

2018 International Conference on Identification, Information and Knowledge in the Internet of Things, IIKI 2018

An Application Research of Kalman Filter Based Algorithms in ECEF Coordinate System for Motion Models of Sensors

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Abstract

Kalman filter based algorithms such as unscented Kalman filter (UKF), and the unbiased conversion measurement Kalman filter (UCMKF) are the most popular nonlinear filters which used in tracking, navigation, estimation and information fusion. Applying those filters directly in East-North-Up (ENU) coordinates with motion models of sensors causes the degradation or even divergence of filter performance. To address this issue, we first analyzed and discussed the motion model consistency of moving sensor with a constant velocity (CV). Next, we proposed to extend the application of common filter algorithms to Earth Centered Earth Fixed (ECEF) coordinates to filter random errors. We verified the validity of our proposed method by filtering random errors in a constant velocity motion model of radar. The theoretical analysis and simulation results show that the extended algorithms provide better efficiency and compatibility in moving sensors.

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Peer-review under responsibility of the scientific committee of the 2018 International Conference on Identification, Information and Knowledge in the Internet of Things.

Keywords: Unscented Kalman filter; Unbiased Conversion Measurement Kalman filter; East-North-Up coordinates ; Earth Centered Earth Fixed coordinates; Motion Model; Sensors; Internet of things

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1. Introduction

With the ability to effectively filter out random errors of measurement, and good real-time performance requiring less storage data with unbiased estimates, the Kalman filter (KF) has been widely used in the field of communication, navigation, guidance and control [1-2], especially in linear systems. However, not all systems are linear. For example, in target tracking, the motion model and the measurement model are established in different coordinate system. Coordinate transformation is a necessary procedure during the data processing. Then nonlinear problems occur and the Kalman filter is not applicable directly to deal with this type of issues.

Researchers and engineers developed different types of methods to address this problem. Some of them attempt to modify the Kalman filter model by linearizing the nonlinear equations, such as the Extended Kalman filter (EKF), and the Unscented Kalman filter (UKF) [4]. The others focus on the conversion of measurement information from a spherical coordinate system to a Cartesian coordinate system, such as the converted measurement Kalman filter (CMKF) [5] and the unbiased converted measurement Kalman filter (UCMKF) [4].

Tracking and location is an important issue for space-time registration in Internet of Things [3]. Kalman filter and its variants are widespread applied tools for motion models to filter random errors. In [10], the authors proposed an automatic parameter setting method for an accurate Kalman filter tracker with the CV model, and achieved better accuracy than the traditional empirical model of process noise. Authors in [6] extended a previous study to a broader range of Constant Turn (CT) models and investigated UKF as well as EKF variants in terms of their performance and sensitivity to noise parameters. Motion models are involved in the previous mentioned literatures, but they only focus on moving targets, not moving sensors. In practical application, the sensors may move in different motion models. Researchers should pay more attention on this issue. Authors in [12] used CT model to discuss the performance degradation problem when UCMKF was directly used in the ENU coordinate with motion radars and extended it to ECEF coordinate to filter random errors. However, more filtering algorithms need to be considered for massive nonlinear applications with motion sensors. The choice of the state coordinate in motion models is very important because it affects the estimation performance [6]. In traditional state estimation problems, the filtering is carried out in the local coordinate system of the sensor; usually it is the East-North-Up (ENU) coordinate system. However, the filtering in the ENU coordinate system causes the degradation or even divergence of filter performance in motion sensors.

In our study, we proposed to solve the filter deviation problem by using nonlinear filtering algorithms in Earth Centered Earth Fixed (ECEF) coordinate system. We discuss the consistency of motion model and analyze the causes of filter performance degradation. Then we use a concrete process with a CV model to test the validity of our proposed method. Matlab simulation tool is used to verify the performance of the extended algorithms. Our proposed method can be applied to avoid degradation or divergence in filter performance in sensor motion models.

