The Frechet Distribution

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The Frechet Distribution

PDF:

$$\frac{\alpha}{s}\left(\frac{x-m}{s}\right)^{-1-\alpha} e^{-\frac{x-m}{s}^{-\alpha}}$$

Inverse CDF:

$$F^{-1}(x) = -log(x)^{-1/a}s + m,$$

Loglikelihood:

$$l(\alpha, m, s|\vec{x}) = nlog(\alpha) - nlog(s) - \sum_{i=1}^{n}(1+\alpha)log(\frac{x_i - m}{s}) + \left(\frac{x_i - m}{s}\right)^{-\alpha}$$

Maximum Likelihood

Hessian:

$$\begin{pmatrix} l_{aa} & l_{am} & l_{as} \\ l_{am} & l_{mm} & l_{sm} \\ l_{as} & l_{sm} & l_{ss} \end{pmatrix}$$

where l is the log likelihood.

Gradient:

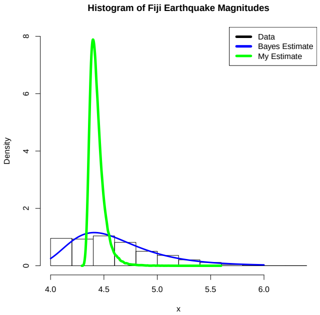
$$\nabla l = \begin{pmatrix} \frac{n}{\alpha} + \sum_{i=1}^{n} log\left(\frac{x_i-m}{s}\right)^2 \left(\frac{x_i-m}{s}\right)^{-\alpha} \\ \sum_{i=1}^{n} \left[\alpha + 1 - \alpha\left(\frac{x_i-m}{s}\right)^{-\alpha}\right] / (x_i - m) \\ -\frac{n}{s^2} + \frac{1}{s} \sum_{i=1}^{n} \alpha + 1 - \alpha \left(\frac{x_i-m}{s}\right)^{-\alpha} \end{pmatrix}$$

My Own Estimator

Since x $>$ m (the location parameter), I estimated m with min(x) (i.e. the smallest value in my dataset.)

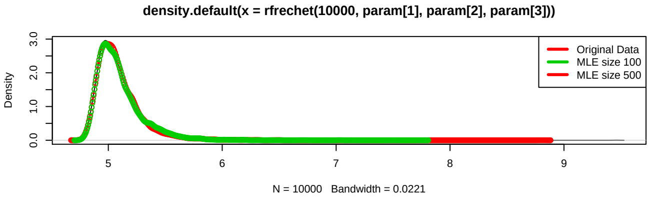
$$
\begin{aligned}
m &= min(x) \\
s &= sd(x) \\
a &= mean(x) + min(x)
\end{aligned}
$$

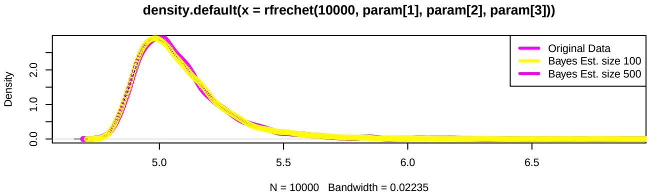
Parameter Estimation for Frechet Dataset

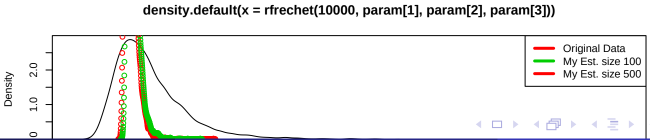


Unfortunately, the MLE's did not converge when I used MY Newton Raphson algorithm in C (or R), so I could not generate a plot for my MLE for this data set.

Graphs


density.default(x = rfrechet(10000, param[1], param[2], param[3]))
Density
0.0
1.0
2.0
3.0
5
6
7
8
9
N = 10000 Bandwidth = 0.0221
Original Data
MLE size 100
MLE size 500


density.default(x = rfrechet(10000, param[1], param[2], param[3]))
Density
0.0
1.0
2.0
5.0
5.5
6.0
6.5
N = 10000 Bandwidth = 0.02235
Original Data
Bayes Est. size 100
Bayes Est. size 500


density.default(x = rfrechet(10000, param[1], param[2], param[3]))
Density
1.0
2.0
Original Data
My Est. size 100
My Est. size 500