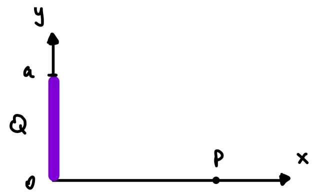
Problem set 9 (due May 22)

1. Positive charge Q is distributed uniformly along the positive y-axis between $y = 0$ and $y = a$.



   (a) (2pt) Calculate the x and y-components of the electric field produced by the charge distribution Q at the point P on the positive x-axis.

   (b) (1pt) Compute the leading contribution to the norm of the electric field in the limit $x \gg a$. Does the result fit your expectations? Explain why.

2. The electric field of a point particle in two dimensions is (for a particle located at the origin)

$$\vec{E}(\vec{r}) = \frac{\tilde{k}q}{r}\hat{r}$$

   where the constant $\tilde{k}$ has units of Nm/C^2.

   (a) (1pts) Consider a test particle of charge q_0 at a location $\vec{r}$. What is the potential energy of the system? How much work does it take to move the test particle to infinity?

   (b) (2pts) Compute the electric field of an infinite long line with uniform linear charge density λ. How does this result compare to the electric field of an infinite long plane in three dimensions?

3. Let $f(x, y, z) = x^2y^2 + y^2z^2 + z^2x^2$.

   (a) (2pt) Compute the vector field $\vec{F} = \vec{\nabla} f$. Sketch the vector field on the xy-plane with $z = 0$.

   (b) (1pt) Compute the divergence and the curl of $\vec{F}$.

   (c) (1pt) Consider an arbitrary scalar field $\Phi(x, y, z)$. Assuming that Φ is a C^2 function (such that all of its first and second partial derivatives exist and are continuous), show that the curl of the gradient of Φ vanishes