

Problema 2-01

Expresiones de circuitos

A partir de las tablas de verdad de las siguientes funciones, obtenga las expresiones algebraicas de dichas funciones y los circuitos lógicos que las realizan:

| XY | F1 | F2 | F3 |
|----|----|----|----|
| 00 | 1 | 0 | 1 |
| 01 | 0 | 1 | 1 |
| 10 | 1 | 1 | 1 |
| 11 | 0 | 0 | 0 |

$$F1 = Y'$$

$$F2 = X \oplus Y$$

$$F3 = (XY)'$$

| XYZ | F4 | F5 | F6 | F7 | F8 | F9 |
|-----|----|----|----|----|----|----|
| 000 | 0 | 1 | 0 | 1 | 1 | 1 |
| 001 | 1 | 0 | 0 | 1 | 0 | 0 |
| 010 | 0 | 0 | 0 | 1 | 1 | 1 |
| 011 | 0 | 1 | 0 | 0 | 0 | 0 |
| 100 | 0 | 1 | 1 | 1 | 1 | 1 |
| 101 | 1 | 1 | 1 | 1 | 1 | 0 |
| 110 | 0 | 0 | 0 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 1 | 1 | 0 |

$$F4 = \Sigma(1, 5) = X'Y'Z + XY'Z = Y'Z(X' + X) = Y'Z$$

$$F5 = \Sigma(0, 3, 4, 5) = \dots$$

$$F6 = \Sigma(4, 5) = XY'Z' + XY'Z = XY'(Z' + Z) = XY'$$

$$F7 = \Pi(3) = (X + Y' + Z')$$

$$F8 = \Pi(1, 3) = (X + Y + Z')(X + Y' + Z') = (X + Z') + \cancel{YY'}$$

$$F9 = Z'$$

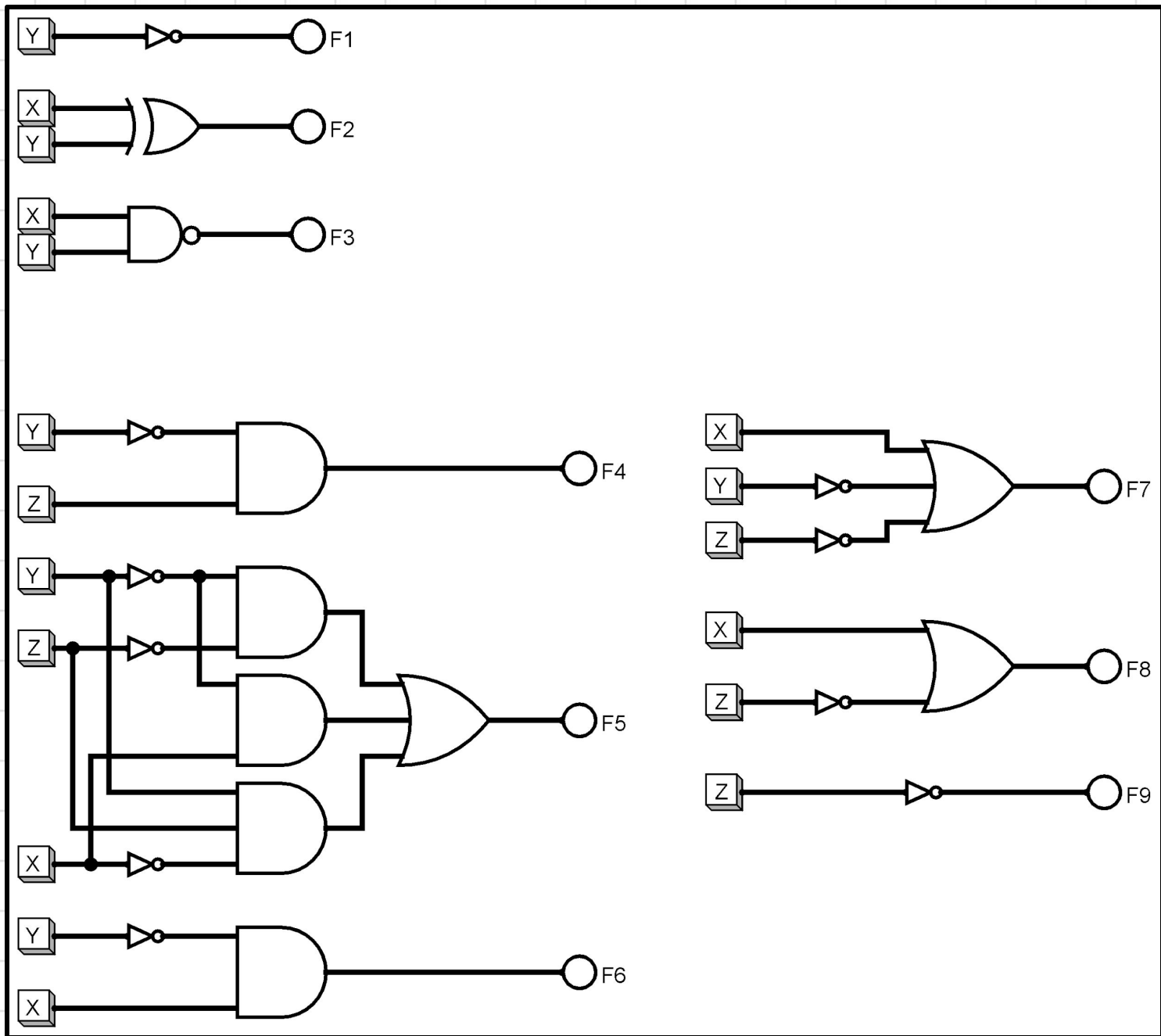
$$F5 = \Sigma(0, 3, 4, 5)$$

$$F = Y'Z' + XY' + X'YZ$$

| | | YZ | | | |
|---|---|-----|-----|-----|----|
| | | 00 | 01 | 11 | 10 |
| X | 0 | 0 1 | 1 | 3 1 | 2 |
| | 1 | 4 1 | 5 1 | 7 | 6 |

Diagram illustrating the Karnaugh map for the function $F = Y'Z' + XY' + X'YZ$. The map shows the function value (1 or 0) for each combination of variables X, Y, and Z. The groups of 1s are circled in blue, red, and green, corresponding to the terms in the expression:

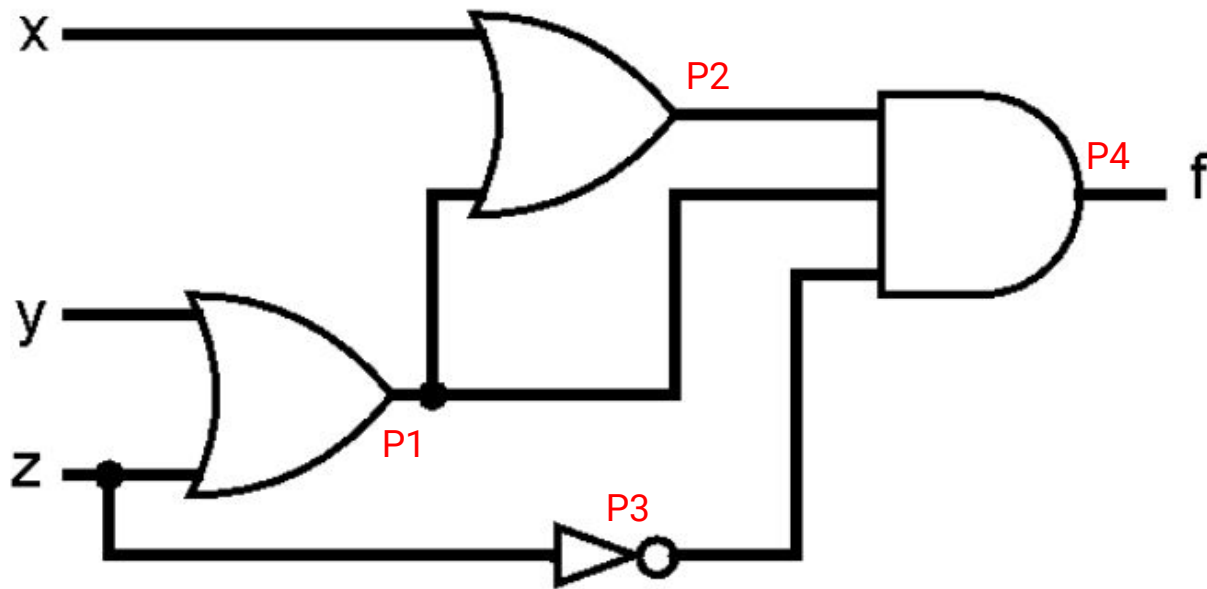
- Blue circle: $Y'Z'$ (cells 0, 4)
- Red circle: XY' (cells 4, 5)
- Green circle: $X'YZ$ (cell 3)



Problema 2-02

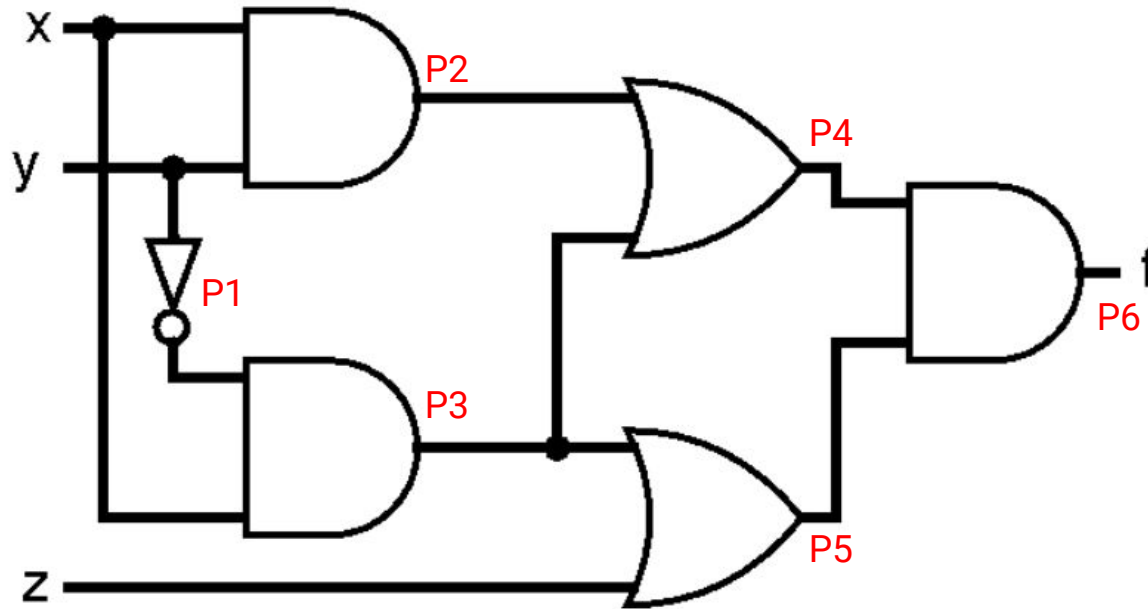
Análisis

a)