2. Analysis of Motion Model Consistency

2.1. The Applied Coordinate Systems

The consistency of the motion models during coordinate system transformation is an important issue, which usually happens in Multi-source information fusion. Different coordinate systems are applied in target tracking systems due to the diversity of motion scenes and measurement platforms. In this paper, we apply 4 different coordinate systems: spherical coordinate system, ENU coordinate system [7], ECEF coordinate system, and WGS 84 coordinate system [8], one of the widely used geodetic coordinate system.

2.2. Problem Statement

In this section, we will explain how filtering random errors directly in ENU coordinate system is not adequate and not consistent. Most systems can be described by the following state equation and measurement equation:

$$\begin{cases} X_{k+1} = F_k X_k + G_k w_k \\ Z_k = H_k X_k + v_k \end{cases} \quad (1)$$

Where X_k is the state vector of process at the time k ; F_k is the state transition matrix of the process from the state at k to the state at $k+1$, and is assumed stationary over time; G_k denotes the process noise covariance matrix; Z_k is the actual measurement of X at time k ; H_k is measurement matrix; w_k and v_k are the process noise vector and measurement noise vector, respectively. They are assumed to be Gaussian white noise with a zero mean. Their covariance matrices can be represented as follows:

$$\begin{cases} \text{cov}(w_k) = E[w_k w_k^T] = Q_k \\ \text{cov}(v_k) = E[v_k v_k^T] = R_k \end{cases} \quad (2)$$

In our study, we take the constant velocity (CV) model to analyze the consistency problem during the coordinate transformation. For ease of description, assuming that the sensor is installed on a gyro stabilized platform that can steadily follow the ENU coordinate and offset biases. The attitude biases are already removed. When the sampling interval of sensor is assumed as T , the state transition matrix F_k of motion model, the process noise covariance matrix G_k and the measurement matrix H_k are as follows:

$$F_k = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_k = \begin{bmatrix} \frac{T}{2} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{T}{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{T}{2} & 1 \end{bmatrix}^T, \quad H_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

The state vector $X_k = (x, \dot{x}, y, \dot{y}, z, \dot{z})^T$ contains the position and velocity components along each axis. The measurement vector $Z_k = (x, y, z)^T$ represents the position of the x, y, z axes.

Next, the consistency of the CV model in the coordinate transformation will be analyzed. For ease of the analysis, we assume that the sensor moves with a constant velocity. Its velocity component is $(V_{xl}(k), V_{yl}(k), V_{zl}(k))$, and the location component in geodetic coordinates is $(L(k), \lambda(k), H(k))$ at time k .

The velocity component of the target, which moves with the CV model in the ECEF coordinate system, is set as (V_{x1}, V_{y1}, V_{z1}) . After transformation to the ENU coordinate; the velocity (V_x, V_y, V_z) can be expressed as follows:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = B^{-1} \left\{ \begin{bmatrix} V_{x1} \\ V_{y1} \\ V_{z1} \end{bmatrix} - \begin{bmatrix} V_{xl} \\ V_{yl} \\ V_{zl} \end{bmatrix} \right\} = B^{-1} V_p, \quad B^{-1} = \begin{bmatrix} -\sin \lambda(k) & -\sin L(k) \cos \lambda(k) & \cos L(k) \cos \lambda(k) \\ \cos \lambda(k) & \sin L(k) \sin \lambda(k) & \cos L(k) \sin \lambda(k) \\ 0 & \cos L(k) & \sin L(k) \end{bmatrix}^{-1} \quad (4)$$

V_p can be viewed as a constant when random noise is ignored. From (4), we can derive that the model in the ENU coordinate system is the same as that in the ECEF coordinate system, when the sensor is motionless. The velocity component is still constant despite the change of its value. It means that the CV model remains unchanged. However, when the sensor is moving, each element in B^{-1} will no longer be a constant, and thereby (V_x, V_y, V_z) varies, it means that the model will not be a CV model during the sensor motion and the consistency of the motion models is interrupted. Changes that are more drastic will be observed with the increase of velocity. The model changes will inevitably result in the deviation of filtering, and even become divergent, which will cause system degradation. Thus, filters cannot be used in ENU coordinate system when applied to CV models.

