Holography and the TT deformation

Quantum gravity and TT

- TT → can be understood as a flow of QG → QG
- TT → " " " in AdS (holography)

Quantum gravity

- gravity is optional, EM, weak, strong formulated in terms of QFT
  "Standard Model"

- non-renormalizable to all $\mathcal{O}(\ell_P)$ Problems:

$$[G_N] = L^2 \qquad S_{EH} = \int d^4x \sqrt{g}\, R$$

$$S = \frac{1}{16\pi G}\int \sqrt{g}\,R + \underbrace{\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^3 + \dots}_{\text{non-renormalizable}}$$

$$A = (\ell_{Pl})^2 \quad \ell^2 = \left(\frac{G}{\ell_{Pl}}\right)^2, \qquad M_{Pl} \approx 6\cdot 10^{-8}\,\mathrm{kg} \approx 10^{19}\,\mathrm{GeV}$$

$\Rightarrow$ Mass, $\hbar \neq 1$

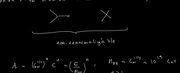
Mass example in Pure theory (e.g. string $m = m_0 + \alpha_1 R$)

$$S(m) = \int d^4x \sqrt{g}\,\Big(\underbrace{m}_{[m]=L^{-1}}\Big)\, \underbrace{g_{\mu\nu}}_{\mathrm{Tr}} \underbrace{g^{\mu\nu}}_{\mathrm{Tr}} \underbrace{g_{\mu\nu}}_{\mathrm{Tr}} \underbrace{g^{\mu\nu}}_{\mathrm{Tr}}$$

$[G_N] = L^2$ non-renormalizable

$m \ll \ell_{Pl}^{-1} \approx m\,\mathrm{GeV}$

$$S_{EH} = \int d^4x\, T_{\mu\nu}\, g^{\mu\nu} g^{\mu\nu}$$



- There is clearly an analogy between $G_N \leftrightarrow G_F$
- Black holes are another reason why gravity is special

Schwarzschild metric:

$$ds^2 = -\left(1-\frac{r_S}{r}\right)dt^2 + \left(1-\frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (= c_0 c_1 c_2 c_3)$$

by $r_S = 2MG_N$, M = mass of the BH

Consider a particle with energy E

$$\lambda = \frac{\hbar}{E}$$

When $G_N \neq 0$

$$\lambda \approx \frac{\hbar}{M_{Pl}}, \qquad r_S = 2G_N \approx \frac{M}{M_{Pl}^2}$$

M_{Pl} → scale where the theory breaks down

⇒ gravity does not fit into the standard Wilsonian paradigm

⇒ gravity has a minimum length scale $\frac{\hbar}{M_{Pl}} = l_{Pl}$

- Top model of QG = TT
  - non-renormalizable int.
  - minimum length scale (l_{Pl})
  - black holes absent by BHs

  ⇒ does not fit within the standard Wilsonian paradigm

  } true scalar field $G = \int d^4x\, \partial_\mu\Phi\, \partial^\mu\Phi$ $+ \frac{\Phi^4}{M^2}$

⇒ any QFT$_d$ + TT = theory of 2d gravity

- Entropy of a cohesion = BH

$S = \ln d = e^{\mathcal{O}(1)}$, QFT, # states
↓ ↳ temperature independent

- Entropy of Black holes

$$S = \frac{A_{BH}}{4G}, \qquad A_{BH} = R^2$$

- Holography: QG in (d+1) dimensions ⇒ theory in lower dimensions

AdS/CFT:

CFT$_d$ → Poincaré invariance → SO(d,1)

$$ds^2 = (-dt^2 + dx^2) + \frac{dz^2}{z^2} + z^2 dx^2 \rightarrow SO(d,1)$$

AdS/CFT: top-down string theory on $AdS_5 \times S^5 \Leftrightarrow$ $\mathcal{N}=4$ SYM in $3+1$

type IIB " " on $AdS_5 \times S^5 \Leftrightarrow$ deformation of $(\mathrm{Sym}^N\, T^4)$

- holographic duality is that space, matter?
- Calculate AdS holography

- non-holographic duality for nearly asymptotically flat spacetimes

⇒ it admits a string theory formulation

The glue of this class is:

I. The TT deformation ⇒ 2d gravity

II. Crude AdS holography ✓

III. Non AdS holography were string theory

The TT deformation

Consider a Poincaré invariant QFT in 2+1. The symmetry

$$\mathrm{ISO}(1,1) \Rightarrow \mathrm{SO}(1,1) \ltimes \mathbb{R}^2$$

↳ Lorentz boosts, ↳ translations

Columns: Plus dilation ⇒ largest operator symmetry group in 2d.

