, changes in coordinates and heights are significant. This means that when entering a new geocentric coordinate system, it is necessary to provide a procedure for recalculating the coordinates and heights for all points of land parcels.

It should be noted that only in 2022 the height system Baltic77 for UCS2000 was replaced by a high-precision modern European system EVRS2000 [49], which, according to Ref. [50], is tied to the geocentric system ETRS89. There will be the same differences between the heights of the EVRS2000 geocentric system and the UCS2000 non-geocentric height system, as the difference between the heights of two Krassovsky and GRS80 ellipsoids shown in Fig. 3. The geocentricity of the new national system of Ukraine will allow much faster and more accurate integration of the national height system into EVRS2000.

According to the right part of Table 1, the changes in line lengths are very small. For example, a line length of 1 km can increase to an extreme by no more than 0.03 mm ($Va_{max} = 2.8 \times 10^{-8}$), or decrease to an extreme by no more than 0.05 mm ($Vb_{min} = -4.7 \times 10^{-8}$).

A similar situation occurs with a change in the area of the land parcels. In particular, the parcel of the area of 10000 m^2 on ellipsoid Krassovsky when the transition to the geocentric coordinate system with the ellipsoid GRS80 may decrease by no more than 0.02 m^2 ($Vp_{min} = -2.3 \times 10^{-8}$).

According to the Eq. (15), a change of the line length of 1 km in the Gauss-Kruger projection, caused by offset in the prime meridian, will not exceed 0.002 m, a change of the parcel area of 10000 m^2 for the same reason will not exceed 0.03 m^2.

6. Conclusions

Thus, the transition to the geocentric ellipsoid GRS80 will not create serious problems related to the quantitative characteristics of land parcels. Problems with the transition from CS63 to UCS2000 arose, during the transition from one variant of the Gauss-Kruger projection to another as shown in Ref. [28]. The prime meridians of the zones were changed, resulting in the land parcel area changes. In the regions close to the boundaries of the zones, both CS63 and UCS2000, areas of 10000 m^2 either decreased or increased by 2 m^2. Since, according to tradition, the areas of the parcels were determined with an accuracy of 1 m^2, the change in the coordinate system would inevitably lead to a change in all documentation for parcels (ownership, lease, etc.). In order not to change the documentation of the parcels formed based on CS63, the areas of the parcels created based on UCS2000 are still calculated in CS63.

Considering European countries' experience discussed above in creating coordinate systems for Ukraine, there are two variants of coordinate systems described by Eqs. (2) and (4), in which a geocentric ellipsoid is used. The variant described by Eq. (4) with a new geocentric ellipsoid and an old projection is the least traumatic, with minimal risk.

For Ukraine, this variant is associated with a return to the Gauss-Kruger projection with the prime meridians, which completely coincide with the prime meridians of CS63 [48]:

$$\lambda_{0,CS63} \ : \ 23.5^\circ, 26.5^\circ, 29.5^\circ, 32.5^\circ, 35.5^\circ, 38.5^\circ.$$

This variant can be written as:

New Geocentric Ellipsoid (GRS80) + Old Projection (G-K from CS63) (16)

Such a return to the old projection will make it possible to legitimize the area of the land parcels, which are now calculated according to this projection, but in this case, semi-officially. The prime meridians of CS63 are not standard. However, as shown in section 2 above, many national coordinate systems in Europe have used Gauss-Kruger projection with prime meridians different from the standard ones.

The second variant of the new geocentric coordinate system based on Eq. (2) for Ukraine looks like this:

New Geocentric Ellipsoid (GRS80) + Projection (G-K from UCS2000) (17)

The prime meridians of this variant are standard, that are Ref. [48]:

$$\lambda_{0,UCS2000} \ : \ 21^\circ, 24^\circ, 27^\circ, 30^\circ, 33^\circ, 36^\circ, 39^\circ.$$

The choice of this variant of the new system will require recalculating not only the coordinates and heights, but also the area of the parcels, which will require updating all parcel documents. A comparison of the parcel areas of both versions of the Gauss-Kruger projection used in Eqs. (13) and (14) is presented in Appendix B.