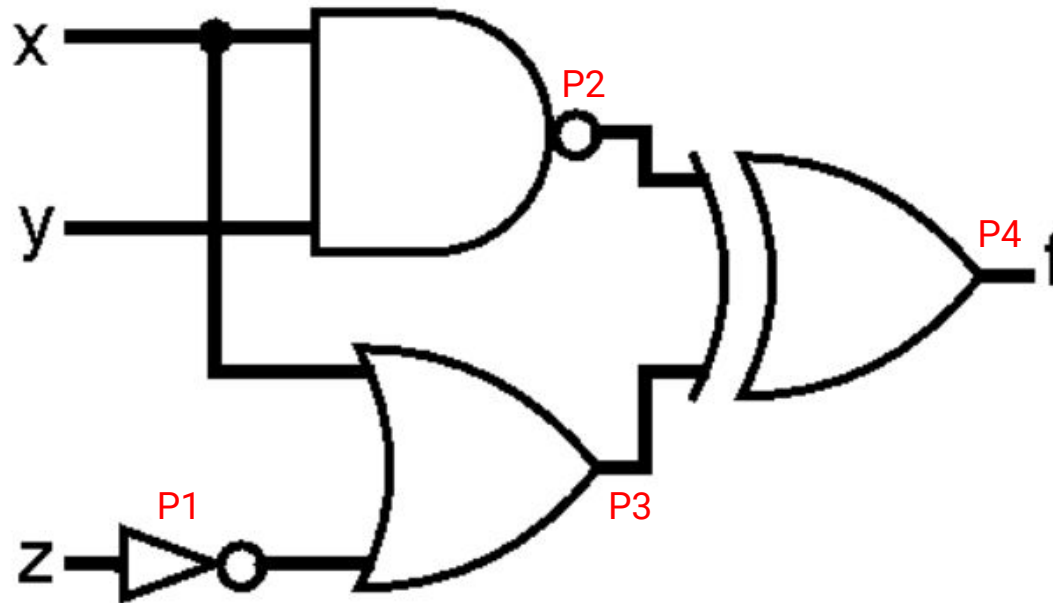
$$\begin{aligned} F = P4 &= P1 \cdot P2 \cdot P3 = (X + P2) \cdot P2 \cdot Z' = X \cdot P2 \cdot Z' + P2 \cdot Z' = \\ &= P2 \cdot Z' = (Y + Z)Z' = YZ' + ZZ' = YZ' \end{aligned}$$

b)



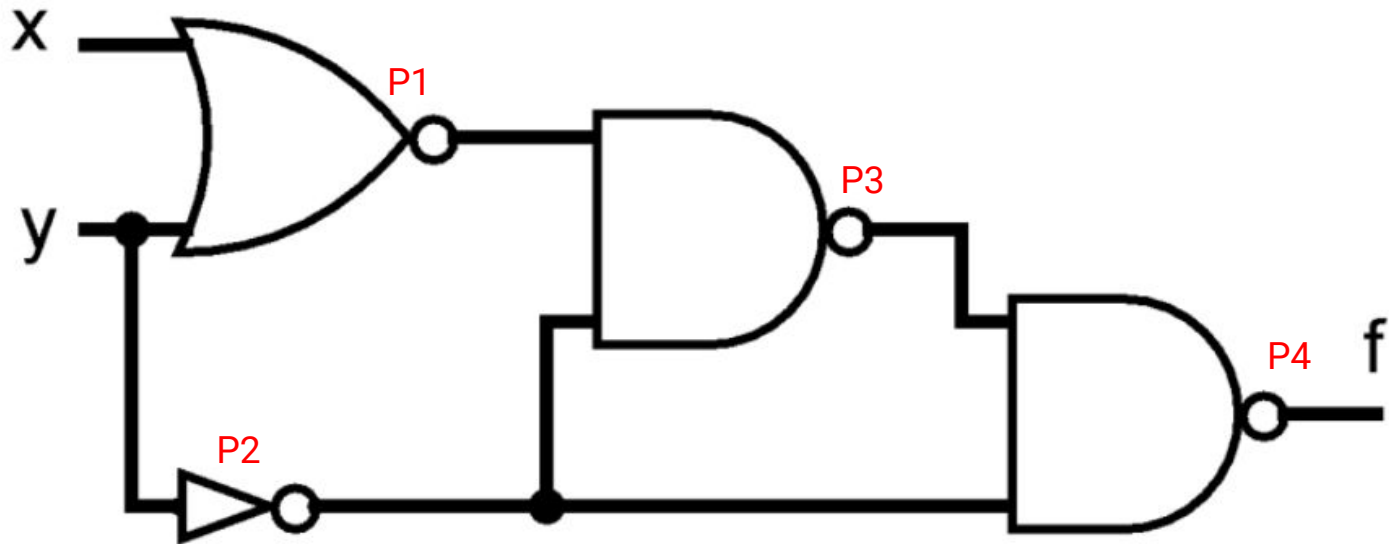
$$\begin{aligned}
 F &= P6 = P4 \cdot P5 = (P2 + P3)(P3 + Z) = \\
 &= (XY + Y'X)(Y'X + Z) = \\
 &= XY Y'X + XYZ + Y'X Y'X + Y'XZ = \\
 &= \cancel{XY Y'X} + XYZ + XY' + XY'Z = \\
 &= XYZ + XY' + XY'Z = \\
 &= XYZ + XY' = X(YZ + Y') = \\
 &= X(Z + Y') = XZ + XY'
 \end{aligned}$$

c)



$$\begin{aligned}
 F &= P4 = P2 \oplus P3 = P2'P3 + P2P3' = \\
 &= (XY)(X + Z') + (XY)'(X + Z')' = \\
 &= XYX + XYZ' + (X' + Y')(X'Z) = \\
 &= XYX + XYZ' + X'X'Z + Y'X'Z = \\
 &= XY + \cancel{XYZ'} + X'Z + \cancel{X'ZY'} = XY + X'Z
 \end{aligned}$$

d)

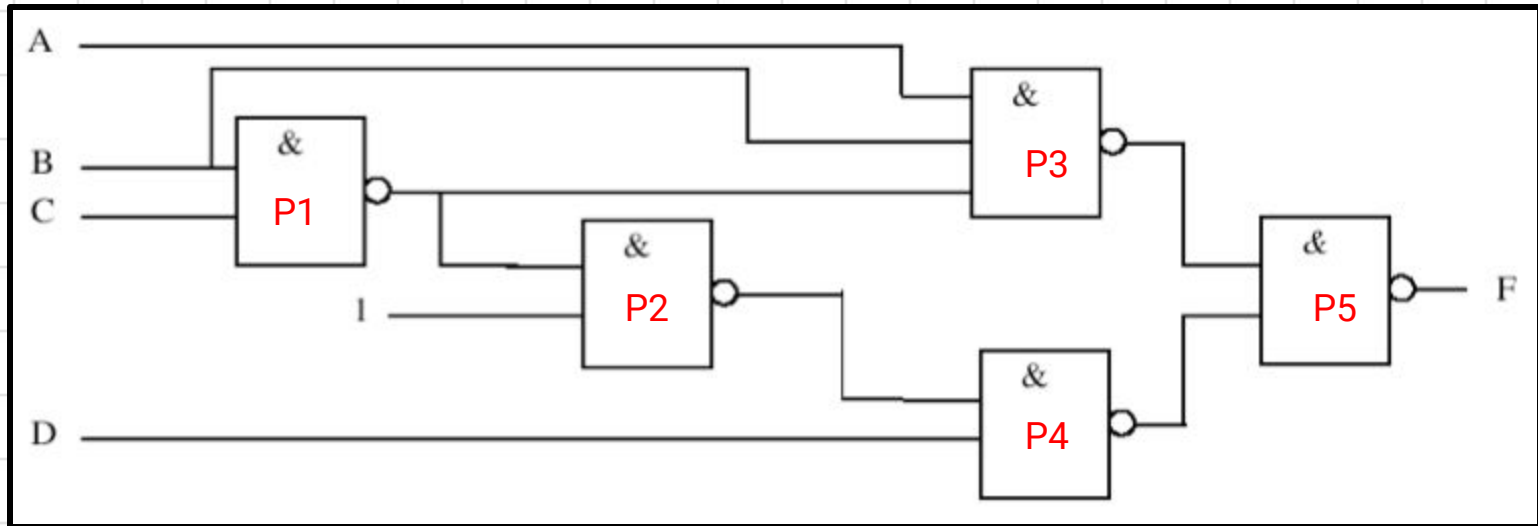


$$\begin{aligned}
 F &= P4 = P2 \odot P3 = P2 \odot (P1 \cdot P2)' = \\
 &= P2 \odot ((X + Y)' \cdot P2)' = \\
 &= Y' \odot ((X + Y)' \cdot Y')' = Y' \odot (X'Y'')' = Y' \odot (X + Y) = \\
 &= Y'(X + Y) + Y(X + Y)' = Y'X + \cancel{Y'Y} + \cancel{YX'Y'} = \\
 &= XY'
 \end{aligned}$$

Problema 2-03

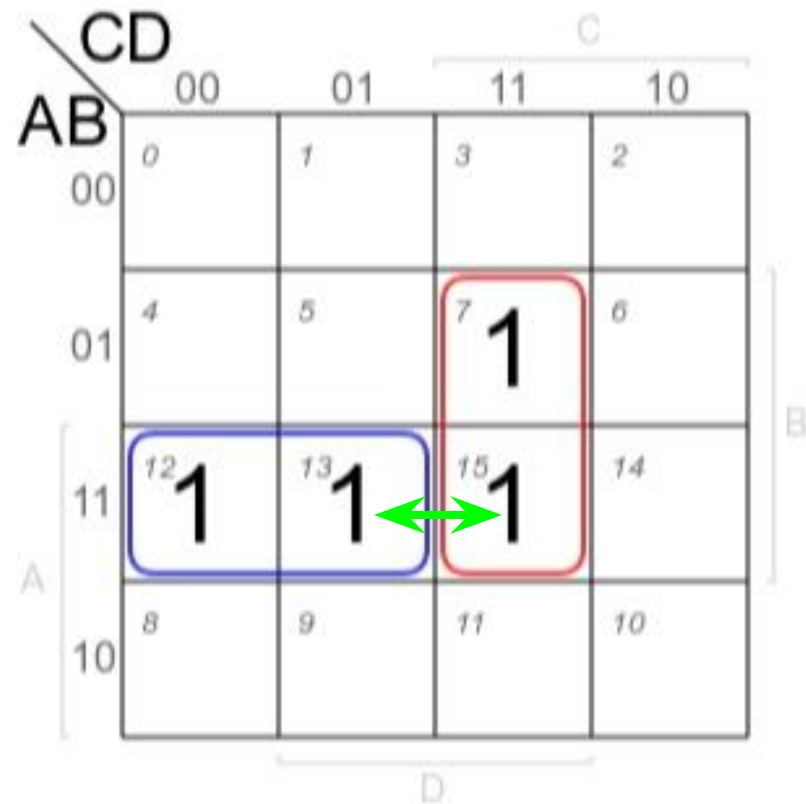
Azares

a)



$$\begin{aligned}
 F = P5 &= (P3 \cdot P4)' = P3' + P4' = A \cdot B \cdot P1 + P2 \cdot D = \\
 &= AB(BC)' + P1'D = AB(B' + C') + BCD = \\
 &= \cancel{ABB'} + ABC' + BCD = ABC' + BCD
 \end{aligned}$$

$$F = ABC' + BCD$$



b)

```
module prob203b(input A, B, C, D, output F);  
  
    wire w1, w2, w3, w4;  
  
    nand #2 p1(w1, B, C);  
    nand #2 p2(w2, w1, 1);  
    nand #2 p3(w3, A, B, w1);  
    nand #2 p4(w4, w2, D);  
    nand #2 pF(F, w3, w4);  
  
endmodule
```

c)

```
module prob203c;
```

```
    reg A, B, C, D;
```

```
    wire F;
```

```
    prob203b dut(A, B, C, D, F);
```

```
    initial begin
```

```
        $dumpfile("prob203c.vcd");
```

```
        $dumpvars;
```

