#### **Comparing Groups: Statistical Tests**

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**FAEDIS** 

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#### Please Read Me

• This presentation is based on (Chapman and Feit 2019, chap. 6)

#### **Purpose**

 Understand the use of statistical tests to identify differences between groups in data

#### Import data

```
<dbl> <chr> <dbl> <dbl> <dbl> <chr>
                               <chr>>
                                        <chr>>
47.3 Male 49483.
                      2 ownNo
                               subNo
                                        Suburb mix
31.4 Male 35546. 1 ownYes
                               subNo
                                        Suburb mix
43.2 Male 44169. 0 ownYes subNo
                                        Suburb mix
37.3 Female 81042. 1 ownNo
                               subNo
                                        Suburb mix
41.0 Female 79353. 3 ownYes subNo
                                        Suburb mix
```

#### Chi-squared test

```
segmentation |> count(Segment)
# A tibble: 4 x 2
 Segment
                 n
 <chr>>
             <int>
1 Moving up
                70
2 Suburb mix
               100
3 Travelers
               80
4 Urban hip
                50
segmentation |>
 count(subscribe, ownHome) |>
 pivot_wider(id_cols = subscribe,
              names from = ownHome,
              values from = n)
```

```
# A tibble: 2 x 3
subscribe ownNo ownYes
<chr> <int> <int> <int> <int>
1 subNo 137 123
2 subYes 22 18
```

Chi-squared test for given probabilities

$$\begin{split} H_0: p_1 &= \tfrac{1}{4} \wedge p_2 = \tfrac{1}{4} \wedge p_3 = \tfrac{1}{4} \wedge p_4 = \tfrac{1}{4} \\ H_1: p_1 &\neq \tfrac{1}{4} \vee p_2 \neq \tfrac{1}{4} \vee p_3 = \tfrac{1}{4} \vee p_4 \neq \tfrac{1}{4} \\ \chi^2 &= \sum_{i=1}^n \frac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{70 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{100 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{80 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{50 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} \end{split}$$

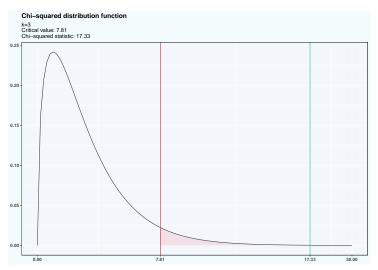
#### Base R way

```
chi_statistic <- table(segmentation$Segment) |>
  chisq.test(p = c(1/4, 1/4, 1/4, 1/4))
chi statistic
```

Chi-squared test for given probabilities

```
data: table(segmentation$Segment)
X-squared = 17.333, df = 3, p-value = 0.0006035
```

• Chi-squared test for given probabilities



Chi-squared test for given probabilities

$$\begin{split} H_0: p_1 &= \tfrac{1}{4} \wedge p_2 = \tfrac{1}{4} \wedge p_3 = \tfrac{1}{4} \wedge p_4 = \tfrac{1}{4} \\ H_1: p_1 &\neq \tfrac{1}{4} \vee p_2 \neq \tfrac{1}{4} \vee p_3 = \tfrac{1}{4} \vee p_4 \neq \tfrac{1}{4} \\ \chi^2 &= \sum_{i=1}^n \frac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{70 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{100 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{80 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} + \tfrac{50 - 300\tfrac{1}{4}}{300\tfrac{1}{4}} \end{split}$$

