Tulips - Nonlinear Logistic Regression

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December 6, 2014

Introduction

Tulip Germination Experiment

- Goal: Understand the effect of chilling time on germination of tulip bulbs.
- Data:
  - 12 populations each with 210 tulips (2005-2009)
  - Each population randomly and evenly split into 7 groups and assigned to one of 7 chilling times (0, 2, 4, . . . , 12 weeks).
  - Response Variable: Indicator (bulb germinated or not).
  - Population 12 did not germinate at all, so it was removed from the analysis.

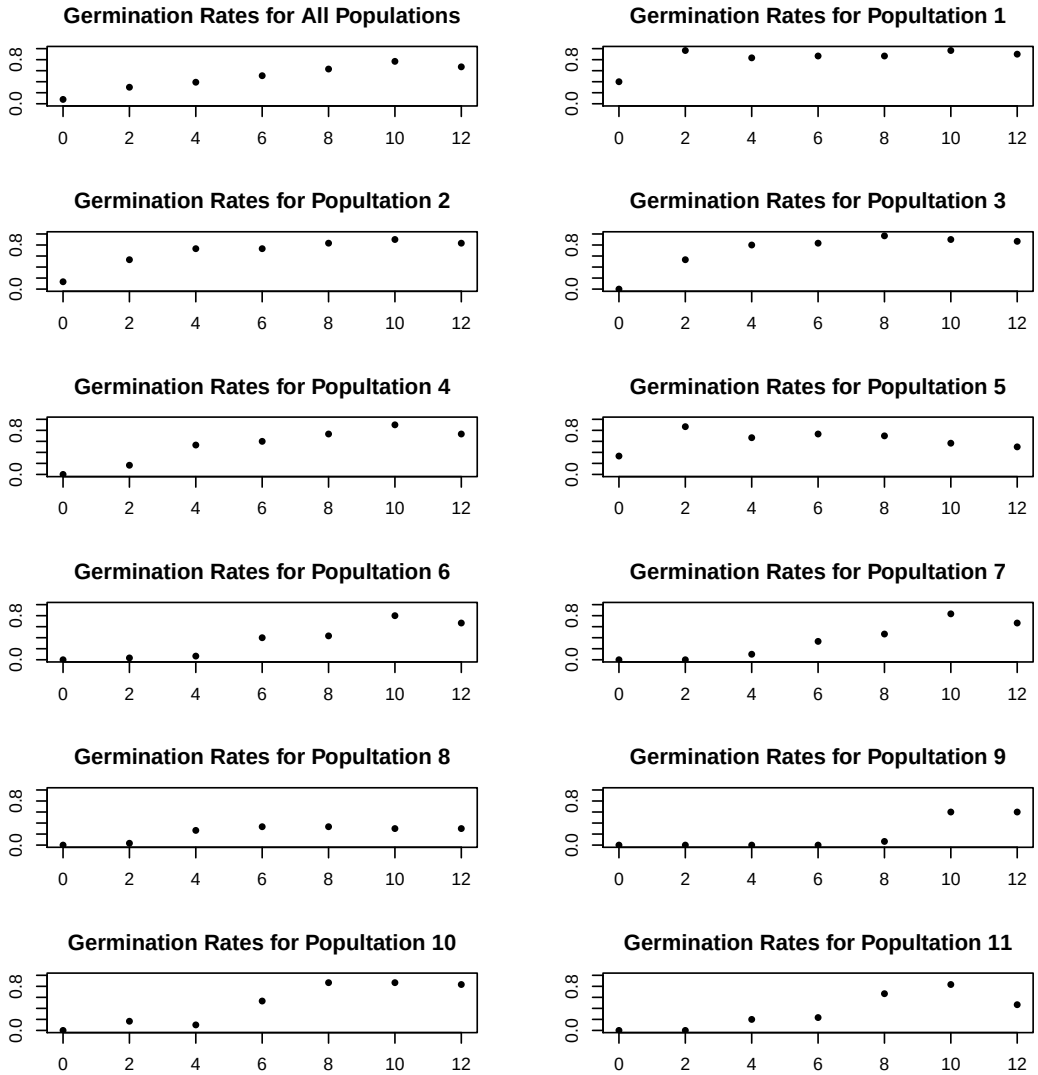
Research Questions

- Is the effect of chilling time the same across all Populations?
  Which populations are the same / different?
- Is there an "ideal" chilling time?
  Does this ideal chilling time vary by population?
- What effect will a decrease from 10 to 8 weeks of chilling time have for tulips?