In target tracking problems, the motion model is often established in ECEF coordinate system. Filtering in ENU coordinate system is not applicable by the above derivation. We will solve it by extending the common filters to ECEF coordinate system.

3. Algorithms Based on ECEF Coordinate System

To ensure the accuracy of the model, this study attempted to successfully carry out the filtering in the ECEF coordinate system, and accomplish the derivation process to convert common filtering methods from the ENU coordinate system to the ECEF coordinate system.

Providing that the measurement data obtained in the spherical coordinate system is represented by $(\tilde{r}, \tilde{\theta}, \tilde{\eta})$, the real measurement data represented by (r, θ, η) , the random measurement errors represented by $(r_\sigma, \theta_\sigma, \eta_\sigma)$, and the

standard deviation of measurement represented by $(\sigma_r, \sigma_\theta, \sigma_\eta)$ are characterized by a Gaussian distribution with a zero mean. The system errors can be eliminated by the registration. The positions of the sensor in the geodetic coordinates and ECEF coordinate system are $(L(k), \lambda(k), H(k))$ and $(x_s(k), y_s(k), z_s(k))$, respectively.

3.1. Extending UKF to ECEF Coordinate System

EKF is based on the first order Taylor series expansion, and it requires the computation of the Jacobian matrix. However, this approximation of EKF may cause divergence, and the calculation of the Jacobian matrix will result in a huge computation burden. Therefore, authors in [7] proposed an unscented conversion based on unscented Kalman filter algorithm.

The linearization of the nonlinear equation in this algorithm is accomplished by calculating the assigned σ sample point by unscented conversion. The final measurement prediction is determined by the measurement sampling and weighting of the σ sample point. This procedure avoids the heavy calculation of the Jacobian matrix. In [7], the authors discussed the UKF filter in the ENU coordinate system.

The state equation is established in the geocentric coordinate system after it is deduced from the ECEF coordinate. To determine the σ sampling of the measurement vector, the sampling point is calculated first in the ECEF coordinates and then it is converted to the ENU coordinates system. Finally, the sampling of a one-step prediction will be achieved. The conversion includes the following 3 steps:

1. Calculating the σ sampling of the measurement vector

$$\begin{cases} \xi_k^0 = X_{k|k-1} \\ \xi_k^i = X_{k|k-1} + \left(\sqrt{(n+\lambda)P_{k|k-1}} \right)_i, i=1, \dots, n \\ \xi_k^i = X_{k|k-1} - \left(\sqrt{(n+\lambda)P_{k|k-1}} \right)_i, i=n+1, \dots, 2n \end{cases} \quad (7)$$

Here, n means the dimension of state X_k , n equals 6, λ determines the dispersive degree of the sampling point. It is usually set as a small positive value (for example, 0.01). k is usually set as 0. $\left(\sqrt{(n+\lambda)P_{k|k-1}} \right)_i$ denotes line i of the square-rooting matrix of matrix $(n+\lambda)P_{k|k-1}$.

2. Converting the sample point into the ENU coordinate

$$\tilde{\xi}_k^i = B^{-1} \left\{ \begin{bmatrix} \xi_k^i(1) \\ \xi_k^i(3) \\ \xi_k^i(5) \end{bmatrix} - \begin{bmatrix} x_s(k) \\ y_s(k) \\ z_s(k) \end{bmatrix} \right\} \quad (8)$$

$\xi_k^i(1)$, $\xi_k^i(3)$, $\xi_k^i(5)$ refer to the 1st, 3rd, and 5th components of ξ_k^i .