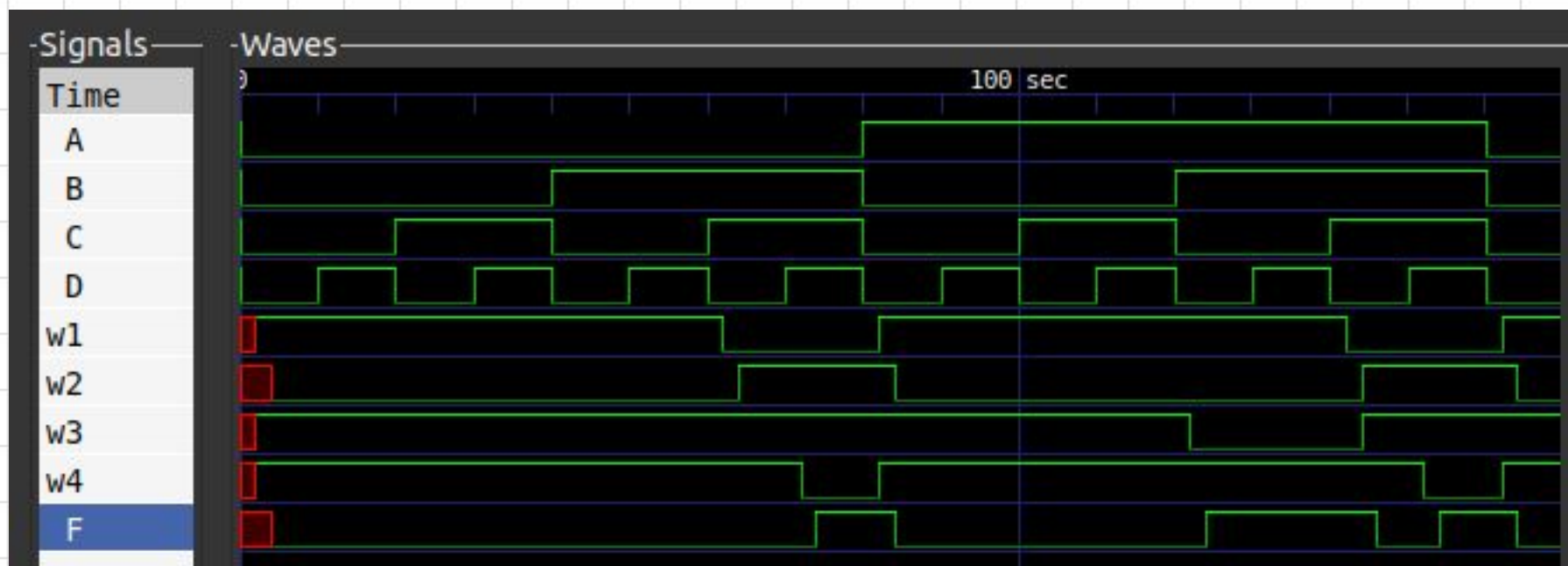
```
        {A, B, C, D} = 0;
```

```
        #170 $finish;
```

```
    end
```

```
    always #10 {A, B, C, D} = {A, B, C, D} + 1;
```

```
endmodule
```



d)

```
module prob203d;

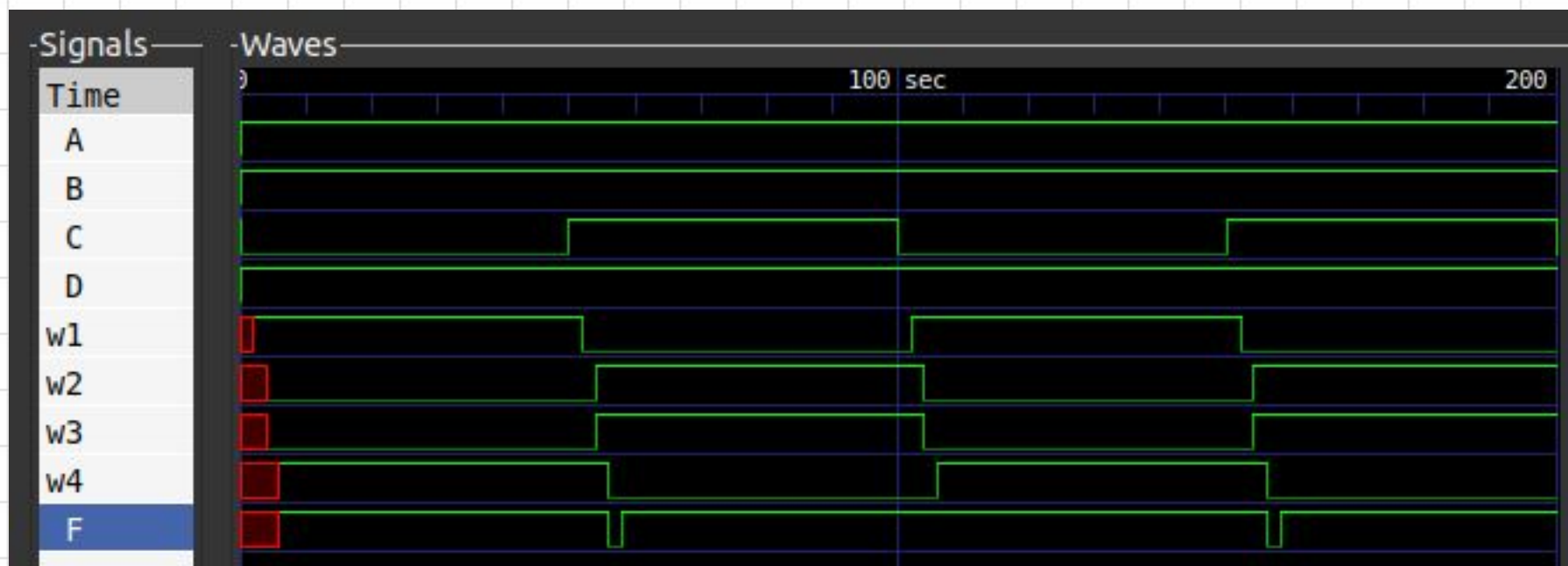
    reg A, B, C, D;
    wire F;

    prob203b dut(A, B, C, D, F);

    initial begin
        $dumpfile("prob203d.vcd");
        $dumpvars;
        {A, B, C, D} = 4'b1101;
        #200 $finish;
    end

    always #50 C = ~C;

endmodule
```



e)

```
module prob203e;
```

```
    reg A, B, C, D;
```

```
    wire F;
```

```
    prob203b dut(A, B, C, D, F);
```

```
    initial begin
```

```
        $dumpfile("prob203e.vcd");
```

```
        $dumpvars;
```

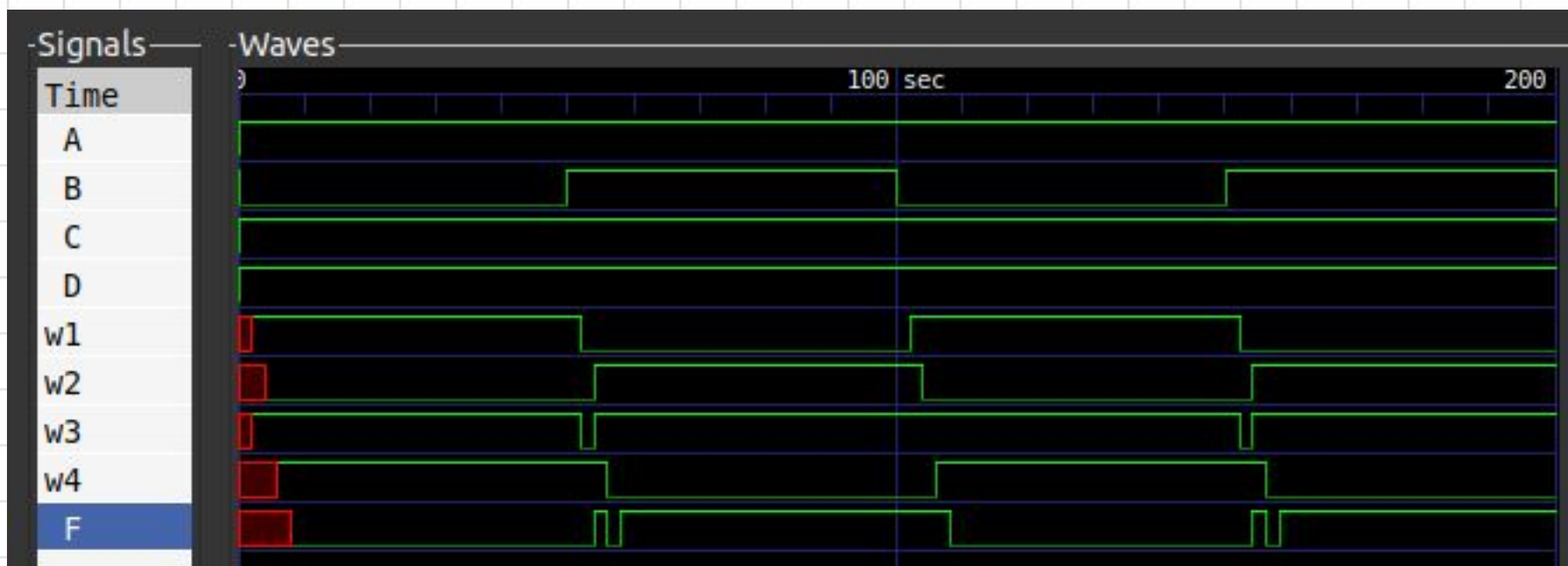
```
        {A, B, C, D} = 4'b1011;
```

```
        #200 $finish;
```

