Segmentation: Clustering

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Table of contents I

Please Read Me

2 Purpose

3 Consumer segmentation survey

Acknowledgments



• This presentation is based on (Chapman and Feit 2019, chap. 11)



 Find groups of customers that differ in different dimensions to engage in more effective promotion



- age: age of the consumer in years
- gender: if the consumer is male of female
- income: yearly disposable income of the consumer
- kids: number of children of the consumer
- ownHome: if the consumer owns a home
- subscribe: if the consumer is subscribed or not



Import data

```
segmentation <- read_csv(file = "http://goo.gl/qw303p") |>
select(-Segment) # Remove Segment column to understand how it was build
segmentation |> head(n = 5)
```



Inspect data

```
segmentation |> glimpse()
```



Transform data

```
segmentation <- segmentation |>
mutate(gender = factor(gender, ordered = FALSE),
    kids = as.integer(kids),
    ownHome = factor(ownHome, ordered = FALSE),
    subscribe = factor(subscribe, ordered = FALSE))
segmentation |> head(n = 5)
```

```
# A tibble: 5 x 6
    age gender income kids ownHome subscribe
    <dbl> <fct> <fct > <fct
```



Summarize data

• Ups the table is really big!!! Try it in your console to see the complete table

```
segmentation |> skim()
```



Segmentation

- Classification (We will not cover this topic)
 - Supervised learning
 - Dependent variable is known and the goal is to predict the dependent variable from the independent variables
 - Naive bayes, Random Forest
- Clustering (This topic will be covered)
 - Unsupervised learning
 - Dependent variable is unknown and the goal is to discover it from the independent variables
 - Model-based clustering, Latent Class Analysis (We will not cover these methods)
 - Hierarchical clustering, k-means (These methods will be covered)



Clustering

- Grouping a set of observations in such a way that observations in the same group (cluster) are more similar to each other than to those in other groups (clusters).
- A notion of how "close" 2 observations is necessary to group objects where this is formalized using the concept of distance (known as metric¹ in mathematics)
 - There are many notions of distance (Deza and Deza 2016) where in this chapter the **Euclidean** and the **Gower** distance will be used



- Euclidean distance: it can only be used for numerical data
 - $\bullet \ x=(x_1,x_2,\dots,x_n)$
 - $\bullet \ y=(y_1,y_2,\dots,y_n)$

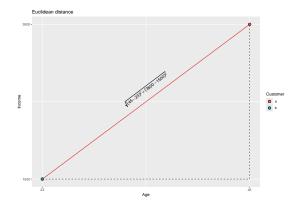
$$\begin{split} d(x,y) &= \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + \ldots + (x_n-y_n)^2} \\ &= \sqrt{\sum_{k=1}^n (x_k-y_k)^2} \end{split}$$

- An example:
 - 2 customers characteristic by age and income
 - a = (45, 3500)
 - b = (23, 1500)



Manual calculation

•
$$d(a,b) = \sqrt{(45-23)^2 + (3500-1500)^2} = 2000.121$$





Using R

```
customers <- tibble(Customer = c("a", "b"),
                   Age = c(45, 23),
                   Income = c(3500, 1500))
customers
# A tibble: 2 x 3
  Customer
            Age Income
  <chr> <dbl> <dbl>
           45 3500
1 a
2 h
             23 1500
library(cluster)
customers |>
  select(-Customer) |>
  daisy(metric = "euclidean")
Dissimilarities :
```



2 2000.121

Metric : euclidean Number of objects : 2 Gower distance: it can be used for categorical, numerical data and missing values

$$\bullet \ x=(x_1,x_2,\dots,x_n)$$

•
$$y = (y_1, y_2, \dots, y_n)$$

$$\begin{split} d(x,y) &= \left[\frac{w_1 \delta_{x_1 y_1}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_1 y_1}^1 + \left[\frac{w_2 \delta_{x_2 y_2}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_2 y_2}^2 + \ldots + \left[\frac{w_n \delta_{x_n y_n}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_n y_n}^n \\ &= \frac{\sum_{k=1}^n w_k \delta_{x_i y_i}^k d_{x_i y_i}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k} \end{split}$$

Where:

$$w_k \in \mathbb{R}$$
 for $k = 1, 2, \dots, n$

$$\sum_{k=1}^n w_k \delta_{x_i y_i}^k = w_1 \delta_{x_1 y_1}^1 + w_2 \delta_{x_2 y_2}^2 + \ldots + w_n \delta_{x_n y_n}^n$$



