Comparing Groups: Statistical Tests

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• This presentation is based on (Chapman and Feit 2019, chap. 6)



 Understand the use of statistical tests to identify differences between groups in data



Import data

37.3 Female 81042.

41.0 Female 79353.

```
segmentation <- read_csv(file = "http://goo.gl/qw303p")</pre>
segmentation |> head(n = 5)
# A tibble: 5 x 7
    age gender income kids ownHome subscribe Segment
 <dbl> <chr>
               <dbl> <dbl> <chr>
                                   <chr>>
                                             <chr>>
  47.3 Male
              49483.
                         2 ownNo
                                   subNo
                                             Suburb mix
  31.4 Male 35546.
                      1 ownYes
                                   subNo
                                             Suburb mix
 43.2 Male
                         0 ownYes
              44169.
                                   subNo
                                             Suburb mix
```

Suburb mix

Suburb mix

1 ownNo

3 ownYes

subNo

subNo



Chi-squared test

```
segmentation |> count(Segment)
# A tibble: 4 x 2
 Segment
 <chr>>
             <int>
1 Moving up
                70
2 Suburb mix
               100
3 Travelers
                80
4 Urban hip
                50
segmentation |>
 count(subscribe, ownHome) |>
 pivot_wider(id_cols = subscribe,
              names_from = ownHome,
              values_from = n)
# A tibble: 2 x 3
 subscribe ownNo ownYes
```



<int> <int>

22

123

18

<chr>

2 subYes

1 subNo 137

Chi-squared test for given probabilities

$$\begin{split} H_0: p_1 &= \tfrac{1}{4} \wedge p_2 = \tfrac{1}{4} \wedge p_3 = \tfrac{1}{4} \wedge p_4 = \tfrac{1}{4} \\ H_1: p_1 &\neq \tfrac{1}{4} \vee p_2 \neq \tfrac{1}{4} \vee p_3 = \tfrac{1}{4} \vee p_4 \neq \tfrac{1}{4} \\ \chi^2 &= \sum_{i=1}^n \frac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{70 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{100 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{80 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{50 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} \end{split}$$

Base R way

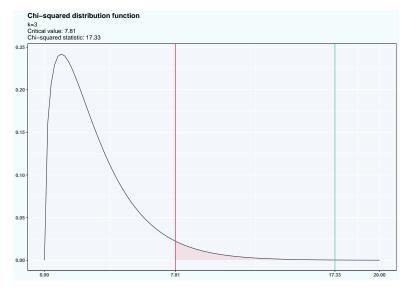
```
chi_statistic <- table(segmentation$Segment) |>
  chisq.test(p = c(1/4, 1/4, 1/4, 1/4))
chi statistic
```

Chi-squared test for given probabilities

```
data: table(segmentation$Segment)
X-squared = 17.333, df = 3, p-value = 0.0006035
```



• Chi-squared test for given probabilities





Chi-squared test for given probabilities

$$\begin{split} H_0: p_1 &= \tfrac{1}{4} \wedge p_2 = \tfrac{1}{4} \wedge p_3 = \tfrac{1}{4} \wedge p_4 = \tfrac{1}{4} \\ H_1: p_1 &\neq \tfrac{1}{4} \vee p_2 \neq \tfrac{1}{4} \vee p_3 = \tfrac{1}{4} \vee p_4 \neq \tfrac{1}{4} \\ \chi^2 &= \sum_{i=1}^n \frac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{70 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{100 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{80 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{50 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} \end{split}$$

