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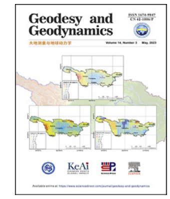

The improved artificial bee colony algorithm for mixed additive and multiplicative random error model and the bootstrap method for its precision estimation

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ABSTRACT

To solve the complex weight matrix derivative problem when using the weighted least squares method to estimate the parameters of the mixed additive and multiplicative random error model (MAM error model), we use an improved artificial bee colony algorithm without derivative and the bootstrap method to estimate the parameters and evaluate the accuracy of MAM error model. The improved artificial bee colony algorithm can update individuals in multiple dimensions and improve the cooperation ability between individuals by constructing a new search equation based on the idea of quasi-affine transformation. The experimental results show that based on the weighted least squares criterion, the algorithm can get the results consistent with the weighted least squares method without multiple formula derivation. The parameter estimation and accuracy evaluation method based on the bootstrap method can get better parameter estimation and more reasonable accuracy information than existing methods, which provides a new idea for the theory of parameter estimation and accuracy evaluation of the MAM error model.

1. Introduction

With the development of modern surveying and mapping technology, various high-tech measurement technologies have emerged, such as Synthetic Aperture Radar (SAR), Global Navigation Satellite System (GNSS), Electronic Distance Measurement (EDM), and Very Long Baseline Interferometry Technology (VLBI) [1–4]. The random errors of observations obtained by these techniques are usually manifested as multiplicative errors or mixed additive and multiplicative (MAM) random errors [5–7], while the traditional adjustment theory only considers additive errors. Therefore, to meet the ever-developing needs of modern observational data processing, it is particularly important to study the processing methods of models with multiplicative errors or MAM errors. At present, in the field of geodesy, there are a few researches on the processing methods of multiplicative errors or MAM errors.

Xu and Shimada first applied the principle of the least squares to the parameter estimation theory of the multiplicative error model and proposed the bias-corrected weighted least squares method of parameter estimation, which proved that this method is equivalent to the quasi-likelihood method [8]. On the basis above, Shi gave the accuracy evaluation formulas of these three methods and constructed the corresponding variance of unit weight [9]. Based on these studies, considering the limitations of the weighted least squares and the total least squares methods for solving EIV models, Xu constructed an EIV stochastic model with multiplicative errors from the perspective of multiplicative error models [10]. Wang and Zou combined the Sterling interpolation method with the weighted least squares method for the parameter estimation and accuracy evaluation of the multiplicative error model and got better results [11]. Wang et al. studied the problem of parameter estimation and

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accuracy assessment of the ill-posed MAM error model [12–14]. Considering that in real-world, the observations contain not only multiplicative errors but also additive errors, Xu et al. and Shi expanded the multiplicative error model to the MAM error model [15,16]. Wang and Han took into account the effect of the priori information and studied the solution of the MAM with inequality constraints [17]. Shi et al. compared the likelihood method and the least squares method of MAM model parameter estimation and extended the least squares method to the case of prior information [18]. Shi and Xu extended the MAM error model to a more general generalized MAM error model with a deterministic trend [19].

To solve the optimization problem of multivariate functions, Karaboga proposed the artificial bee colony algorithm, which is an optimized search algorithm based on the idea of cluster intelligence [20]. The biggest advantage of artificial bee colony is that only one objective function needs to be given without many requirements and other external information [21]. Since the algorithm has not been proposed for a long time, this method still has some defects. Therefore, many scholars have put forward improvement methods. Pampará proposed three binary artificial bee colony algorithms applied to the binary domain, and proved the effectiveness of the method on an unconstrained optimization problem [22]. Santana improved on this basis and proposed a new binary artificial bee colony algorithm [23]. Gao et al. improved the behavior of the scout bees and the dimensional update strategy in the ABC algorithm and solved the problem of separable and non-separable functions effectively [24].

To solve the problem that the artificial bee colony algorithm has a slow convergence speed and is easy to fall into local optimization, Bi improved the selection and crossover strategy in the algorithm and adopted a mutation strategy based on the idea of reverse learning to replace the scout bees' behavior in the original algorithm [25]. Gao et al. introduced an intelligent learning scheme to improve the behavior of employed and onlooker bees, while the turbulence operator was used to balance the global and local search of the algorithm [26].

Zhao proposed an artificial bee colony algorithm based on the proposed affine transformation, improved the position update strategy in the original algorithm, and constructed a new position update strategy to improve inter-particle synergy using the idea of affine transformation [27]. At present, in the field of geodesy, artificial bee colony algorithms are rarely used. Compared to other optimization algorithms, the artificial bee colony algorithm has a better ability to jump out of the local optimum with higher accuracy [28].

The bootstrap method is a resampling method with replacement [29–33]. This method approximates the probability distribution of the output of the nonlinear function through bootstrap samples, which can avoid complex derivative operations. At present, this method is widely used in various fields [34–37], but in the field of geodesy, the research on the bootstrap method is limited [38–40].

Most of the parameter estimation methods for the existing MAM error model are based on the least squares criterion. The analytical expressions of the weight matrix and matrix of weight coefficient are nonlinear functions of parameters or observations, which need many iterative calculations in the process of solving. After the iterative process, the final parameter estimation will show a complex multiple nonlinear form [11], introducing a large number of model errors. Meanwhile, complex derivative operations are required to assess the accuracy of the parameters. Therefore, the improved artificial bee colony algorithm and bootstrap method for parameter estimation and accuracy evaluation are used to estimate and evaluate the unknown parameters effectively without derivation.

2. Mixed additive and multiplicative random error model and its least squares solution

The mathematical expression of the MAM random error model can be expressed as [16].

$$\boldsymbol{y} = f(\beta) \odot (\mathbf{1} + \boldsymbol{\varepsilon_m}) + \boldsymbol{\varepsilon_a} \tag{1}$$

where $\boldsymbol{y} \in \boldsymbol{R}^{n\times 1}$ is an observation; $\beta \in \boldsymbol{R}^{n\times 1}$ means unknown parameter; $f(\beta)$ is a function with unknown parameters; $\odot$ is expressed as the Hadamard product of a vector or matrix; $\mathbf{1} \in \boldsymbol{R}^{n\times 1}$ represents a column vector of all ones; $\boldsymbol{\varepsilon_m} \in \boldsymbol{R}^{n\times 1}$ means zero mean multiplicative random error; $\boldsymbol{\varepsilon_a} \in \boldsymbol{R}^{n\times 1}$ means zero mean additive random error.