#### tidymodels way

#### Pearson's Chi-squared test

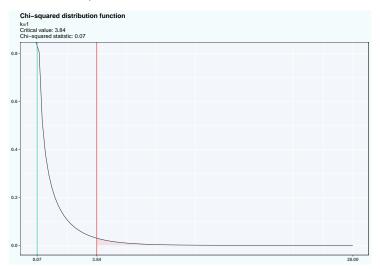
$$\begin{split} H_0: p_{11} &= \tfrac{260}{300} \tfrac{159}{300} \land p_{12} = \tfrac{260}{300} \tfrac{141}{300} \land p_{21} = \tfrac{40}{300} \tfrac{159}{300} \land p_{22} = \tfrac{40}{300} \tfrac{141}{300} \\ H_1: p_{11} &\neq \tfrac{260}{300} \tfrac{159}{300} \lor p_{12} \neq \tfrac{260}{300} \tfrac{141}{300} \lor p_{21} \neq \tfrac{40}{300} \tfrac{159}{300} \lor p_{22} \neq \tfrac{40}{300} \tfrac{141}{300} \\ \chi^2 &= \sum_{i=1}^n \tfrac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{(137 - 300 \tfrac{260}{300} \tfrac{159}{300}}{300} + \tfrac{(123 - 300 \tfrac{260}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(123 - 300 \tfrac{260}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{400}{300} \tfrac{140}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{40}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{40}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{40}{300} \tfrac{140}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{140}{300})^2}{300 \tfrac{4$$

#### Base R way

chi statistic <- chisq.test(table(segmentation\$subscribe. segmentation \$ own Home).

correct = FALSE)

#### • Pearson's Chi-squared test



Pearson's Chi-squared test

$$\begin{split} H_0: p_{11} &= \tfrac{260}{300} \tfrac{159}{300} \land p_{12} = \tfrac{260}{300} \tfrac{141}{300} \land p_{21} = \tfrac{40}{300} \tfrac{159}{300} \land p_{22} = \tfrac{40}{300} \tfrac{141}{300} \\ H_1: p_{11} &\neq \tfrac{260}{300} \tfrac{159}{300} \lor p_{12} \neq \tfrac{260}{300} \tfrac{141}{300} \lor p_{21} \neq \tfrac{40}{300} \tfrac{159}{300} \lor p_{22} \neq \tfrac{40}{300} \tfrac{141}{300} \\ \chi^2 &= \sum_{i=1}^n \tfrac{(Observed_i - Expected_i)^2}{Expected_i} = \\ \tfrac{(137 - 300 \tfrac{260}{300} \tfrac{159}{300})}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(123 - 300 \tfrac{260}{300} \tfrac{141}{300})^2}{300 \tfrac{260}{300} \tfrac{159}{300}} + \tfrac{(18 - 300 \tfrac{40}{300} \tfrac{141}{300})^2}{300 \tfrac{400}{300} \tfrac{140}{300}} \end{split}$$

#### tidymodels way

Exact binomial test

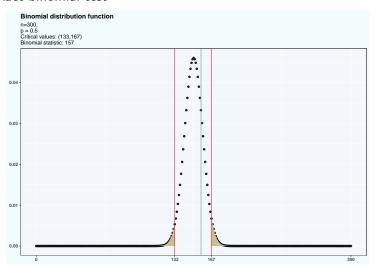
$$\begin{split} H_0: p &= 0.5 \ H_1: p \neq 0.5 \\ B &= \sum_{i=1}^n x_i = 157 \ \text{where} \ x_i \in 0, 1 \end{split}$$

#### R base way

```
Exact binomial test
```

```
data: 157 and 300
number of successes = 157, number of trials = 300, p-value = 0.453
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
0.4651595 0.5810418
sample estimates:
probability of success
0.5233333
```

#### Exact binomial test



- Exact binomial test
  - Confidence interval:

$$p_L$$

•  $p_L$  and  $p_U$  are random variables but p is not a random variable. Therefore  $[p_L,p_U]$  is a random interval where we have that:

$$P(0.4651595 \approx p_L$$

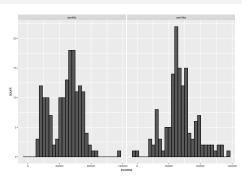
Exact binomial test

$$\begin{split} H_0: p &= 0.5 \ H_1: p \neq 0.5 \\ B &= \sum_{i=1}^n x_i = 157 \ \text{where} \ x_i \in 0,1 \end{split}$$