tidymodels way



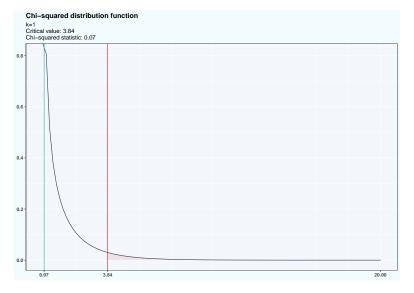
Pearson's Chi-squared test

$$\begin{split} H_0: p_{11} &= \tfrac{260}{300} \tfrac{159}{300} \land p_{12} = \tfrac{260}{300} \tfrac{141}{300} \land p_{21} = \tfrac{40}{300} \tfrac{159}{300} \land p_{22} = \tfrac{40}{300} \tfrac{141}{300} \\ H_1: p_{11} &\neq \tfrac{260}{300} \tfrac{159}{300} \lor p_{12} \neq \tfrac{260}{300} \tfrac{141}{300} \lor p_{21} \neq \tfrac{40}{300} \tfrac{159}{300} \lor p_{22} \neq \tfrac{40}{300} \tfrac{141}{300} \\ \chi^2 &= \sum_{i=1}^n \tfrac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{(137 - 300 \tfrac{260}{300} \tfrac{159}{300})^2}{300 \tfrac{260}{200} \tfrac{159}{200}} + \tfrac{(123 - 300 \tfrac{260}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{200} \tfrac{159}{200}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{40}{200} \tfrac{159}{200}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{40}{200} \tfrac{159}{200}} \end{split}$$

Base R way



• Pearson's Chi-squared test





Pearson's Chi-squared test

$$\begin{split} H_0: p_{11} &= \tfrac{260}{300} \tfrac{159}{300} \land p_{12} = \tfrac{260}{300} \tfrac{141}{300} \land p_{21} = \tfrac{40}{300} \tfrac{159}{300} \land p_{22} = \tfrac{40}{300} \tfrac{141}{300} \\ H_1: p_{11} &\neq \tfrac{260}{300} \tfrac{159}{300} \lor p_{12} \neq \tfrac{260}{300} \tfrac{141}{300} \lor p_{21} \neq \tfrac{40}{300} \tfrac{159}{300} \lor p_{22} \neq \tfrac{40}{300} \tfrac{141}{300} \\ \chi^2 &= \sum_{i=1}^n \tfrac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{(137 - 300 \tfrac{260}{300} \tfrac{159}{300})}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(123 - 300 \tfrac{260}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} \end{split}$$

tidymodels way



Exact binomial test

$$\begin{split} H_0: p &= 0.5 \ H_1: p \neq 0.5 \\ B &= \sum_{i=1}^n x_i = 157 \ \text{where} \ x_i \in 0, 1 \end{split}$$

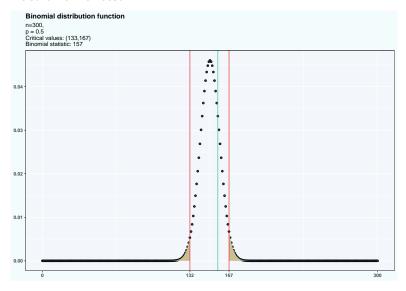
R base way

```
Exact binomial test

data: 157 and 300
number of successes = 157, number of trials = 300, p-value = 0.453
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
0.4651595 0.5810418
sample estimates:
probability of success
0.5233333
```



Exact binomial test





- Exact binomial test
 - Confidence interval:

$$p_L$$

• p_L and p_U are random variables but p is not a random variable. Therefore $[p_L,p_U]$ is a random interval where we have that:

$$P(0.4651595 \approx p_L$$



Exact binomial test

$$H_0: p = 0.5 \ H_1: p \neq 0.5$$

$$B = \sum_{i=1}^n x_i = 157 \ \text{where} \ x_i \in 0, 1$$

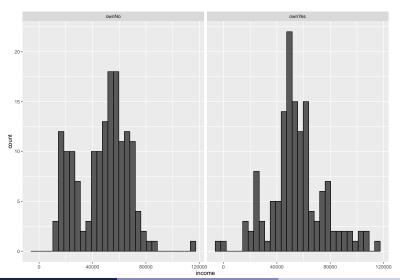
tidymodels way

```
# A tibble: 1 x 8
 estimate statistic p.value parameter conf.low conf.high method
                                                                       alternative
     <dbl>
               <dbl>
                       <dbl>
                                  <dh1>
                                           <db1>
                                                     <dhl> <chr>>
                                                                       <chr>>
     0.523
                 157
                       0.453
                                    300
                                           0.465
                                                     0.581 Exact bin~ two.sided
```



• 2 sample t-test: independent samples

```
segmentation |> ggplot() +
  geom_histogram(aes(x = income), color='black') +
  facet_wrap(facets = vars(ownHome))
```