When $f(\beta)$ is a linear function of β [19], which is $f_i(\beta) = \boldsymbol{a}_i^{\mathrm{T}}\beta$, then the MAM error model can be expressed as

$$\boldsymbol{y} = \boldsymbol{A}\beta \odot (\mathbf{1} + \boldsymbol{\varepsilon_m}) + \boldsymbol{\varepsilon_a} \tag{2}$$

where $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n]^{\mathrm{T}}$ is a known design matrix, and $\boldsymbol{a}_i \in \boldsymbol{R}^{1\times t}$, $\beta = [\beta_1, \beta_2, \cdots, \beta_n]^{\mathrm{T}}$.

When the least squares criterion to estimate the parameters of the MAM error model is used, the objective function can be expressed as

$$\min : F(\beta) = (\boldsymbol{y} - E(\boldsymbol{y}))^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{y} - E(\boldsymbol{y})) \tag{3}$$

where

$$E(\boldsymbol{y}) = \boldsymbol{A}\beta \tag{4}$$

$$\boldsymbol{P} = \left(\left(diag(\boldsymbol{a}_i\beta)^2\right)\boldsymbol{\sigma}_{\boldsymbol{m}}^2 + \boldsymbol{I}\boldsymbol{\sigma}_{\boldsymbol{a}}^2\right)^{-1} \tag{5}$$

$\boldsymbol{\sigma}_{\boldsymbol{m}}^2$ and $\boldsymbol{\sigma}_{\boldsymbol{a}}^2$ denote the variance of unit weight of $\boldsymbol{\varepsilon_m}$ and $\boldsymbol{\varepsilon_a}$, respectively.

When the ordinary least squares method is used, the parameter estimation of the MAM error model is expressed as

$$\widehat{\beta}_{LS} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} \tag{6}$$

When the weighted least squares method is applied, the iterative formula for parameter estimation of the MAM error model is [15].

$$\widehat{\beta}_{WLS}^{k+1} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}^k\boldsymbol{A}\right)^{-1}\left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}^k\boldsymbol{y} + \boldsymbol{G}^k\left(\boldsymbol{A}\beta^k - \boldsymbol{y}\right)\right) \tag{7}$$

where k is the number of iterations; $\boldsymbol{G} = \begin{bmatrix} (\boldsymbol{A}\widehat{\beta} - \boldsymbol{y})\boldsymbol{P}\boldsymbol{E}_{ae_1}\boldsymbol{D}_m\boldsymbol{E}_{a\beta}\boldsymbol{P} \\ (\boldsymbol{A}\widehat{\beta} - \boldsymbol{y})\boldsymbol{P}\boldsymbol{E}_{ae_2}\boldsymbol{D}_m\boldsymbol{E}_{a\beta}\boldsymbol{P} \\ \vdots \\ (\boldsymbol{A}\widehat{\beta} - \boldsymbol{y})\boldsymbol{P}\boldsymbol{E}_{ae_t}\boldsymbol{D}_m\boldsymbol{E}_{a\beta}\boldsymbol{P} \end{bmatrix}$; $\boldsymbol{D}_m$ is the variance matrix of multiplicative random error; $\boldsymbol{E}_{ae_i}$ is the diagonal matrix with the k_{th} diagonal element equal to $a_k\boldsymbol{e}_i$; $\boldsymbol{e}_i$ is a t-dimensional natural basis vector.

3. Artificial bee colony algorithm for parameter estimation of mixed additive and multiplicative random error model

The basic artificial bee colony algorithm can be described as [20]:

(a) Determine the parameters, including the number of food sources SN, the populations of employed bees, onlooker bees and scout bees, the maximum number of iterations MG, the maximum number of individual updates Lm, the feasible solution space $[\boldsymbol{L}_{\min}, \boldsymbol{L}_{\max}]$, and the problem dimension D. In the D-dimensional solution space, SN food sources are randomly generated as the initial population, and the position of the i_{th} food source is

$$\boldsymbol{L}_i = \boldsymbol{L}_{\min} + rand(\boldsymbol{L}_{\max} - \boldsymbol{L}_{\min}) \quad i = 1, 2, \ldots, SN \tag{8}$$

Meanwhile, calculate the corresponding fitness value

$$\boldsymbol{fit}_i(\boldsymbol{L}_i) = \begin{cases} \dfrac{1}{1 + \boldsymbol{f}_i(\boldsymbol{L}_i)} & \boldsymbol{f}_i(\boldsymbol{L}_i) \geq 0 \\ 1 + |\boldsymbol{f}_i(\boldsymbol{L}_i)| & \boldsymbol{f}_i(\boldsymbol{L}_i) < 0 \end{cases} \tag{9}$$

where $\boldsymbol{f}_i(\boldsymbol{L}_i)$ is the value of the function to be optimized at L_i.

(b) The employed bees search for a new source near the initialized food source. The location of the new food source is

$$\boldsymbol{NL}_{ij} = \boldsymbol{L}_{ij} + \phi\left(\boldsymbol{L}_{ij} - \boldsymbol{L}_{kj}\right) \quad i, k = 1, 2, \ldots, SN; j = 1, 2, \ldots D \tag{10}$$

where $i \neq k$, ϕ is a random number within the interval [-1,1]. The better solution in $\boldsymbol{NL}_i = [nl_{i1}, nl_{i2}, \ldots, nl_{iD}]$ and $\boldsymbol{L}_i = [l_{i1}, l_{i2}, \ldots, l_{iD}]$ is selected according to the greedy selection mechanism.