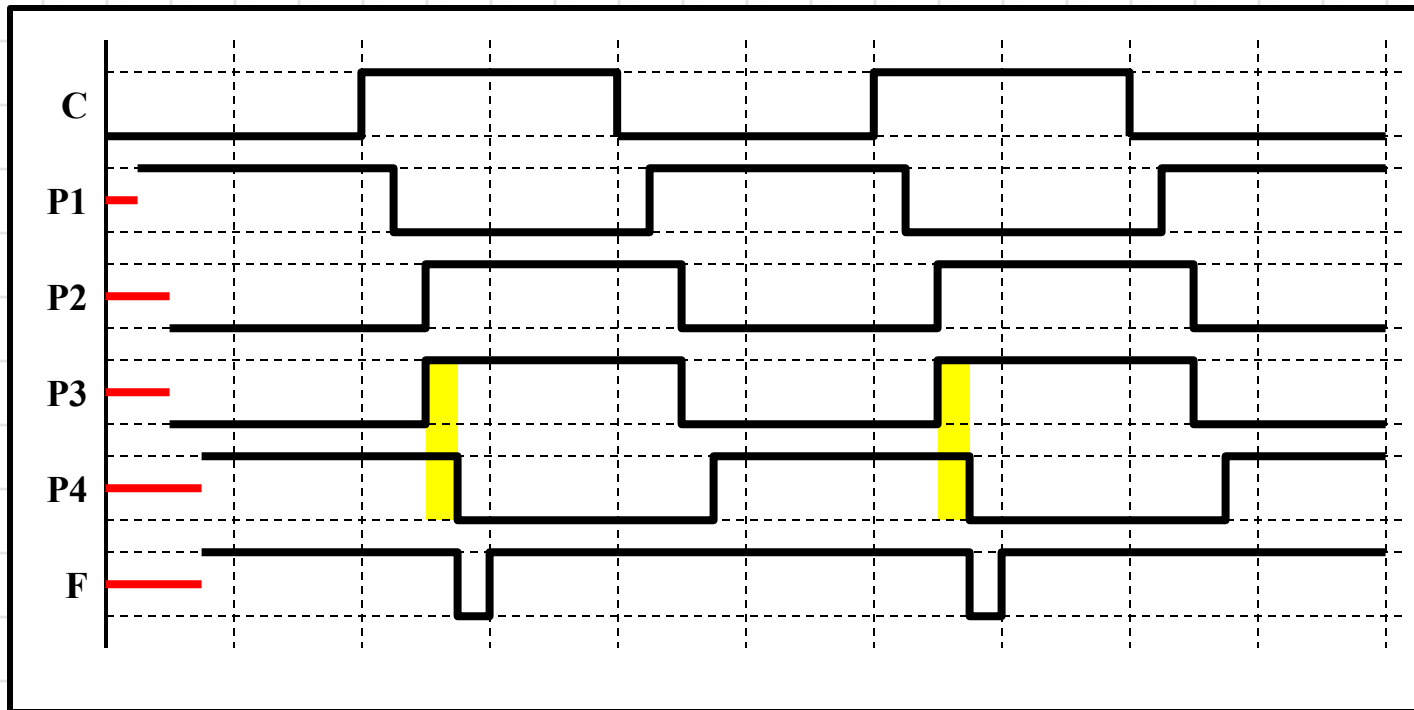
```
    end
```

```
    always #50 B = ~B;
```

```
endmodule
```

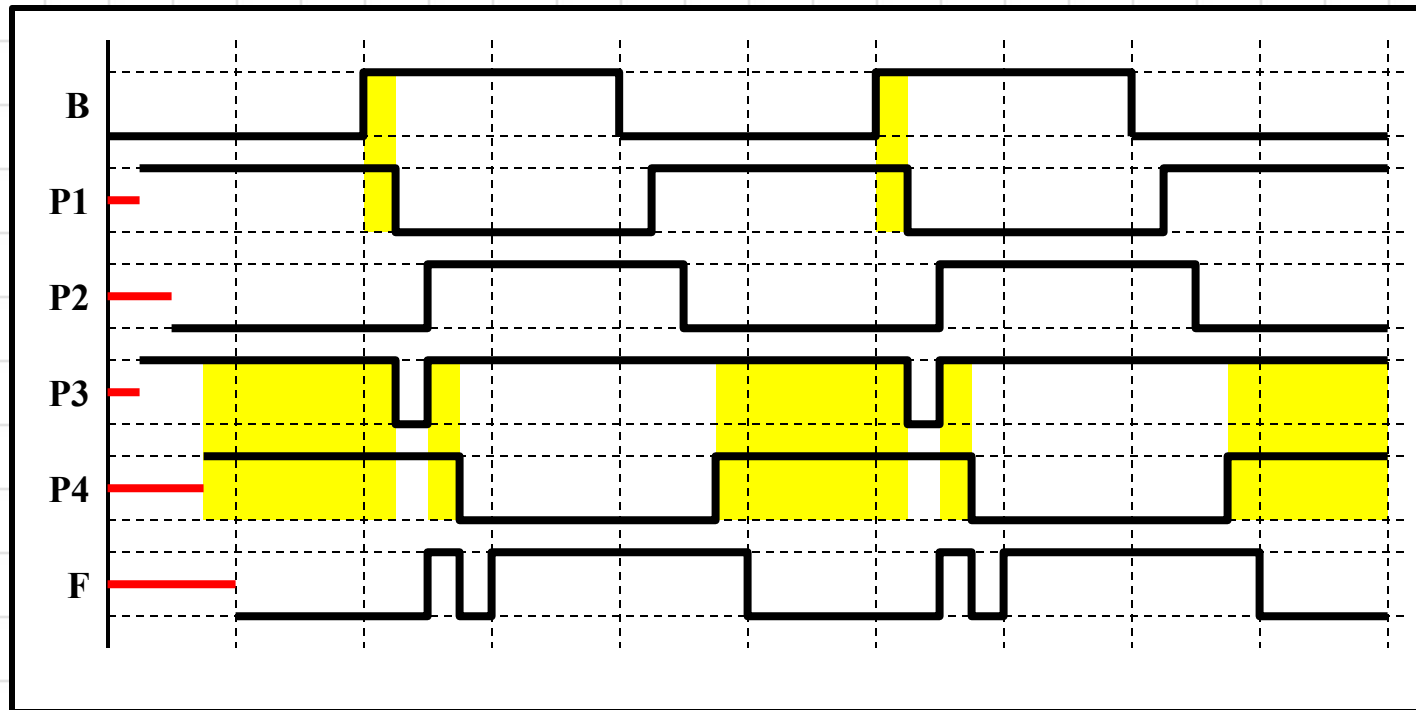


b) $A = B = D = 1$



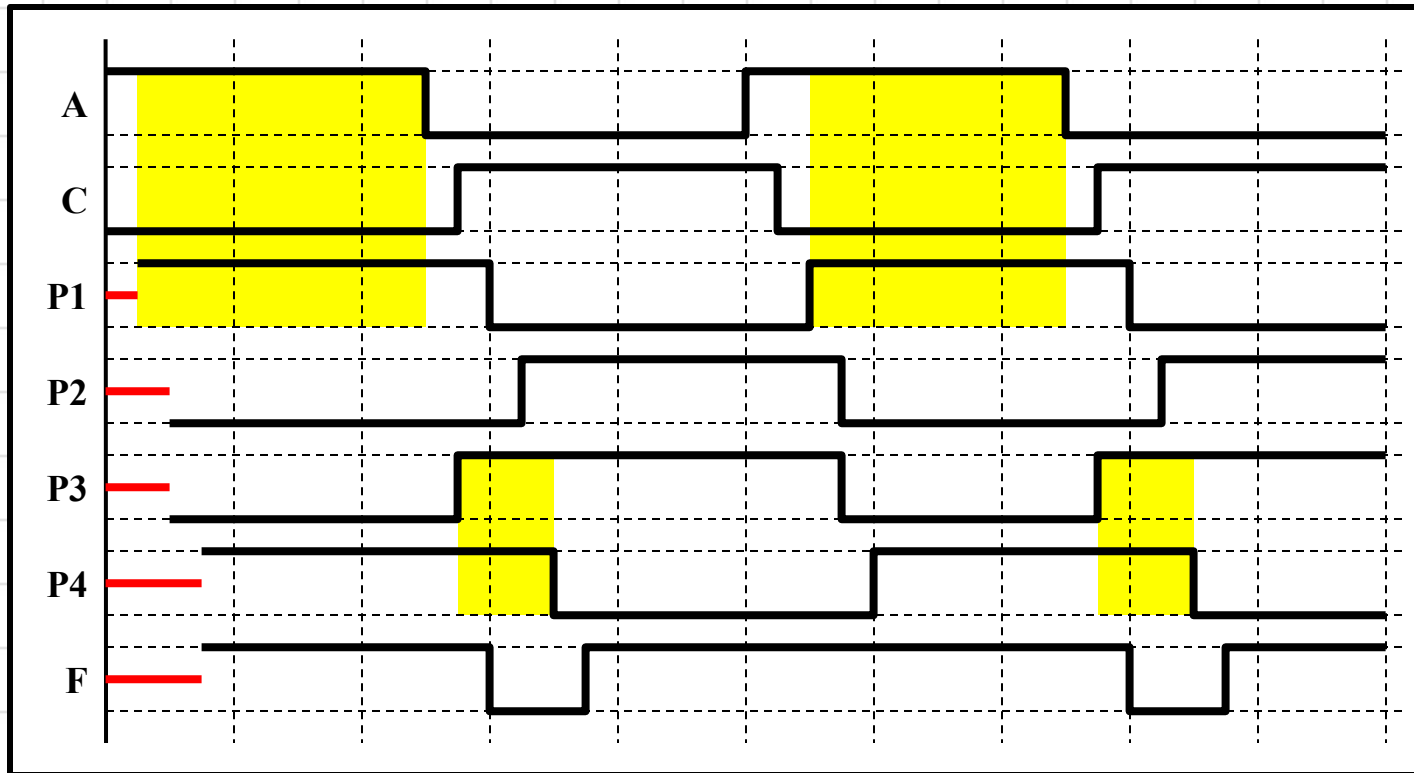
$$F = ABC' + BCD = 1 \cdot 1 \cdot C' + 1 \cdot C \cdot 1 = C' + C = 1$$

c) $A = C = D = 1$



$$F = ABC' + BCD = 1 \cdot B \cdot 0 + B \cdot 1 \cdot 1 = B$$

d) $B = D = 1$



$$F = ABC' + BCD = AC' + C = A + C$$

Problema 2-04

Relaciones mínimas

a)

Suma de Productos:

ZU

XY

| | 00 | 01 | 11 | 10 |
|----|------|------|------|------|
| 00 | 0 | 1 | 3 1 | 2 |
| 01 | 4 1 | 5 | 7 1 | 6 |
| 11 | 12 1 | 13 1 | 15 | 14 1 |
| 10 | 8 1 | 9 | 11 1 | 10 1 |

$$F = XU' + YZ'U' + X'ZU + XY'Z + XYZ'$$

Productos de sumas:

ZU

XY

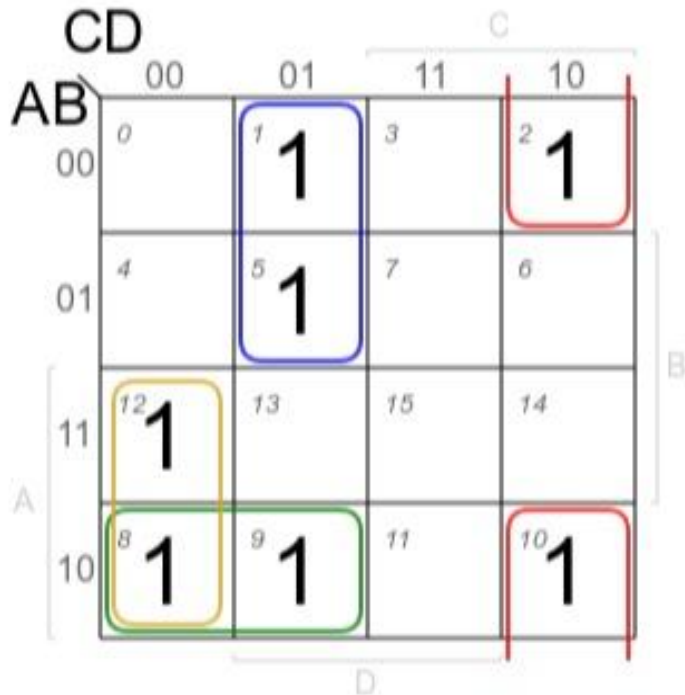
| | 00 | 01 | 11 | 10 |
|----|------|------|------|------|
| 00 | 0 0 | 1 0 | 3 1 | 2 0 |
| 01 | 4 1 | 5 0 | 7 1 | 6 0 |
| 11 | 12 1 | 13 1 | 15 0 | 14 1 |
| 10 | 8 1 | 9 0 | 11 1 | 10 1 |

$$F = (X+Y+U)(X+Z+U')(Y+Z+U') \cdot (X+Z'+U)(X'+Y'+Z'+U')$$

f)