3. Calculating the measurement prediction

$$\hat{\zeta}_{k|k-1}^i = h[\tilde{\xi}_k^i], Z_{k|k-1} = \sum_{i=0}^{2n} \omega_i^m \zeta_{k|k-1}^i, i=0, 1, \dots, 2n \quad (9)$$

After conversion, the common UKF filter algorithm can accomplish the filtering.

3.2. Extending UCMKF to ECEF Coordinate System

The expression for conversion of the spherical coordinate system to the ENU coordinate system is:

$$\begin{aligned} x &= (r + r_\sigma) \cos(\eta + \eta_\sigma) \cos(\theta + \theta_\sigma) \\ y &= (r + r_\sigma) \cos(\eta + \eta_\sigma) \sin(\theta + \theta_\sigma) \\ z &= (r + r_\sigma) \sin(\eta + \eta_\sigma) \end{aligned} \quad (10)$$

The presence of random measurement error will result in coupling of each component, when data transferred from the spherical coordinate system to the ENU coordinate system. Therefore, removing the deviation caused by the coupling is an important issue. Previous studies [9] proposed an unbiased converted measurement Kalman filter

(UCMKF) algorithm in three-dimensional space to correct the conversion measurements. The corrected measurements (x^u, y^u, z^u) and covariance matrix R^u are:

$$\begin{aligned} x^u &= \tilde{r} \cos \tilde{\eta} \cos \tilde{\theta} (1 + \lambda_\theta \lambda_\eta - \lambda_\theta^{-1} \lambda_\eta^{-1}) \\ y^u &= \tilde{r} \cos \tilde{\eta} \sin \tilde{\theta} (1 + \lambda_\theta \lambda_\eta - \lambda_\theta^{-1} \lambda_\eta^{-1}) \\ z^u &= \tilde{r} \sin \tilde{\eta} (1 + \lambda_\eta - \lambda_\eta^{-1}) \end{aligned}, \quad R^u = \begin{bmatrix} R^{11} & R^{12} & R^{13} \\ R^{21} & R^{22} & R^{23} \\ R^{31} & R^{32} & R^{33} \end{bmatrix} \quad (11)$$

$$\lambda_\theta = e^{-\sigma_\theta^2/2}, \lambda'_\theta = \lambda_\theta^4, \lambda_\eta = e^{-\sigma_\eta^2/2}, \lambda'_\eta = \lambda_\eta^4$$

The definition of R^u can be referred to in the literature [11].

The corrected position of the target in the ENU coordinate system can be set as (x^u, y^u, z^u) . To maintain the consistency of the motion model, we transform measurement data into the ECEF coordinate system. Then, we can obtain the position (x_t, y_t, z_t) of the target in the ECEF coordinate system as:

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} x_s(k) \\ y_s(k) \\ z_s(k) \end{bmatrix} + B \begin{bmatrix} x^u \\ y^u \\ z^u \end{bmatrix}, \quad B = \begin{bmatrix} -\sin \lambda(k) & -\sin L(k) \cos \lambda(k) & \cos L(k) \cos \lambda(k) \\ \cos \lambda(k) & \sin L(k) \sin \lambda(k) & \cos L(k) \sin \lambda(k) \\ 0 & \cos L(k) & \sin L(k) \end{bmatrix} \quad (12)$$

According to properties of covariance, the corresponding covariance matrix R_p of measurement in the ECEF coordinate system is obtained as follows:

$$R_p = B R^u B^T \quad (13)$$

The consistency of the model is ensured after the data measurement and the covariance matrix are converted to the ECEF coordinate. Therefore, the standard Kalman filter can be applied for filtering the measured data after the conversion. The related formula can be found in the literature [1].

4. Simulation Experiment and Data Analysis

The algorithms deduced in previous section are conducted by Matlab simulation software to verify their validity. To facilitate comparison, the UKF algorithm and UCMKF algorithm are applied in the ENU coordinate and ECEF coordinate systems.

