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An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis

We ask whether relations of the IS-LM type can sensibly be used for the aggregate demand portion of a dynamic optimizing general equilibrium model intended for analysis of issues regarding monetary policy and cyclical fluctuations. The main result is that only one change—the addition of a term regarding expected future income—is needed to make the IS function match a fully optimizing model, whereas no changes are needed for the LM function. This modification leads to a dynamic, forward-looking model of aggregate demand that is tractable and usable with a wide variety of aggregate supply specifications. Theoretical applications concerning price level determinacy and gradual price adjustment are included.

ALTHOUGH A FEW SCATTERED WORDS of defense can be found in the literature,[1] the once-dominant IS-LM framework for macroeconomic analysis has been sharply criticized by many leading researchers over the last twenty years or more. Among the critics it is possible to list authors as prominent and diverse as Barro (1984), Brunner and Meltzer (1974, 1993), Friedman (1976), King (1993), Leijonhufvud (1983), Lucas (1994), Sims (1992), Tobin (1969), and Wallace (1980). There is also a considerable amount of diversity in the reasons or logical bases for the criticisms, with at least six distinct failings being mentioned. Nevertheless, most undergraduate macroeconomics textbooks continue to feature IS-LM models,[2] and variants are frequently utilized in both theoretical and empirical analyses by a substantial number of workers.

From a historical perspective it might be regarded as unlikely that the IS-LM con-

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1. See for example Mankiw (1990), Ball and Mankiw (1994), Gali (1992), Hall (1980), Taylor (1993), and McCallum (1989, 1995). See also footnote 7 below.

2. See, for example, recent texts by Abel and Bernanke (1992), Mankiw (1994), or Blanchard (1996). A notable exception is Barro (1994), whose IS-LM section comes at the very end of the book.

3. This is, of course, an understatement, as students of his *Value and Capital* (1939) will know.

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struct would be truly incoherent, since its creator, J. R. Hicks, was unquestionably competent in general equilibrium analysis.[3] But Hicks's (1937) famous article was intended as an exposition of Keynes's *General Theory* views, and it has been firmly established that these involve numerous logical inconsistencies,[4] so it is conceivable that a wholesale criticism might be warranted. In any event, it has long been the belief of the first-named author of the present paper that sensible macro and monetary analysis of many issues[5] can be conducted using IS-LM relations, provided that they are accompanied by "aggregate supply" or "price adjustment" sectors that may reflect temporary price stickiness, but have reasonable—that is, classical—long-run properties. To put the matter bluntly, it is suggested in McCallum (1989, pp. 102–107) that useful insights into monetary policy and business cycle behavior may be provided by a macroeconomic structure such as

$$\log y_t = b_0 + b_1[R_t - E_t(\log P_{t+1} - \log P_t)] + v_t \tag{IS}$$

$$\log M_t - \log P_t = c_0 + c_1 \log y_t + c_2 R_t + \eta_t \tag{LM}$$

$$\log y_t = a_0 + a_1(\log P_t - E_{t-1} \log P_t) + a_2 \log y_{t-1} + u_t \tag{AS}$$

plus a policy rule for M_t (or R_t). Here y_t, P_t, and M_t are measures of real output, the price level, and nominal money balances, respectively, while R_t is a nominal interest rate and $E_t(\bullet) \equiv E(\bullet|\Omega_t)$, with Ω_t representing the set of information available in period t. In such a system, the IS and LM relations pertain to the demand side of the macroeconomic system, while (AS) represents aggregate supply behavior.[6] But the defense of IS-LM demand-side specifications offered in those pages can be regarded as successful only for the LM half of the combination (at best); in the case of the IS function, a much weaker justification is provided. Similarly, Blanchard and Fischer (1989, p. 532) find the LM function to be "quite consistent with the demand for money that emerges" from their more detailed optimizing treatment, but conclude that the IS function "is only a pale reflection of our analysis of optimal [saving and investment] behavior under uncertainty."

In the present paper, we seek to provide a reconsidered and reasonably wide-ranging analysis of the issue. In particular, we will in section 1 review the main criticisms of the IS-LM approach, concluding that most of them are not crippling in the context of monetary policy and business cycle (as opposed to growth) analysis. For consistency with optimizing behavior, however, one simple but crucial modification to the usual IS specification is needed.[7] This central point is developed in section 2. Two

4. See Patinkin (1976). Patinkin himself paid tribute to Keynes's work as a great contribution, but his cited book details a large number of significant theoretical errors in the *General Theory.*

5. Not including those related to economic growth.

6. We do not mean to suggest that the particular specification given by (*AS*) is an appropriate one. There is an enormous amount of professional disagreement over the proper specification of this sector of the model. We wish merely to indicate that the IS and LM relations do not themselves constitute a complete macroeconomic structure.

7. This statement does not refer to the absence of fiscal variables from equation (*IS*) above. Instead, the modification involves the inclusion of an additional variable reflecting expected future income. A similar

illustrative applications of the resulting specification to issues including price level determinacy and gradual price adjustment are then included in sections 3 and 4. The paper concludes with a recapitulation in section 5.

1. WEAKNESSES OF IS-LM MODELS

Let us begin by cataloging some of the principal objections to IS-LM models that have been expressed over the years. Among these are the following:

(i) IS-LM analysis presumes a fixed, rigid price level;
(ii) It does not distinguish between real and nominal interest rates;
(iii) It does not recognize enough distinct assets;
(iv) It permits only short-run analysis;
(v) It treats the capital stock as fixed;
(vi) It is not derivable from explicit maximizing analysis of rational economic agents.[8]

We now discuss each objection in turn.

It is certainly true that textbook-style IS-LM analysis of the 1950s and 1960s was often guilty of charges (i) and (ii), and we share the opinion that such analysis is fundamentally misguided—in part because it creates the impression that real output movements and levels are readily manipulatable by the monetary authorities. But models with such weaknesses are not the type under discussion. Instead, as indicated above, we are concerned with the use of IS and LM functions to represent aggregate demand behavior, as opposed to aggregate supply, in macroeconomic models that do recognize price level variability and the real versus nominal interest rate distinction. Such usage was emphasized by Bailey (1962), and is utilized in the famous paper of Sargent and Wallace (1975), as well as textbooks by Sargent (1979) and McCallum (1989).