Unfortunately, neither the first nor the second variants solve the problem associated with the area of land parcels.

An exact solution to the parcel area problem was developed in Germany. According to Ref. [51], when the official area (the area for which the regulatory value, taxes and rents) are calculated, it is determined not on the plane in the projection, but on the ellipsoid. Moreover, a correction for the height of the parcel location is added to the area, which takes into account the fact that two parcels with the same area on the ellipsoid in the space above the ellipsoid have areas depending on the height. It is this solution to the area problem that is preferable. It is independent of projection. Replacing the projection will not change the official area of the parcels.

As Marko Ollikainen & Matti Ollikainen pointed out in Ref. [45], replacing the national coordinate system with a new one is an intricate problem that will take decades to solve completely. Therefore, before introducing a new coordinate system in geodesy and cadastre, it is necessary to conduct comprehensive studies of all the consequences of this input. In particular, changes in all quantitative characteristics of land parcels should be studied in detail. A procedure for updating the cadastral database and documentation on land management should be developed at the legislative level.

Currently, Ukrainian geodesists are faced with the task of creating a new coordinate system. What characteristics this system will have is still unknown. But there is hope that this system will not repeat the mistakes of previous coordinate systems.

As the esteemed reviewers of this article pointed out quite rightly, regardless of which of the two options for the geocentric system is chosen by the Ukrainian surveyors, this will be only the first step in building a new high-precision national coordinate system.

Author statement

Elena Novikova: designed the research and contributed to all phases to the writing of the manuscript.

Alena Palamar: contributed to all phases of data acquisition and the proofreading.

Rostislav Lopunov: contributed to the data acquisition.

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Conflicts of interest

The authors declare that there is no conflicts of interest.

Appendix A. Working formulae for calculating changes in line lengths and areas of parcels during the transition from one ellipsoid to another

The process of transition from the surface of one ellipsoid to another consist of the following parts (Helmert's scheme):

$$\boxed{\varphi_1, \lambda_1, H_1 = 0} \xrightarrow{A.1} \boxed{X_1, Y_1, Z_1} \xrightarrow{A.2} \boxed{X_2, Y_2, Z_2} \xrightarrow{A.3} \boxed{\varphi_2, \lambda_2, H_2}$$

Fig. A.1. Scheme of obtaining geodetic coordinates of points in the transition from one ellipsoid to another.

According to Fig. A1, this process consists of three sub-processes, each of which is described by a separate group of Eqs. (A1-A3), namely [46]:

$$X_1 = N_1 \cos\varphi_1 \cos\lambda_1,\ Y_1 = N_1 \cos\varphi_1 \sin\lambda_1,\ Z_1 = N_1\left(1-e_1^2\right)\sin\varphi_1 \tag{A.1}$$

$$X_2 = (1+\mu)X_1 - R_Z Y_1 + R_Y Z_1 + \Delta X,\ Y_2 = R_Z X_1 + (1+\mu)Y_1 - R_X Z_1 + \Delta Y$$

$$Z_2 = -R_Y X_1 + R_X Y_1 + (1+\mu)Z_1 + \Delta Z \tag{A.2}$$

$$\tan\varphi_2 = \frac{Z_2}{\sqrt{X_2^2+Y_2^2}}\left[1 - e_2^2\frac{N_2}{N_2+H_2}\right]^{-1},\ \tan\lambda_2 = \frac{Y_2}{X_2},\quad H_2 = \frac{\sqrt{X_2^2+Y_2^2}}{\cos\varphi_2} - N_2 \tag{A.3}$$

where N_1, N_2 - radii of curvature in the prime vertical normal section of the original (Krassovsky) and geocentric (GRS80) ellipsoids, respectively [15]; e_1^2, e_2^2 - the square of the first eccentricities of these ellipsoids; $\varphi_1, \lambda_1, H_1$ - geodetic latitude, longitude and height of a point in the UCS2000; X_1, Y_1, Z_1 - spatial rectangular coordinates of a point in the UCS2000; $\varphi_2, \lambda_2, H_2$ - geodetic latitude, longitude and height of a point in the WGS84; X_2, Y_2, Z_2 - spatial rectangular coordinates of a point in the WGS84; $\Delta X, \Delta Y, \Delta Z, R_X, R_Y, R_Z, \mu$ - the Helmert's transformation parameters from the UCS2000 system to the WGS84 system, defined in Ref. [48].[1]