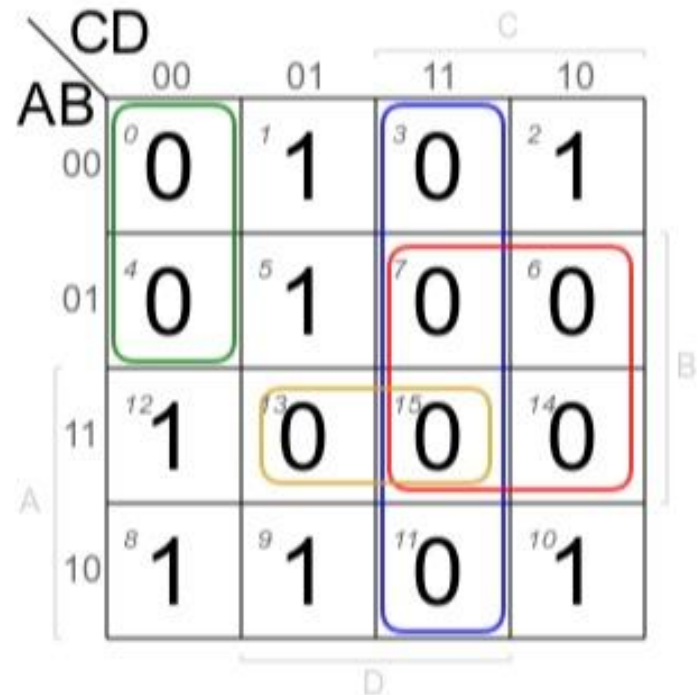
Suma de Productos:

$$F = A'C'D + B'CD' + AB'C' + AC'D'$$



Productos de sumas:

$$F = (C' + D')(B' + C')(A + C + D)(A' + B' + D')$$



g) Suma de Productos:

$$F = C'E + CE' + A'BE' + BD'E' + AB'C$$

| DE \ BC | | D | | | |
|---------|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| BC | 00 | 0 | 1 | 1 | 2 |
| | 01 | 4 | 1 | | 6 |
| | 11 | 12 | | | 14 |
| | 10 | 8 | 1 | 1 | 10 |

$$A = 0$$

| DE \ BC | | D | | | |
|---------|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| BC | 00 | 16 | 17 | 19 | 18 |
| | 01 | 20 | 21 | 23 | 22 |
| | 11 | 28 | 29 | 31 | 30 |
| | 10 | 24 | 25 | 27 | 26 |

$$A = 1$$

Productos de sumas:

$$F = (B + C + E)(A + C' + E')(B' + C' + E')(A' + C + D' + E)$$

| DE \ BC | | D | | | |
|---------|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| BC | 00 | 0 | 1 | 1 | 0 |
| | 01 | 1 | 0 | 0 | 1 |
| | 11 | 1 | 0 | 0 | 1 |
| | 10 | 1 | 1 | 1 | 1 |

$$A = 0$$

| DE \ BC | | D | | | |
|---------|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| BC | 00 | 16 | 17 | 19 | 18 |
| | 01 | 20 | 21 | 23 | 22 |
| | 11 | 28 | 29 | 31 | 30 |
| | 10 | 24 | 25 | 27 | 26 |

$$A = 1$$

Problema 2-05

Valor absoluto

a)

ABC Valor XYZ Valor

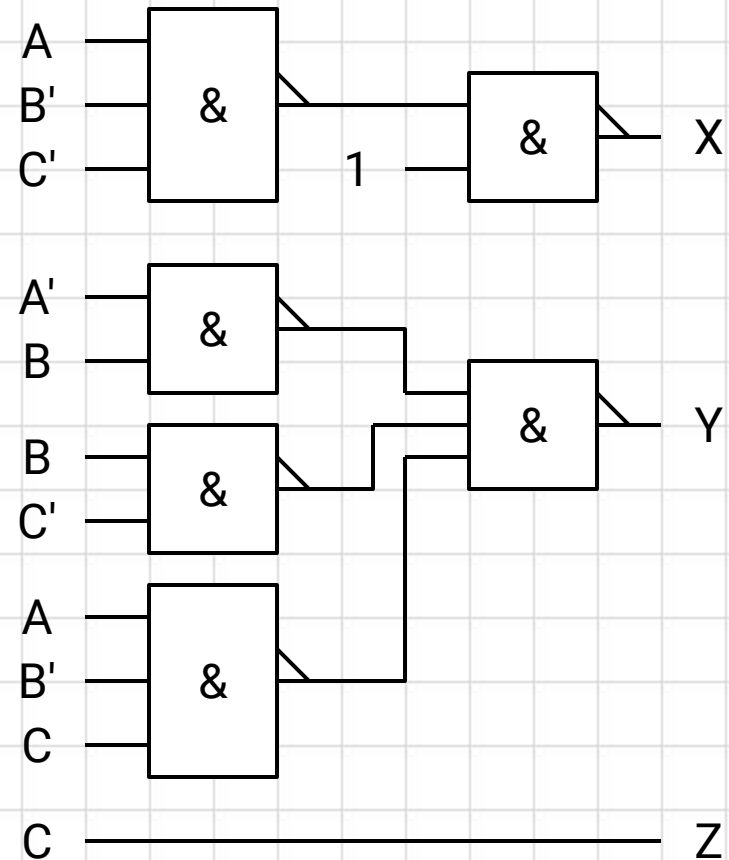
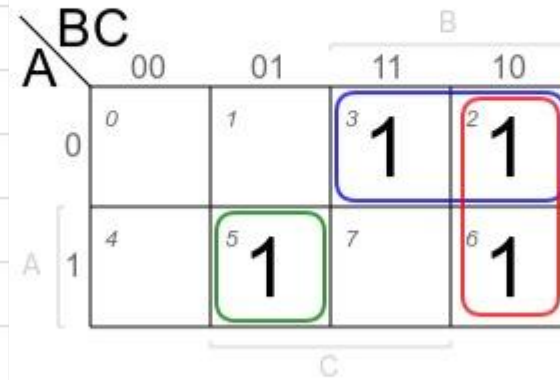
| | | | |
|-----|----|-----|---|
| 000 | 0 | 000 | 0 |
| 001 | 1 | 001 | 1 |
| 010 | 2 | 010 | 2 |
| 011 | 3 | 011 | 3 |
| 100 | -4 | 100 | 4 |
| 101 | -3 | 011 | 3 |
| 110 | -2 | 010 | 2 |
| 111 | -1 | 001 | 1 |

$$X = \Sigma(4) = AB'C'$$

$$Y = \Sigma(2, 3, 5, 6)$$

$$Z = C$$

$$Y = A'B + BC' + AB'C$$



b)

```
module prob205b(input A, B, C, output X, Y, Z);  
  
    assign X = A & ~B & ~C;  
    assign Y = ~A & B | B & ~C | A & ~B & C;  
    assign Z = C;  
  
endmodule
```

c)

```
module prob205c;
```

```
    reg A, B, C;
```

```
    wire X, Y, Z;
```

```
    prob205b dut(A, B, C, X, Y, Z);
```

```
    initial begin
```

```
        $dumpfile("prob205c.vcd");
```

```
        $dumpvars;
```

```
        {A, B, C} = 0;
```

```
        #90 $finish;
```

```
    end
```

```
    always #10 {A, B, C} = {A, B, C} + 1;
```

```
endmodule
```

0 1 2 3 -4 -3 -2 -1 0

-Signals

-Waves

Time

A

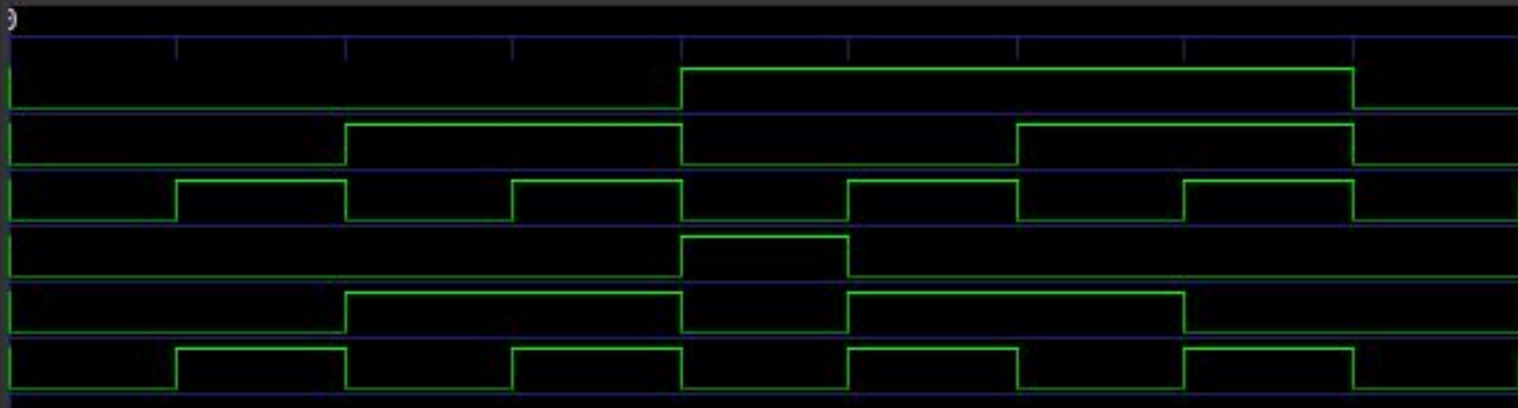
B

C

X

Y

Z



0 1 2 3 4 3 2 1 0

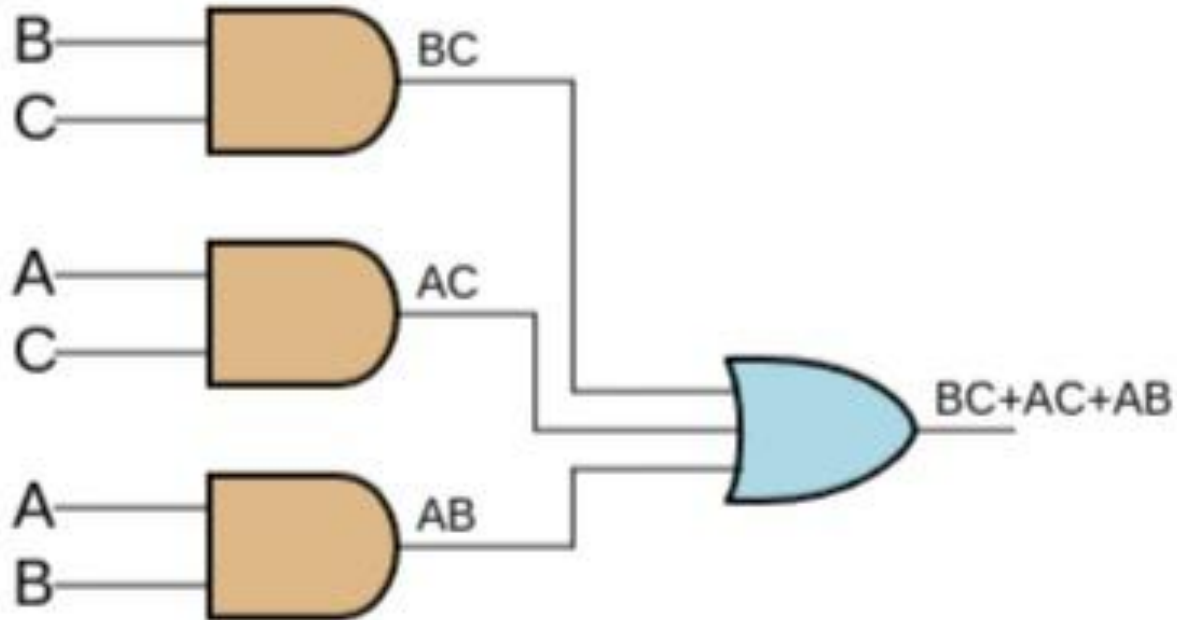
Problema 2-06

Circuito votador

a)