Criticism (iii), expressed by Brunner and Meltzer (1974, 1993) and Tobin (1969), is clearly correct for some purposes. Since the usual IS-LM model recognizes only one (nominal) interest rate, it implicitly lumps all assets into two categories, termed "money" and "bonds." Thus there is no distinction between treasury bills, commercial paper, long-term private and government bonds, or physical capital—all are simply treated as perfectly substitutable components of a single interest-bearing nonmonetary asset. Accordingly, many interesting macro or monetary issues cannot be addressed by such IS-LM models. But for a considerable range of problems in these areas, it would appear that the two-asset restriction is not critical. Many critics of IS-LM analysis are evidently willing to use models of other types with two (or fewer!) as-

modification has recently been suggested in a useful paper by Kerr and King (1996), and was outlined in McCallum (1995). Fane (1985) and Koenig (1989, 1993) represent previous efforts with objectives similar to those of the present paper, but they only show that some comparative-static properties of their models are like those of an IS-LM setup. In particular, they do not develop dynamic equations analogous to IS and LM functions, as is done below. Auerbach and Kotlikoff (1995, pp. 312–313) derive IS and LM equations from an overlapping generations framework under the highly restrictive assumption of rigid prices.

8. The point stressed by King (1993) and Blanchard (1996, ch. 10), that the standard IS-LM model omits important expectational influences, is a significant special case of this objection.

sets for a variety of issues—see, for example, Wallace (1980), Lucas (1972), or King (1993). Or, to put the point in another way, disaggregation provides benefits but also costs, so two-asset models will often prove convenient and satisfactory.

Criticisms (iv) and (v) are closely related, since the traditional justification for the fixed capital stock assumption is that short-run analysis was originally the model's reason for existence. For the purpose of business cycle analysis, however, it is clearly unsatisfactory to maintain the short-run limitation that was common in the literature of the 1940s and 1950s. The problem is not only that the duration of a typical cycle seems too long to justify a "short-run" assumption; in addition, it is the case that macroeconomics in the era of rational expectations is inherently dynamic. The emphasis of recent IS-LM supporters is therefore directed toward expectational phenomena, gradual adjustment to various shocks, and the consequences of alternative maintained policy rules.[9] Thus the short-run assumption will not be a feature of the framework to be constructed below. Instead, it will be presumed that the model is designed for quarterly time series data over sample periods of many years' duration (for example, ten to fifty years).

What, then, becomes of the original fixed-capital assumption? In principle it might be possible to incorporate an endogenously determined capital stock, as in Sargent and Wallace (1975). Our interest, however, is in obtaining relationships from optimizing behavior, and it appears unlikely that such an equation for capital accumulation can be derived that is both analytically tractable and empirically successful. Lucas (1975), for example, gave "analytical necessity" as the reason why he simply postulated a log-linear demand function for capital, instead of obtaining it from optimization as he did for consumption.

For the present paper, therefore, we will instead proceed by treating capital's time path as exogenous. A special case of this would be to hold the capital stock constant, as is done by Woodford (1995) and Goodfriend and King (1997) in theoretical papers or by Rotemberg and Woodford (1997) in an empirical study. But our assumption is somewhat more general. Specifically, we adopt the strategy that in a theoretical analysis, one would assume a constant or steadily growing capital stock, while in an empirical application, the behavior of log investment is approximated by a random walk. This strategy implies that in both theoretical and empirical analysis, movements in capital will not be explained endogenously, and investment is assumed to have a constant expected growth rate: $E_t \log i_{t+1} - \log i_t = \xi$. In fact, this random walk assumption for $\log i_t$ is not grossly inconsistent with the U.S. quarterly data: using gross fixed investment in 1992 prices for i_t, a good ARMA model for $\Delta \log i_t$ over 1954:1–1997:1 is an AR(1) with a residual standard deviation of 0.0225, as compared with 0.0255 when $\Delta \log i_t$ is modeled as a constant plus white noise. Furthermore, this assumption implies that output and investment are positively correlated at cyclical frequencies.[10]

9. Taylor's work (for example, 1993) in this area is outstanding. Although he tends not to use the term IS-LM, his models amount to IS-LM structures with some disaggregation in the IS portion, with staggered nominal contracts in the AS portion, and with open-economy influences recognized.

10. It is widely recognized that capital and output are not highly correlated at cyclical frequencies; see, for example, Aiyagari (1994) or Cooley and Prescott (1995, p. 11).

Nevertheless, despite these supportive facts, the main justification for our exogenous-capital assumption is analytical simplicity.

2. BASIC RESULT

Let us now turn our attention to the final criticism mentioned above, concerning the compatibility of IS and LM relations with explicit analysis of the maximizing behavior of rational economic agents. For this purpose, we shall consider an economy consisting of numerous individual households, beginning with a deterministic setting and then extending the analysis so as to accommodate stochastic shocks. Each household seeks at time t to maximize the time-separable utility function $\Sigma_{\tau=0}^{\infty} \beta^{\tau} u(c_{t+\tau}, m_{t+\tau})$, where $\beta \in (0, 1)$ is the household's discount factor, c_t denotes the household's consumption during t, and m_t is the stock of real money balances held at the start of the period. The rationale for the inclusion of m_t is, of course, that holdings of the economy's medium of exchange provide transaction services that reduce the time or other resources needed in "shopping" for the numerous distinct consumption goods whose aggregate is represented by c_t.[11] This method of introducing money is less restrictive, and more convenient for present purposes, than adoption of a cash-in-advance constraint.

Although households consume many goods, they specialize in production. Each one produces a single good as restricted by the production function $y_t = f(n_t, k_t)$, where y_t is output, n_t is labor input, and k_t is the stock of capital held by the household at the start of t. The functions $f(\bullet)$ and $u(\bullet)$ are assumed to be well-behaved, that is, to satisfy the Inada conditions.

Each household inelastically supplies one unit of labor per period to a labor market, from which the households as producers purchase labor inputs at the real wage rate w_t.[12] In addition, there is a market for one-period government bonds on which the real rate of interest is r_t, where $(1 + r_t)^{-1}$ is the real purchase price of a bond that is redeemed for one unit of output in the next period. Letting $\pi_t = (P_t - P_{t-1})/P_{t-1}$ denote the inflation rate, where P_t is the money price of goods, a typical household's budget constraint is

$$f(n_t, k_t) - tx_t = c_t + k_{t+1} - (1 - \delta)k_t + w_t(n_t - 1) + (1 + \pi_{t+1})m_{t+1} - m_t + b_{t+1}(1 + r_t)^{-1} - b_t \qquad (1)$$

11. This venerable and familiar assumption can be rationalized by drawing on the argument developed in Lucas (1980), which supposes that technology, preferences, and markets are such that relative prices of goods are constant, so that aggregation is possible. An alternative aggregation setup, utilized by Blanchard and Kiyotaki (1987), is the Dixit-Stiglitz (1977) method of aggregating the outputs of monopolistically competitive producers. This procedure could be applied to our setup and would leave our optimality conditions unchanged, except that w_t in (5) below would be multiplied by a constant equal to the aggregate markup of price over marginal cost.

