$v_k \mid \alpha \sim Be(\alpha/K, 1)$, $\alpha \sim Ga(a_\alpha, b_\alpha)$ K : fixed $\boxed{K \geq 2}$ 4/17/18

$Z = [z_1; z_2; \cdots ; z_K]$

$\uparrow$

$(J \times K)$ binary matrix

z_k, $k=1, \ldots, K$; J-dim binary vector (each defines a latent cell phenotype)

$$Z \sim Rep_{J,K}(f_0, C_0, \rho)$$

$$f_{0k} = \prod_{j=1}^{J} (v_k)^{z_{jk}} (1-v_k)^{1-z_{jk}}$$

$$C_0(r) = \exp\left\{-\frac{r^2}{2\nu}\right\}, \quad \text{fixed } \nu \in \mathbb{R}^+ \quad \boxed{0 < C_0(r) \leq 1}$$

$C_0(r) = 1$ if $r = 0$

$$r_{k_1,k_2} = \sqrt{\sum_{j=1}^{J} |z_{jk_1} - z_{jk_2}|}$$: difference between z_{k_1} and z_{k_2}

$$\Rightarrow p(Z) \propto \prod_{k=1}^{K} f_{0k} \cdot \underbrace{\prod_{k_1=1}^{K-1} \prod_{k_2=k_1+1}^{K} \left\{1 - C_0(r_{k_1,k_2})\right\}}_{A}$$

$$\propto \prod_{k=1}^{K} \prod_{j=1}^{J} (v_k)^{z_{jk}} (1-v_k)^{1-z_{jk}} \cdot \prod_{k_1=1}^{K-1} \prod_{k_2=k_1+1}^{K} \left\{1 - \exp\left(-\frac{\sum_{j=1}^{J} |z_{jk_1} - z_{jk_2}|}{2\nu}\right)\right\}$$

① If Z has identical columns, $\underline{P(Z) = 0}$

② If z_{k_1} and z_{k_2} are very different, $\rho(z_{k_1}, z_{k_2})$ is big and

and $\left\{1 - \exp\left(-\frac{r^2_{k_1,k_2}}{2\nu}\right)\right\}$ is large (close to 1) $\Rightarrow$ increase $P(Z)$

$\Rightarrow$ $Rep_{J,K}$ promotes diverse z_k's.

③ Due to Ⓐ, $P(z_{jk} = 1) \neq v_k$ $\Rightarrow$ no closed form.

$P(z_{jk} = 1)$ depends on $\underline{\nu}$, $\underline{K}$, J as well as $\underline{(v_k, k=1, \ldots, K)}$

We can numerically evaluate $\underline{P(z_{jk}=1)}$ for any given $\underline{(\nu, K, J, \{V_k; k=1,\dots,K\})}$

(4) Other functions can be used for (A) for example, ...

Posterior updating

(1) z_{jk}

$$P(z_{jk}=0 \mid -) \propto (1-V_k)\cdot \prod_{k' \neq k} \left\{ 1 - \exp\left(- \frac{\sum_{j=1}^{J} |z_{jk} - z_{jk'}|}{2\nu} \right) \right\} \times P(\text{---} \mid z_{jk}=0)$$

$$P(z_{jk}=1 \mid -) \propto V_k \cdot \prod_{k' \neq k} \left\{ 1 - \exp\left(- \frac{\sum_{j=1}^{J} |z_{jk} - z_{jk'}|}{2\nu} \right) \right\} \times P(\text{---} \mid z_{jk}=1)$$

(2) Entire column $\boxed{z_k}$

Propose z_k' by letting $z_{jk}' \neq z_{jk}$ w/p a (diff)

$z_{jk}' = z_{jk}$ $(1-a)$ (same)

$$\alpha = \min\left(1, \frac{\prod_{j=1}^{J} (V_k)^{z_{kj}'} (1-V_k)^{1-z_{kj}'}}{\prod_{j=1}^{J} (V_k)^{z_{kj}} (1-V_k)^{1-z_{kj}}} \times \frac{\prod_{k' \neq k} \left\{ 1 - \exp\left(- \frac{\sum_{j=1}^{J} |z_{jk}' - z_{jk'}|}{2\nu} \right) \right\}}{\prod_{k' \neq k} \left\{ 1 - \exp\left(- \frac{\sum_{j=1}^{J} |z_{jk} - z_{jk'}|}{2\nu} \right) \right\}} \times \frac{P(\text{---} \mid z_k')}{P(\text{---} \mid z_k)} \right)$$