#### tidymodels way

#### • 2 sample t-test: independent samples

```
segmentation |> ggplot() +
  geom_histogram(aes(x = income), color='black') +
  facet_wrap(facets = vars(ownHome))
```



• 2 sample t-test: independent samples

```
# A tibble: 2 x 4
ownHome mean_income var_income n
<chr> <dh> <dh> <dh> <dh> <int> < dh > int> 1
ownNo 47391. 358692875. 159
2 ownYes 54935. 430890091. 141
```

2 sample t-test: independent samples

$$\begin{split} H_0: \mu_{ownNo} - \mu_{ownYes} &= 0 \ H_1: \mu_{ownNo} - \mu_{ownYes} \neq 0 \\ t &= \frac{\overline{ownNo} - \overline{ownYes}}{\sqrt{\frac{s_{ownNo}}{n_{ownNo}} - \frac{s_{ownYes}^2}{n_{ownYes}}}} = \frac{47391.01 - 54934.68}{\sqrt{\frac{358692875}{159} - \frac{430890091}{141}}} \approx -3.273094 \end{split}$$

#### R base way

```
Welch Two Sample t-test
```

```
data: income by ownHome

t = -3.2731, df = 285.25, p-value = 0.001195

alternative hypothesis: true difference in means between group ownNo and group ownYes is not equal to 0

95 percent confidence interval:

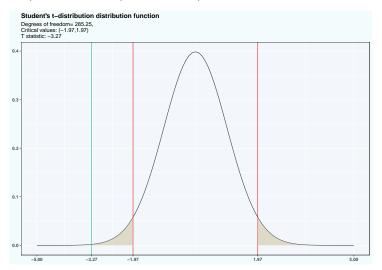
-12080.155 -3007.193

sample estimates:

mean in group ownNo mean in group ownYes

47301.01 54934.68
```

• 2 sample t-test: independent samples



- 2 sample t-test: independent samples
  - Confidence interval:

$$c_L < \mu_{ownNo} - \mu_{ownYes} < c_U$$

•  $\mu_{ownNo} - \mu_{ownYes}$  is not a random variable so we need to use a random variable

$$P\Bigg(t_L < \frac{\overline{x}_{ownNo} - \overline{x}_{ownYes} - (\mu_{ownNo} - \mu_{ownYes})}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_{ownYes}^2}{n_{ownYes}}}} < t_U\Bigg) = 0.95$$

•  $\overline{x}_{ownNo} - \overline{x}_{ownYes}$  is a random variable

- 2 sample t-test: independent samples
  - Confidence interval:
    - $\begin{array}{l} \bullet \quad & \frac{\overline{x}_{ownNo} \overline{x}_{ownYes} (\mu_{ownNo} \mu_{ownYes})}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_{ownYes}^2}{n_{ownYes}}}} \quad \text{is also a random variable with} \\ \text{student's t-distribution and} \quad & \nu \approx \frac{(\frac{s_{ownNo}^2}{n_{ownNo}} + \frac{s_2^2}{n_{ownNo}^2})^2}{(\frac{s_{ownNo}^2}{n_{ownNo}^2})^2} + \frac{s_2^2}{(\frac{s_{ownNe}^2}{n_{ownYes}})^2}}{(\frac{s_{ownNo}^2}{n_{ownNo}^2})^2} + \frac{s_2^2}{(\frac{s_{ownNe}^2}{n_{ownYes}})^2} \\ \text{degrees of freedom}. \end{array}$

degrees of freedom

 $\bullet$  Also we need to specify  $t_L$  and  $t_U$ 

```
t_L <- qt(p = 0.025, df = 285.25, lower.tail = TRUE)
t_L

[1] -1.968315
t_U <- qt(p = 0.975, df = 285.25, lower.tail = TRUE)
t_U
```