12. A rental market for capital goods could also be recognized but, with households treated (for simplicity) as alike, this would not alter the workings of the model. We abstract from population growth to avoid unnecessary notational clutter.

for period t, with similar constraints for all future periods. In (1), b_{t+1} is the number of real bonds purchased in t and tx_t is the magnitude of lump-sum taxes levied on the household, while δ is the capital stock depreciation rate.

In this setup, the household's optimality conditions include (1) and[13]

$$u_1(c_t, m_t) - \lambda_t = 0 \tag{2}$$

$$\beta u_2(c_{t+1}, m_{t+1}) - \lambda_t(1 + \pi_{t+1}) + \beta\lambda_{t+1} = 0. \tag{3}$$

$$-\lambda_t + \beta\lambda_{t+1}[f_2(n_{t+1}, k_{t+1}) + (1-\delta)] = 0. \tag{4}$$

$$f_1(n_t, k_t) - w_t = 0. \tag{5}$$

$$-\lambda_t(1+r_t)^{-1} + \beta\lambda_{t+1} = 0. \tag{6}$$

These difference equations determine the household's choices of sequences for c_t, b_{t+1}, m_{t+1}, n_t, k_{t+1}, and λ_t in response to paths for w_t, r_t, π_t, and tx_t that it faces.[14]

For competitive equilibrium, we also have the government's budget constraint, written in per-household terms as

$$g_t - tx_t = (1 + \pi_t)m_{t+1} - m_t + (1 + r_t)^{-1}b_{t+1} - b_t, \tag{7}$$

and the market-clearing conditions

$$n_t = 1 \tag{8}$$

and

$$m_t = M_t / P_t. \tag{9}$$

In (7), g_t is government purchases per household, and in (9), M_t is the per-household nominal money supply. Thus equations (1)–(9) plus the identity

$$\pi_{t+1} = (P_{t+1} - P_t)/P_t \tag{10}$$

determine paths for w_t, r_t, π_t, and P_t plus the six variables mentioned above, in response to government-chosen sequences for g_t, M_t, and tx_t.

The foregoing is clearly a flexible-price model. But our objective is to obtain from it a pair of relations that are analogous to IS and LM functions, which could then be used sensibly in a setting with slow price adjustment. To that end, we first use (6) in (3) to obtain

$$\beta u_2(c_{t+1}, m_{t+1}) - (1 + \pi_{t+1})\beta\lambda_{t+1}(1 + r_t) + \beta\lambda_{t+1} = 0, \tag{11}$$

13. Here and below, for any function $g(\bullet)$ that possesses multiple arguments, the notation $g_i(\bullet)$ denotes the partial derivative of g with respect to its ith argument.

14. We assume that three transversality conditions—pertaining to the household's accumulation of capital, money, and bonds—are satisfied.

which with $\lambda_{t+1} = u_1(c_{t+1}, m_{t+1})$ implies

$$\beta u_2(c_{t+1}, m_{t+1}) = \beta u_1(c_{t+1}, m_{t+1})[(1 + \pi_{t+1})(1 + r_t) - 1]. \tag{12}$$

But with real and nominal (R_t) interest rates related by $1 + r_t = (1 + R_t)/(1 + \pi_{t+1})$, equation (12) collapses to

$$u_2(c_{t+1}, m_{t+1}) \,/\, u_1(c_{t+1}, m_{t+1}) = R_t\,. \tag{13}$$

And under reasonably standard assumptions, the latter can be solved for m_{t+1}, as in

$$m_{t+1} = L(c_{t+1}, R_t). \tag{14}$$

Thus we have a relation expressing end-of-period real money balances as a function of the upcoming period's consumption spending and the current nominal interest rate. Under reasonable specifications for $u(\bullet)$ and $f(\bullet)$, $L(\bullet, \bullet)$ will be increasing in c_{t+1} and decreasing in R_t. Thus equation (14) describes essentially the same type of behavior as that of the standard LM equation—real money balances are positively related to a transactions variable and negatively related to an opportunity-cost variable.[15]

The timing in (14) is not quite the same as in (LM) of the introduction, but could be made to coincide exactly by specifying that it is end-of-period real money balances that facilitate transactions.[16] To some, that might seem a questionable proposition, but we would suggest that neither the end-of-period nor the start-of-period specification is fully "correct"; indeed, some average over the period might arguably be more appropriate (as in the Baumol-Tobin model).[17] But each of these specifications is actually just an approximation or a metaphor designed to represent the transaction-facilitating services of the medium of exchange. Accordingly, we contend that the model of equations (1)–(14) provides adequate justification for the use of a money-demand relation taking a form such as (LM). We will discuss the issue of the relationship between c_t and y_t shortly.

Arguments similar to the foregoing have, as stated above, been developed previously by several writers including McCallum and Goodfriend (1987) and Blanchard and Fischer (1989). The more novel portion of our current task is to attempt an analogous derivation of a relation implied by the model at hand that represents behavior of the sort described by IS functions. Proceeding toward that goal, we find that substitution of (6) into (2) gives

15. It is emphasized by McCallum and Goodfriend (1987) that relations such as (14) are not properly termed "demand functions" since the spending variable is not exogenous to the individual economic unit in question. But such relations are typically referred to as money demand functions in the literature.

16. The analysis is conducted in that fashion in McCallum and Goodfriend (1987). If we interpret m_t as end-of-period real money balances, then the budget constraint (4) includes $m_t - m_{t-1}(1 + \pi_{t+1})^{-1}$ on the right-hand side and, following the same steps as above, we obtain $u_2(c_t, m_t)/u_1(c_t, m_t) = R_t(1 + R_t)^{-1}$ in place of (13).