• 2 sample t-test: independent samples

```
segmentation |>
 group_by(ownHome) |>
 summarise(mean_income = mean(income),
           var_income = var(income),
           n = n()
# A tibble: 2 x 4
 ownHome mean_income var_income
 <chr>
              <db1>
                           <dbl> <int>
1 ownNo
              47391. 358692875.
                                 159
2 ownYes
              54935. 430890091.
                                  141
```



2 sample t-test: independent samples

$$\begin{split} H_0: \mu_{ownNo} - \mu_{ownYes} &= 0 \ H_1: \mu_{ownNo} - \mu_{ownYes} \neq 0 \\ t &= \frac{\overline{ownNo} - \overline{ownYes}}{\sqrt{\frac{s_{ownNo}^2}{n_{ownYes}}} - \frac{s_{ownYes}^2}{n_{ownYes}}} = \frac{47391.01 - 54934.68}{\sqrt{\frac{358692875}{159} - \frac{430890091}{141}}} \approx -3.273094 \end{split}$$

R base way

```
Welch Two Sample t-test

data: income by ownHome

t = -3.2731, df = 285.25, p-value = 0.001195

alternative hypothesis: true difference in means between group ownNo and group ownYes is not equal to 0

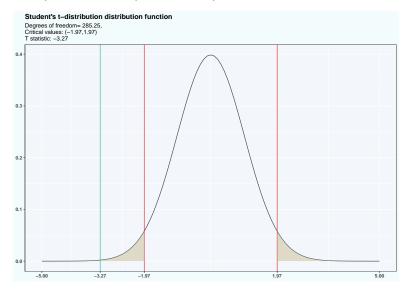
95 percent confidence interval:
-12080.155 -3007.193

sample estimates:
mean in group ownNo mean in group ownYes

47301.01 54034.68
```



• 2 sample t-test: independent samples





- 2 sample t-test: independent samples
 - Confidence interval:

$$c_L < \mu_{ownNo} - \mu_{ownYes} < c_U$$

• $\mu_{ownNo} - \mu_{ownYes}$ is not a random variable so we need to use a random variable

$$P\Bigg(t_L < \frac{\overline{x}_{ownNo} - \overline{x}_{ownYes} - (\mu_{ownNo} - \mu_{ownYes})}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_{ownYes}^2}{n_{ownYes}}}} < t_U\Bigg) = 0.95$$

• $\overline{x}_{ownNo} - \overline{x}_{ownYes}$ is a random variable



- 2 sample t-test: independent samples
 - Confidence interval:
 - $\bullet \ \ \, \frac{\overline{x}_{ownNo} \overline{x}_{ownYes} (\mu_{ownNo} \mu_{ownYes})}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_{ownYes}^2}{n_{ownYes}}}} \ \, \text{is also a random variable with} \\$

student's t-distribution and
$$\nu \approx \frac{(\frac{s_{ownNo}^2 + \frac{s_2^2}{n_{ownNo}} + \frac{s_2^2}{n_{ownNo}})^2}{\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_2^2}{\frac{s_2^2}{n_{ownYes}})^2}}{\frac{s_2^2}{n_{ownNo} + \frac{s_2^2}{n_{ownYes}}}} = 285.2521$$

degrees of freedom

ullet Also we need to specify t_L and t_U

```
t_L <- qt(p = 0.025, df = 285.25, lower.tail = TRUE) t_L
```

[1] -1.968315
$$t_U < -qt(p = 0.975, df = 285.25, lower.tail = TRUE)$$
 $t_U < -qt(p = 0.975, df = 285.25, lower.tail = TRUE)$

[1] 1.968315



- 2 sample t-test: independent samples
 - Confidence interval:

$$P(-7543.674-1.968315\times2304.753<\mu_{ownNo}-\mu_{ownYes}<-7543.674-1.968315\times2304.753)=0.95$$

$$P(-12080.16<\mu_{ownNo}-\mu_{ownYes}<-3007.193)=0.95$$

• In the long run 95% of confidence intervals constructed in this manner will contain the true parameter