[1] 1.968315

- 2 sample t-test: independent samples
  - Confidence interval:

$$P(-7543.674 - 1.968315 \times 2304.753 < \mu_{ownNo} - \mu_{ownYes} < -7543.674 - 1.968315 \times 2304.753) = 0.95$$
 
$$P(-12080.16 < \mu_{ownNo} - \mu_{ownYes} < -3007.193) = 0.95$$

• In the long run 95% of confidence intervals constructed in this manner will contain the true parameter

2 sample t-test: independent samples

$$\begin{split} H_0: \mu_{ownNo} - \mu_{ownYes} &= 0 \ H_1: \mu_{ownNo} - \mu_{ownYes} \neq 0 \\ t &= \frac{\overline{ownNo} - \overline{ownYes}}{\sqrt{\frac{s_{ownNo}^2}{n_{ownNo}} - \frac{s_{ownYes}^2}{n_{ownYes}}}} = \frac{47391.01 - 54934.68}{\sqrt{\frac{358692875}{159} - \frac{430890091}{141}}} \approx -3.273094 \end{split}$$

#### tidymodels way

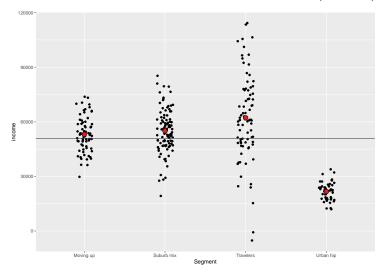
```
segmentation |>
t_test(formula = income ~ ownHome,
    alternative = "two-sided",
    order = c("ownNo", "ownYes"),
    mu = 0,
    conf_level = 0.95)
```

```
# A tibble: 1 x 7
statistic t_df p_value alternative estimate lower_ci upper_ci
ddbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> = 7544. -12080. -3007.
```

Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
segmentation |>
 group_by(Segment) |>
 summarise(mean = mean(income),
           variance = var(income),
           n = n()
# A tibble: 4 x 4
 Segment
            mean
                     variance
 <chr>>
       <db1>
                        <dbl> <int>
1 Moving up 53091.
                    92862689.
2 Suburb mix 55034, 142761527,
                                100
3 Travelers 62214, 564173979.
                                 80
4 Urban hip 21682.
                    23885953.
                                 50
```

• Testing Multiple Group Means: Analysis of Variance (ANOVA)



Testing Multiple Group Means: Analysis of Variance (ANOVA)

$$H_0: \mu_{Moving\;up} = \mu_{Suburb\;mix} = \mu_{Travelers} = \mu_{Urban\;hip}$$

 $H_1: At$  least one group mean is different from the rest

$$n = \sum_{j=1}^{4} n_j = n_1 + \dots + n_4 = 70 + 100 + 80 + 50 = 300$$

$$\overline{income} = \frac{1}{n} \sum_{j=1}^{4} \sum_{i=1}^{n_j} income_{ij}$$

$$\overline{income}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} income_{ij}$$

$$F = \frac{\frac{\sum_{j=1}^{4} \sum_{i=1}^{n_{j}} \frac{(income}{j} - \overline{income})^{2}}{\frac{4-1}{\sum_{j=1}^{4} \sum_{i=1}^{n_{j}} \frac{(income}{ij} - \overline{income})^{2}}{300-4}} = \frac{\frac{54969675428}{3}}{\frac{66281072794}{296}} = \frac{18323225143}{223922543} = 81.82841$$

• Testing Multiple Group Means: Analysis of Variance (ANOVA)

#### R base way

```
anova_table <- aov(data = segmentation, formula = income ~ Segment) |>
anova()
anova_table

Analysis of Variance Table

Response: income
```

```
Response: income

Df Sum Sq Mean Sq F value Pr(>F)

Segment 3 5.4970e+10 1.8323e+10 81.828 < 2.2e-16 ***

Residuals 296 6.6281e+10 2.2392e+08
---
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

• Testing Multiple Group Means: Analysis of Variance (ANOVA)



Testing Multiple Group Means: Analysis of Variance (ANOVA)