17. In addition, we would point out that in several recent papers, it is assumed that end-of-period money balances are relevant for facilitating transactions. See, for example, Obstfeld and Rogoff (1995) and Woodford (1995).

$$u_1(c_t, m_t) = \beta u_1(c_{t+1}, m_{t+1})(1 + r_t). \tag{15}$$

Now we add one new assumption, namely, that the functional form of $u(c_t, m_t)$ is separable[18] and of the form

$$u(c_t, m_t) = \theta\sigma(\sigma - 1)^{-1} c_t^{(\sigma-1)/\sigma} + (1 - \theta)\psi(m_t), \tag{16}$$

where $\theta \in (0, 1)$, $\sigma > 0$ with $\psi'(\bullet) > 0$ and $\psi''(\bullet) < 0$ over the empirically relevant range.[19] For the case $\sigma = 1$ in (16), we take preferences to be logarithmic in consumption. With that convention, it is the case that for all $\sigma > 0$, $u_1(c_t, m_t) = \theta c_t^{-1/\sigma}$ and we have

$$\theta c_t^{-1/\sigma} = \theta c_{t+1}^{-1/\sigma} \beta(1 + r_t) \tag{17}$$

or

$$c_t = c_{t+1}[\beta(1 + r_t)]^{-\sigma} \tag{18}$$

so that, upon taking natural logarithms,

$$\log c_t = \log c_{t+1} - \sigma \log(1 + r_t) - \sigma \log \beta. \tag{19}$$

The crucial thing to realize about this relation is that, behaviorally, it represents the typical household's choice in period t of c_t in response to r_t and expectations concerning c_{t+1}—*not* the choice of c_{t+1} in response to the lagged values c_t and r_t. Thus the analysis provides justification for a consumption equation such as

$$\log c_t = b_0' + b_1' r_t + E_t \log c_{t+1}, \qquad b_1' < 0, \tag{20}$$

where we have used the common approximation $\log(1 + x) = x$ (for x small relative to 1.0). Next, the economy's resource constraint, $y_t = c_t + i_t + g_t$, can be log-linearized to yield

$$\log y_t = d_1 \log c_t + d_2 \log i_t + d_3 \log g_t, \tag{21}$$

where the d_j are steady-state ratios of consumption, investment, and government purchases to output. Inserting (20) into (21), we obtain

$$\begin{aligned}\log y_t &= d_1(b_0' + b_1' r_t + E_t \log c_{t+1}) + d_2 \log i_t + d_3 \log g_t \\ &= d_1 b_0' + d_1 b_1' r_t + E_t(\log y_{t+1} - d_2 \log i_{t+1} - d_3 \log g_{t+1}) \\ &\quad + d_2 \log i_t + d_3 \log g_t.\end{aligned} \tag{22}$$

18. We do not claim that separability is theoretically an appropriate assumption. But we believe that for many purposes an approximation that neglects interaction effects will be satisfactory. Such approximation are certainly quite common in the literature.

19. For some purposes, such as optimal inflation rate analysis, it would be appropriate to assume that there exists a satiation level of real money balances.

But under our assumption that, instead of being determined by (4), investment has a constant expected growth rate ξ, we have $E_t(\log i_{t+1} - \log i_t) = \log(1 + \xi) \approx \xi$. Thus we end up with

$$\log y_t = b_0 + b_1 r_t + E_t \log y_{t+1} + b_3 \log g_t - b_3 E_t \log g_{t+1}. \tag{23}$$

This is the IS function we propose when government spending is a non-negligible contributor to cyclical fluctuations. In many applications, however, one might wish to abstract from government spending, in which case the last two terms would vanish from (23). Thus our basic conclusion, as in McCallum (1995), is that a relation of the form

$$\log y_t = b_0 + b_1 r_t + E_t \log y_{t+1}, \qquad b_1 < 0, \tag{24}$$

is justifiable by the foregoing analysis of a maximizing model. Equation (24) is, clearly, of the common IS form that we set out to consider—but with one significant difference. Specifically, the expected value of next period's output is an important determinant of the quantity of output demanded in the current period. This extra term gives a forward-looking aspect to (24) that is not present in typical IS-LM analysis, and which could possibly have a major effect on the dynamic properties of a macroeconomic system. Some examples of models with expectational IS functions of this type will be investigated below, in sections 3 and 4.

The identity (21) also indicates how we can move from (14), which has c_t as the scale variable, to (LM) in the introduction, where y_t appears instead. A semi-logarithmic approximation to (14), with timing adjusted, is

$$\log M_t - \log P_t = f_0 + f_1 \log c_t + f_2 R_t, \qquad f_1 > 0, f_2 < 0. \tag{25}$$

Using (21) to eliminate c_t and assuming no government spending, we have

$$\log M_t - \log P_t = f_0 + (f_1/d_1)\log c_t + f_2 R_t - (f_1/d_2)\log i_t. \tag{26}$$

If, as we recommend for theoretical exercises, the capital stock is treated as growing smoothly, the log i_t term in (26) becomes a constant, and the equation corresponds to a nonstochastic version of (LM), with $c_1 \equiv (f_1/d_1)$. In an empirical study, on the other hand, we recommend, as discussed in section 1, that the observations on investment be treated as exogenous. In that case, investment would not disappear from (26), and relative to (LM), the money demand equation would involve the extra term log i_t, whose behavior would be modeled as generated by an exogenous random walk process.

The preceding maximizing analysis took place in a deterministic setting. Analogous results can be obtained in a stochastic version of the model, provided that we employ some commonly made approximations. To be specific, let us assume that preference shocks appear in the household's utility function, so that $u(c_t, m_t)$ is stochastic.[20] The household's problem is now to maximize $E_t\Sigma_{\tau=0}^{\infty} \beta^{\tau} u(c_{t+\tau}, m_{t+\tau})$, and its optimality conditions include (1), (2), (5), and the stochastic analogues to (3), (4), and (6); that is,

$$\beta E_t u_2(c_{t+1}, m_{t+1}) - \lambda_t(1 + E_t\pi_{t+1}) + \beta E_t\lambda_{t+1} = 0. \tag{27}$$

$$-\lambda_t + \beta E_t\lambda_{t+1}[f_2(n_{t+1}, k_{t+1}) + 1 - \delta] = 0. \tag{28}$$

$$-\lambda_t(1 + r_t)^{-1} + \beta E_t\lambda_{t+1} = 0. \tag{29}$$

The competitive equilibrium conditions again include (10)–(13). We assume preferences are given by

$$u(c_t, m_t) = \theta\sigma(\sigma - 1)^{-1} c_t^{(\sigma-1)/\sigma} \exp(\omega_t) + (1 - \theta)\psi(m_t). \tag{30}$$

where ω_t is a disturbance to preferences over consumption. Then (29) becomes

$$\theta c_t^{-(1/\sigma)} \exp(\omega_t) = \beta\theta E_t c_{t+1}{}^{-(1/\sigma)} \exp(\omega_{t+1})(1 + r_t) \tag{31}$$

or, approximately

$$\omega_t - \sigma^{-1}\log c_t = -\sigma^{-1}E_t \log c_{t+1} + \log\beta + \log(1 + r_t) + E_t\omega_{t+1}. \tag{32}$$