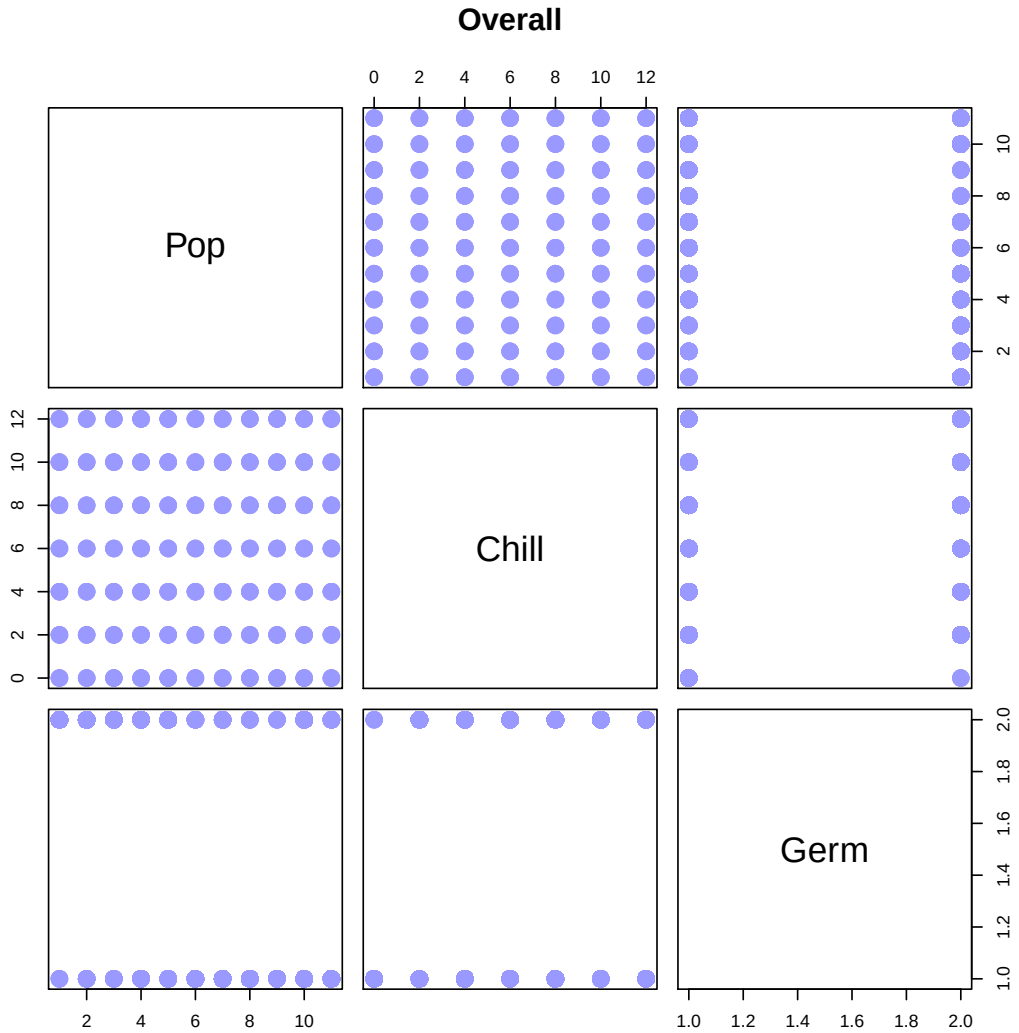
Germination Rates

	Chilling Time (Weeks)						
Population	0	2	4	6	8	10	12
1	0.40	0.97	0.83	0.87	0.87	0.97	0.90
2	0.13	0.53	0.73	0.73	0.83	0.90	0.83
3	0.00	0.53	0.80	0.83	0.97	0.90	0.87
4	0.00	0.17	0.53	0.60	0.73	0.90	0.73
5	0.33	0.87	0.67	0.73	0.70	0.57	0.50
6	0.00	0.03	0.07	0.40	0.43	0.80	0.67
7	0.00	0.00	0.10	0.33	0.47	0.83	0.67
8	0.00	0.03	0.27	0.33	0.33	0.30	0.30
9	0.00	0.00	0.00	0.00	0.07	0.60	0.60
10	0.00	0.17	0.10	0.53	0.87	0.87	0.83
11	0.00	0.00	0.20	0.23	0.67	0.83	0.47