(c) Each onlooker bee randomly selects a food source location using roulette with a certain probability and searches within its domain according to Eq. (10). The probability of each food source being selected is expressed by the following formula

$$\boldsymbol{Pr}_i = \frac{\boldsymbol{fit}_i}{\sum_{i=1}^{SN} \boldsymbol{fit}_i} \tag{11}$$

When the weighted least squares method is used to solve the MAM error model, the weight matrix is shown in Eq. (5). It can be seen that the weight matrix $\boldsymbol{P}$ is a nonlinear function of parameter β. In the actual derivation process, the weight matrix $\boldsymbol{P}$ needs to be differentiated many times, increasing unnecessary calculation cost. To solve this problem, the quasi-affine transformation artificial bee colony algorithm (QABC) [27] is introduced to estimate the parameters of nonlinear MAM error model. The aim is to avoid the derivation of complex nonlinear weight matrices and directly estimate the optimal parameter solution through the objective function under the least squares criterion.

The QABC algorithm built a new search equation based on the idea of quasi-affine transformation, so that it can update individuals in multiple dimensions, improve the collaboration ability between individuals, and accelerate the speed and accuracy of convergence. The position update formula is

$$\begin{cases} \boldsymbol{B}_i = \boldsymbol{L}_{\text{best}} + q(\boldsymbol{L}_{r1} - \boldsymbol{L}_{r2}) \\ \boldsymbol{NL}_i = \overline{\boldsymbol{H}}_i \boldsymbol{L}_i + \boldsymbol{H}_i \boldsymbol{B}_i \end{cases} \tag{12}$$

where $\boldsymbol{L}_{\text{best}}$ denotes the global optimal position of the current iteration; q is a constant as control parameter (according to reference [27], q is set to 0.6); i is the current number of iterations; $r1$ and $r2$ are two randomly selected individuals, and $i \neq r1 \neq r2$; $\boldsymbol{H}_i$ represents the collaborative search matrix generated during the i_{th} iteration, which is constructed as follows: first, set a lower triangular matrix $\boldsymbol{H}_{i0}$ with $SN \times D$, then randomly sort the elements of each row in $\boldsymbol{H}_{i0}$ to get $\boldsymbol{H}_{ir0}$, and finally randomly arrange each row vector in $\boldsymbol{H}_{ir0}$ to obtain the synergy matrix $\boldsymbol{H}_i$. $\overline{\boldsymbol{H}}_i$ denotes the anti-binary matrix of $\boldsymbol{H}_i$, that is, all elements in $\boldsymbol{B}$ are processed in the anti-binary method, non-zero elements are set to 0, and zero elements are taken as 1.

The steps of the QABC algorithm are as follows [27]:

(1) Initialize the population, calculate the fitness values of various groups, and set various parameters;
(2) Get the current global optimal position $\boldsymbol{L}_{\text{best}}$;
(3) Generate a collaborative search matrix $\boldsymbol{H}_i$ in each iteration;
(4) Update the population by Eq. (12) and then choose the better one by greedy selection mechanism;
(5) Select an individual through roulette, update through Eq. (10), and select a better population according to greedy selection mechanism;
(6) If a population with more than Lm iterations but no update occurs in steps (4) and (5), regenerate the population through Eq. (8) and select a better population;
(7) Repeat steps (2) to (6) until the iteration termination condition is met.

Since the objective function Eq. (3) is always non-negative in the feasible region. Thus, based on the objective function value in step (4) and taking the minimum objective function value as greedy selection mechanism, then $\boldsymbol{fit}_i$ in Eq. (11) can be expressed as

$$\boldsymbol{fit}_i = \exp(-(\boldsymbol{c}_i / \overline{c})) \tag{13}$$

where $\boldsymbol{c}_i$ denotes the i_{th} objective function value and $\overline{c}$ represents the mean of all the values of objective function.

Meanwhile, to further accelerate the convergence speed and accuracy of the algorithm, the approximate interval of parameters can be given according to the prior information when the feasible solution range of the parameters is set. This paper assumes that the model to be solved is non-ill-conditioned. Hence, the parameters estimated by the least squares method have a certain reference significance. Therefore, the ordinary least squares solution of the MAM error model that is easy to calculate is considered the prior information. On this basis, an approximate range can be set for the parameters of each dimension to improve the efficiency of the algorithm. When estimating the parameters of the MAM error model, Eq. (3) is used as the objective function. Since the weight matrix $\boldsymbol{P}$ is a nonlinear function of the parameter β, $\boldsymbol{P}$ will change with each iteration. Therefore, in the actual solution process, the weight of the objective function in the next iteration needs to be calculated through the results of the previous iteration.

In summary, the complete calculation process of the QABC algorithm for parameter estimation of the non-ill-conditioned MAM error model is obtained:

(1) Determine every basic parameter, initializing the population and calculating the fitness value of the population by Eqs. (8) and (13), then calculating the ordinary least squares solution of the MAM error model by Eq. (6). On this basis, setting the parameter interval of the model as $[\beta_{LSi} - r, \beta_{LSi} + r]$, where r is an empirical value;
(2) Set Eq. (3) as the objective function of the QABC algorithm;
(3) The same as steps (2) to (6) in the technical process of the QABC algorithm above;
(4) Substitute the global optimal position obtained after one iteration into the weight matrix of the objective function as the weight matrix of the objective function in the next iteration;
(5) Repeat steps (2) to (4) until the output is the same as the result of the previous two iterations or the number of iterations reaches Lm. In other words, the number of iterations of

the QBAC algorithm is at least 3 (Avoid the algorithm falling into local optimization).

(6) The optimal position, i.e., optimum parameters β_{best}.

Thus, we can draw the specific steps of the QABC algorithm in Fig. 1.

4. The bootstrap method for parameter estimation and accuracy evaluation

The bootstrap method is a method of re-sampling the original sample with replacement. By taking multiple samples of the original data sample to obtain multiple bootstrap samples with the same volume as the original sample, each bootstrap sample is calculated to get the corresponding statistical information and finally, make an overall inference on this basis [33]. The main advantage of this method is that it can obtain reasonable accuracy information of unknown quantity only by testing the change of statistics in samples without understanding too much about the unknown model and its distribution and deriving specific expressions for the estimated parameters. Considering that when the bee colony algorithm is used to estimate the parameters of the MAM error model, the objective function is set based on the least squares criterion, that is, Eq. (3). The optimal result of parameter estimation can only reach the result of weighted least squares method. That is, the parameter estimation obtained by deriving Eq. (3). Therefore, on this basis, the accuracy evaluation of the parameters of the MAM error model by using the algorithm is not ideal. Meanwhile, considering that when using the conventional iterative method to solve the MAM error model, the weight matrix and parameter estimates in each iteration are always nonlinearly related to the observations, as shown in Eqs. (5) and (7). With the increase in the number of iterations, there is a very complex multi nonlinear relationship between the final parameter estimation and observed values, so it is difficult to carry out the complex derivative operation in the accuracy evaluation of parameters. In response to this problem, combined with the QABC algorithm, the bootstrap method without formula derivation is introduced, aiming to obtain reasonable accuracy information and parameter estimation.