The projection of the surface of one ellipsoid (Krassovsky) onto the surface of another ellipsoid (GRS80) is generally written as in the Ref. [47]:

$$\varphi_2 = f_1(\varphi_1, \lambda_1),\ \lambda_2 = f_2(\varphi_1, \lambda_1) \tag{A.4}$$

In Eq. (A.4), $f_1(\varphi_1, \lambda_1), f_2(\varphi_1, \lambda_1)$ - mapping projection functions.

The maximum a and minimum b scale factors at a point of this projection are determined using the well-known formula in Ref. [47]:

$$a = \frac{A+B}{2},\ b = \frac{A-B}{2} \tag{A.5}$$

where

$$A = \sqrt{m^2+n^2+2mn\sin i}\ B = \sqrt{m^2+n^2-2mn\sin i} \tag{A.6}$$

$$m = \frac{\sqrt{e_{el}}}{M_1},\ = \frac{\sqrt{g_{el}}}{r_1}\ \sin i = \frac{h_{el}}{\sqrt{e_{el}g_{el}}} \tag{A.7}$$

In Eqs. (A.6-A.7) m, n - linear scale factors along with the images of the meridians and parallels of the first ellipsoid on the surface of the second ellipsoid, respectively; i - the angle between images of meridians and parallels of the first ellipsoid on the surface of the second ellipsoid; M_1, r_1 - radii of curvature of meridian and parallel at a given point on the first ellipsoid (Krassovsky).

Gaussian coefficients e_{el}, g_{el} and Jacobian h_{el} for the case of mapping from one ellipsoid to another are determined by the formulae:

$$e_{el} = M_2^2\left(\frac{\partial\varphi_2}{\partial\varphi_1}\right)^2 + r_2^2\left(\frac{\partial\lambda_2}{\partial\varphi_1}\right)^2\ g_{el} = M_2^2\left(\frac{\partial\varphi_2}{\partial\lambda_1}\right)^2 + r_2^2\left(\frac{\partial\lambda_2}{\partial\lambda_1}\right)^2 \tag{A.8}$$

$$h_{el} = M_2 r_2\left(\frac{\partial\lambda_2}{\partial\lambda_1}\frac{\partial\varphi_2}{\partial\varphi_1} - \frac{\partial\varphi_2}{\partial\lambda_1}\frac{\partial\lambda_2}{\partial\varphi_1}\right) \tag{A.9}$$

where M_2, r_2 - radii of curvature of meridian and parallel at a given point on the second ellipsoid (GRS80).

Considering the scheme of Fig. A1 and Eqs. (A1-A3), the partial derivative of the mapping function φ_2 concerning the variable φ_1 is written as follows:

$$\begin{aligned}\frac{\partial\varphi_2}{\partial\varphi_1} = {} & \frac{\partial\varphi_2}{\partial X_2}\left(\frac{\partial X_2}{\partial X_1}\frac{\partial X_1}{\partial\varphi_1} + \frac{\partial X_2}{\partial Y_1}\frac{\partial Y_1}{\partial\varphi_1} + \frac{\partial X_2}{\partial Z_1}\frac{\partial Z_1}{\partial\varphi_1}\right)\\ & + \frac{\partial\varphi_2}{\partial Y_2}\left(\frac{\partial Y_2}{\partial X_1}\frac{\partial X_1}{\partial\varphi_1} + \frac{\partial Y_2}{\partial Y_1}\frac{\partial Y_1}{\partial\varphi_1} + \frac{\partial Y_2}{\partial Z_1}\frac{\partial Z_1}{\partial\varphi_1}\right)\\ & + \frac{\partial\varphi_2}{\partial Z_2}\left(\frac{\partial Z_2}{\partial X_1}\frac{\partial X_1}{\partial\varphi_1} + \frac{\partial Z_2}{\partial Y_1}\frac{\partial Y_1}{\partial\varphi_1} + \frac{\partial Z_2}{\partial Z_1}\frac{\partial Z_1}{\partial\varphi_1}\right)\end{aligned} \tag{A.10}$$

Similar formulae also hold for other derivatives of Eqs. (A.8-A.9).