| A \ BC | B | | | |
|--------|----|-----|-----|-----|
| | 00 | 01 | 11 | 10 |
| 0 | 0 | 1 | 3 1 | 2 |
| 1 | 4 | 5 1 | 6 1 | 7 1 |

$$F = BC + AC + AB$$

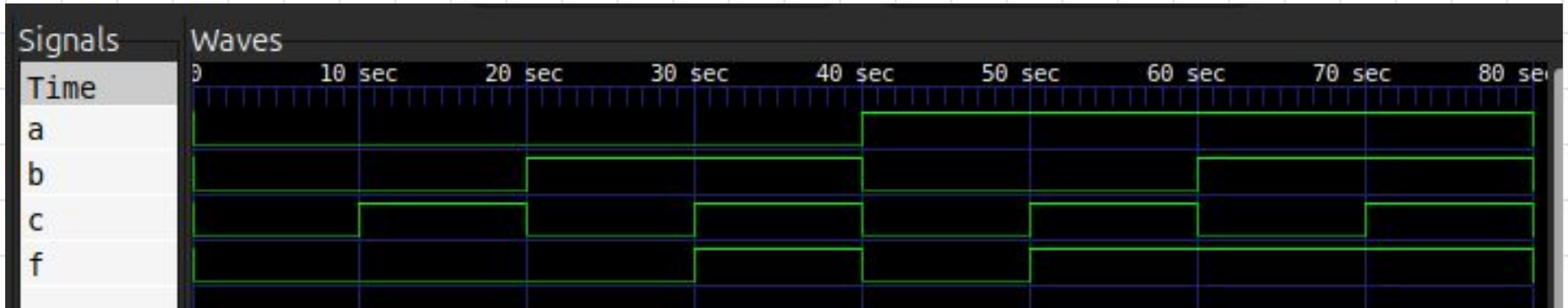


b)

```
module votador (  
    input  a,  
    input  b,  
    input  c,  
    output f  
);  
    assign f = b & c | a & c | a & b;  
endmodule
```

c)

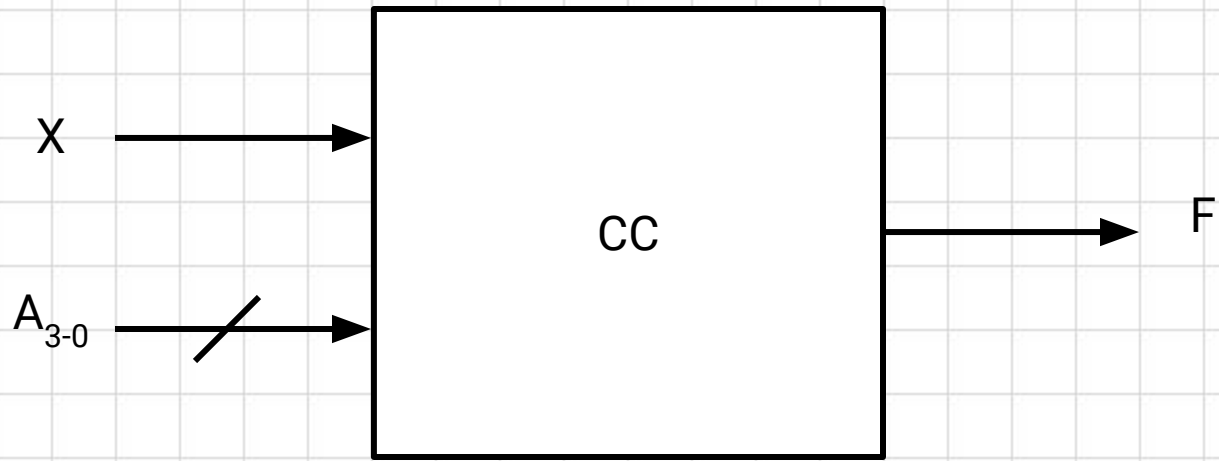
```
module votador_tb;
    reg a, b, c;
    wire f;
    votador dut (
        a,
        b,
        c,
        f
    );
    initial begin
        $dumpfile("sim.vcd");
        $dumpvars;
        {a, b, c} = 0;
        #80 $finish;
    end
    always #10 {a, b, c} = {a, b, c} + 1;
endmodule
```



Problema 2-07

Función a partir de BCD

a)



| Posición | X A | F | Posición | X A | F |
|----------|--------|---|----------|--------|---|
| 0 | 0 0000 | 0 | 16 | 1 0000 | 0 |
| 1 | 0 0001 | 1 | 17 | 1 0001 | 0 |
| 2 | 0 0010 | 1 | 18 | 1 0010 | 0 |
| 3 | 0 0011 | 0 | 19 | 1 0011 | 1 |
| 4 | 0 0100 | 1 | 20 | 1 0100 | 0 |
| 5 | 0 0101 | 0 | 21 | 1 0101 | 0 |
| 6 | 0 0110 | 0 | 22 | 1 0110 | 1 |
| 7 | 0 0111 | 1 | 23 | 1 0111 | 0 |
| 8 | 0 1000 | 1 | 24 | 1 1000 | 0 |
| 9 | 0 1001 | 0 | 25 | 1 1001 | 1 |

$$F = \Sigma(1,2,4,7,8,19,22,25) + d(10,11,12,13,14,15,26,27,28,29,30,31)$$

$$F = \Sigma(1,2,4,7,8,19,22,25) + d(10,11,12,13,14,15,26,27,28,29,30,31)$$

| F | | A_2, A_1, A_0 | | | | | | | |
|----------|----|-----------------|-----|-----|-----|-----|-----|-----|-----|
| | | 000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 |
| X, A_3 | 00 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| | 01 | 1 | 0 | - | - | - | - | - | - |
| | 11 | 0 | 1 | - | - | - | - | - | - |
| | 10 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |

$$F(X, A_3, A_2, A_1, A_0) = X'A_3'A_2'A_1'A_0 + X'A_2'A_1A_0' + X'A_2A_1'A_0' + X'A_2A_1A_0 + X'A_3A_0' + XA_2'A_1A_0 + XA_2A_1A_0' + XA_3A_0$$

$F(X, A3, A2, A1, A0) =$

$X'A3'A2'A1'A0 +$

$X'A2'A1A0' +$

$X'A2A1'A0' +$

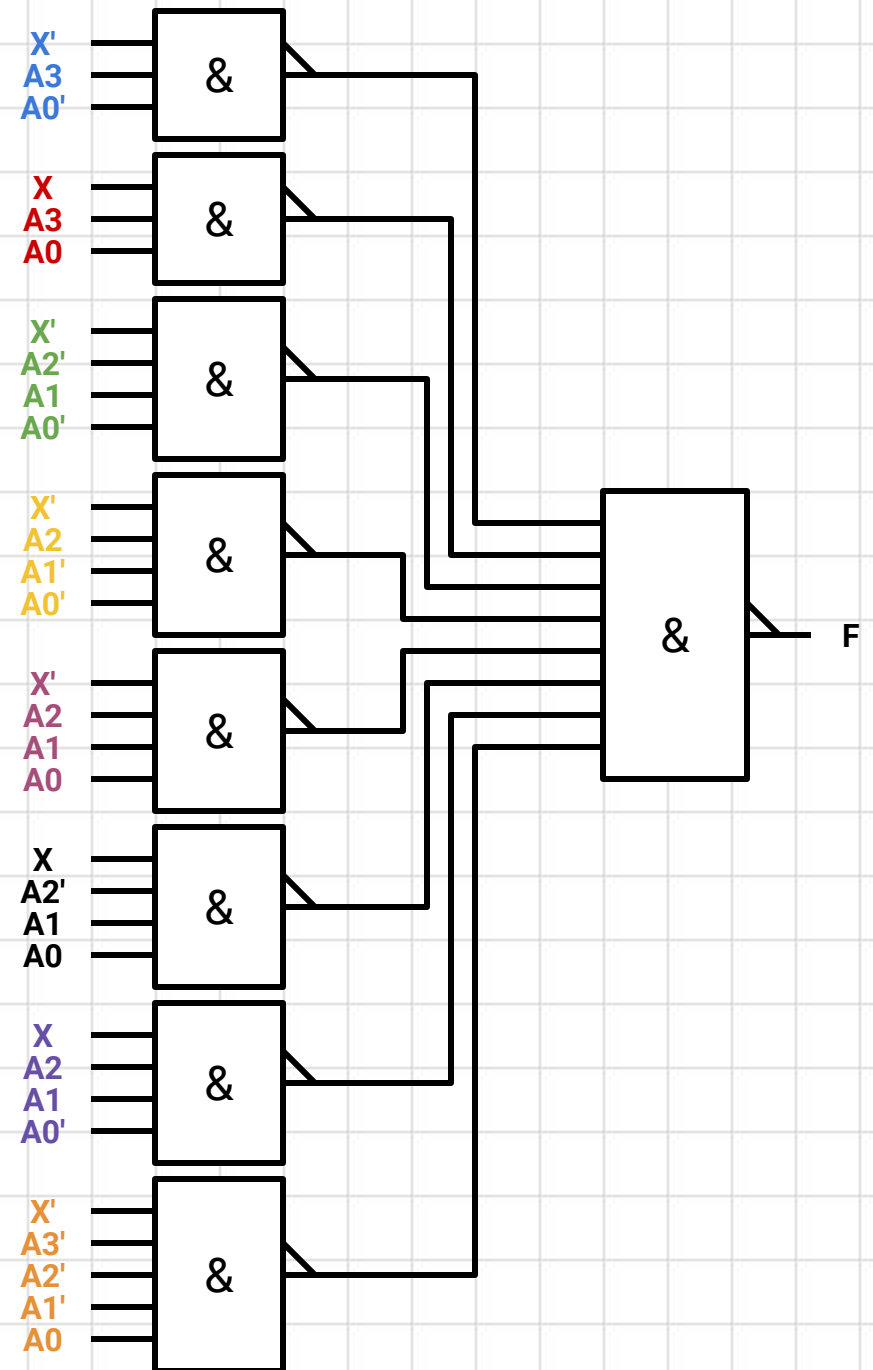
$X'A2A1A0 +$

$X'A3A0' +$

$XA2'A1A0 +$

$XA2A1A0' +$

$XA3A0$



b)

```
module prob207b(input X, input [3:0] A, output F);  
  
    assign F = (~X & ~A[3] & ~A[2] & ~A[1] & A[0])  
               | (~X & ~A[2] & A[1] & ~A[0])  
               | (~X & A[2] & ~A[1] & ~A[0])  
               | (~X & A[2] & A[1] & A[0])  
               | (~X & A[3] & ~A[0])  
               | ( X & ~A[2] & A[1] & A[0])  
               | ( X & A[2] & A[1] & ~A[0])  
               | ( X & A[3] & A[0])  
  
endmodule
```

c)

```
module prob207c;
```

```
    reg X;
```

```
    reg [3:0] A;
```

```
    wire F;
```

```
    prob207b dut(X, A, F);
```

```
    initial begin
```

```
        $dumpfile("sim.vcd"); $dumpvars;
```

```
        X = 0; A = 0;
```

```
        #110 X = 1; A = 0;
```

```
        #110 $finish;
```

```
    end
```

```
    always #10 A = A + 1;
```

```
endmodule
```

