1. Define or state:
   a) Sufficient Statistic
   b) Minimal Sufficient Statistic
   c) Likelihood Function for θ
   d) Exponential Family
   e) Ancillary Statistic
   f) Complete Statistic
   g) Factorization Theorem
   h) Basu's Theorem
   i) Cramer Rao Lower Bound
   j) Fisher's Information for θ
   k) Minimum Variance Unbiased Estimator for θ
   l) Maximum Likelihood Estimator for θ
   m) Rao-Blackwell Theorem
   n) Lehmann-Scheffe Theorem
   o) Unbiased Estimator for θ
2. Suppose Y_i, $i = 1, \ldots, n$, are a random sample with $(Y_i|X_i = x_i) \sim \text{Poisson}(\lambda x_i)$.
   a) (10 points) Using the factorization criterion, find a non-trivial sufficient statistic for λ.
   b) (10 points) Find the maximum likelihood estimator of λ.
   c) (10 points) Calculate the bias for $\hat{\lambda}_{MLE}$.
   d) (10 points) Calculate the variance of $\hat{\lambda}_{MLE}$.
   e) (10 points) Calculate the Cramer-Rao Lower Bound for unbiased estimators of λ.
   f) (10 points) Is $\hat{\lambda}_{MLE}$ the UMVUE for μ? Justify your answer.

1. Define or state:
   a) Statistical hypothesis
   b) Hypothesis test
   c) Likelihood ratio test
   d) Power function
   e) Size α test
   f) Level α test
   g) Unbiased test
   h) Uniformly most powerful test
   i) Neyman-Pearson Lemma
   j) Karlin-Rubin Theorem
   k) p-value
   l) Valid p-value
   m) Interval estimator
   n) Coverage probability
   o) Confidence coefficient
   p) Pivotal quantity
   q) Credible set
   r) HPD credible set

2. Let X be a random variable with

θ	0	1	2
$f(x\|\theta)$	$2xI(0 \le x \le 1)$	$3x^2 I(0 \le x \le 1)$	$xe^x I(0 \le x \le 1)$

   a) Find the most powerful level 0.0975 test of $H_0 : \theta = 0$ versus $H_1 : \theta = 1$.
   b) What is the power of the test you found in 2a?
   c) Is the test obtained in part 2a the UMP level .0975 test for $H_0 : \theta = 0$ versus $H_1 : \theta \in \{1, 2\}$? Justify your answer.

3. $X_1, \ldots, X_n \sim U(\theta, 1)$
   a) Show that $X_{(1)}/(1 - \theta)$ is a pivotal quantity.
   b) Find a $1 - \alpha$ confidence interval for θ using the pivot.
   c) Find a $1 - \alpha$ confidence interval for θ by pivoting the CDF of $X_{(1)}$.

In addition to the material covered on previous exams, you should be able to:

1. Define or state:
   a) Consistency
   b) Asymptotic variance
   c) Asymptotic efficiency
   d) Asymptotic relative efficiency
   e) Asymptotic properties of the MLE
   f) Asymptotic distribution of the median
   g) Asymptotic distribution of the LRT
   h) Wald test
   i) Score test
2. Pick your favorite distribution:
   a) Is it a member of the exponential family?
   b) Find a minimal sufficient statistic.
   c) Find the MLE.
   d) Find the CRLB.
   e) Find the CRLB for $\tau(\theta)$
   f) Is the MLE UMVUE?
   g) Find the LRT for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$
   h) Find the UMP test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$.
   i) Find a pivotal quantity. Construct a confidence interval based on this pivotal quantity.
   j) Create a pivot from the CDF. Construct a confidence interval based on this pivotal quantity.
   k) Create an HPD credible set using a conjugate prior.
   l) What is the exact distribution of the MLE?
   m) What is the asymptotic distribution of the MLE?
   n) What is the asymptotic distribution of the LRT test you found above?