Assuming that the observation sample $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n)$ of the MAM error model is an independent and identically distributed sample, where n is the sample size; $y_i (i = 1, 2, \ldots, n)$ denotes the value of the observation value, assuming $\boldsymbol{\beta}$ is the unknown parameter; and the estimated parameter is $\widehat{\boldsymbol{\beta}}$. The bootstrap method resamples the observed value samples in $\boldsymbol{Y}$ to obtain bootstrap samples with a sample size of n in K groups so that the empirical distribution of the bootstrap samples can approximate the distribution of the original sample to obtain more reasonable accuracy information [29].

In practical application, the bootstrap method resamples the observed values in the overall sample in the form of equal probability. After sampling K times, each bootstrap sample is calculated, and the corresponding parameter estimation $\widehat{\boldsymbol{\beta}}_i^b$ by the QABC algorithm is obtained. Finally, K groups of bootstrap parameter estimation $(\widehat{\boldsymbol{\beta}}_1^b, \widehat{\boldsymbol{\beta}}_2^b, \ldots, \widehat{\boldsymbol{\beta}}_K^b)$ are obtained. According to $(\widehat{\boldsymbol{\beta}}_1^b, \widehat{\boldsymbol{\beta}}_2^b, \ldots, \widehat{\boldsymbol{\beta}}_K^b)$, the parameter mean value is calculated, and its expression is

$$\overline{\boldsymbol{\beta}} = \frac{1}{K} \sum_{i=1}^{K} \widehat{\boldsymbol{\beta}}_i^b \tag{14}$$

where $\overline{\boldsymbol{\beta}}$ is the estimation of the mean value of parameters. Under the least squares criterion, the QABC algorithm can obtain the result consistent with the weighted least squares method directly according to the given objective function, and the resampling with return in bootstrap can effectively reduce the deviation of the estimated value [40]. The combination of QABC algorithm and bootstrap method can improve the accuracy of parameter

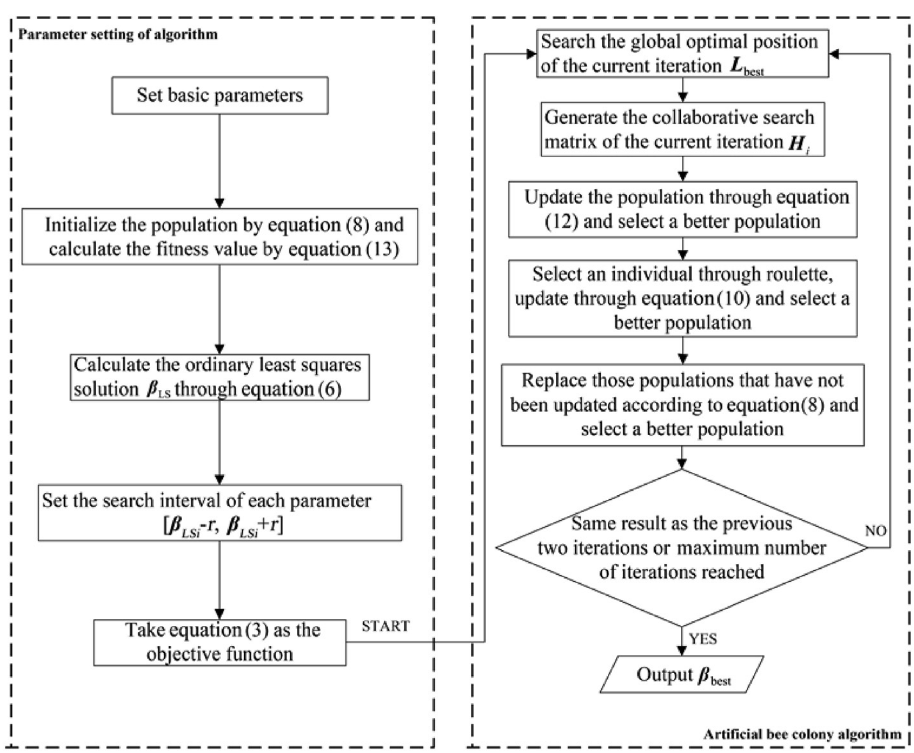

Fig. 1. The quasi-affine transformation artificial bee colony algorithm.

estimation. Therefore, $\overline{\beta}$ is used as the parameter estimation of bootstrap method in this paper.

Then, the parameter accuracy information of the model can be calculated:

$$\boldsymbol{\sigma}_{\beta}^{2} = \frac{1}{K-1}\sum_{i=1}^{K}\left[\left(\widehat{\beta}_i^b - \overline{\beta}\right)^{\mathrm{T}}\left(\widehat{\beta}_i^b - \overline{\beta}\right)\right] \tag{15}$$

where $\boldsymbol{\sigma}_{\beta}^{2}$ denotes estimation of parameter variance, and $\widehat{\beta}_i^b$ is the parameter estimation obtained in the i_{th} bootstrap sample.

Resampling observations of bootstrap method are carried out on the original samples. Therefore, no new random error will be generated in the sampling process, and the bootstrap samples are derived from the original samples. Therefore, no new random error will be generated in the sampling process. Bootstrap samples are all derived from the original samples. However, due to the randomness of sampling, the bootstrap samples may contain multiple duplicate data samples, or some samples in the original samples may not be included. Therefore, there are differences between different bootstrap samples under multiple sampling, and the parameter estimates calculated through the bootstrap samples are also different. Since the bootstrap method is sampling with replacement, after a large batch of sampling, the distribution of the overall bootstrap sample obtained can fit the original sample well. Therefore, the final results are more convincing.