Germination Rates



Germination Rates



Logistic Regression Model (Linear)

$$Y_i \overset{ind}{\sim} \text{Bernoulli}(p_i)$$

$$log\left(\frac{p_i}{1-p_i}\right) = \mathbf{x_i'}\boldsymbol{\beta}$$

$$\Rightarrow p_i = \frac{e^{\mathbf{x_i'}\boldsymbol{\beta}}}{1+e^{\mathbf{x_i'}\boldsymbol{\beta}}}$$

where $p_i = P(Y_i = 1)$

Logistic Regression Model (Nonlinear)

$$Y_i \overset{ind}{\sim} \text{Bernoulli}(p_i)$$

$$log\left(\frac{p_i}{1-p_i}\right) = (ns(\mathbf{x}_i))'\beta$$

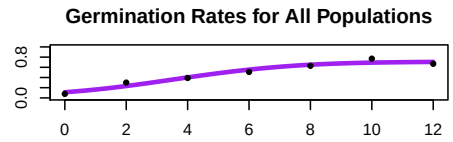
$$\Rightarrow p_i = \frac{e^{\mathbf{ns(x_i)'\beta}}}{1+e^{\mathbf{ns(x_i)'\beta}}}$$

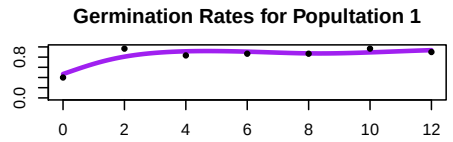
where $p_i = P(Y_i = 1)$

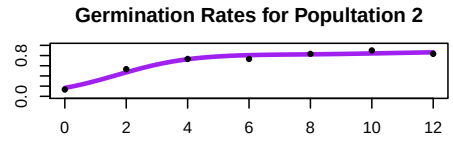
Model

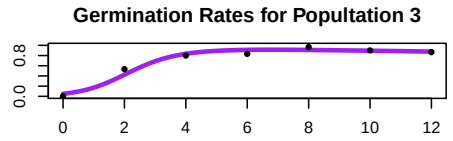
+ Model is flexible
- Number of knots needs to be predetermined

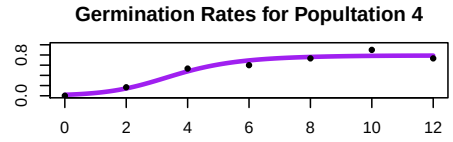
Model

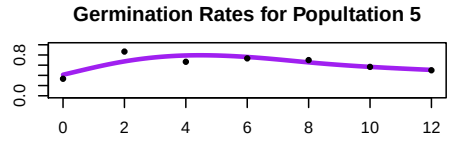


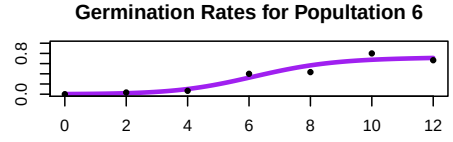


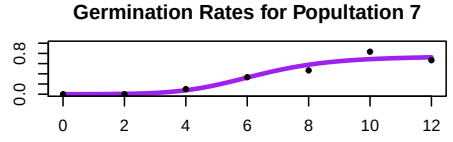


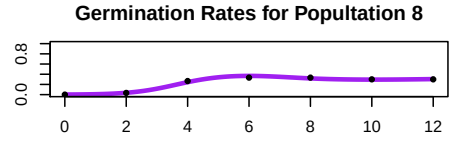


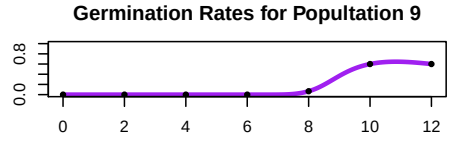


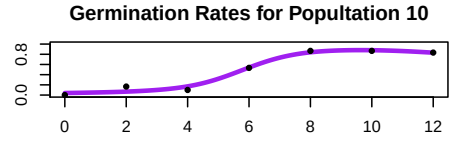


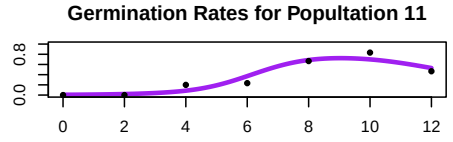












Error Rates

Population 1	Population 2	Population 3	Population 4	Population 5	Population 6	Population 7	Population 8	Population 9	Population 10	Population 11	Overall
0.17	0.23	0.17	0.25	0.35	0.24	0.21	0.22	0.13	0.18	0.22	0.31