GTKWave - sim.vcd

File Edit Search Time Markers View Help

From: 0 sec To: 220 sec Marker: - | Cursor: 1 sec

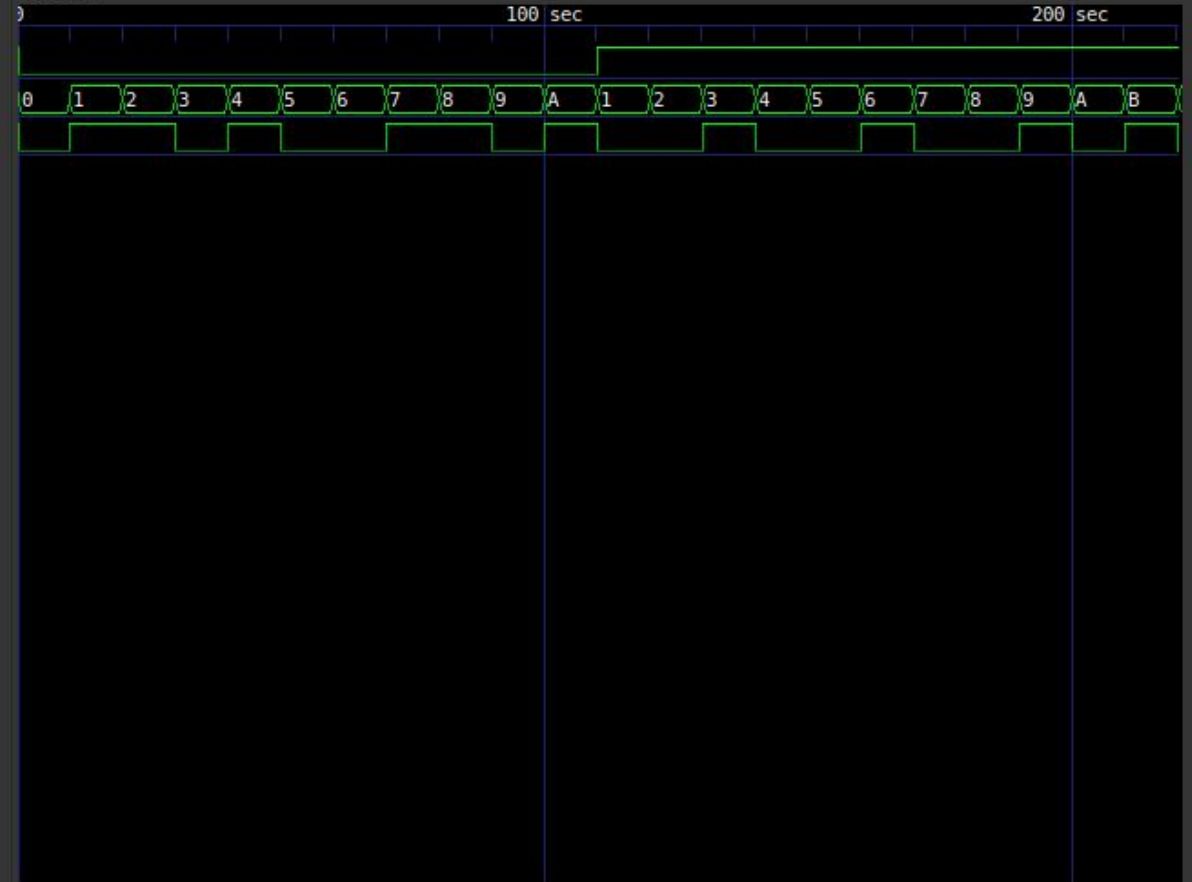
SST

prob207c
└ dut

Signals

| Time | Signal |
|------|--------|
| | X |
| | A[3:0] |
| | F |

Waves



Type Signals

| | |
|------|--------|
| wire | A[3:0] |
| wire | F |
| wire | X |

Filter:

Append

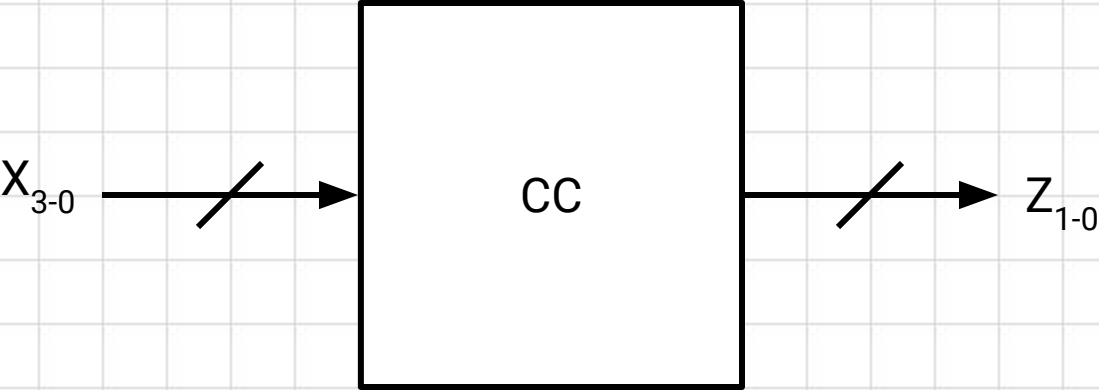
Insert

Replace

Problema 2-08

División BCD

a) Interfaz:



| X | X3X2X1X0 | Z | Z1Z0 |
|---|--------------|---|------|
| 0 | 0000 | 0 | 00 |
| 1 | 0001 | 0 | 00 |
| 2 | 0010 | 0 | 00 |
| 3 | 0011 | 1 | 01 |
| 4 | 0100 | 1 | 01 |
| 5 | 0101 | 1 | 01 |
| 6 | 0110 | 2 | 10 |
| 7 | 0111 | 2 | 10 |
| 8 | 1000 | 2 | 10 |
| 9 | 1001 | 3 | 11 |
| | en otro caso | | ** |

$$Z1 = \Sigma(6,7,8,9) + d(10,11,12,13,14,15)$$

$$Z0 = \Sigma(3,4,5,9) + d(10,11,12,13,14,15)$$

| Z1 | X1,X0 | | | |
|----------|-------|----|----|----|
| | 00 | 01 | 11 | 10 |
| X3,X2 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | - | - | - | - |
| 10 | 1 | 1 | - | - |

$$Z1 = X2X1 + X3$$

| Z0 | X1,X0 | | | |
|----------|-------|----|----|----|
| | 00 | 01 | 11 | 10 |
| X3,X2 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | - | - | - | - |
| 10 | 0 | 1 | - | - |

$$Z0 = X2'X1X0 + X2X1' + X3X0$$

| Z1 | X1,X0 | | | |
|----------|-------|----|----|----|
| | 00 | 01 | 11 | 10 |
| X3,X2 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | - | - | - | - |
| 10 | 1 | 1 | - | - |

$$Z1 = (X3 + X2)(X2' + X1)$$

| Z0 | X1,X0 | | | |
|----------|-------|----|----|----|
| | 00 | 01 | 11 | 10 |
| X3,X2 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | - | - | - | - |
| 10 | 0 | 1 | - | - |

$$Z0 = (X3 + X2 + X1) \cdot (X2' + X1')(X2 + X0)$$

b)

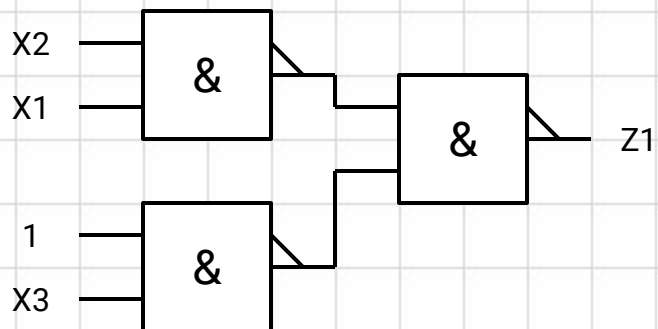
$$\begin{aligned} Z1 &= X2X1 + X3 \\ &= [X2X1 + X3]'' \\ &= [(X2X1)'(1 \cdot X3)]' \end{aligned}$$

$$\begin{aligned} Z1 &= (X3 + X2)(X2' + X1) \\ &= [(X3 + X2)(X2' + X1)]'' \\ &= [(X3 + X2)' + (X2' + X1)]' \end{aligned}$$

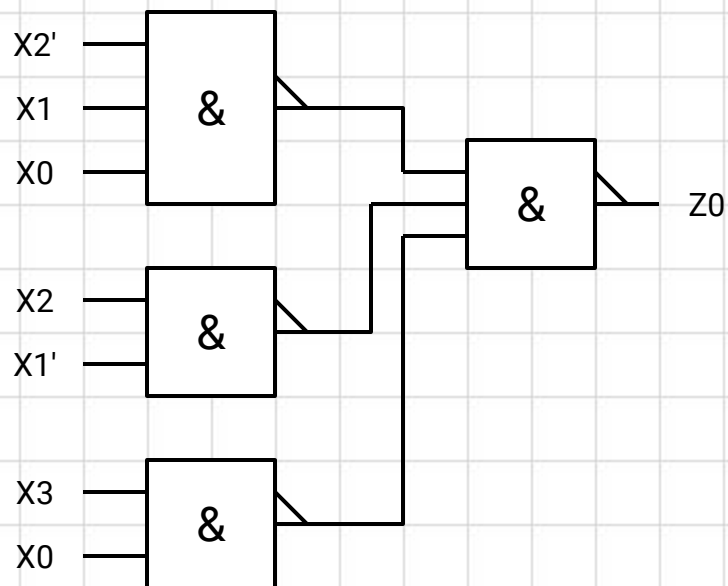
$$\begin{aligned} Z0 &= X2'X1X0 + X2X1' + X3X0 \\ &= [X2'X1X0 + X2X1' + X3X0]'' \\ &= [(X2'X1X0)'(X2X1')'(X3X0)]' \end{aligned}$$

$$\begin{aligned} Z0 &= (X3 + X2 + X1) \cdot \\ &\quad (X2' + X1')(X2 + X0) \\ &= [(X3 + X2 + X1) \cdot \\ &\quad (X2' + X1')(X2 + X0)]'' \\ &= [(X3 + X2 + X1)' + \\ &\quad (X2' + X1')' + (X2 + X0)]' \end{aligned}$$

$$Z1 = [(X2X1)'(1 \cdot X3)']'$$



$$Z0 = [(X2'X1X0)'(X2X1)'(X3X0)']'$$



c)

```
module prob208c(input [3:0] X, output [1:0] Z);  
  
    assign Z[1] = (X[2] & X[1]) | (X[3]);  
    assign Z[0] = (~X[2] & X[1] & X[0])  
                  | (X[2] & ~X[1]) | (X[3] & X[0]);  
  
endmodule
```

d)

```
module prob208d;  
  
    reg [3:0] X;  
    wire [1:0] Z;  
  
    prob208c dut(X, Z);  
  
    initial begin  
        $dumpfile("sim.vcd"); $dumpvars;  
        X = 0;  
        #160 $finish;  
    end  
  
    always #10 X = X + 1;  
  
endmodule
```

