Gibbs Sampling

In order to perform a Gibbs sampling to explore the posterior distribution of the parameters in the SAT example we need to obtain the full conditionals.

$$p(\theta_j|\mu,\tau,Y) = N(\hat{\theta}_j, V_j)$$

$$p(\mu|\theta,\tau,Y) \propto \exp\left\{-\frac{1}{2\tau^2}\sum_{j=1}^{8}(\mu-\theta_j)^2\right\} \propto \exp\left\{-\frac{8}{2\tau^2}(\mu-\bar{\theta})^2\right\}$$

then $p(\mu|\theta,\tau,Y) = N(\mu|\bar{\theta},\tau^2/8)$.

$$p(\tau|\theta,\mu,Y) \propto \left(\frac{1}{\tau^2}\right)^{8/2} \exp\left\{-\frac{1}{2\tau^2}\sum_{j=1}^{8}(\mu-\theta_j)^2\right\}$$

changing variables we obtain that the full conditional of τ^2 is an inverse gamma density with parameters $5/2$ and $\sum(\theta_j-\mu)^2/2$.

An Approximation

$$E(\theta_i|\overline{y}_{\cdot}, s^2) \approx \overline{y}_{i\cdot} - \frac{(n_i-1)s_i^2 b(2a+J-1)}{2n_i(n_i-3)(1+bRSS_B/2)}(\overline{y}_{i\cdot} - \overline{y}_{\cdot\cdot})$$

and

$$V(\theta_i|\overline{y}_{\cdot}, s^2) \approx \frac{s_i^2(n_i-1)}{n_i(n_i-3)}$$

$$\left\{1 + \frac{(n_i-1)s_i^2 b^2(2a+J-1)(n_i-4+2a+J)}{2n_i(n_i-3)(n_i-5)(1+b/2RSS_B)^2}(\overline{y}_{i\cdot} - \overline{y}_{\cdot\cdot})^2 \right.$$

$$\left. + \frac{(n_i-1)s_i^2 b(2a+J-1)^2}{2Jn_i(n_i-5)(1+bRSS_B/2)}\right\} \quad ,$$

where $RSS_B = \sum \overline{y}_{i\cdot}^2 - J\overline{y}_{\cdot\cdot}^2$, $\tau^2 \sim IG(a,b)$ and s_i^2 is the observational variance for group i.

Approximations for the first and second moments of τ and μ can be found is Abrams and Sansó (1998), *Stats. in Med.*, 17, 201–218.

Meta-Analysis

Meta-Analysis usually refers to the methods used to summarize evidence from a number of studies, performed for a given purpose, of the same type of design. A typical example is that of combining evidence from various randomized controlled trials that compare two treatments for a given disease. Notice that in these studies very rarely we have availability of the original data sets. Only summaries are provided.

Dentifrice example (A&S '98)

We consider 9 studies to compare sodium fluoride with sodium monofluorophosphate dentifrices in terms of differences from baseline in DMFS dental index.

We assume that the data are normally distributed and that the variances are known. Thus

$$y_i \sim N(\theta_i, \sigma_i^2/n_i), \quad \theta_i \sim N(\mu, \tau^2)$$

is the kind of model that we consider. Here y_i is the difference in means responses for the i-th study. $n_i = n_{1i}n_{2i}/(n_{1i} + n_{2i})$, where n_{1i} and n_{2i} are the number of patients in the treatment and the control group in study i.

Infection example (A&S '98)

We investigate the evidence of clinical benefits for the selective decontamination of the digestive tract for patients in intensive care units. There are 22 randomized trials. In each trial the patients where randomized to received a combination of antibiotics (treatment) or no treatment (control).

Here the observations are binary outcomes. So we take

$$y_i = \log\left\{\frac{p_{1i}}{1-p_{1i}}\frac{1-p_{2i}}{p_{2i}}\right\}$$

the so called log-odds-ratio. We take

$$\sigma_i^2 = \frac{1}{p_{1i}(1-p_{1i})} + \frac{1}{p_{2i}(1-p_{2i})}$$

(see Cox and Snell '89). We take $n_i = \min(n_{i1}, n_{i2})$. See A&S '98 for results.