In summary, the specific steps of the bootstrap method for the MAM error model can be given:

(1) Obtain the observed value information $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n)$ of the target area, assuming that the sample is independent and identically distributed, and its corresponding weight matrix is $\boldsymbol{P} = diag(p_1, p_2, \ldots, p_n)$;
(2) Sample the elements in $\boldsymbol{Y}$ with equal probability. In actual operation, it can be realized by generating n random numbers satisfying uniform distribution, and the value of random number i satisfies $1 \le i \le n$;
(3) Obtain the bootstrap sample $\boldsymbol{Y}_q^b = (y_{q_{i_1}}^b, y_{q_{i_2}}^b, \ldots, y_{q_{i_n}}^b)$ and its weight matrix $\boldsymbol{P}_q^b = diag(p_{q_{i_1}}^b, p_{q_{i_2}}^b, \ldots, p_{q_{i_n}}^b)$ where q is the number of current samples and $1 \le q \le K$;
(4) Substitute $\boldsymbol{Y}_q^b$ and $\boldsymbol{P}_q^b$ into the QABC algorithm to obtain a set of bootstrap parameter estimates $\widehat{\beta}_q^b$;
(5) Repeat steps (2) to (4) K times to obtain K groups of bootstrap parameter estimates;
(6) According to Eqs. (14) and (15), the mean of parameters $\overline{\beta}$ and variance $\boldsymbol{\sigma}_{\beta}^{2}$ are obtained, respectively.

Similarly, the flow chart of the bootstrap method with the QABC algorithm can be drawn in Fig. 2.

5. Case studies

In this paper, by simulating a straight-line fitting model with low nonlinear intensity and a digital elevation model with high nonlinear intensity, the performance evaluation of the QABC algorithm and bootstrap method combined with the QABC algorithm in parameter estimation and accuracy evaluation is compared with the least squares method, weighted least squares method and bias-correction weighted least squares method, respectively, to prove the effectiveness and applicability of the proposed method. These five schemes and corresponding methods are shown in Table 1.

5.1. Case I: two-dimensional straight-line fitting model

Firstly, Case I uses a simple straight-line fitting model to verify the feasibility of the algorithm in this paper in the MAM error model.

The straight-line fitting model contains two unknown parameters and, as is shown in the formula

$$y = \beta_1 x + \beta_2 \tag{16}$$

where β_1 and β_2 are the unknown parameters to be estimated.

The model disturbed by multiplicative errors and additive errors are:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \odot \left(\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} \varepsilon_{m_1} \\ \varepsilon_{m_2} \\ \vdots \\ \varepsilon_{m_n} \end{bmatrix} \right) + \begin{bmatrix} \varepsilon_{a_1} \\ \varepsilon_{a_2} \\ \vdots \\ \varepsilon_{a_n} \end{bmatrix} \tag{17}$$

where (x_i, y_i) is the coordinate point of the observation point on the straight line; ε_{m_i} and ε_{a_i} are independent identically distributed multiplicative random errors and additive random errors, respectively.

In this case, the true values of β_1 and β_2 are 7 m and 14 m, respectively. x takes 100 points at an interval of 1 m within 0–99 m, and the standard deviations of multiplicative random errors and additive random errors are 0.05 m and 0.3 m [11].

In the QABC algorithm, the basic parameters are set as follows: the number of food source SN is 100; the number of employed bees, onlooker bees, and scout bees are 100, 100 and 5, respectively. The maximum number of iterations of the algorithm MG is 50, individual maximum update limit Lm, the dimension of objective function is 2 and feasible solution space $[L_{\min}, L_{\max}]$ is set as $[-100, 100]$.

The bootstrap method needs enough sampling time. When the sampling time reaches about 200 times, it can obtain more reasonable accuracy information. When the sampling time exceeds 200 times, the accuracy information obtained is not improved and will tend to be a more stable result with the increase of time. Therefore, to improve the rationality and stability of accuracy information, in this case, the sampling time K is set to 1000 [40].

The fitting line of coordinate points not affected by the error and the coordinate points affected by MAM errors are drawn in Fig. 3.

It can be seen that under the influence of the error, the points on the straight line seriously deviate from the original position. In order to reconstruct the straight line, the error model is solved using the various schemes in Table 1, and the estimated parameters are listed in Table 2.

As is shown in Table 2, LS only considers the equal weight of the observations, the results deviate seriously from the true value, and the 2-norm with the true value reaches 2.6574. Accordingly, the mean square error of unit weight is 16.4266. WLS takes into account the influence of weights on observations, so it gets better results than LS. BCWLS method eliminates the deviation term of the WLS and obtains an approximately unbiased result. Therefore, the parameter estimation is closer to the true value. QABC algorithm takes Eq. (3) as the optimization objective function, and the optimal solution that can be searched theoretically is the solution of WLS. Numerical experiments show that the results of QABC are consistent with WLS, which proves that it can ensure the accuracy of search without derivation and verifies the effectiveness of this method. By sampling the observations in multiple batches and combining the QABC algorithm without destroying the original sample distribution information, the bootstrap method can obtain