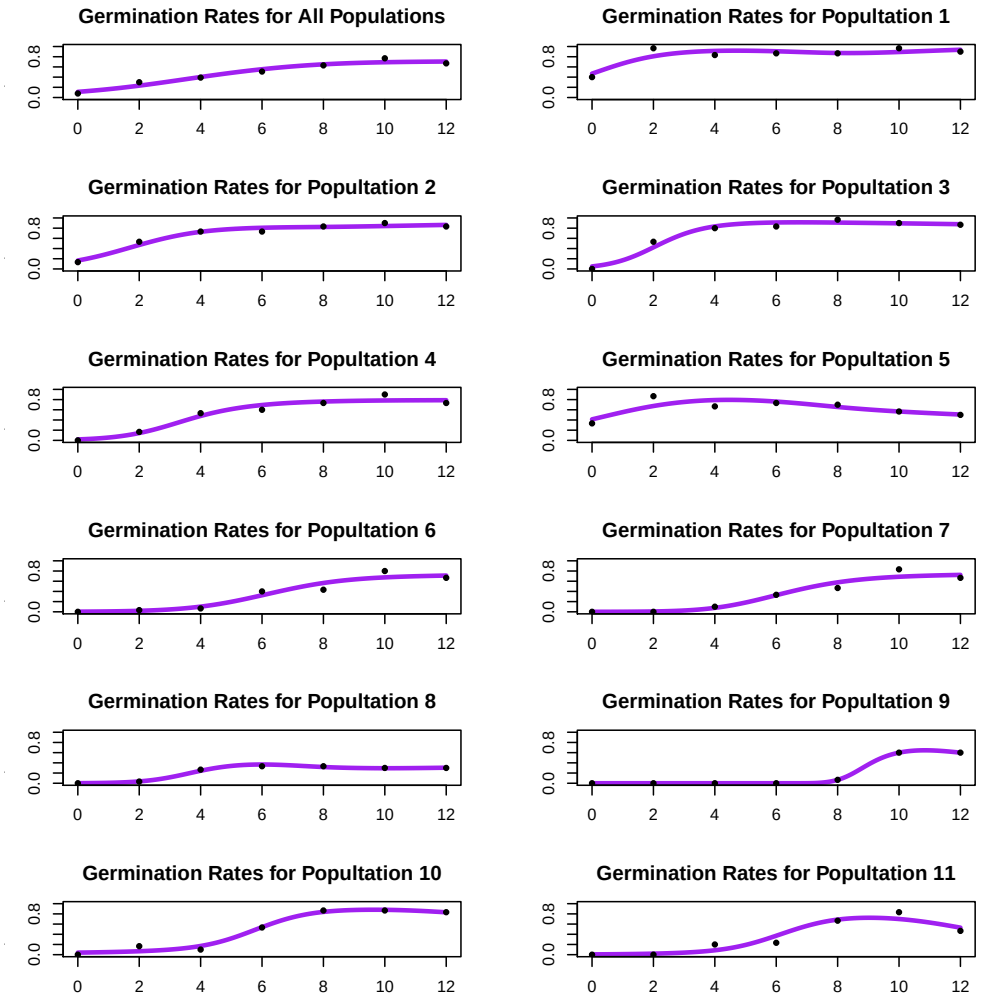
Effect of Chilling Time

Models to compare for every pair of populations:

$$log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + pop_i\beta_1 + ns(chill_i)\beta_2 + pop_i ns(chill_i)\beta_3$$