- Gower distance: it can be used for categorical, numerical data and missing values
 - $\bullet \ x = (x_1, x_2, \dots, x_n)$
 - $y = (y_1, y_2, \dots, y_n)$

$$d(x,y) = \frac{\sum_{k=1}^n w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^n w_k \delta_{x_k y_k}^k}$$

Where²:

$$\delta_{x_ky_k}^k = \begin{cases} 0 & \text{if } x_k \text{ or } y_k \text{ is a missing value} \\ 0 & \text{if } x_k, y_k \text{ represent an asymmetric binary variable and } x_k = y_k = 0 \\ 1 & \text{otherwise} \end{cases}$$

²See (Kaufman and Rousseeuw 1990, 25–27) for a definition of asymmetric binary variable



- Gower distance: it can be used for categorical, numerical data and missing values
 - $x = (x_1, x_2, \dots, x_n)$
 - $y = (y_1, y_2, \dots, y_n)$

$$d(x,y) = \frac{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^k}$$

Where:

$$d^k_{x_ky_k} = \begin{cases} 0 & \text{if } x_k,y_k \text{ represent a nominal or binary variable and } x_k = y_k \\ 1 & \text{if } x_k,y_k \text{ represent a nominal or binary variable and } x_k \neq y_k \\ \frac{|x_k - y_k|}{max(x_k,y_k) - min(x_k,y_k)} & \text{otherwise} \end{cases}$$

If x_k,y_k represent an ordinal variable they are replaced by their integer codes. For example if $x_k \precsim y_k$ then 1 is assigned to x_k and 2 is assigned to y_k



An example:

- 2 customers characteristic by sex (nominal), income (numerical), satisfaction (ordinal with levels $Low \preceq Medium \preceq High$) and age (with a missing value (NA))
 - a = (Female, 3500, Medium, 45)
 - $\bullet \ b = (Male, 1500, High, NA)$

Manual calculation:

- \bullet In R $w_k=1$ for every k as a default value where in this example k=1,2,3,4
- $\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k = 1 * 1 + 1 * 1 + 1 * 1 + 1 * 0 = 1 + 1 + 1 + 0 = 3$
- $\bullet \ \, \sum_{k=1}^4 w_k \delta^k_{x_k y_k} d^k_{x_k y_k} = 1*1+1* \frac{|3500-1500|}{3500-1500} + 1* \frac{|2-3|}{3-2} + 0 = 3$
- $d(x,y) = \frac{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k} = \frac{3}{3} = 1$



Gower distance range:

- $\bullet \ d(x,y) \in [0,1]$
- If $d(x,y) \longrightarrow 0$ is more similar
- If $d(x,y) \longrightarrow 1$ is more dissimilar

Using R

```
# A tibble: 2 x 5
Customer Sex Income Satisfaction Age
<ch>< chr> <fct> <dbl> <ord> <dch> <dch> <ddb> <ord> <ddb> </dd> </dr>
1 a Female 3500 Medium 45
b Male 1500 High NA
```



Using R

Dissimilarities :

```
customers2 |>
  select(-Customer) |>
  daisy(metric = "gower")
```

```
1
2 1

Metric: mixed; Types = N, I, O, I
Number of objects: 2
```

- In this case:
 - Metric: mixed because it includes categorical and numerical data
 - For Types = N, I, O, I check out
 ?cluster::dissimilarity.object3
 - N: Nominal (factor)
 - I: Interval scaled (numeric)
 - D: Ordinal (ordered factor)



³See (Stevens 1946) and Level of measurement

Using R

```
customers2 |>
  select(-Customer) |>
  daisy(metric = "gower")
```

```
Dissimilarities:
1
2 1
Metric: mixed; Types = N, I, O, I
Number of objects: 2
```

- In this case:
 - Number of objects : 2
 - There are 2 observations that correspond to customers a and b:

```
\begin{split} a &= (Female, 3500, Medium, 45) \text{ and } \\ b &= (Male, 1500, High, NA) \end{split}
```



- ullet The original dissimilarity matrix is of dimension 300 imes 300
 - ullet Showing only the relation between the first 5 observations
 - \bullet The position (i,j) means the dissimilarity between the observations i and j
 - \bullet For example (4,3), which is equal to 0.425, is the dissimilarity between the observations 4 and 3

```
segmentation_dist <- segmentation |>
daisy(metric = "gower")

segmentation_dist |>
as.matrix() |>
as_tibble() |>
select('1':'5') |>
slice(1:5)
```



```
# A tibble: 5 x 5
 Customer Sex Income Satisfaction
                                   Age
 <chr> <fct> <dbl> <ord>
                                 <db1>
1 a
        Female 3500 Medium
                                   45
2 b
        Male 1500 High
                                   NΑ
        Female 200 Low
                                   34
3 c
4 d
       Female 450 Low
                                   23
5 e
        Male 5000 Medium
                                   55
```