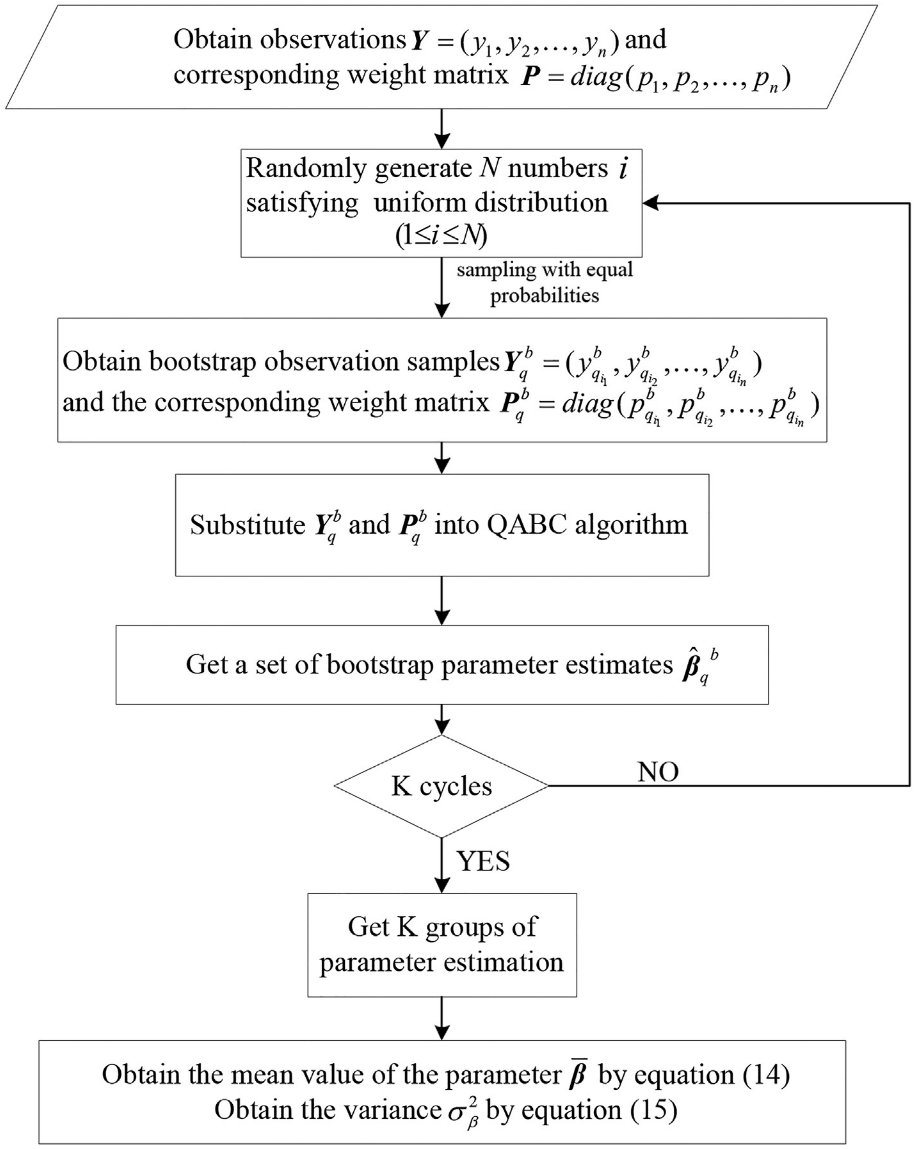

Fig. 2. Bootstrap method with QABC algorithm.

Table 1
Five schemes and corresponding methods.

Scheme	Method
1	Least squares method (LS)
2	Weighted least squares method (WLS)
3	Bias-corrected weighted least squares method (BCWLS)
4	Improved artificial bee colony algorithm (QABC)
5	Bootstrap method combined with QABC(QBSP)

more stable and accurate parameter estimates. The second norm is 0.7078, which is the closest to the true value. The mean square errors of unit weight obtained from scheme 2 to scheme 5 are all better than LS. They are almost the same, and the difference is only reflected in the fourth decimal place.

As is shown in Table 3, compared with the other four methods, LS does not consider the influence of weight and obtains the worst result. Due to the low nonlinear strength of the straight-line fitting model, the parameter accuracy information obtained by BCWLS is almost consistent with WLS. The QABC algorithm gets the

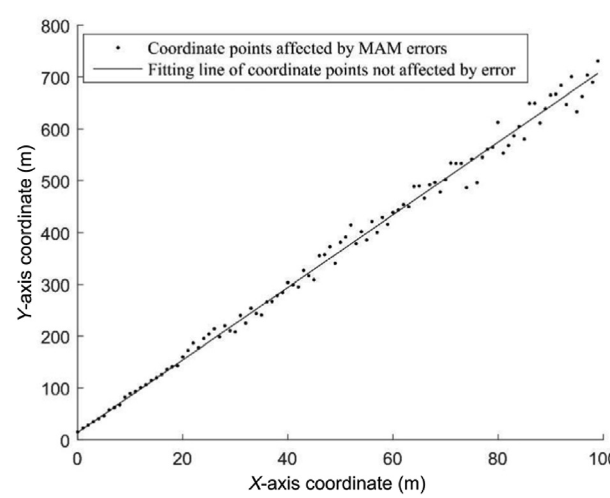

Fig. 3. Unaffected lines and coordinate points affected by mixed additive and multiplicative random errors.

Table 2
Parameter estimation, two norm and mean square error of unit weight of line fitting.

Parameter	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	True value
$\hat{\beta}_1$ (m)	6.9761	7.0385	7.0259	7.0385	7.0277	7.0000
$\hat{\beta}_2$ (m)	16.6573	14.7868	14.7770	14.7868	14.7072	14.0000
$\hat{\sigma}_0$ (m)	16.4266	0.2496	0.2498	0.2496	0.2499	0.3000
2-Norm	2.6574	0.7877	0.7774	0.7877	0.7078	

parameter same standard deviation as WLS due to the same parameter estimation obtained. The bootstrap method adopts resampling instead of derivation for the calculation to avoid the influence of the nonlinear iteration on parameters. Therefore, the minimum parameter standard deviation is obtained and the accuracy of the parameters is effectively improved. It is proved that the bootstrap method has better applicability in the straight-line fitting model with MAM errors. In order to fully demonstrate the effectiveness of this method, a digital elevation model with higher nonlinear strength is used to verify it.

5.2. Case II: four-dimensional digital elevation model

Nowadays, DEM is widely used in hydrology, soil, meteorology, and other fields [41]. Therefore, it is important to restore the real DEM under the influence of various errors. Based on this, Case II uses a more complex digital elevation model with higher nonlinear strength to further validate the effectiveness of the method in this paper.

