

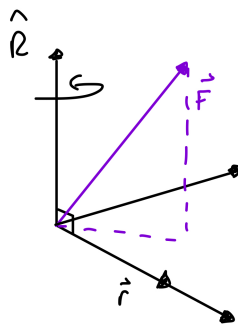
## Problem set 4 (due March 27)

1. Use the Levi-Civita symbol to prove the following properties of the cross product

(a) (1pt)  $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$

(b) (1pt)  $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$  (note that using this identity it's easy to prove the Jacobi identity).

(d) (0.5pt) Show that  $(\vec{r} \times \vec{F})\hat{R} = \vec{r} \times \vec{F}_\perp$  where  $\hat{R}$  is the unit vector along the axis of rotation and  $\vec{F}_\perp$  is the component of  $\vec{F}$  perpendicular to  $\hat{R}$  and  $\vec{r}$ . You must use one of the identities above.



2. (2.5pts) A small block with mass  $0.0400\text{kg}$  slides in a vertical circle of radius  $R = 0.500\text{m}$  on the inside of a circular track. During one of the revolutions of the block, when the block is at the bottom of its path, point  $A$ , the normal force exerted on the block by the track has magnitude  $3.95\text{N}$ . In this same revolution, when the block reaches the top of its path, point  $B$ , the normal force exerted on the block has magnitude  $0.680\text{N}$ . How much work is done on the block by friction during the motion of the block from point  $A$  to point  $B$ ?
3. (2.5pts) Consider an object of mass  $m$  in free fall in a fluid with linear drag  $\vec{f} = -b\vec{v}$  (we discussed this problem in class) Compute the energy as a function of time and show that  $\Delta E = E(t) - E(0) = \int_0^{y(t)} \vec{f} \cdot d\vec{y}$  for any arbitrary time  $t$ .
4. Consider a one dimensional version of the "Mexican hat" potential that is given by  $U(x) = -\alpha x^2 + \beta x^4$  (a higher dimensional version of this potential is responsible for the "Higgs mechanism" that gives particles mass)
  - (a) (1pt) Find the location of the local maximum and minima of this potential
  - (b) (0.5pt) If you release the particle from the local maximum (by giving it a small perturbation), what is its speed at the local minima? How far does the particle get before turning back?
  - (c) (1pt) Find the period of oscillation for a particle stuck around one of the local minima assuming that  $x - x_0$  is small where  $x_0$  is the location of the local minima.