Let us assume that ω_t is an AR(1) process with AR parameter $|\rho_v| < 1$. Rearranging (32), substituting in (21), and assuming $E_t(\log i_{t+1} - \log i_t) = \xi$ and that g_t is constant, we obtain the counterpart to result (24):

$$\log y_t = b_0 + b_1 r_t + E_t \log y_{t+1} + v_t, \tag{33}$$

where $b_1 = -\sigma d_1$ and $v_t = \sigma d_1(1 - \rho_v)\omega_t$. Since ω_t has AR(1) parameter ρ_v, so does v_t. (32) and (33) involve approximations of the form $\log E_t d_{t+1} \approx E_t \log d_{t+1}$. These are standard first-order approximations, arising from passing nonlinear functions through the linear expectations operator.[21]

The derivation of (33) shows that by introducing uncertainty, we may include an additive disturbance in (24). By including a shock term in the specification of $\psi(m_t)$ in (30), it is also possible to derive log-linearized versions of (14) where a money demand shock enters explicitly, just as in (*LM*) in the introduction.

It might be noted that the optimizing model that we have utilized is one in which there are no adjustment costs associated with the process of investing in physical capital. Since standard IS-LM frameworks are frequently regarded as implying the existence of capital adjustment costs—see, for example, Sargent (1979)—this feature of our setup might be questioned. It is our intention, however, to demonstrate that rela-

20. We could introduce additional uncertainty by allowing the production function to be stochastic.

21. Gaspar and Judd (1997) argue for an alternative procedure for log-linearizing Euler equations, which allows for departures from certainty equivalence.

tions such as (14) and (24) can be obtained via maximizing analysis, so use of the simplest and most standard version of the Sidrauski (1967)-Brock (1974) model seems appropriate. And our treatment of the capital stock, as smoothly changing at an exogenously given rate for a typical household, will keep the model from possessing nonstandard implications that might otherwise obtain.

Recently, capital adjustment costs have become a popular modification of sticky-price quantitative general equilibrium models (for example, Kimball 1995). Evidently, models without this modification exhibit very strong contemporaneous responses of investment to monetary shocks. These responses tend to produce some counterfactual model properties; for example, real and nominal interest rates increase in reaction to monetary expansion. The presence of capital adjustment costs dampens the investment response, and thereby tends to produce smooth behavior of the capital stock. In their practical effect, therefore, capital adjustment costs appear to have much the same effect on an optimizing model as our constant-growth assumption.[22]

We do not wish to overstate the similarity of our expectational IS function (33) with the traditional version in which the determinant $E_t \log y_{t+1}$ does not appear. Clearly, almost any real shock will induce a different dynamic pattern of spending responses and the presumed unit coefficient on $E_t \log y_{t+1}$ may result in an additional unit root in the time series properties of some endogenous variables. In addition, a reader has pointed out that several recent papers have utilized IS functions of the form

$$\log y_t = b_0 + b_1 r_{t-1} + b_2 \log y_{t-1} + v_t , \qquad (34)$$

where the real interest rate is lagged one period and a lagged output term appears. The motivation for specification (34) is evidently empirical; demand seems to respond to interest rates with a lag and to possess inertia. Obviously our (33) differs even more from (34) than from the traditional specification, but the lags in (34) need to be rationalized, perhaps by appeal to information delays and preferences that reflect habit formation. Recent papers by Rotemberg and Woodford (1997) and Fuhrer (1998) show that such modifications can be incorporated into analyses that are basically of the same type as represented by our derivation of (33).

3. PRICE LEVEL DETERMINACY

To illustrate the usefulness of the foregoing, let us now reconsider an issue of fundamental importance in monetary theory, namely, the possible analytical indeterminacy of prices and other nominal variables in a setting in which all private agents are free of money illusion and form their expectations rationally. In a famous article, Sargent and Wallace (1975) put forth the claim that in a model with those properties, nominal magnitudes would be formally indeterminate if the central bank used an interest rate as its instrument variable, that is, if it set the interest rate R_t each period by means of a policy feedback rule that specifies R_t as a linear function of (any) data from

22. Indeed, Leeper and Sims (1994, p. 83) give the desire for a model with smooth capital growth as one reason for their inclusion of adjustment costs and other modifications of the standard optimizing model.

previous periods.[23] Sargent (1979, p. 362) summarized the conclusion as follows: "There is no interest rate rule that is associated with a determinate price level." The specific model used by Sargent and Wallace (1975) was of the IS-LM-AS type, similar to the one exhibited above in the introduction.[24] Subsequently, however, McCallum (1981, 1986) showed that the Sargent-Wallace claim was actually incorrect in such a model; instead, all nominal variables will be fully determinate provided that the policy rule utilized for the interest rate instrument involves some nominal variable, just as suggested previously by Parkin (1978) and in the classic discussion of Patinkin (1965, pp. 295–310). Our objective here, consequently, will be to reconsider that result in the context of our modified IS-LM framework, that is, using the expectational version of the IS function. It will be seen that the analysis is much simpler than in other optimizing models.[25]

At this point it will be useful to adopt a *change in notation* relative to preceding sections. Specifically, from this point onward we will let y_t, p_t, and m_t denote the logarithms of real output, the price level, and the nominal money stock with R_t representing the (nominal) rate of interest. Thus the modified IS-LM portion of the model to be used in the present section can be written as

$$y_t = b_0 + b_1(R_t - E_t p_{t+1} + p_t) + b_2 E_t y_{t+1} + v_t \tag{35}$$

$$m_t - p_t = c_0 + c_1 y_t + c_2 R_t + \eta_t , \tag{36}$$

where v_t and η_t are stochastic disturbances, and where $b_2 = 1.0$ is a prominent possibility.[26]

For the aggregate supply portion of this model it would be possible to use some sticky-price specification, but in order to maximize the possibility of nominal indeterminacy we shall assume fully flexible prices, with y_t being determined exogenously according to

$$y_t = \rho_0 + \rho_1 y_{t-1} + u_t , \tag{37}$$

where $|\rho_1| < 1.0$, and u_t is white noise.[27]

23. Linearity of the feedback rule is not of central importance, but was assumed because the analysis was being conducted in a linear model.