Hierarchical clustering

Method: Complete Linkage Clustering

```
customers3_dist <- daisy(x = select(customers3, -Customer),</pre>
                        metric = "gower")
customers3_dist
Dissimilarities :
2 0.63888889
3 0.38281250 0.75694444
4 0.45572917 0.73958333 0.09895833
5 0.40625000 0.40972222 0.78906250 0.86197917
Metric: mixed; Types = N, I, O, I
Number of objects: 5
customers3 hc <- hclust(d = customers3 dist.
                        method = "complete")
customers3 hc
```

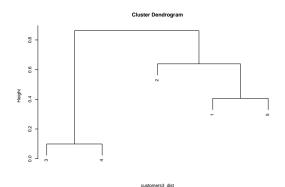
```
Call:
hclust(d = customers3_dist, method = "complete")
Cluster method : complete
Number of objects: 5
```



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- Hierarchical clustering
 - Method: Complete Linkage Clustering

plot(customers3_hc)



hclust (*, "complete")

• Compare each observation and find the pair that is more similar

	1	2	3	4	5
1	0.0000000	0.6388889	0.3828125	0.4557292	0.4062500
2	0.6388889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.4557292	0.7395833	0.09895833	0.0000000	0.8619792
5	0.4062500	0.4097222	0.7890625	0.8619792	0.0000000



- \bullet Now we have the first cluster that includes the observations 3 and 4 : C(3,4)
- \bullet Then we need to create clusters with observations $1,\,2$ and 5 and the cluster C(3,4)
 - How we compare a cluster with an observation
 - Complete Linkage Clustering: Use the maximum distance between an observation and an observation that belongs to the cluster



- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case 1, 2, 5 and C(3,4)
 - The distance between 1 and C(3,4) is 0.45572917
 - ullet The distance between 2 and C(3,4) is 0.7569444
 - ullet The distance between 5 and C(3,4) is 0.8619792

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000



- Now we have the second cluster that includes the observations 1 and $5\colon C(1,5)$
- \bullet Then we need to create clusters with observation 2 and clusters C(3,4) and C(1,5)
 - How we compare a cluster with another cluster
 - Complete Linkage Clustering: Use the maximum distance between an observation that belongs to the first cluster and an observation that belongs to the second cluster



- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case 2, C(3,4) and C(1,5)
 - ullet The distance between 2 and C(3,4) is 0.7569444
 - \bullet The distance between 2 and C(1,5) is 0.6388889

	-				
	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000



- Now we have the third cluster that includes the observation 2 and the cluster $C(1,5)\colon C(2,C(1,5))$
- \bullet Then we need to create clusters with cluster C(2,C(1,5)) and cluster C(3,4)
 - This is the cluster that includes all the observations



- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case C(3,4) and C(2,C(1,5))
 - \bullet The distance between C(3,4) and C(2,C(1,5)) is 0.86197917

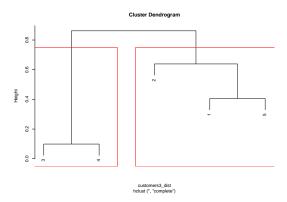
	1	2	3	4	5
1	0	0.6388889	0.3828125	0.45572917	0.4062500
2	0.63888889	0.0000000	0.75694444	0.73958333	0.4097222
3	0.3828125	0.7569444	0	0.09895833	0.7890625
4	0.45572917	0.7395833	0.09895833	0	0.8619792
5	0.40625	0.4097222	0.7890625	0.86197917	0.0000000

 \bullet The heights of the **Cluster Dendrogram** are: $0.09895833,\,0.40625,\,0.63888889$ and 0.86197917



• Select a number of clusters, for example: 2 clusters

```
plot(customers3_hc)
rect.hclust(customers3_hc, k = 2, border = "red")
```





