AMS 206B: Intermediate Bayesian Inference
EXAM 2, TAKE HOME, WINTER 2016

Please work individually on this problem. Do not share with anyone any information or comments about your findings or the models and methods you use. Please turn in your solutions by Monday 03/07/2016 at 10am. Please print your solution and bring it to my office. I will not be grading online or printing the solutions myself so please do not e-mail me your file. You may turn in your solution before the deadline. Please organize and present the material in the best possible way answering all the questions below. Be informative but concise and annotate all relevant figures and/or tables.

Problem: The data `dataexam2th.txt` contains $I = 5$ time series with $T = 300$ observations each. Consider a model of the form

$$\begin{aligned} y_{t,i} &= \phi_i y_{t-1,i} + \epsilon_{t,i}, \;\; \epsilon_{t,i} \sim N(0, v), \\ \phi_i &\sim N(\phi, \tau^2), \\ p(\phi, v) &\propto 1/v, \end{aligned}$$

with $\tau^2 = 0.1$. Let

$$p(y_{1:T,i}|\phi_i, v) = \frac{(1-\phi_i^2)^{1/2}}{(2\pi v)^{T/2}} \exp\left\{-\frac{Q^*(\phi_i)}{2v}\right\},$$

with

$$Q^*(\phi_i) = y_{1,i}^2(1-\phi_i^2) + \sum_{t=2}^{T}(y_{t,i} - \phi_i y_{t-1,i})^2,$$

be the full likelihood, and

$$p(y_{2:T,i}|y_{1,i}, \phi_i, v) = \frac{1}{(2\pi v)^{T/2}} \exp\left\{-\frac{Q(\phi_i)}{2v}\right\},$$

with

$$Q(\phi_i) = \sum_{t=2}^{T}(y_{t,i} - \phi_i y_{t-1,i})^2,$$

be the conditional likelihood.

1. (18 points) Using the conditional likelihood as an approximation, summarize the posterior distributions of $\phi_1, \ldots, \phi_5, \phi$ and v given the data in `dataexam2th.txt`. In order to do this, implement a MCMC algorithm to obtain samples from the posterior distribution.

2. (10 points) Using the full likelihood, summarize the posterior distributions of $\phi_1, \ldots, \phi_I, \phi$, and v given the data. Again, implement a MCMC algorithm to obtain samples from the posterior distribution.

3. (5 points) Compare the results from the 2 approaches described above.

4. (7 points) Prior sensitivity: Repeat the analyses listed above with $\tau^2 = 1$ and $\tau^2 = 10$ and comment on how this affects your posterior results.