2 sample t-test: independent samples

$$\begin{split} H_0: \mu_{ownNo} - \mu_{ownYes} &= 0 \ H_1: \mu_{ownNo} - \mu_{ownYes} \neq 0 \\ t &= \frac{\overline{ownNo} - \overline{ownYes}}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} - \frac{s_{ownYes}^2}{n_{ownYes}}}} = \frac{47391.01 - 54934.68}{\sqrt{\frac{358692875}{159} - \frac{430890091}{141}}} \approx -3.273094 \end{split}$$

tidymodels way

```
segmentation |>
t_test(formula = income - ownHome,
    alternative = "two-sided",
    order = c("ownNo", "ownYes"),
    mu = 0,
    conf_level = 0.95)
```

```
# A tibble: 1 x 7

statistic t_df p_value alternative estimate lower_ci upper_ci

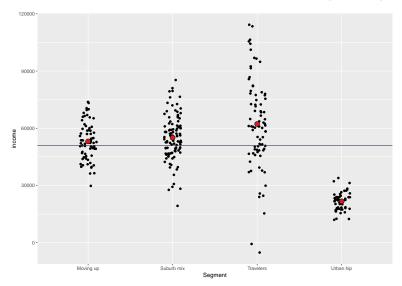
dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> < -7544. -12080. -3007.
```



```
group_by(Segment) |>
 summarise(mean = mean(income),
            variance = var(income).
            n = n()
# A tibble: 4 x 4
 Segment
                      variance
               mean
                                   n
 <chr>>
              <dbl>
                         <dbl> <int>
1 Moving up 53091.
                     92862689.
2 Suburb mix 55034. 142761527.
                                 100
3 Travelers 62214. 564173979.
                                  80
4 Urban hip 21682.
                     23885953.
                                  50
```

segmentation |>







$$H_0: \mu_{Moving\; up} = \mu_{Suburb\; mix} = \mu_{Travelers} = \mu_{Urban\; hip}$$

 H_1 : At least one group mean is different from the rest

$$n = \sum_{j=1}^{4} n_j = n_1 + \dots + n_4 = 70 + 100 + 80 + 50 = 300$$

$$\overline{income} = \frac{1}{n} \sum_{j=1}^{4} \sum_{i=1}^{n_j} income_{ij}$$

$$\overline{income}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} income_{ij}$$

$$F = \frac{\frac{\sum_{j=1}^{4} \sum_{i=1}^{n_{j}} (\overline{income_{j}} - \overline{income})^{2}}{\frac{4-1}{\sum_{j=1}^{4} \sum_{i=1}^{n_{j}} (income_{ij} - \overline{income_{j}})^{2}}}{\frac{300-4}{296}} = \frac{\frac{54969675428}{3}}{\frac{66281072794}{296}} = \frac{18323225143}{223922543} = 81.82841$$



R base way

```
anova_table <- aov(data = segmentation, formula = income ~ Segment) |>
anova()
anova_table

Analysis of Variance Table
```

```
Response: income

Df Sum Sq Mean Sq F value Pr(>F)

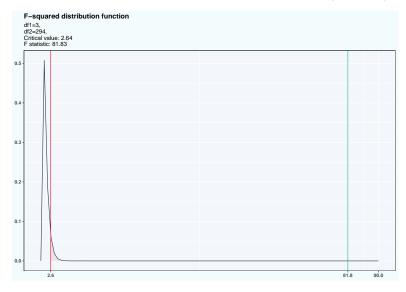
Segment 3 5.4970e+10 1.8323e+10 81.828 < 2.2e-16 ***

Residuals 296 6.6281e+10 2.2392e+08

---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```







tidymodels way

```
anova_table <- aov(data = segmentation, formula = income ~ Segment) |>
anova() |>
tidy()
anova_table
```

```
# A tibble: 2 x 6
              df
                                    meansq statistic
                                                        p.value
  term
                         sumsq
 <chr>
            <int>
                         <dh1>
                                     <dh1>
                                                <db1>
                                                         <dh1>
1 Segment
                3 54969675428. 18323225143.
                                                81.8 1.41e-38
2 Residuals 296 66281072794.
                                223922543.
                                                NA
                                                     NA
```



```
segmentation |>
  distinct(Segment) |>
  arrange(Segment) |>
  rowid_to_column(var = 'i')