24. Capacity output was treated as endogenous, but that is of no relevance for the issue at hand.

25. Woodford (1995) has recently suggested that the determinacy issue should be considered in a model with explicitly optimizing agents. His own analysis is in large part supportive of the results in McCallum (1981, 1986), despite some differences that are discussed in McCallum (1997). Earlier, Sargent and Wallace (1982) put forth arguments quite different from those of their 1975 paper, and attributed this difference to the use of an optimizing model. In fact, however, the main relevant difference is that their 1982 analysis is based on a model in which monetary and nonmonetary assets cannot be distinguished—and indeterminacy does not actually prevail in any case. On this, see McCallum (1986, pp. 144–45).

26. As noted in the discussion following equation (26), i_t is absent from (36) under the simplifying assumption that capital moves exactly along its steady state path, but would appear in (36) if we instead assumed only that investment had a constant expected growth rate. In both of our applications of our IS-LM model in this paper, the monetary authority operates a rule that does not refer explicitly to the money stock, and consequently the solutions for variables beside m_t are invariant to whether (26) or (36) is used to describe money demand behavior.

27. Here the process generating y_t is more general than that adopted in McCallum (1981), where output

To illustrate the notion of nominal indeterminacy, initially suppose that in an economy represented by (35)–(37), the monetary authority conducts policy by manipulating R_t according to the following rule:

$$R_t = \mu_0 + \mu_1 y_{t-1}. \tag{38}$$

As in section 2, the IS shock y_t is assumed to be an AR(1) process:

$$v_t = \rho_v v_{t-1} + e_{vt}, \tag{39}$$

with $|\rho_v| < 1.0$, and e_{vt} white noise. Then we can substitute from (37)–(39) into (35) to obtain

$$\begin{aligned}\rho_0 + \rho_1 y_{t-1} + u_t = b_0 &+ b_1(\mu_0 + \mu_1 y_{t-1}) - b_1 E_t p_{t+1} + b_1 p_t \\ &+ b_2[\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1} + u_t)] + \rho_v v_{t-1} + e_{vt}.\end{aligned} \tag{40}$$

Since y_{t-1}, u_t, v_{t-1}, and e_{vt} are apparently the only relevant state variables, we conjecture that the minimal-state-variable (MSV) solution for p_t will be of the form

$$p_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 u_t + \phi_3 v_{t-1} + \phi_4 e_{vt}, \tag{41}$$

and thus that $E_t p_{t+1} = \phi_0 + \phi_1(\rho_0 + \rho_1 y_{t-1} + u_t) + \phi_3 \rho_v v_{t-1}$. Substitution of these into (39) gives an expression that will hold for all values of y_{t-1}, u_t, and v_t [thereby making (41) a solution] only if the following "undetermined coefficient" restrictions hold:

$$\begin{aligned}\rho_0 &= b_0 + b_1\mu_0 - b_1\phi_1\rho_0 + b_2\rho_0(1 + \rho_1) \\ \rho_1 &= b_1\mu_1 - b_1\phi_1\rho_1 + b_1\phi_1 + b_2\rho_1^2 \\ 1 &= b_1\phi_2 - b_1\phi_1 + b_2\rho_1 \\ 0 &= b_1\phi_3(1-\rho_v) + \rho_v \\ 0 &= b_1\phi_4 + 1.\end{aligned} \tag{42}$$

From the latter three conditions we see that $\phi_1 = (\rho_1 - b_1\mu_1 - b_2\rho_1^2)/b_1(1 - \rho_1)$, $\phi_2 = (1 - b_1\mu_1 - b_2\rho_1)/b_1(1 - \rho_1)$, $\phi_3 = -\rho_v/[b_1(1 - \rho_v)]$, and $\phi_4 = -1/b_1$. But the coefficient ϕ_0 does not appear in any of the conditions, so its value is not determined. In this sense, p_t is indeterminate in the model at hand. Then from (36) it follows that m_t, the system's other nominal variable, is indeterminate as well.

But the hypothetical policy rule (38) does not meet the proviso mentioned in the first paragraph of this section, namely, that of involving some nominal variable in an essential way.[28] Accordingly, let us now consider a policy rule that does involve a

is treated as a constant. It is that property that has led us to use a different example here, because with a constant output the modified expectational IS function (35) cannot be distinguished from the unmodified special case in which $b_2 = 0$. Logically, then, it follows that the analysis in McCallum (1981) is valid even under the assumption that the modified IS function is relevant, but only because output does not vary.

28. To illustrate the meaning of the last phrase, consider a specification that includes $m_{t-1} - p_{t-1}$. This

nominal variable. Suppose then that R_t is set each period so as to make the expected value of p_t equal to an exogenously chosen target value p^*. Thus R_t is set according to

$$\begin{aligned} R_t &= E_{t-1}[E_t\, p_{t+1} - p^* + (1/b_1)(y_t - b_0 - b_2 E_t y_{t+1}) - (1/b_1)v_t)] \\ &= E_{t-1}p_{t+1} - p^* + (1/b_1)[\rho_0 + \rho_1 y_{t-1} - b_0 \\ &\quad - b_2\{\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1})\} - (1 - b_1)\rho_v v_{t-1}]. \end{aligned} \tag{43}$$

Since $E_{t-1}p_{t+1} = \phi_0 + \phi_1(\rho_0 + \rho_1 y_{t-1}) + \phi_3\rho_v v_{t-1}$ under the conjecture that (41) is the solution form, rule (43) is a feedback rule for R_t of the class considered by Sargent and Wallace (1975).