Differentiation of Eqs. (A1-A3) and substitution of the obtained derivatives in Eq. (A.9) after identical transformations will give the following equation:

$$\begin{aligned}\frac{\partial\varphi_2}{\partial\varphi_1} \cong {} & (1+\mu)\frac{M_1}{N_2}\left[\sin\varphi_2\sin\varphi_1\cos(\lambda_2-\lambda_1) + \frac{\cos\varphi_2}{(1-e_2^2)}\cos\varphi_1\right]\\ & + R_X\frac{M_1}{N_2}\left[\sin\varphi_2\cos\varphi_1\sin\lambda_2 - \frac{\cos\varphi_2}{(1-e_2^2)}\sin\varphi_1\sin\lambda_1\right]\\ & + R_Y\frac{M_1}{N_2}\left[-\sin\varphi_2\cos\varphi_1\cos\lambda_2 + \frac{\cos\varphi_2}{(1-e_2^2)}\sin\varphi_1\cos\lambda_1\right]\\ & + R_Z\frac{M_1}{N_2}\sin\varphi_2\sin\varphi_1\sin(\lambda_2-\lambda_1)\end{aligned} \tag{A.11}$$

The Eqs. for the rest of the derivatives look like this:

$$\begin{aligned}\frac{\partial\varphi_2}{\partial\lambda_1} \cong \frac{r_1}{N_2}\Bigg[& -(1+\mu)\sin\varphi_2\sin(\lambda_2-\lambda_1) + \frac{\cos\varphi_2}{(1-e_2^2)}(R_X\cos\lambda_1\\ & + R_Y\sin\lambda_1) + R_Z\sin\varphi_2\cos(\lambda_2-\lambda_1)\Bigg]\end{aligned} \tag{A.12}$$

$$\begin{aligned}\frac{\partial\lambda_2}{\partial\varphi_1} \cong \frac{M_1}{r_2}[& (1+\mu)\sin\varphi_1\sin(\lambda_2-\lambda_1) - \cos\varphi_1(R_X\cos\lambda_2\\ & + R_Y\sin\lambda_2) - R_Z\sin\varphi_1\cos(\lambda_2-\lambda_1)]\end{aligned} \tag{A.13}$$

$$\frac{\partial\lambda_2}{\partial\lambda_1} \cong \frac{r_1}{r_2}[(1+\mu)\cos(\lambda_2-\lambda_1) + R_Z\sin(\lambda_2-\lambda_1)] \tag{A.14}$$

Eqs. (A.11-A.14) are approximate because they do not take into account the effect of height H_2 on the latitude φ_2.

[1] The transformation parameters from UCS2000 to WGS84, presented in Ref. [48], are calculated based on the Helmert transformation parameters from ITRF-2000 to UCS-2000, adopted at the state level [52], using GNSS network stations and stations of the state geodetic network of Ukraine. The total number of GNSS stations as of May 2019 in Ukraine was 297 [53]. The relative accuracy of the position of these stations is 1:10^{-8}. The horizontal state geodetic network of Ukraine consists of 519 stations of class 1 (relative error 1:10^{-7}), 5376 stations of class 2 (relative error 1:300,000) and 13633 stations of classes 3–4 (relative error for class 3–1:200,000, for class 4–1:100,000). Total station density: 1 station per 30.5 km^2, which is sufficient to carry out a survey at a scale of 1:5000 [54].