4.1. Experiment Setup

As one of most widely used kinds of sensors, radar is taken as an example to verify our proposed method. In the simulation, the radar is installed on a moving platform, and tracks a moving target. The radar moves with a CV model along the meridian. Its initial geographical coordinates are [N30°, E121.5°, 100m], and its geographical coordinates at time k can be expressed as:

$$(L(k), \lambda(k), H(k)) = (L(k-1) + 0.001v, \lambda(k), H(k)) \quad (14)$$

The velocity coefficient v equals 0, 10, and 20, which denote the stationary state and two different velocities of the motion state, respectively. The sampling interval of the radar is 1s, and the standard deviations of the random measurement noises are [100m, 0.08°, 0.08°]. The offset and attitude biases of the radar have been removed by registration already. The initial ECEF coordinates of the target are [-2921 km, 4726 km, 3120 km].

The simulation steps are 50, and the number of Monte Carlo runs is set as 100. The root mean square error (RMSE) at time instant k is defined as:

$$RMSE(k) = \sqrt{\frac{1}{N} \sum_{i=1}^N M(k)}, \quad M(k) = [\tilde{x}(k) - x(k)]^2 + [\tilde{y}(k) - y(k)]^2 + [\tilde{z}(k) - z(k)]^2 \quad (15)$$

In the above expression, $\tilde{x}(k), \tilde{y}(k), \tilde{z}(k)$ denotes the position of the x, y, z directions after filtering at time instant k . In addition, $x(k), y(k), z(k)$ denote the real position of the x, y, z directions at time k . In addition, N is the number of Monte Carlo simulations. The root mean square error (RMSE) is used to evaluate the filter capacity. When it is smaller, the filter capacity will be better.

4.2. Simulation Results and Discussion

The simulation results are shown in Fig.1-3. The solid line depicts the location RMSE of the raw data. The line with dots shows the location RMSE of filtered data in the ECEF coordinate system. Moreover, the line with circles describes the location RMSE of filtered data in the ENU coordinate system.

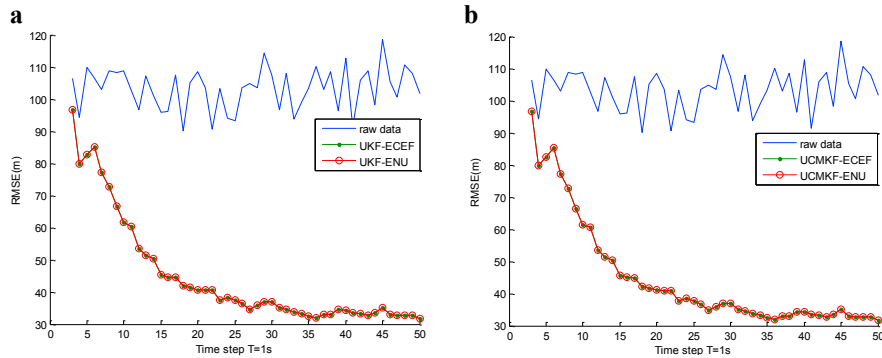


Fig. 1. The location RMSE when V is set as 0. (a) UKF algorithm, (b) UCMKF algorithm

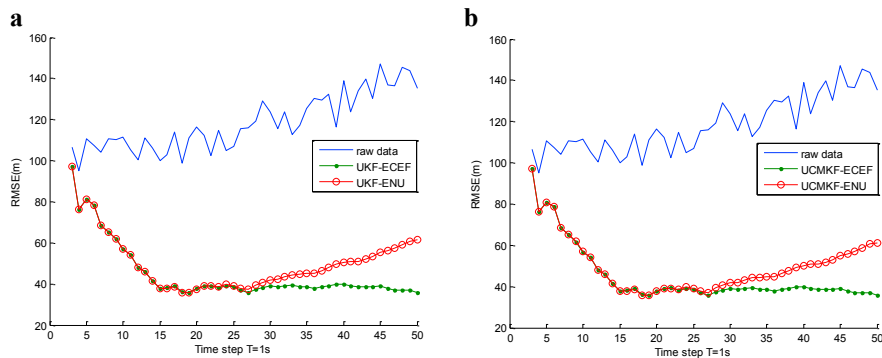


Fig. 2. The location RMSE when V is set as 10. (a) UKF algorithm, (b) UCMKF algorithm