The non-ill-conditioned digital elevation model with higher nonlinear strength contains four unknown parameters, and its model is as follows:

$$\boldsymbol{G}(x,y)=\boldsymbol{F}(x,y)\odot(\mathbf{1}+\boldsymbol{\varepsilon_m})+\boldsymbol{\varepsilon_a} \tag{18}$$

where

$$\boldsymbol{F}(x,y)=\sum_{i=1}^{4}\boldsymbol{\beta_i f}_i(x,y) \tag{19}$$

$$\begin{cases} f_1(x,y)=\exp\left(-\dfrac{(x-30)^2+(y-30)^2}{500}\right) \\ f_2(x,y)=\exp\left(-\dfrac{(x-50)^2+(y-50)^2}{500}\right) \\ f_3(x,y)=\exp\left(-\dfrac{(x-35)^2+(y-35)^2}{500}\right) \\ f_4(x,y)=\exp\left(-\dfrac{(x-55)^2+(y-55)^2}{500}\right) \end{cases} \tag{20}$$

where $\boldsymbol{G}(x,y)$ represents the elevation of each point and β_i is the unknown parameter.

Both x and y are taken evenly at intervals of 1.5 m within 0–80 m. We set the mean square error of unit weight at 0.3 m; $\boldsymbol{\varepsilon_m}$ and $\boldsymbol{\varepsilon_a}$ are independent identically distributed multiplicative random errors and additive random errors, and we set their standard deviations as 0.05 m and 0.3 m, respectively [11,15]. In the MAM error model, the additive error does not vary with the signal strength, while the multiplicative error is correlated with the signal strength, so it is more important to determine the value of the multiplicative error for this model. In order to further verify the applicability and validity of the QABC and bootstrap method in the MAM error model with different error criteria, Case II uses three sets of MAM error models with different standard deviations of multiplicative errors to determine them.

In the QABC algorithm, the dimension of the objective function is 4; other parameter settings are the same as those in case I. In order to accelerate the convergence speed, for the non-ill-conditioned model, when setting the feasible solution space, the ordinary least squares parameter solution of the MAM model is taken as the prior information. On this basis, an approximate interval range of each dimension is given, and the feasible solution space $[L_{\min}, L_{\max}]$ of each dimension is set as $[\beta_{LSi}-r, \beta_{LSi}+r]$, where r is set as 10. The number of bootstrap samples sampled by the bootstrap method is set to 1000.

In order to illustrate the influence of MAM error on the digital elevation model, the simulated unaffected digital elevation model is drawn in Fig. 4, and the model affected by the MAM error (the standard deviation of multiplicative errors is 0.05 m) is drawn in Fig. 5.

The minimum output of each iteration of the QABC algorithm in different solution spaces is listed in Table 4. Among them, the QABC algorithm that does not consider prior information is recorded as Algorithm 1, and the QABC algorithm that considers prior information is recorded as Algorithm 2. The five schemes in Table 1 are used to estimate the parameters of the digital elevation model with MAM errors, and the results are shown in Table 5.

Table 3
Standard deviations of linear fitting parameter estimation.

Standard deviation	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5
$\hat{\sigma}_1$ (m)	0.0569	0.0349	0.0349	0.0349	0.0340
$\hat{\sigma}_2$ (m)	3.2608	0.4333	0.4333	0.4333	0.4023

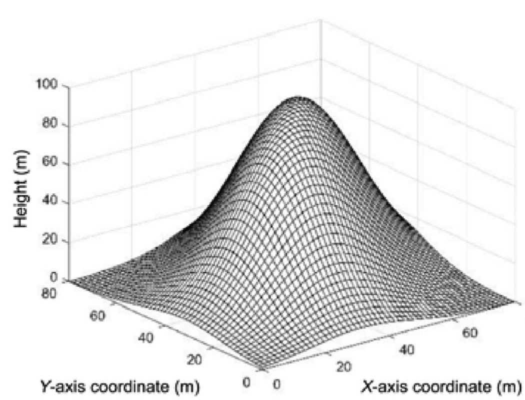

Fig. 4. Unaffected digital elevation model.

It can be seen from Table 4 that the QABC algorithm without prior information requires 19 iterative calculations to search for the global optimal solution, which shows that the algorithm has high efficiency when estimating the parameters of the MAM error model. However, the QABC algorithm with the solution space set on the basis of the model LS solution can search the optimal solution in only 6 iterations, which proves that making full use of the prior information can improve the efficiency of the algorithm to a certain extent.

As shown in Table 5, since LS does not consider the influence of weight on the observed value, the parameter estimation and the calculated mean square error of unit weight deviate greatly from the true value of the parameter. WLS considers the influence of weight, so its parameter estimation is better and closer to the true value. BCWLS discards the variance-covariance matrix of the observations in WLS and obtains an approximately unbiased result, which is better than WLS [9]. The results obtained by the QABC algorithm are consistent with case I. This algorithm can obtain the same parameter estimation as WLS without derivation, which proves the accuracy of its search. The bootstrap method obtains bootstrap samples through resampling and uses the QABC algorithm to solve each sample, which can obtain better results than the existing methods on the basis of the accurate search.

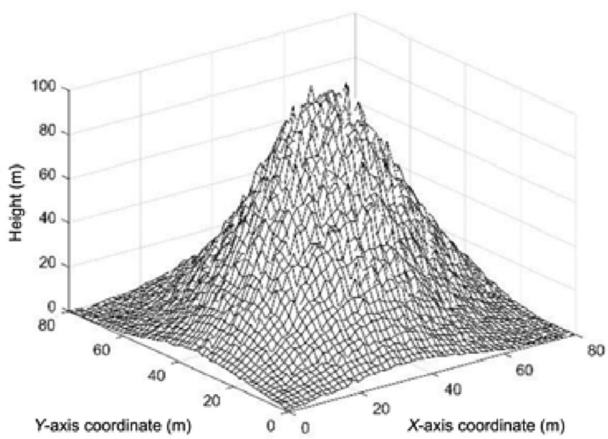

Fig. 5. Digital elevation model affected by MAM error.

It can be seen from Table 6 that both WLS and BCWLS consider the influence of weight in the observed values, and the accuracy results are better than LS ignoring the influence of weight. Among them, BCWLS eliminates the deviation term in WLS, which improves the accuracy to a certain extent. Since the QABC algorithm and WLS obtain the same parameter estimation, the parameter accuracy information obtained by the two methods also remains the same. In the digital elevation model with higher nonlinear intensity, the errors caused by Taylor's higher-order terms are more significant. The bootstrap method fully considers the influence of high-order terms in parameter estimation. Therefore, the standard deviation obtained by the bootstrap method is slightly larger than WLS and BCWLS but better than LS. It is proved that this method is effective to evaluate the accuracy of MAM error model with high nonlinear strength. The results of the three sets of experiments in Tables 5 and 6 all fit the description above.