Eqs. (A.5-A.14) solve the problem of finding the Gaussian coefficients and Jacobian (Eqs. (A.8-A.9)), and together with them the scale factors of the projection (Eqs. (A.5-A.7)) from the surface of one ellipsoid to the surface of another.

Table B.1
Coordinates of a land parcel of 10000 m^2 in CS63 ($\lambda_0 = 32.5^\circ$, $n = 4$).

N	Northing (m)	Easting (m)	φ (Degrees, Decimal)	λ (Degrees, Decimal)
1	4939000	4418800	44.65887633	33.99783026
2	4939100	4418800	44.65977589	33.99785342
3	4939100	4418900	44.65975935	33.99911381
4	4939000	4418900	44.65885979	33.99909063
P (m^2)		10000		

Appendix B. Practical example of calculating the area of a land parcel in different coordinate systems of Ukraine

As indicated in Ref. [28], despite the official transition to UCS2000 [52], the areas of land parcels continue to be calculated in CS63. It is this area that appears in the official documents for the parcel. Therefore, the initial parcel in the example is a virtual parcel with an area of 10000 m^2 in CS63, the coordinates of which are presented in Table B1.

The parcel is located in a zone with the longitude of prime meridian $\lambda_0 = 32.5^\circ$, and number , $n = 4$. The area of the parcel was determined by the well-known Gauss Equation in Ref. [55]:

$$P = \frac{1}{2}\sum_{i=1}^{I} \text{Northing}_i(\text{Easting}_{i+1} - \text{Easting}_{i-1}) \tag{B.1}$$

where I - the total number of points of the parcel.

Geodetic coordinates calculated from rectangular coordinates in the Gauss-Kruger projection of the points of the parcel on the surface of the Krassovsky ellipsoid were calculated by the formulae [47]:

$$\varphi = \varphi_M - d_\varphi, \lambda = \lambda_0 + d_\lambda \tag{B.2}$$

$$\varphi_M = \left[\frac{x}{a(1-e^2)} + \frac{B_G}{2}\sin(2\varphi_M) - \frac{C_G}{4}\sin(4\varphi_M) + \frac{D_G}{6}\sin(6\varphi_M)\right]\frac{1}{A_G} \tag{B.3}$$

$$d_\varphi = \frac{y^2 t}{2MN}\left[1 - \frac{y^2}{12N^2}\left(5 + 3t^2 + \eta^2 - 9\eta^2 t^2\right) + \frac{y^4}{360N^4}\left(61 + 90t^2 + 45t^4\right)\right] \tag{B.4}$$

$$d_\lambda = \frac{y}{r}\left[1 - \frac{y^2}{6N^2}\left(1 + 2t^2 + \eta^2\right) + \frac{y^4}{120N^4}\left(5 + 28t^2 + 24t^4 + 6\eta^2 + 8\eta^2 t^2\right)\right] \tag{B.5}$$

Constants of Gauss-Kruger projection were calculated using the formulae in Ref. [47]:

$$A_G = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 \ldots,\ B_G = \frac{3}{4}e^2 + \frac{15}{16}e^4 + \frac{525}{512}e^6 \ldots,\ C_G = \frac{15}{64}e^4 + \frac{105}{256}e^6 \ldots,\ D_G = \frac{35}{512}e^6 \ldots \tag{B.6}$$

The latitude functions t, η^2 are:

$$t = tan\varphi\ \eta^2 = e'^2\cos^2\varphi \tag{B.7}$$

$$x = \text{Northing} - \text{FalseNorthing},\ y = \text{Easting} - \text{FalseEasting} \tag{B.8}$$

For CS63:

$$\text{FalseNorthing} = -9214.688\ \text{(m)},\ \text{FalseEasting} = 300\,000 + 100000n\ \text{(m)} \tag{B.9}$$

where n is number of zones. According to the data in Table. B1, for CS63.

For UCS2000:

FalseNorthing = 0 (m), FalseEasting = 300 000 (m) (B.10).