Putting (43) instead of (38) into (35) yields

$$\begin{aligned} \rho_0 + \rho_1 y_{t-1} + u_t = {} & b_0 - b_1 p^* + \rho_0 + \rho_1 y_{t-1} - b_0 - b_2\rho_0 \\ & - b_2\rho_0(\rho_0 + \rho_1 y_{t-1}) - \rho_v v_{t-1} - b_1\phi_1 u_t - b_1\phi_3\rho_v e_{vt} \\ & + b_1[\phi_0 + \phi_1 y_{t-1} + \phi_2 u_t + \phi_3 v_{t-1} + \phi_4 e_{vt}] \\ & + b_2[\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1} + u_t)] + \rho_v v_{t-1} + e_{vt}. \end{aligned} \tag{44}$$

Consequently, the conditions analogous to (42) are:

$$\begin{aligned} 0 &= -b_1 p^* + b_1\phi_0 \\ \rho_1 &= \rho_1 + b_1\phi_1 \\ 1 &= -b_1\phi_1 + b_1\phi_2 + b_2\rho_1 \\ 0 &= b_1\phi_3 \\ 0 &= -b_1\phi_3\rho_v + b_1\phi_4 + 1. \end{aligned} \tag{45}$$

Here the solutions for ϕ_2 and ϕ_4 are as before whereas those for ϕ_1 and ϕ_3 are now equal to zero. But the important difference is that the coefficient ϕ_0 appears precisely once in the first of equations (45) so its value is clearly and uniquely determined. Thus the conjectured solution form (41) is shown to be justified, and to yield a fully determinate solution for p_t. This remains true, moreover, if the value 1.0 is assigned to b_2, as the discussion of section 2 would suggest, or if $b_2 = 0$ (as in the unmodified IS function). Furthermore, if the price level target was a *path*, p_t^*, instead of a constant value, then ϕ_0 would equal zero and p_t^* would appear in its place in the solution; thus determinacy would again result.

The foregoing example provides an illustration of price level determinacy with an interest rate rule that is rather different than the one used in McCallum (1981), $E_{t-1}p_t$ rather than $E_{t-1}m_t$ being the targeted nominal "anchor" and interest rate smoothing being absent. Because of the latter property, there is here no need to decide which value of ϕ_1 represents the bubble-free or "fundamentals" MSV solution, as is necessary

represents the inclusion of the log of the previous period's real money balances, so would not actually involve any nominal variable.

for the counterpart of ϕ_1 in the 1981 paper. The important aspect of the comparison, however, is the similarity of the two examples with regard to the issue of nominal determinacy with a R_t policy instrument. In both cases, a nominal magnitude enters the policy rule and determinacy prevails. The basic reason is that indeterminacy results only when the monetary authority totally neglects to provide a nominal anchor, in which case there is no economic actor that is in any way concerned with nominal magnitudes—rational private actors being concerned only with real variables such as y_t or $m_t - p_t$.

If interest rate smoothing—that is, a tendency by the monetary authority to keep R_t close to R_{t-1}—were present in our current example, there would be two values of some coefficients that would satisfy the undetermined coefficient conditions analogous to (45). Thus, there might be a multiplicity of rational expectation solutions to the model, if one of the values was not ruled out by an assumed transversality condition. But even if a multiplicity were to occur, there would be no implication of nominal indeterminacy. In this regard it is important to distinguish between solution multiplicities and nominal indeterminacies, that is, between bubbles and the absence of any nominal anchor. The former phenomenon typically involves multiple solution paths for real variables (for example, multiple solution paths for p_t even with m_t paths given), whereas the latter refers to situations in which the model provides a single solution for all real variables but simply fails to determine a solution path for any nominal variable.[29] These two types of "aberrant" behavior are conceptually quite distinct, a fact that is sometimes masked by an unsatisfactory terminology that refers to both merely as "indeterminacies."[30] In particular, nominal indeterminacy is a static concept relating to the difference between real and nominal variables whereas solution multiplicity is a dynamic concept involving arbitrary but rational expectational behavior. In McCallum (1997), it is argued that failure to distinguish between these two concepts, as in Kerr and King (1996) and Woodford (1995), is apt to lead to analytical confusion. A third concept is dynamic instability.[31]

From the standpoint of the present paper's main issue, our conclusion is that use of the modified IS function does not alter results pertaining to potential nominal indeterminacy. Just as with conventional IS-LM specifications, nominal prices and money-stock values are determinate even with interest rate policy rules provided that these rules involve some nominal magnitude in an essential way.

29. This conceptual distinction was emphasized by McCallum (1986, p. 137).

30. In a recent paper that uses a modified IS function similar to ours, Kerr and King (1996) develop several interesting results, one of which may appear to conflict with our finding since it suggests that, for some parameter values, an interest rate rule that involves a nominal variable will not result in a "unique equilibrium." But there is, in fact, no conflict, since a non-unique equilibrium is not the same form of aberrant behavior as nominal indeterminacy.

31. Howitt (1992) finds that an interest rate peg would lead to dynamic instability in an IS-LM model that includes a sticky-price Phillips curve and a generalized adaptive form of dynamic learning behavior rather than rational expectations. Irrespective of this latter difference, Howitt's results do not pertain to the issue at hand since the type of pegging that he is concerned with involves keeping R_t at some fixed value indefinitely, not varying R_t period by period in an instrument capacity.

4. GRADUAL PRICE ADJUSTMENT

Our second application is designed to illustrate the use of the modified IS-LM formulation in a model with sticky prices. As a simple and convenient example, let us consider an issue involving nominal income targeting, a policy advocated by a number of economists who have found that under a variety of model specifications, such a policy delivers satisfactory results, including stable behavior of inflation and output. Recently, however, Ball (1997) has claimed that rules directed at achieving either a target path or growth rate of nominal income would be "disastrous: they imply that output and inflation have infinite variances." Ball's result is also presented and discussed by Svensson (1997). In the particular model utilized, a nominal income targeting rule produces a solution equation for log output (y_t) whose dynamics contain a (negative) unit root, producing undamped oscillations and an unbounded unconditional variance for y_t and also output growth Δy_t. But with a nominal income growth rate rule producing stability of the sum of Δy_t and Δp_t, inflation Δp_t must have unstable dynamics too, so both inflation and output have unbounded variances.

In the model of Ball's paper, the specification of the IS function is entirely backward looking as it relates y_t to y_{t-1} and r_{t-1}, where r_t denotes the real interest rate, rather than the determinants $E_t y_{t+1}$ and r_t suggested by the analysis of the present paper. In addition, Ball's price adjustment relation is also backward looking. Concerning this specification, there exists considerable professional disagreement, but probably the closest thing at present to a "standard" or "consensus" model of aggregate supply is one that incorporates expectational elements, by specifying inflation as a function of its own expected future value (as well as an output gap variable). Roberts (1995), for example, demonstrates that several popular models of nominal rigidities—including the Calvo (1983), Rotemberg (1982), and Taylor (1980) setups—imply an inflation equation of the form $\Delta p_t = \beta E_t \Delta p_{t+1} + a y_t + u_t$, where $a > 0$ and $0 < \beta < 1$.[32,33] Accordingly, it is of interest to consider whether an instability result obtains in a model with forward-looking IS and price-adjustment relations.