$$log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + ns(chill_i)\beta_2$$

Effect of Chilling Time

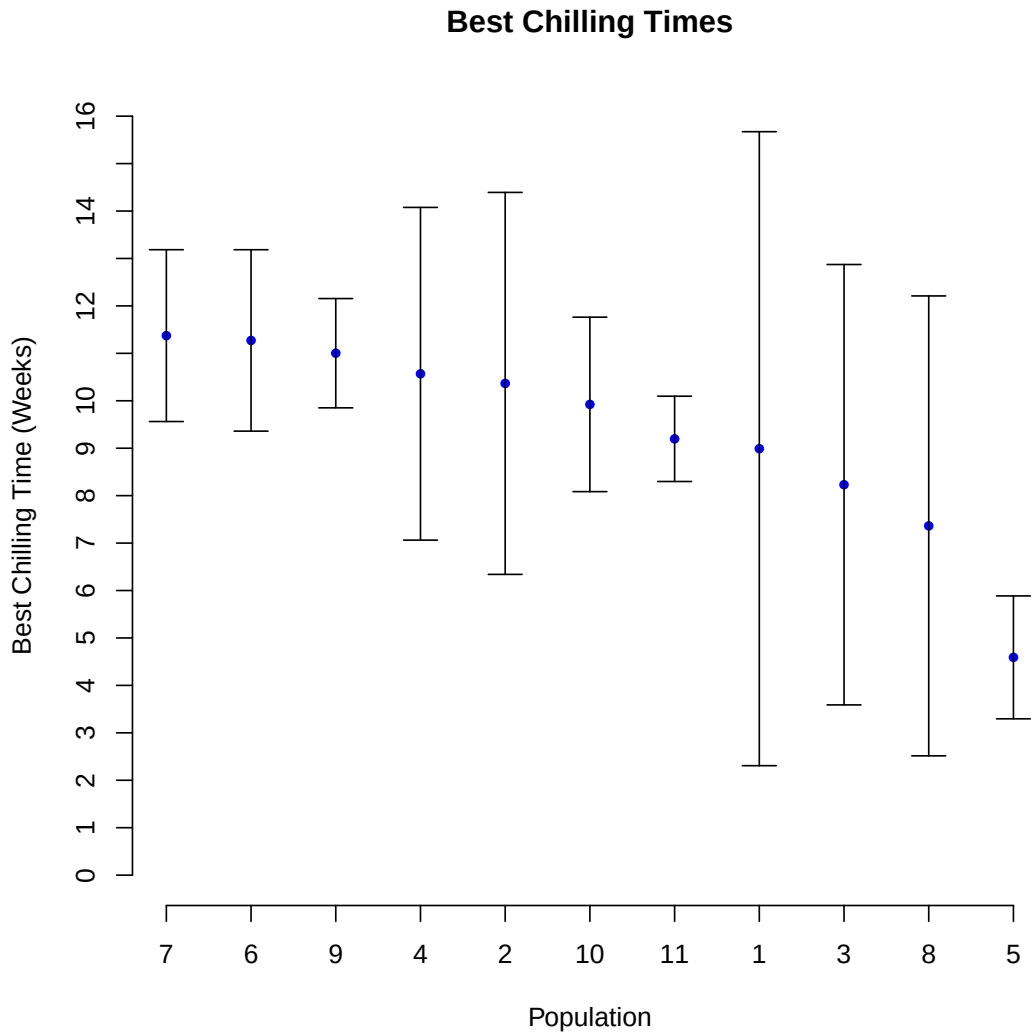


Populations that respond similarly to chilling time: (3,2), (4,2), (10,4), (7,6), (10,6), (11,6), (10,7), (11,7), (11,10)

Ideal Chilling Time

	Estimate	95% Lower.CI	95% Upper.CI
Population 1	8.99	2.31	15.67
Population 2	10.37	6.34	14.39
Population 3	8.23	3.59	12.87
Population 4	10.57	7.06	14.08
Population 5	4.59	3.30	5.89
Population 6	11.27	9.36	13.18
Population 7	11.37	9.56	13.18
Population 8	7.36	2.52	12.21
Population 9	11.00	9.85	12.15
Population 10	9.92	8.08	11.76
Population 11	9.20	8.30	10.10
All Populations	11.46	9.76	13.15

