

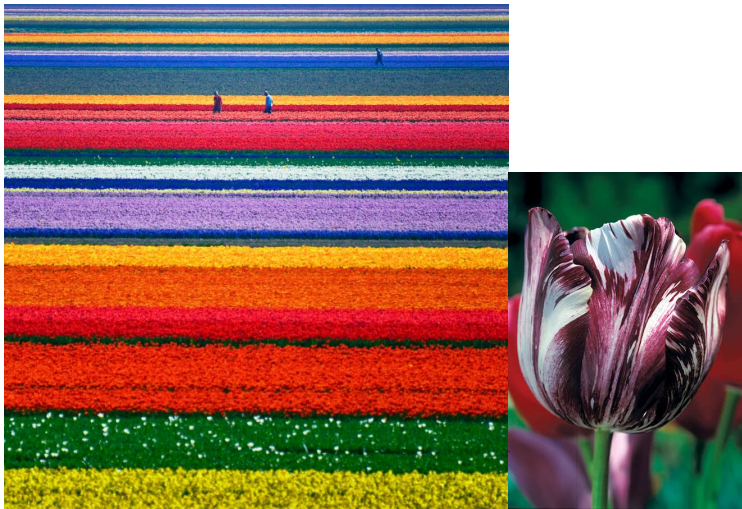
# Predicting Germination Rates of Different Tulip Populations at Various Chilling Times

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## Tulips in the Netherlands



**Figure:** Tulips popularized in the sixteenth century in Holland. During the tulipomania, a viceroy (right) bulb could allegedly be exchanged for a basket of goods, some furniture, *and* some live stock. Today, 9M bulbs are produced annually, and tulips account for 25% of agricultural exports.

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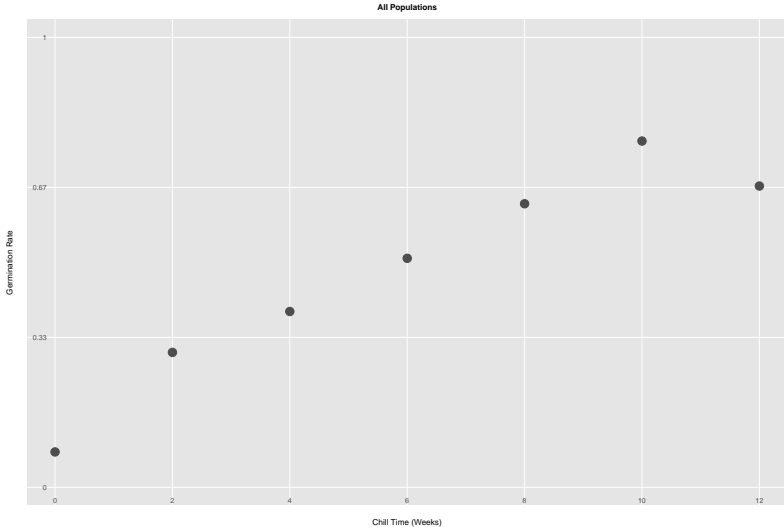
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  - ▶ How do the chill times differ by population?

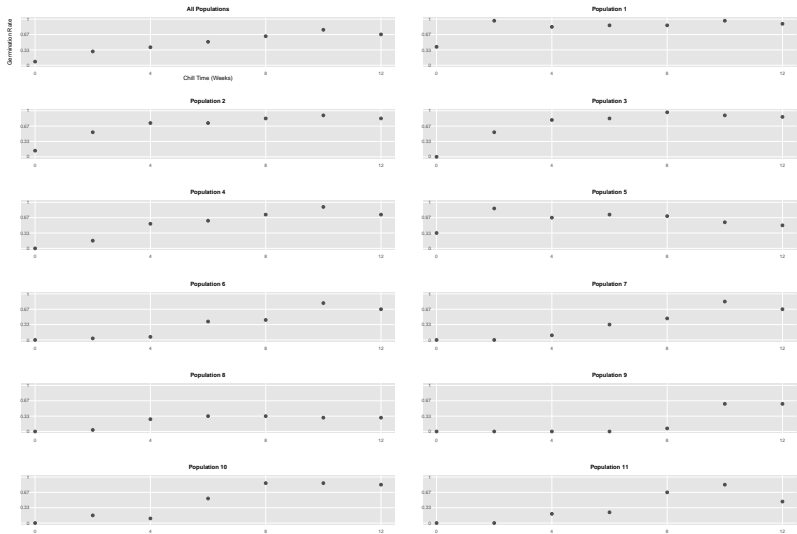
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**Figure:** Germination rates across all tulips for 7 chilling time (weeks). Thirty measurements were made at each chilling time for each tulip population



**Figure:** Germination rates of each tulip population by chilling times (weeks)

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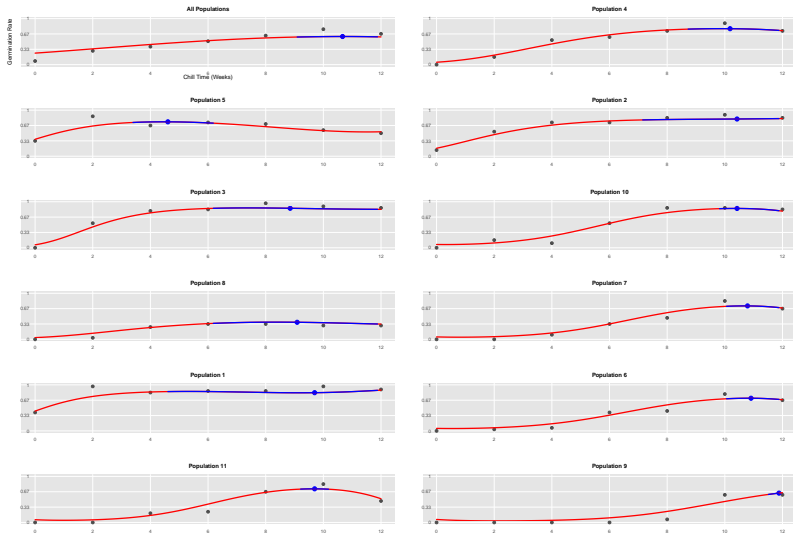
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Note that for this Bayesian probit model, we can make use of the Albert and Chib (1993) algorithm to obtain the posterior for  $\boldsymbol{\beta}$ .



# Results



**Figure:** Predicted germination rates of each population of tulips by chilling times (weeks)

The End

Questions?