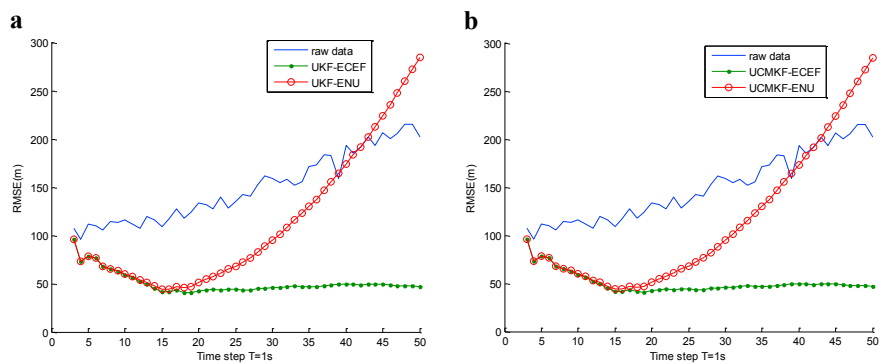


Fig. 3. The location RMSE when V is set as 20. (a) UKF algorithm, (b) UCMKF algorithm

As shown in Fig. 1, when motion coefficient is 0, i.e., the radar is static. The filtering results by UKF, and UCMKF algorithms are all good enough in the ENU coordinate. As observed in Fig. 1(a) and (b), the filtering results in the ECEF coordinate system and the ENU coordinate system are almost the same.

In the case of high speed moving of radars, performance degradation is imminent. As shown in Fig. 2 when the motion coefficient is set to 10, the filtering performance degradation occurs due to the UKF, and UCMKF algorithms in the ENU coordinate. However, Fig. 2 (a) and (b) show that the filter performance by the UKF and UCMKF algorithms in the ECEF coordinate system is better than in ENU coordinate system. When the motion coefficient is set as 20, i.e., radar is moving with a higher speed, the filtering performance will diverge by the UKF, and UCMKF algorithms in the ENU coordinate system. The data accuracy has a lower precision than the raw data, which is easy to be observed in Fig. 3. This performance degradation makes the filtering useless. However, the filtering by the UKF and UCMKF in the ECEF coordinate system still maintains a good performance. It is helpful in filtering random noise.

We can conclude that in the CV model (i.e., targets and radars are moving at a constant speed) with low speed, the filtering works well in both the ENU and ECEF coordinate systems. In the other hand, when the speed is high enough the filtering performance will degrade and could even diverge in the ENU coordinate system, which is theoretically discussed in section 2. However, the filtering in the ECEF coordinate system gives accurate filter results regardless of the speed of the radar. That means the effectiveness of proposed extension algorithms has been validated.

5. Conclusion

In this article, theoretical motivation shows that using nonlinear filters derived from Kalman filter in the ENU coordinate system causes the degradation or even divergence of filter performance in motion sensors. We proposed a new method to accomplish the filter task in the ECEF coordinate system instead of the ENU coordinate.

The proposed method guarantees the stability and accuracy of the filter and keeps the precision of the model. The application and extension of the frequently used filtering algorithms are derived and a series of simulations are performed to verify the validity of the proposed method. This method will significantly help in filtering tasks where motion models are more complicated. Our Future work will focus on the deduction and analysis of other motion models. More specifically, we want to extend our method with more motion models, such as constant turn model, constant acceleration model, variable acceleration model and so on to make it more reliable in practical engineer applications.

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