In addition, it can be seen that the experimental results of three sets of different multiplicative errors can reflect the effectiveness of the algorithm in this paper. Therefore, it can be seen that the algorithm in this paper has some advantages for solving the models with higher nonlinear strength.

Table 4
Iterative results of improved artificial bee colony algorithm in different solution spaces.

Iteration	1	2	3	4	5	6	…	19	…	50
Algorithm 1	1279.9833	565.9818	375.3915	307.9124	278.2202	266.6159	…	**258.6076**	…	258.6076
Algorithm 2	263.3755	260.2915	258.7213	258.6109	258.6081	**258.6076**	…	258.6076	…	258.6076

Table 5
Parameter estimation results of five method.

ε_m	Parameter	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	True value
0.20	$\widehat{\beta}_1$ (m)	26.577	29.668	29.156	29.668	29.196	30.000
	$\widehat{\beta}_2$ (m)	21.620	24.680	23.570	24.680	23.752	25.000
	$\widehat{\beta}_3$ (m)	40.452	38.801	37.067	38.801	36.947	35.000
	$\widehat{\beta}_4$ (m)	51.286	52.280	50.788	52.280	50.701	50.000
	$\widehat{\sigma}_0$ (m)	7.958	1.027	1.042	1.027	1.042	0.300
	2-Norm	7.384	4.457	2.766	4.457	2.547	
0.10	$\widehat{\beta}_1$ (m)	30.664	30.907	30.622	30.907	30.592	30.000
	$\widehat{\beta}_2$ (m)	26.170	26.318	25.994	26.318	25.926	25.000
	$\widehat{\beta}_3$ (m)	33.471	34.010	33.747	34.010	33.795	35.000
	$\widehat{\beta}_4$ (m)	49.551	50.064	49.670	50.064	49.708	50.000
	$\widehat{\sigma}_0$ (m)	4.021	0.543	0.545	0.543	0.545	0.300
	2-Norm	2.086	1.883	1.747	1.883	1.657	
0.05	$\widehat{\beta}_1$ (m)	29.927	30.423	30.314	30.423	30.300	30.000
	$\widehat{\beta}_2$ (m)	25.842	25.483	25.393	25.483	25.362	25.000
	$\widehat{\beta}_3$ (m)	34.815	34.474	34.441	34.474	34.463	35.000
	$\widehat{\beta}_4$ (m)	49.108	49.731	49.623	49.731	49.647	50.000
	$\widehat{\sigma}_0$ (m)	2.019	0.298	0.298	0.298	0.298	0.300
	2-Norm	1.243	0.873	0.841	0.873	0.796	

Table 6
Standard deviations obtained by five methods.

ε_m	Standard deviation	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5
0.20	$\hat{\sigma}_1$ (m)	2.528	1.305	1.295	1.305	1.316
	$\hat{\sigma}_2$ (m)	3.433	2.430	2.398	2.430	2.477
	$\hat{\sigma}_3$ (m)	3.371	2.064	2.043	2.064	2.079
	$\hat{\sigma}_4$ (m)	2.606	1.737	1.714	1.737	1.752
0.10	$\hat{\sigma}_1$ (m)	1.278	0.677	0.675	0.677	0.688
	$\hat{\sigma}_2$ (m)	1.735	1.257	1.252	1.257	1.257
	$\hat{\sigma}_3$ (m)	1.703	1.067	1.064	1.067	1.077
	$\hat{\sigma}_4$ (m)	1.317	0.899	0.896	0.899	0.900
0.05	$\hat{\sigma}_1$ (m)	0.642	0.370	0.369	0.370	0.386
	$\hat{\sigma}_2$ (m)	0.871	0.684	0.684	0.684	0.704
	$\hat{\sigma}_3$ (m)	0.855	0.583	0.582	0.583	0.601
	$\hat{\sigma}_4$ (m)	0.661	0.489	0.489	0.489	0.500

6. Conclusion

Firstly, based on the existing research, this paper analyzes the least squares solution of the MAM error model and introduces an improved artificial bee colony algorithm to estimate the parameters to solve the problem of derivation when the weighted least squares method is used. Meanwhile, considering the complex nonlinear relationship between observations and parameters in the existing iterative methods, the bootstrap method is applied to the accuracy evaluation of the MAM error model. Combined with the case studies, the following conclusions are given:

(1) When the QABC algorithm is used to estimate the parameters of the MAM error model, it can get the same results as the weighted least squares method without deriving the complex weight matrix, which improves the efficiency to a certain extent.
(2) Because the MAM error model studied in this paper is non-ill conditioned, although the solution effect of its ordinary least squares method is poor, it still has a certain reference value. Thus, making full use of the ordinary least squares solution of the model and setting the feasible solution space on this basis may accelerate the convergence speed and improve the efficiency of the QABC algorithm.
(3) Aiming at the fact that the QABC algorithm can obtain the same result as the weighted least squares method under the least squares criterion, the bootstrap method combined with the QABC algorithm can obtain better parameter estimation results than the existing methods. Moreover, because it considers the high-order terms of the model, the accuracy evaluation result is more reasonable.
(4) In this paper, the objective function of QABC is determined based on the weighted least squares method, so the upper limit of parameter estimation accuracy can only reach the result of the weighted least squares. Therefore, it remains to be studied to build a more reasonable objective function to obtain better parameter information.

Author statement

Leyang Wang: Conceptualization, Methodology, Supervision, Formal analysis, Investigation, Funding acquisition.
Shuhao Han: Conceptualization, Methodology, Software, Writing – Original Draft, Writing – Review & Editing.

Conflicts of interest

The authors declare that there is no conflicts of interest.

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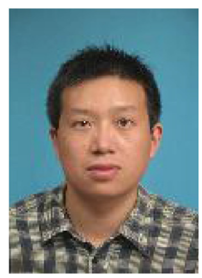

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