Exam C Formulas:

(Note: These are equations that I think are useful. You may want to add other equations to this list. If you would like the tex code for this, just email me at luiarthur@gmail.com)

Lesson 1:

$$h(x) = f(x)/S(x)$$
$$S(x) = e^{-H(x)}$$
$$C.V. = \sigma/\mu$$
$$Skewness = \mu_3/\sigma^3$$
$$Kurtosis = \mu_4/\sigma^4$$
$$n^{th} Central Moment : E[(X-\mu)^n]$$
$$n^{th} Raw Moment : E[X^n]$$

Lessons 2-3:

For Scaled Distributions:

$$X \sim Scaled(\theta) \Rightarrow Y = cX \sim Scaled(c\theta)$$

(Note that for this exam, the *continuous* distributions in your table use θ as the scale parameter.)

Transformations (Only Univariate):

$$f_Y(y) = f_X(g^{-1}(y))\ |g^{-1\,\prime}(y)|$$

What is e?

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

Bernoulli Shortcut:

If $X \sim$ Bern(p) then, for

$$\text{Y} = \begin{cases} a, \text{with probability p} \\ b, \text{with probability (1-p)} \end{cases}$$

$$Y = (a-b)X + b$$
$$Var(Y) = (a-b)^2 p(1-p)$$

Lesson 4:

Discrete Mixture Distributions:

If X is a Mixture Distribution then,

$$f_X(x) = \sum w_i f_{X_i}(x),$$

where $w_i \geq 0$ and $\sum w_i = 1$.

Review Continuous Mixture Distributions, Frailty Models, and Splices on your own.

Conditional Variance:

$$Var(X) = Var(E[X|Y]) + E(Var[X|Y])$$

Lesson 5:

$$E[X] = \int_0^\infty S(x)dx$$
$$E[X \wedge u] = \int_0^u S(x)dx$$
$$Y = (1+r)X \Rightarrow E[Y \wedge u] = (1+r)E[X \wedge \frac{u}{1+r}]$$

Lesson 6:

$$Ordinary:\quad Y = (X-d)_+ = max(0, x-d)$$
$$Franchise:\quad Y = \begin{cases} x, & \text{if } x > d \\ 0, & \text{otherwise.} \end{cases}$$

Pmt. per Loss: $E[Y^L] = E[(X-d)_+] = \int_d^\infty S(x)dx$

Pmt. per Pmt. (mean excess loss): $e_x(d) = \dfrac{E[(X-d)_+]}{S_X(d)}$

$$E[X] = E[X \wedge d] + E[(X-d)_+]$$

The expected pmt per pmt for a FRANCHISE deductible is $e_X(d) + d$.

Lesson 7:

$$LossEliminationRatio = LER(d) = \frac{E[X \wedge d]}{E[X]}$$

Lesson 8:

Let c be a constant, and X,Y be R.V's.
Translational Invariance: $\rho(X+c) = \rho(X) + c$
Positive Homogeneity: $\rho(cX) = c\rho(X)$
Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
Monotonicity: $\rho(X \leq Y) \Rightarrow \rho(X) \leq \rho(Y)$

Value at Risk:
$VaR_p(X) = F_X^{-1}(p)$
Tail Value at Risk:
$TVaR_p(X) = E[X|X > VaR_p(X)]$
$= VaR_p(X) + e_X(VaR_p(X))$
(See Tables)

Lesson 11:

ab0,ab1: See Tables

Lesson 12:

Poisson-Gamma
Given:

1. $X \sim NB(r, \beta)$
2. $X|\lambda \sim POI(\lambda)$

Then:
$\lambda \sim GAM(r, \beta)$

Sum of NB's in NB:
$X_i \sim NB(r_i, \beta)$
$\Rightarrow Y = \sum X_i = NB(\sum r_i, \beta)$

Lesson 13:

Exposures can refer to # of members in insured group OR # of time units they're insured for.

Let N_i be a frequency distribution.

1) $N_1 \sim POI(\lambda)$, with n_1 exposures
$\Rightarrow N_2 \sim POI(\lambda \frac{n_2}{n_1})$

2) $N_1 \sim NB(r, \beta)$, with n_1 exposures
$\Rightarrow N_2 \sim NB(r\frac{n_2}{n_1}, \beta)$

3) $N_1 \sim BIN(m, p)$, with n_1 exposures
$\Rightarrow N_2 \sim BIN(m\frac{n_2}{n_1}, p)$
where the new m must be an integer.

For (ab0,ab1) classes, suppose the probability of paying a claim $= P[X > d] = \nu$. Then if the model for loss frequency =

1) $N_1 \sim POI(\lambda)$
$\Rightarrow N_2 \sim POI(\nu\lambda)$

2) $N_1 \sim NB(r, \beta)$
$\Rightarrow N_2 \sim NB(r, \nu\beta)$

3) $N_1 \sim BIN(m, p)$
$\Rightarrow N_2 \sim BIN(m, \nu p)$

Lesson 14:

Collective Risk Model (N is R.V.)

$$E(S) = E[\sum X_i] = E[N]E[X]$$
$$Var(X) = E[N]Var(X) + E[X]^2 Var(N)$$

Lesson 16 (Agg. Loss - Severity Modf.):

Exp. Annual Pmts:

1. $E[Y^L]E[N^L]$ (easier for discrete losses)
2. $E[Y^P]E[N^P]$ (easier for continuous losses)

Lesson 18 (Agg. Loss - Agg. Ded.):

$$E[(S-d)_+] = E[S] - E[S \wedge d]$$

Lesson 21

$$Bias_{\hat{\theta}} = E[\hat{\theta} - \theta \mid \theta]$$
$$Consistent: \lim_{n \to \infty} P(|\hat{\theta} - \theta < \epsilon) = 1, \ \forall \theta > 0$$

Sufficient (but not necessary conditions for consistency:
1) $\hat{\theta_n}$ asymptotically unbiased for θ
2) $Var(\hat{\theta_n}) \to 0$.

UMVUE: Uniformly Minimum Variance Unbiased Estimator
$\Rightarrow$ An unbiased estimator with smallest possible variance.

MSE: Mean Squared Error.
$bias_{\hat{\theta}}(\theta)^2 + Var(\hat{\theta})$

95% C.I. Computed Using:
1) Approximate Variance: $\hat{\beta} \pm 1.96 * s.e.(\hat{\beta})$
2) True Variance: $-1.96 < \frac{x-\mu}{\sqrt{\frac{Var(X)}{n}}} < 1.96$

Lesson 22

For complete, individual data:

$$f_n(x) = \frac{n_j}{n}$$

For complete, grouped data:

$$f_n(x) = \frac{n_j}{n(c_j - c_{j-1})}$$

(Refer to the book to understand the notation.)