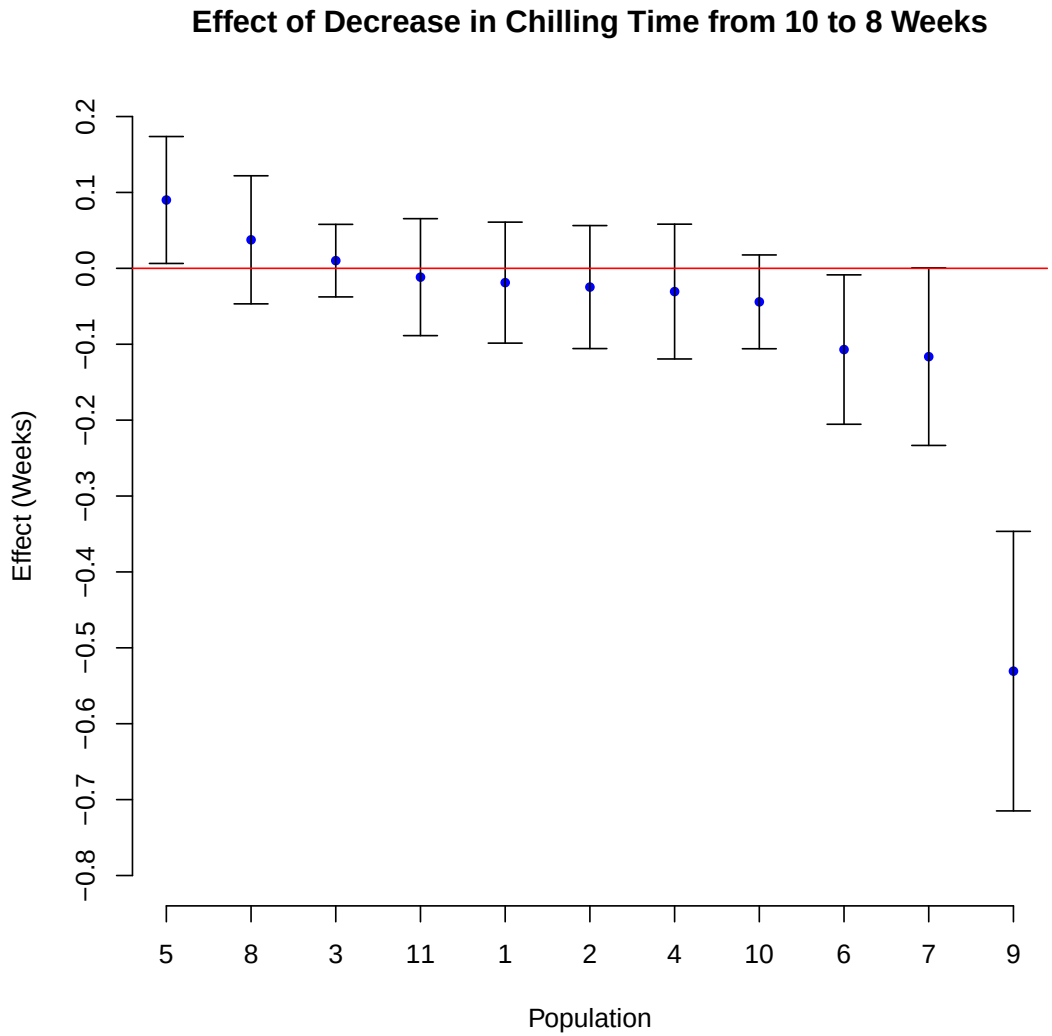
Ideal Chilling Time



Effect of Decrease in Chilling Time from 10 to 8 Weeks

	Estimate	95% CI.Lo	95% CI.Hi
Population 1	-0.02	-0.10	0.06
Population 2	-0.02	-0.11	0.06
Population 3	0.01	-0.04	0.06
Population 4	-0.03	-0.12	0.06
Population 5	0.09	0.01	0.17
Population 6	-0.11	-0.21	-0.01
Population 7	-0.12	-0.23	0.00
Population 8	0.04	-0.05	0.12
Population 9	-0.53	-0.71	-0.35
Population 10	-0.04	-0.11	0.02
Population 11	-0.01	-0.09	0.07
All Populations	-0.04	-0.07	-0.01

Effect of Decrease in Chilling Time from 10 to 8 Weeks



Conclusions

- Populations: (3,2), (4,2), (10,4), (7,6), (10,6), (11,6), (10,7), (11,7), (11,10) are the same.
- Ideal Chilling Time: 11.5 (9.908898, 13.097589)
- Effect of Chilling Time Decrease on Germination Rates: -0.041 (-0.066 -0.017)

Future

- Try Smoothing Splines

The End

Thanks for the great semester!