GTKWave - sim.vcd

File Edit Search Time Markers View Help

From: 0 sec To: 160 sec Marker: 0 sec | Cursor: 0 sec

SST

prob208d
dut

Type Signals

wire X[3:0]

wire Z[1:0]

Filter:

Append

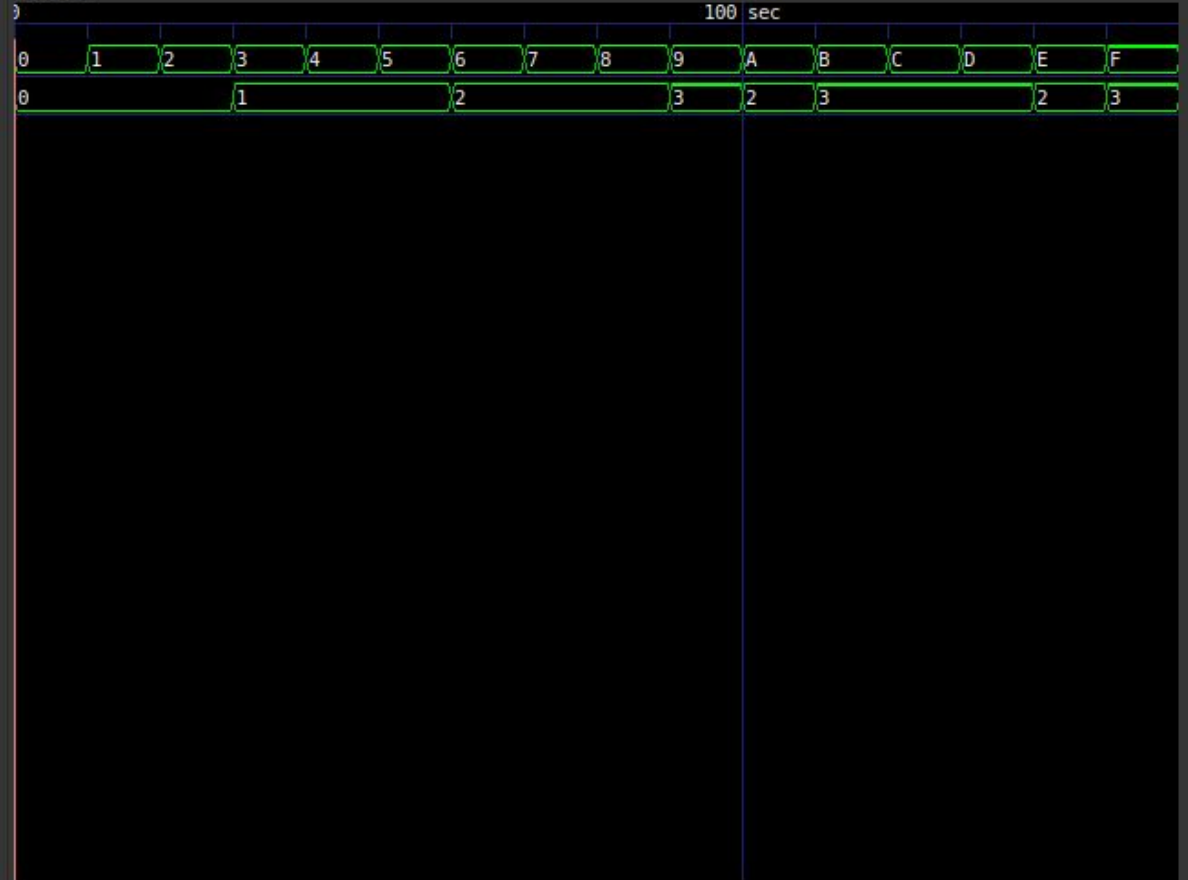
Insert

Replace

Signals

Time
X[3:0] = 0
Z[1:0] = 0

Waves

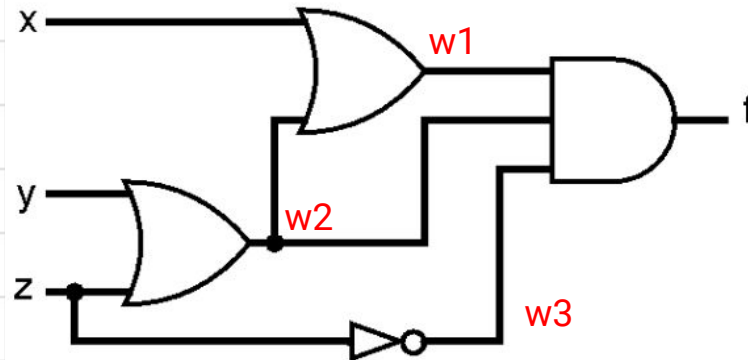


Problema 2-09

Descripción estructural

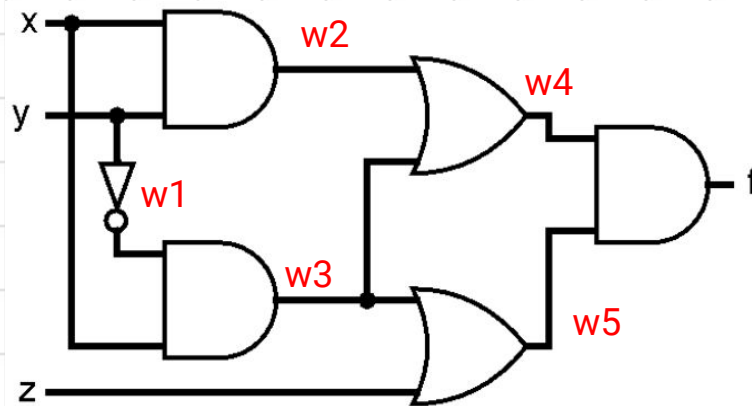
a)

```
module prob209a(input x, y, z, output f);  
  wire w1, w2, w3;  
  or p1(w1, x, w2);  
  or p2(w2, y, z);  
  not p3(w3, z);  
  and pF(f, w1, w2, w3);  
endmodule
```



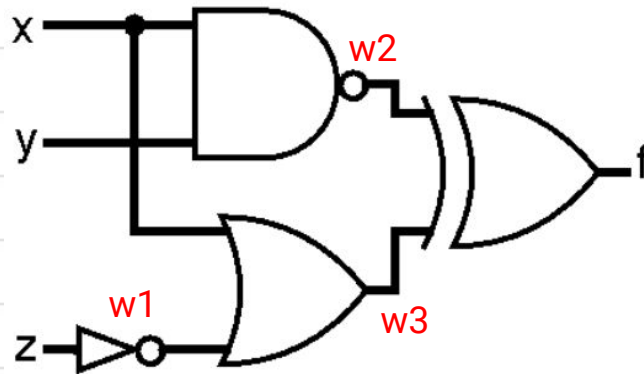
b)

```
module prob209b(input x, y, z, output f);  
  wire w1, w2, w3, w4, w5;  
  not p1(w1, y);  
  and p2(w2, x, y);  
  and p3(w3, w1, x);  
  or p4(w4, w2, w3);  
  and pF(f, w4, w5);  
endmodule
```



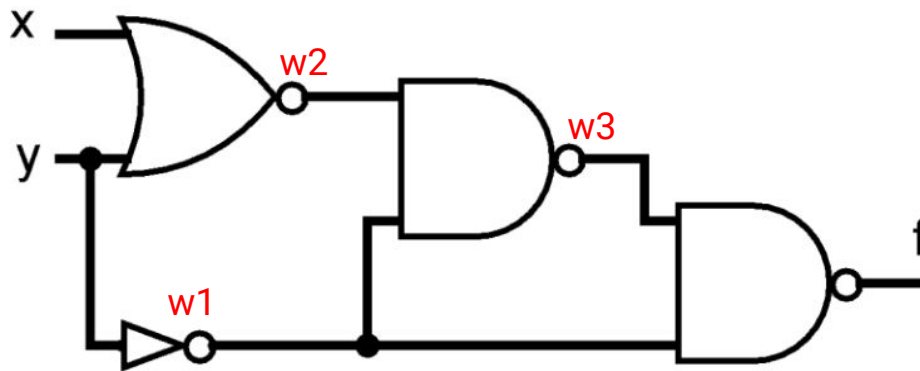
c)

```
module prob209c(input x, y, z, output f);  
  wire w1, w2, w3;  
  not p1(w1, z);  
  nand p2(w2, x, y);  
  or p3(w3, x, w1);  
  xor pF(f, w2, w3);  
endmodule
```



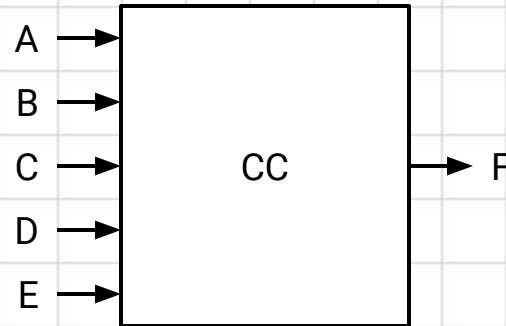
d)

```
module prob209d(input x, y, z, output f);  
  wire w1, w2, w3;  
  not p1(w1, y);  
  nor p2(w2, x, y);  
  nand p3(w3, w2, w1);  
  nand pF(f, w3, w1);  
endmodule
```



Problema 2-18

Directiva de fútbol



| <i>F</i> | | <i>C,D,E</i> | | | | | | | |
|------------|----|---------------------------------|---|---|---|---|---|---|---|
| | | 000 001 011 010 110 111 101 100 | | | | | | | |
| <i>A,B</i> | 00 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| | 01 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| | 11 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| | 10 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |

$$F = A'C'DE' + A'CD'E' + A'B'CDE + A'C'D'E + ACDE' + ABC'D'E' + AC'DE + ACD'E + A'BDE' + B'C'D'E$$

Problema 2-20

BCD x5

| A3-A0 | X3-X0 | Y3-Y0 |
|--------------|-------|-------|
| 0000 | 0000 | 0000 |
| 0001 | 0000 | 0101 |
| 0010 | 0001 | 0000 |
| 0011 | 0001 | 0101 |
| 0100 | 0010 | 0000 |
| 0101 | 0010 | 0101 |
| 0110 | 0011 | 0000 |
| 0111 | 0011 | 0101 |
| 1000 | 0100 | 0000 |
| 1001 | 0100 | 0101 |
| en otro caso | ---- | ---- |

$$X3 = 0$$

$$X2 = A3$$

$$X1 = A2$$

$$X0 = A1$$

$$Y3 = 0$$

$$Y2 = A0$$

$$Y1 = 0$$

$$Y0 = A0$$