Math/ Physics Summary

Src: MIT-Path-Integral-paper.pdf

- Vector Represenataion
  - Dirac Bra-ket Notation
    - 1 dim complex Hilbert space
      - is a collection of vectors with n dim basis vectors
      - Probability curve of positional dim of 2 dim vector in hilbert space:
        - $\phi(x) = \langle x|\Psi\rangle$
    - Ket Vecotr state
      - $\phi\rangle$
      - e.g. basis vectors of a 2dim Hilbert space
        - $\phi_1\rangle, \phi_2\rangle$
    - ϕ scaled by c_1
      - $c_1 \mid \phi\rangle$
      - akin to: ϕ has an associated probability amplitude or coefficient of c_1
    - Braket - Complex Dot product
      - In this notation, a Bra Ket vector represents the dot product between the two which is equal to the probability curve from one to the other state
      - $\langle\phi_1\ |\phi_2\rangle = \phi_1 \cdot \phi_2$
      - Complex dot product:
        - $A \cdot B = \sum_i^n a_i \overline{b_i}$
    - Outer product, creates new Operator
      - $\hat{A}_n = \phi_1\rangle\langle\phi_2$
    - Combining States
      - $|\phi_1\rangle\ |\phi_2\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$
      - The resulting combined state contains all possible combinations of the states $|\phi_1\rangle$ and $|\phi_2\rangle$, where the state of each individual system remains unchanged.
      - Also called Identity Operator $\hat{I} = \sum_{i=1}^n |\phi_i\rangle\langle\phi_i$
    - Operator: $\hat{A}$
      - $\hat{A} \mid \psi\rangle = \mid \phi\rangle$ where ψ and ϕ are in Hilberspace
      - Some Eigenmatrix that transforms that's applied on the state vector(ψ) that returns an eigenvalue "the measurement"
        - $\text{measurement} = \langle\psi \mid \hat{A}|\psi\rangle$
    - Probability Amplitude of a vector ϕ
      - $\|\phi\|^2$
    - Linear Operations
      - $\hat{A}[c_1 \mid \phi_1\rangle + c_2 \mid \phi_2\rangle] = c_1\hat{A}|\phi_1\rangle + c_2\hat{A}|\phi_2\rangle$
    - Examples of 2 Dim Hilbertspace
      - State with momentum: $p\ |p\rangle$
      - State with definite position: $x\ |x\rangle$
      - Probability amplitude for state ϕ_1 to ϕ_2:
        - $\langle\phi_1 \mid \phi_2\rangle = \int_{-\infty}^{+\infty} \phi_1 \cdot \phi_2 dx$
      - Probability amplitude for a particle to be at position x
        - ASSUMPTION: in the original version the Ψ below also is ϕ but withou an index
        - $\phi(x) = \langle x \mid \Psi\rangle$
      - src

- Path Integral Formula
  - Formula: $|\psi(x,t')\rangle = \int_{-\infty}^{\infty} \langle\psi(x',t')|\psi(x_0,t_0)\rangle dx' |\psi(x',t')$
  - This formulation has proven crucial to the subsequent development of theoretical physics, because manifest Lorentz covariance (time and space components of quantities enter equations in the same way) is easier to achieve than in the operator formalism of canonical quantization. Unlike previous methods, the path integral allows one to easily change coordinates between very different canonical descriptions of the same quantum system. Another advantage is that it is in practice easier to guess the correct form of the Lagrangian of a theory, which naturally enters the path integrals (for interactions of a certain type, these are coordinate space or Feynman path integrals), than the Hamiltonian). *Source*
  - Propagator: $U(x',t';x_0,t_0) = \langle\psi(x',t') \mid \psi(x_0,t_0)\rangle$
    - The Propagator represents the probability amplitude for a particle to travel from one point in space and time to another
    - with elapsed time written as: $U(x',t;x_0)$
    - Propagator and an initial state Ket can fully describe the evolution of a system over time
    - Action: $S[x(t)]$
      - An infinite continuum of trajectories $x(t)$(time indipendent) are possible, each with a classical action
    - $\rightarrow$ Every possible path contributes with equal amplitude to the Propagator, but with a phase related to the classical action (action $\rightarrow$ complex phase). Summing over all possible trajectories $\rightarrow$ Propagator
    - $$U(x',t;x_0) = A(t) \sum_{\substack{\text{all} \\ \text{trajectories}}} \exp\left[\frac{i}{\hbar} \overbrace{S[x(t)]}^{\substack{\text{action over} \\ \text{trajectory}}}\right]$$
      - This is the heart of the path integral formulation. How the complete formulation is found is subject to the rest of my notes about the path integral.
    - Since all actions for every path contribute to the Propagator one would suspect that it would diverge quite fast. This is not the case since every action for every path will cancel the greate the difference in the action $\Delta S \approx \pi\hbar$.

    Contributions of trajectories far away from the "classical path", in aggregate, cancel.
  - Assume the classical trajectory $x_{\text{cl}}(t)$ as the trajectory with the minimum value of the action $S[x_{\text{cl}}]$, which is stationary to fist order with regard to deciations.
    - trajectory can be observed with high probability(same as little uncerainty? TODO)
    - trajectories close contribute with coherent phase to the intefral
    - trajectories with action $\pi\hbar$ more than the classical action ar out of phase and intefere destructively with each other. Integrating over more of such destructive trajectories cause their contribution to average out to zero
    - $\rightarrow$ the calssical trajectory is qualitatively imoprtant
      - $\pi\hbar$ is frightinly small making the principal contributions trajectories those in a narrow band around the classical one. On quantum scale though $\pi\hbar$ is big enough to cause significant deviations from the classical trajectory
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