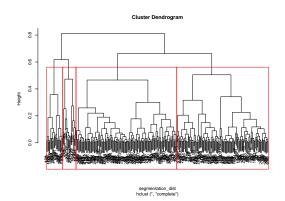
Extract clusters and assign them to observations

```
customers3_hc_clusters <- cutree(customers3_hc, k = 2)
customers3 |>
mutate(cluster = customers3_hc_clusters)
```

```
# A tibble: 5 x 6
 Customer Sex
              Income Satisfaction
                                 Age cluster
 <chr>
       <fct> <dbl> <ord>
                               <dbl>
                                      <int>
        Female 3500 Medium
        Male 1500 High
     Female 200 Low
                                34
     Female
               450 Low
                                 23
        Male
                5000 Medium
                                  55
```



 Select a number of clusters, using segmentation, for example: 4 clusters





 Extract clusters and assign them to observations, using segmentation

```
segmentation |>
 mutate(cluster = segmentation hc clusters)
# A tibble: 300 x 7
    age gender income
                       kids ownHome subscribe cluster
   <dbl> <fct> <dbl> <int> <fct>
                                    <fct>
                                                <int>
 1 47.3 Male 49483.
                          2 ownNo
                                    subNo
   31.4 Male 35546.
                          1 ownYes subNo
  43.2 Male
              44169.
                          0 ownYes
                                    subNo
   37.3 Female 81042.
                          1 ownNo
                                    subNo
   41.0 Female 79353.
                          3 ownYes subNo
   43.0 Male
               58143.
                          4 ownYes subNo
   37.6 Male
              19282
                          3 ownNo
                                    subNo
   28.5 Male
              47245
                          0 ownNo
                                    subNo
   44.2 Female 48333.
                                    subNo
                          1 ownNo
   35.2 Female 52568
                          O ownYes subNo
# i 290 more rows
```

segmentation_hc_clusters <- cutree(segmentation_hc, k = 4)



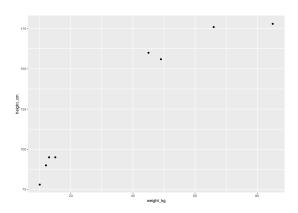
• K-means clustering example (Kaufman and Rousseeuw 1990, 5)

A tibble: 8 x 3 weight_kg height_cm name <chr>> <db1> <db1> 1 Ilan 15 95 2 Jacqueline 49 156 3 Kim 13 95 4 Lieve 45 160 5 Leon 85 178 6 Peter 66 176 7 Talia 12 90 8 Tina 10 78



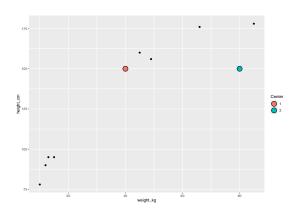
• K-means clustering example (Kaufman and Rousseeuw 1990, 5)

```
kaufman_example |>
ggplot() +
geom_point(aes(x = weight_kg, y = height_cm))
```





- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - \bullet Choose k centers or the computer will choose k centers at random, in our case we choose k=2

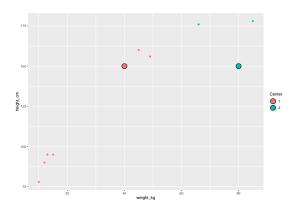




- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - ullet Calculate the squared euclidean distance for each point to the k centers and assign each point to the nearest center
 - \bullet For example for the point Ilan=(15,95) the distance to $Center_1=(40,150) \text{ is } (15-40)^2+(95-150)^2=3650 \text{ and the distance to } Center_2=(80,150) \text{ is } (15-80)^2+(95-150)^2=7250$
 - ullet Therefore Ilan=(15,95) is assigned to $Center_1$



- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm

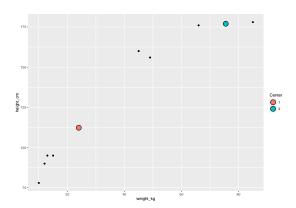




- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - Now calculate new centers using the assigned points by using the mean
 - For example for the new $Center_1$ the new position will be $x=\frac{15+49+13+45+12+10}{6}=24$ and $y=\frac{95+156+95+160+90+78}{6}\approx 112.33$
 - Therefore we update as $Center_1 \approx (24, 112.33)$

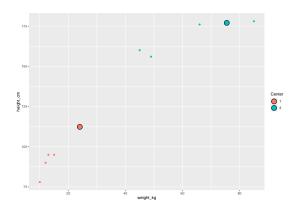


- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm



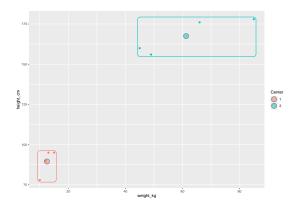


- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - ullet Repeat the process by calculating the squared euclidean distance for each point to the new k centers and assign each point to the nearest center





- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - ullet Repeat the process until the k centers don't change and assign each point to the nearest final center





- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Hartigan-Wong algorithm

```
kaufman_example_kmeans <- kaufman_example |>
 select(weight_kg, height_cm) |>
 kmeans(centers = 2.
         algorithm = "Hartigan-Wong") # R uses this algorithm by default
kaufman_example_kmeans
K-means clustering with 2 clusters of sizes 4, 4
Cluster means:
 weight_kg height_cm
     61.25 167.5
     12.50
              89.5
Clustering vector:
[1] 2 1 2 1 1 1 2 2
Within cluster sum of squares by cluster:
[1] 1371.75 206.00
 (between_SS / total_SS = 91.5 %)
Available components:
[1] "cluster"
                   "centers"
                                  "totss"
                                                 "withinss"
                                                                "tot.withinss"
[6] "betweenss" "size"
                                  "iter"
                                                 "ifault"
```



Extract clusters and assign them to observations

```
kaufman_example_kmeans_clusters <- kaufman_example |>
mutate(cluster = kaufman_example_kmeans$cluster)
kaufman_example_kmeans_clusters
```

```
# A tibble: 8 x 4
             weight_kg height_cm cluster
  name
  <chr>>
                  <dh1>
                            <dbl>
                                     <int>
1 Ilan
                     15
                                95
2 Jacqueline
                              156
3 Kim
                               95
4 Lieve
                     45
                              160
                     85
                              178
5 Leon
                     66
                              176
6 Peter
7 Talia
                     12
                                90
8 Tina
                     10
                               78
```



- Select a number of clusters, using segmentation, for example: 4 clusters
 - k-means only work with numerical data
 - A possible solution is to transform categorical data into numerical data
 - If a variable is nominal only works if you have 2 categories
 - If a variable is ordinal you assume that the notion of distance between them is constant or you need to specify integers to determine what distance is appropriate
 - Also you need to scale the variables taking into account that you are mixing categorical and numerical variables



- Convert binary nominal data to numerical data
 - Only make sense when you have 2 categories

```
# A tibble: 300 x 6
    age gender income kids ownHome subscribe
  <dbl> <int> <dbl> <int>
                           <int>
                                     <int>
1 47 3
            2 49483
  31.4
            2 35546.
3 43.2
            2 44169.
4 37.3
           1 81042.
         1 79353.
5 41.0
6 43.0
        2 58143.
7 37.6
        2 19282.
8 28 5
       2 47245.
9 44 2
            1 48333
   35.2
            1 52568.
# i 290 more rows
```



- Scale data to map each variable to a common scale
 - ullet We are going to scale each variable to [0,1]
 - Use across and rescale

```
# A tibble: 6 x 6
   age gender income kids ownHome subscribe
 <dbl> <dbl> <dbl> <dbl> <dbl> 
                          <dh1>
                                   <dh1>
1 0.458
           1 0.458 0.286
2 0 198
       1 0.341 0.143
       1 0.413 0
3 0.391
4 0.295 0 0.722 0.143
5 0.354
         0 0.708 0.429
6 0 388
       1 0.530 0.571
```



- ullet Apply k-means with k=4 and Hartigan-Wong algorithm
 - \bullet k-means start with k=4 random centers so you need to fix this initial decision using set.seed if the clusters tend to change



Extract clusters and assign them to observations

```
segmentation_kmeans_clusters <- segmentation |>
  mutate(cluster = segmentation_numeric_scale_kmeans$cluster)
segmentation_kmeans_clusters
```

```
# A tibble: 300 x 7
    age gender income kids ownHome subscribe cluster
  <dbl> <fct> <dbl> <int> <fct>
                                 <fct>
                                             <int>
 1 47.3 Male 49483.
                        2 ownNo
                                 subNo
   31.4 Male 35546. 1 ownYes subNo
3 43.2 Male 44169.
                        0 ownYes subNo
4 37.3 Female 81042.
                     1 ownNo
                                 subNo
5 41.0 Female 79353
                        3 own Yes subNo
6 43.0 Male 58143.
                        4 ownYes subNo
   37.6 Male 19282.
                        3 ownNo
                                subNo
   28.5 Male 47245.
                                subNo
                        0 ownNo
                     1 ownNo
9 44 2 Female 48333
                                 subNo
10 35.2 Female 52568.
                        O ownYes subNo
# i 290 more rows
```



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- To the Linux kernel community for allowing me the possibility to use some Linux distributions as my main OS without paying for a license



References I

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