(J, K, ν, V)

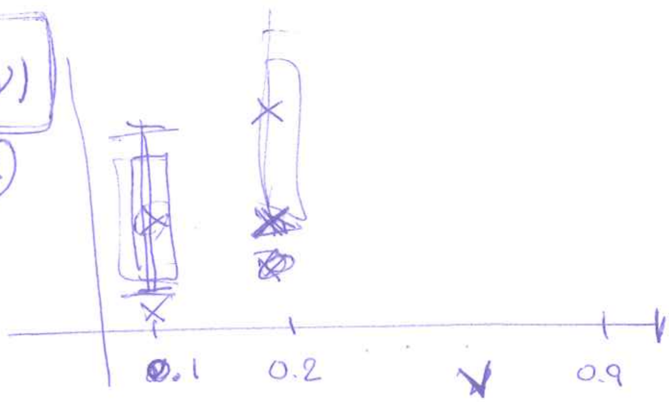
$(\ Z\ ) \longrightarrow p(z_{jk} = 1) \neq V_k$

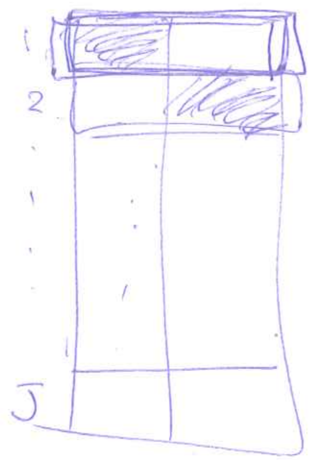
$$\boxed{D_Z = \sum_{\text{all pairs}} r^2_{k_1, k_2} \Big/ \binom{K}{2}} > \boxed{J \times (V_k(1-V_k) \cdot 2)}$$

$$\boxed{V_1 = V_2 = \cdots = V_K}$$

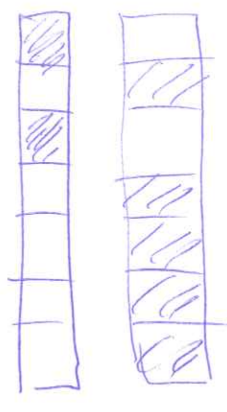
$\boxed{(J, K, \nu)}$

2

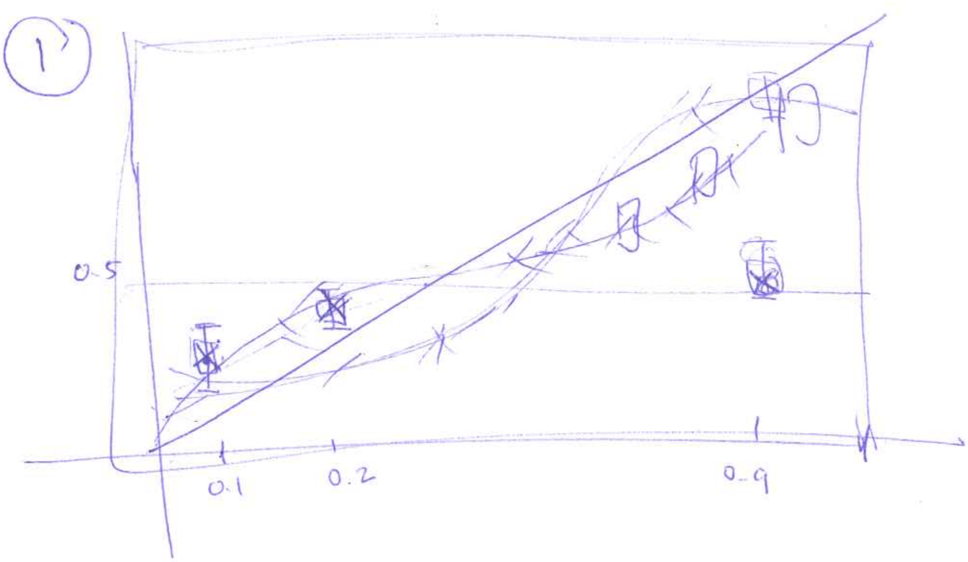




$\boxed{V_k(1-V_k) \times 2}$



1



V