# A tibble: 4 x 2
    i Segment
  <int> <chr>
    1    1 Moving up
    2    2 Suburb mix
    3    3 Travelers
    4    4 Urban hip

segmentation |>
    distinct(ownHome) |>
    rowid_to_column(var = 'j')

# A tibble: 2 x 2
```



j ownHome
<int> <chr>
1 ownNo
2 ownYes

```
count(Segment, ownHome, name = "n_ij")
# A tibble: 8 x 3
             ownHome
 Segment
                      n_ij
 <chr>>
             <chr>
                     <int>
1 Moving up ownNo
                        47
2 Moving up ownYes
                        23
3 Suburb mix ownNo
                        52
4 Suburb mix ownYes
                        48
5 Travelers ownNo
                        20
6 Travelers ownYes
                        60
7 Urban hip ownNo
                        40
```

10

8 Urban hip ownYes

segmentation |>



```
mu_ij <- segmentation |>
  group_by(Segment, ownHome) |>
  summarise(mean = mean(income)) |>
  ungroup()
mu_i1 <- mu_ij$mean[1]
mu_i1

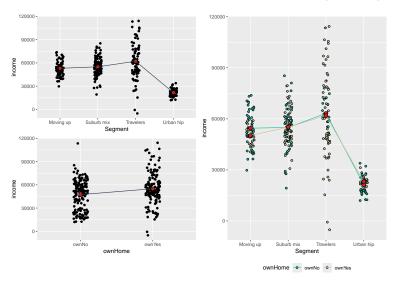
[1] 54497.68
segmentation |>
  select(income, Segment, ownHome) |>
  head(n=5)

# A tibble: 5 x 3
  income Segment  ownHome
  sdbl> schr>  schr>  schr>
```

```
3 44169. Suburb mix ownYes
4 81042. Suburb mix ownNo
5 79353. Suburb mix ownYes
```



1 49483. Suburb mix ownNo 2 35546. Suburb mix ownYes





$$\begin{split} income_{ijk} = & \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \\ & \text{where } + \epsilon_i \sim \mathcal{N}(0, \sigma^2) \\ & \text{and } i = 1, 2, 3, 4 \\ & j = 1, 2 \\ & k = 1, \dots n_{ij} \\ & \mu = \mu_{11} \\ & \alpha_1 = \beta_1 = 0 \\ & (\alpha\beta)_{11} = (\alpha\beta)_{12} = 0 \\ & (\alpha\beta)_{21} = (\alpha\beta)_{31} = (\alpha\beta)_{41} = 0 \end{split}$$



$$\begin{split} \widehat{income}_{ijk} = & \widehat{\mu} + \widehat{\alpha}_i + \widehat{\beta}_j + (\widehat{\alpha\beta})_{ij} + \widehat{\epsilon}_{ijk} \\ \text{and } i = 1, 2, 3, 4 \\ j = 1, 2 \\ k = 1, \dots n_{ij} \\ \widehat{\mu} = & \widehat{\mu}_{11} \\ \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\alpha}_4 \\ \widehat{\beta}_2 \\ \widehat{(\alpha\beta)}_{22}, (\widehat{\alpha\beta})_{32}, (\widehat{\alpha\beta})_{42} \end{split}$$



 $income_{ijk} - \widehat{income_{ijk}} = \hat{\epsilon}_{ijk}$

```
segmentation |>
  select(income, Segment, ownHome) |>
 head(n=2) >
  glimpse()
Rows: 2
Columns: 3
$ income <dbl> 49482.81, 35546.29
$ Segment <chr> "Suburb mix", "Suburb mix"
$ ownHome <chr>> "ownNo", "ownYes"
```