The geodetic coordinates of the points of the parcel, calculated by the method of successive approximations, are shown in Table. B1. Spatial rectangular coordinates of points in the CS63 system were calculated using Eq. (A.1), provided that the heights of all points of the parcel are equal to zero. To calculate the spatial rectangular coordinates in the UCS2000 and WGS84 systems by the Helmert method, the parameters presented in Table B2 were used.

Table B.2
Helmert parameters for the transformation of coordinates between the CS63, UCS2000 and WGS-84 systems, according to Ref. [48,56].

Parameter	CS63 > WGS84	UCS2000 > WGS-84	CS63 > UCS2000
ΔX (m)	25.0	24.3234	0.6766
ΔY (m)	−141.0	−121.3708	−19.6292
ΔZ (m)	−78.5	−75.8275	−2.6725
R_X (arcsec)	0.000	0	0
R_Y (arcsec)	0.350	0	0.350
R_Z (arcsec)	0.736	0	0.736
μ (ppb)	0	−1.74	1.74

In Table B2 Helmert transformation parameters CS63 > UCS2000 are obtained by subtracting the transformation parameters UCS2000 > WGS84 from the corresponding transformation parameters CS63 > WGS84. The theoretical justification for this procedure is presented in Ref. [48].

Geodetic coordinates of the boundary points of the parcel in UCS2000, calculated by Eq. (A.3), are presented in Table B3.

Table B.3
Coordinates of the land parcel in UCS2000.

N	φ (Degrees, Decimal)	λ (Degrees, Decimal)	Northing (m)	Easting (m)
1	44.65884448	33.99777139	4947603.918	379137.943
2	44.65974405	33.99779455	4947703.907	379138.556
3	44.65972751	33.99905494	4947703.294	379238.544
4	44.65882795	33.99903176	4947603.305	379237.931
P (m^2)				9998.1
ΔP (m^2)				−1.9

The geodetic coordinates of the points of the parcel calculated for the WGS84 system using the Eq. (A.3) are presented in Table B4.

Table B.4
Coordinates of the land parcel in WGS84 using the Gauss-Kruger projection with the prime meridians of 32.5° and 33°.

N		φ (Degrees, Decimal)	λ (Degrees, Decimal)	Gauss-Kruger projection with $\lambda_0 =$ 32.5°		Gauss-Kruger projection with $\lambda_0 =$ 33°	
				Northing (m)	Easting (m)	Northing (m)	Easting (m)
1		44.65868515	33.99633123	4948103.981	118679.514	4947497.546	79022.614
2		44.65958473	33.99635437	4948203.980	118679.513	4947597.535	79023.227
3		44.65956821	33.99761478	4948103.981	118779.513	4947596.921	79123.215
4		44.65866863	33.99759162	4948103.981	118779.513	4947496.933	79122.602
	P (m^2)				9999.9		9998.0
	ΔP (m^2)				−0.1		−2.0

As shown in Table B4, coordinates Northing and Easting in the Gauss-Kruger projection for the GRS80 (WGS84) geocentric ellipsoid were calculated using two variants of the prime meridian. The first pair of coordinates was calculated for the prime meridian $\lambda_0 = 32.5°$, coinciding with the prime meridian of the CS63 system, the second pair was calculated for the prime meridian $\lambda_0 = 33°$, coinciding with the prime meridian of the UCS2000 system. As expected, the difference between the area calculated in the first variant is almost zero (-0.1 m^2), and in the second variant this difference reaches 2 m^2 per 10000 m^2.

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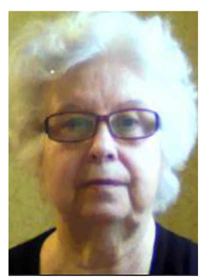

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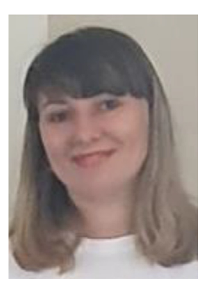

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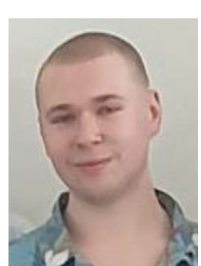

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