For that purpose, our model consists of

$$y_t = b_0 + b_1(R_t - E_t\Delta p_{t-1}) + b_2 E_t y_{t+1} + v_t, \tag{46}$$

$$m_t - p_t = c_0 + c_1 y_t + c_2 R_t + \eta_t, \tag{47}$$

$$\Delta p_t = \beta E_t \Delta p_{t+1} + a y_t + u_t, \quad a > 0,\ 0 < \beta < 1, \tag{48}$$

$$R_t = E_t \Delta p_{t+1} + \mu_0 + \mu_1(\Delta p_t + y_t - y_{t-1}),\ \mu_1 > 0, \tag{49}$$

where v_t behaves according to (39), and u_t is white noise. Equation (47), the LM or money demand function, is actually superfluous in such a model since R_t is used as the central bank's instrument. Its only function, therefore, is to determine the behavior of

32. Roberts approximates β by unity, which is not vital for his derivations.

33. The derivation of our optimizing IS-LM equations in section 2 assumed price flexibility, but it is possible to derive the same equations from a sticky-price general equilibrium model. The labor market clearing condition (8) would then be replaced by the assumption that labor departs from its supply curve temporarily, with producers hiring sufficient labor to produce the quantity of output demanded.

m_t that is required to support policy rule (49). The latter, like Ball's, is a rule that directs the monetary authority to raise the real interest rate $(R_t - E_t\Delta p_{t+1})$ whenever nominal income growth exceeds a constant target value.[34]

Substituting (49) into (46), we obtain

$$y_t = b_0 + b_1(\mu_0 + \mu_1[\Delta p_t + y_t - y_{t-1}]) + b_2 E_t y_{t+1} + v_t\,. \tag{50}$$

The MSV solutions of the system (48), (50) will be of the form

$$y_t = \phi_{10} + \phi_{11}y_{t-1} + \phi_{12}u_t + \phi_{13}v_t; \tag{51}$$

$$\Delta p_t = \phi_{20} + \phi_{21}y_{t-1} + \phi_{22}u_t + \phi_{23}v_t\,. \tag{52}$$

After using (51) and (52) to evaluate $E_t y_{t+1}$ and $E_t\Delta p_{t+1}$, we obtain eight "undetermined coefficient" conditions, including the following pair:

$$\begin{aligned}\phi_{11} &= b_1\mu_1\phi_{21} + b_1\mu_1\phi_{11} - b_1\mu_1 + b_2\phi_{11}^2\\ \phi_{21} &= a\phi_{11} + \beta\phi_{11}\phi_{21}\,.\end{aligned} \tag{53}$$

The second condition implies $\phi_{21} = a\phi_{11}(1 - \beta\phi_{11})^{-1}$ which when substituted into the first condition produces

$$\begin{aligned}&b_1\mu_1 + \phi_{11}(1 - ab_1\mu_1 - b_1\mu_1 - \beta b_1\mu_1) + \phi_{11}^2(\beta b_1\mu_1 - b_2 - \beta)\\ &\quad + \beta b_2\phi_{11}^3 = 0\,.\end{aligned} \tag{54}$$

The left-hand side of (54) is a cubic equation in ϕ_{11}. At $\phi_{11} = 0$, it takes the value $b_1\mu_1 < 0$. At $\phi_{11} = 1$, on the other hand, it takes the value $(1 - \beta) - ab_1\mu_1 > 0$, for $b_2 = 0$ (the parameter choice associated with the traditional IS relationship) and equals $-ab_1\mu_1 > 0$ for $b_2 = 1.0$ (the value associated with our preferred, forward-looking IS specification). Since polynomials are continuous, the cubic therefore must have a root in (0, 1), regardless of which b_2 value is chosen. Accordingly, $0 < \phi_{11} < 1$, so the solution (51) for y_t is dynamically stable. Then (52) shows immediately that Δp_t also must be dynamically stable. Therefore, Ball's instability result does not obtain in a model with forward-looking versions of the IS and aggregate supply relations.

5. CONCLUSIONS

In the preceding sections we have shown that a dynamic, optimizing general equilibrium model of the Sidrauski-Brock type gives rise, when rather orthodox preference and production functions are specified, to a pair of linear equations that are

34. The monetary authority is assumed to observe the values of v_t and u_t when it sets the value of R_t, so that it knows the values of the real interest rate $(R_t - E_t\Delta p_{t+1})$ and output y_t implied by an R_t choice. A more realistic rule would not give the monetary authority knowledge of period t shocks, but we let the central bank have this knowledge in order to maximize comparability with Ball (1997).

analogous to traditional IS and LM functions. One of these equations is similar—except for timing—to a typical money demand specification, and can be made identical by a minor (and frequently utilized) timing modification. The other equation differs from a basic IS specification in one respect: an additional variable reflecting expected next-period income is present.[35] This modification gives a dynamic, forward-looking aspect to saving behavior. Together, the two equations provide a model of aggregate demand behavior that is reasonably tractable and yet usable with a wide variety of aggregate supply specifications—from full price flexibility to ones with sticky prices.

Our specification treats capital (and therefore capacity output) as exogenous, with a constant expected growth rate of investment, so the model is not usable for issues concerning capital accumulation. Subject to that proviso, we in effect argue that traditional IS and LM functions need to be modified, for monetary policy or business cycle issues, only by the addition to the former of one forward-looking term. Consequently, the question naturally arises: Does this one modification result in a framework that implies traditional or nontraditional answers to substantive problems? To this question there can evidently be no single answer, except that "it depends" upon the problem at hand. But some tentative conclusions are possible, at least as conjectures. Thus it seems clear that issues involving details of dynamic behavior will have answers that differ from the traditional ones, possibly sharply, unless income (per capita) has a constant expected future value. On the other hand, issues that hinge on the distinction between real and nominal variables would appear to yield familiar and traditional answers.

35. In section 2 we also presented stochastic versions, which require a few rather common approximations.

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