```
framed <- model frame(formula = income ~
                                 Segment +
                                 ownHome +
                                 Segment: ownHome,
            data = segmentation)
model matrix(terms = framed$terms,
             data = framed$data) |>
 head(n = 2) \mid >
 glimpse()
Rows: 2
Columns: 8
$ `(Intercept)`
                                     <dbl> 1, 1
$ `SegmentSuburb mix`
                                     <dbl> 1, 1
$ SegmentTravelers
                                     <dbl> 0, 0
$ `SegmentUrban hip`
                                     <dbl> 0, 0
$ ownHomeownYes
                                     <dbl> 0, 1
$ `SegmentSuburb mix:ownHomeownYes` <dbl> 0. 1
```

\$ `SegmentTravelers:ownHomeownYes`

\$ `SegmentUrban hip:ownHomeownYes`



<dbl> 0, 0

<dbl> 0, 0

- Testing Multiple Group Means: Analysis of Variance (ANOVA)
 - Model

$$\begin{bmatrix} 49482.81 \\ 35546.29 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Gamma \\ \alpha_2 \\ \beta_2 \\ (\alpha\beta)_{13} \\ (\alpha\beta)_{14} \\ (\alpha\beta)_{22} \\ (\alpha\beta)_{32} \\ (\alpha\beta)_{42} \end{bmatrix}$$

Coefficients to estimate using aov

$$\widehat{\mu} = \widehat{\mu}_{11}, \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\alpha}_4, \widehat{\beta}_2, (\widehat{\alpha\beta})_{22}, (\widehat{\alpha\beta})_{32}, (\widehat{\alpha\beta})_{42}$$



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Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
model_aov <- aov(formula = income ~ Segment + ownHome + Segment:ownHome,
                 data = segmentation)
coef(model aov) |> enframe(name = "coef")
# A tibble: 8 x 2
  coef
                                    value
 <chr>>
                                    <dh1>
1 (Intercept)
                                    54498
2 SegmentSuburb mix
                                     435.
3 SegmentTravelers
                                    8691.
4 SegmentUrban hip
                                  -33160
5 ownHomeownYes
                                   -4281.
6 SegmentSuburb mix:ownHomeownYes
                                    4492.
7 SegmentTravelers:ownHomeownYes
                                    2982
```



8 SegmentUrban hip:ownHomeownYes

- Testing Multiple Group Means: Analysis of Variance (ANOVA)
- Segment

$$H_0: \mu_{Moving\; up} = \mu_{Suburb\; mix} = \mu_{Travelers} = \mu_{Urban\; hip}$$

 H_1 : At least one group mean is different from the rest

ownHome

$$H_0: \mu_{ownNo} = \mu_{ownYes}$$

 $\boldsymbol{H}_1:$ At least one group mean is different from the rest

Segment, ownHome

$$H_0: \mu_{Moving\;up,\;ownNo} - \mu_{Moving\;up,\;ownYes} = \mu_{Suburb\;mix,\;ownNo} - \mu_{Suburb\;mix,\;ownYes} = \mu_{Suburb\;mix,\;ownYes} + \mu_{$$

 $\mu_{Travelers,\ ownNo} - \mu_{Travelers,\ ownYes} = \mu_{Urban\ hip,\ ownNo} - \mu_{Urban\ hip,\ ownYes}$

 H_1 : At least one difference group mean is different from the rest



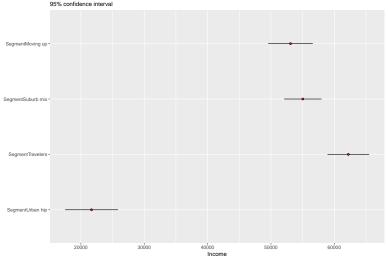
```
model_aov |>
anova()
```

Analysis of Variance Table

```
Response: income
```



Average income by segment





- To my family that supports me
- To the taxpayers of Colombia and the UMNG students who pay my salary
- To the Business Science and R4DS Online Learning communities where I learn R and π -thon
- To the R Core Team, the creators of RStudio IDE, Quarto and the authors and maintainers of the packages tidyverse, skimr, latex2exp, tidymodels, patchwork, hardhat and tinytex for allowing me to access these tools without paying for a license
- To the Linux kernel community for allowing me the possibility to use some Linux distributions as my main OS without paying for a license



References I

Chapman, Chris, and Elea McDonnell Feit. 2019. *R For Marketing Research and Analytics*. 2nd ed. 2019. Use R! Cham: Springer International Publishing: Imprint: Springer. https://doi-org.ezproxy.umng.edu.co/10.1007/978-3-030-14316-9.