#### tidymodels way

```
anova_table <- aov(data = segmentation, formula = income ~ Segment) |>
anova() |>
tidy()
anova_table
# A tibble: 2 x 6
```

```
df
                                     meansq statistic
                                                        p.value
  term
                         sumsq
                         <dh1>
                                      <dh1>
                                                          <dh1>
 <chr>>
            <int>
                                                <dh1>
1 Segment
                3 54969675428 18323225143
                                                 81.8 1.41e-38
2 Residuals
             296 66281072794.
                                                      NA
                                 223922543.
                                                 NA
```

Testing Multiple Group Means: Analysis of Variance (ANOVA)

• Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
segmentation |>
 count(Segment, ownHome, name = "n_ij")
# A tibble: 8 x 3
 Segment ownHome n_ij
 <chr>
          <chr>
                    <int>
1 Moving up ownNo
                       47
2 Moving up ownYes
3 Suburb mix ownNo
                       52
4 Suburb mix ownYes
                       48
5 Travelers ownNo
                       20
6 Travelers ownYes
                       60
7 Urban hip ownNo
                       40
```

10

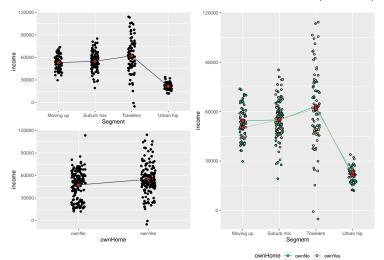
8 Urban hip ownYes

#### Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
mu_ij <- segmentation |>
  group_by(Segment, ownHome) |>
  summarise(mean = mean(income)) |>
  ungroup()
mu_11 <- mu_ij$mean[1]
mu_11
Γ17 54497.68
segmentation |>
  select(income, Segment, ownHome) |>
  head(n=5)
# A tibble: 5 x 3
  income Segment
                    ownHome
   <dhl> <chr>>
                     <chr>>
1 49483 Suburb mix ownNo.
2 35546. Suburb mix ownYes
3 44169. Suburb mix ownYes
4 81042. Suburb mix ownNo
```

5 79353. Suburb mix ownYes

• Testing Multiple Group Means: Analysis of Variance (ANOVA)



Testing Multiple Group Means: Analysis of Variance (ANOVA)

$$\begin{split} income_{ijk} = & \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \\ & \text{where } + \epsilon_i \sim \mathcal{N}(0, \sigma^2) \\ & \text{and } i = 1, 2, 3, 4 \\ & j = 1, 2 \\ & k = 1, \dots n_{ij} \\ & \mu = \mu_{11} \\ & \alpha_1 = \beta_1 = 0 \\ & (\alpha\beta)_{11} = (\alpha\beta)_{12} = 0 \\ & (\alpha\beta)_{21} = (\alpha\beta)_{31} = (\alpha\beta)_{41} = 0 \end{split}$$

Testing Multiple Group Means: Analysis of Variance (ANOVA)

$$\begin{split} \widehat{income}_{ijk} = & \widehat{\mu} + \widehat{\alpha}_i + \widehat{\beta}_j + (\widehat{\alpha\beta})_{ij} + \widehat{\epsilon}_{ijk} \\ \text{and } i = 1, 2, 3, 4 \\ j = 1, 2 \\ k = 1, \dots n_{ij} \\ \widehat{\mu} = & \widehat{\mu}_{11} \\ \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\alpha}_4 \\ \widehat{\beta}_2 \\ (\widehat{\alpha\beta})_{22}, (\widehat{\alpha\beta})_{32}, (\widehat{\alpha\beta})_{42} \\ \\ \widehat{income}_{ijk} - \widehat{income}_{ijk} = \widehat{\epsilon}_{ijk} \end{split}$$

#### Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
framed <- model_frame(formula = income ~</pre>
                                 Segment +
                                 ownHome +
                                 Segment: ownHome,
            data = segmentation)
model matrix(terms = framed$terms,
             data = framed$data) |>
 head(n = 2) >
 glimpse()
Rows: 2
Columns: 8
$ `(Intercept)`
                                     <dbl> 1, 1
$ `SegmentSuburb mix`
                                     <dbl> 1, 1
$ SegmentTravelers
                                     <dbl> 0, 0
$ `SegmentUrban hip`
                                     <dbl> 0, 0
$ ownHomeownYes
                                     <dbl> 0, 1
$ `SegmentSuburb mix:ownHomeownYes` <dbl> 0, 1
```

\$ `SegmentTravelers:ownHomeownYes` <dbl> 0. 0

\$ `SegmentUrban hip:ownHomeownYes`

<dbl> 0, 0

- Testing Multiple Group Means: Analysis of Variance (ANOVA)
  - Model

$$\begin{bmatrix} 49482.81 \\ 35546.29 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \mu \\ \alpha_2 \\ \beta_2 \\ (\alpha\beta)_{13} \\ (\alpha\beta)_{14} \\ (\alpha\beta)_{22} \\ (\alpha\beta)_{32} \\ (\alpha\beta)_{42} \end{bmatrix}$$

Coefficients to estimate using aov

$$\widehat{\mu} = \widehat{\mu}_{11}, \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\alpha}_4, \widehat{\beta}_2, (\widehat{\alpha\beta})_{22}, (\widehat{\alpha\beta})_{32}, (\widehat{\alpha\beta})_{42}$$

#### • Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
coef
                                     value
  <chr>>
                                     <dh1>
1 (Intercept)
                                    54498
2 SegmentSuburb mix
                                     435.
3 SegmentTravelers
                                    8691.
4 SegmentUrban hip
                                   -33160
5 ownHomeownYes
                                    -4281.
6 SegmentSuburb mix:ownHomeownYes
                                    4492.
7 SegmentTravelers:ownHomeownYes
                                     2982
8 SegmentUrban hip:ownHomeownYes
                                     6003
```

- Testing Multiple Group Means: Analysis of Variance (ANOVA)
- Segment

```
H_0: \mu_{Moving\; up} = \mu_{Suburb\; mix} = \mu_{Travelers} = \mu_{Urban\; hip}
```

 $H_1$ : At least one group mean is different from the rest

ownHome

$$H_0: \mu_{own,No} = \mu_{own,Yes}$$

 $H_1:$  At least one group mean is different from the rest

Segment, ownHome

```
H_0: \mu_{Moving\;up,\;ownNo} - \mu_{Moving\;up,\;ownYes} = \mu_{Suburb\;mix,\;ownNo} - \mu_{Suburb\;mix,\;ownYes} = \\ \mu_{Travelers,\;ownNo} - \mu_{Travelers,\;ownYes} = \mu_{Urban\;hip,\;ownNo} - \mu_{Urban\;hip,\;ownYes}
```

 $H_1$ : At least one difference group mean is different from the rest

#### Testing Multiple Group Means: Analysis of Variance (ANOVA)

```
Analysis of Variance Table

Response: income

Df Sum Sq Mean Sq F value Pr(>F)

Segment 3 5.4970e+10 1.8323e+10 81.1305 <2e-16 ***
ownHome 1 6.9918e+07 6.9918e+07 0.3096 0.5784

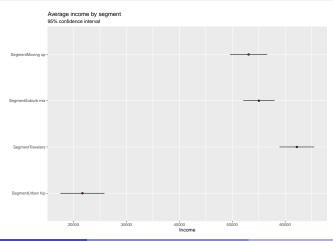
Segment:ownHome 3 2.6329e+08 8.7762e+07 0.3886 0.7613

Residuals 292 6.5948e+10 2.2585e+08

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

model\_aov |>
anova()

#### Testing Multiple Group Means: Analysis of Variance (ANOVA)



#### References

Chapman, Chris, and Elea McDonnell Feit. 2019. *R For Marketing Research and Analytics*. 2nd ed. 2019. Use R! Cham: Springer International Publishing: Imprint: Springer. https://doi.org/10.1007/978-3-030-14316-9.