More on Hypothesis Testing

$\text{BIC} = -2\log(L(\hat{\theta})) + p\log(n)$ $\text{AIC} = -2\log(L(\hat{\theta})) + 2p$

We want BIC,AIC to be small.

Compare M_1, M_2.

$$S = -.5[BIC(M_1) - BIC(M_2)]$$

We can show that $-2S = -2\log(BF(M_1, M_2))$.

Bayes Factors: Computation

- Using Laplace approx
- Using posterior samples

Laplace Approximation

$$p(x|H_i) = \int p_i(x|\theta_i)\pi_i(\theta_i|H_i)d\theta_i \quad = \int \exp\left(-n\{-\frac{1}{n}\log p_i(x|\theta_, H_i) - \frac{1}{n}\log \pi_i(\theta_i|H_i)\}\right) d\theta_i,$$

where H_i can be a model.

$$BF(H_0, H_1) \approx (2n\pi)^{(p_0-p_1)/2} \frac{p_0(x|\hat{\theta}_0)\pi(\hat{\theta}_0)}{p_1(x|\hat{\theta}_1)\pi(\hat{\theta}_1)} \sqrt{\frac{\Sigma_0}{\Sigma_1}}$$

$$\begin{aligned} p(x|H_i) &= \int p_i(x|\theta_i)\pi_i(\theta_i|H_i)d\theta_i \\ \theta_i^{(1)}, ..., \theta_i^{(M)} &\sim \pi_i(\theta_i|H_i) \\ \hat{p}(x|H_i) &= \sum_{i=1}^{M} p_i(x|\theta_i^{(m)}, H_i)/M \end{aligned}$$

If we have posterior samples, and $\dim(\theta_1) = \dim(\theta_0)$

$$BF(M_0, M_1) = \sum_{i=1}^{M} \frac{p_0(x|\theta^{(m)}, M_0)\pi_0(\theta^{(m)})}{p_1(x|\theta^{(m)}, M_1)\pi_1(\theta^{(m)})}/M$$

Case II: Nested (dimension of parameters not the same)

Example:

$$\begin{aligned} M0: \quad y_i &= \beta_0 + \beta_1 x_i + \epsilon_i \\ M1: \quad y_i &= \beta_0^* + \beta_1^* x_i + \beta_2^* z_i + \epsilon_i \end{aligned}$$

Int this case it is also possible to obtain an approximation to the BF using samples from the posterior distribution of M_1. The approximation will have the form

$$\sum_{m=1}^{M} r(\theta^{(m)})$$

Exercise: find $r(\theta^{(m)}$

Hierarchical Models

Putting priors on parameters.