The Minkowski metric is given by

$$ds^2 = -dt^2 + dx^2, \qquad x^\pm = x^0 \pm x^1, \quad \partial^\pm = \partial_\mp$$

translations: $x^\pm \to x^\pm + a^\pm$, $\quad \partial_\pm \to \partial_\pm$ $\quad \Rightarrow \quad$ $C^+ = \int dx\, \partial_+ = \int dx\, \partial_0$, $C^- = \int dx\, \partial_- = \int dx\, \partial_1$

boosts: $x^\pm \to x^\pm + a^\pm x^\pm$, $\quad \partial_\pm \to x^\pm \partial_\pm$ $\quad \Rightarrow \quad$ $B_+ = \int dx\, (x^0 x^1 - x^1 x^0)$, $B_- = \int dx\, (x^0 x^1 - x^1 x^0)$

$$\Rightarrow \partial_+ T^{++} = 0 \quad \Leftrightarrow \quad \partial_+ T^{+-} = \partial_- T^{++} = 0$$

$$\Rightarrow T_{++} = T_{--} = 0$$

Lightcone coordinates: $x^\pm = x \pm t$,

stress tensor: T_{++}, T_{--}, T_{+-}

$$\rightarrow \boxed{TT: \lim_{x^\pm \to 0} \left[T_{++}(x^+) T_{--}(x^-) - T_{+-}(x^+) T_{+-}(x^-) \right]}$$

Exercise: write the conserved currents in terms of T_{++}, T_{--}, T_{+-}. Verify that the currents are conserved. What happens to the translation currents when $T_{+-} \neq 0$? Hint: generate $\mathbb{R}^2$ in the coordinate dependent.

- TT operator is well defined
- scaling dimension 4

⇒ the TT operator is a universal and well defined operator in any 2d QFT.

Deformations of a QFT

$$S[\lambda] = S + \int d^d x\, \mathcal{O}_\Delta, \qquad \Delta = \text{scaling field}$$

↓

$$C \to \lambda^{d-\Delta}$$

I. Relevant deformations, $\Delta < d$

Any local QFT has a finite # of relevant operators ⇒ RG eqs can be solved given some initial data

$d=2$, $S = \int d^2x\, (\partial_\mu \Phi \partial^\mu \Phi)$, $m^2 \int d^2x\, \Phi^2$

II. Marginal deformations, $\Delta = d$

$d=2$, $S = \int d^2x\, \partial_\mu\Phi\, \partial^\mu\Phi$, UV invariant ↳ $\Delta = d$ ⇒ $\Delta = 1$
$\Delta = d$ ⇒ $\Delta = 1$

$$\underbrace{\mathcal{O}_\Delta = i\partial}_{\Delta = 1}, \qquad \Delta S = \lambda \int d^2x\, \mathcal{O}_\Delta$$

III. Irrelevant deformations, $\Delta > d$

irrelevant op in QFT → ∞

two examples:

$$(\mathcal{O}_\Delta)^{\#\mathcal{O}} \int d^2x\, \partial_\mu\Phi\, \partial^\mu\Phi, \qquad G_F \int d^2x\, \bar\psi\gamma^\mu\partial_\mu\psi$$

Another example:

$$\boxed{\mathcal{O}_\Delta = \det(T)}, \quad d=2, \quad \Delta = 4, \quad \int d^2x\, \det(T)$$

$$\boxed{S[\lambda] = \int d^2x\, (\det T)^2 \quad (\lambda) = L^2}$$

TT flow: continuous flow by irrelevant TT

- not an RG flow
- doesn't add new data

The TT deformation of a Poincaré-invariant QFT is

$$\boxed{\partial_\lambda S(\lambda) = \mathrm{EH} \int d^2x\, (TT)}$$

$$S(\lambda) = S(0) + \mathrm{EH} \int d^2x\, (TT) + \mathcal{O}(\lambda^2)$$

$$\Rightarrow S(\lambda) = \int d^2x\, \sqrt{1 - \lambda \mathrm{EH}}$$

I. Generality:

The TT operator is universal and exists among Poincaré invariant QFTs (Lorentz invariance)

II. Solvability:

- universal Diff eq for the spectrum of the deformed theory
  ↳ if the QFT is a CFT, the spectrum can be solved explicitly
- universal expression for the deformed S-matrix

III. UV completeness:

$$\partial_\lambda S = \int TT \quad \Rightarrow \quad S(\lambda) = S(0) + \lambda \int TT + \mathcal{O}(\lambda^2)$$

Lecture 2:

- show that TT is well defined
- show that TT, $\Delta = 4$

} spectrum of TT-deformed QFT.