Segmentation: Clustering

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FAEDIS

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Please Read Me

• This presentation is based on (Chapman and Feit 2019, chap. 11)

Purpose

• Find groups of customers that differ in different dimensions to engage in more effective promotion

- age: age of the consumer in years
- gender: if the consumer is male of female
- income: yearly disposable income of the consumer
- kids: number of children of the consumer
- ownHome: if the consumer owns a home
- subscribe: if the consumer is subscribed or not

Import data

```
segmentation <- read_csv(file = "http://goo.gl/qw303p") |>
select(-Segment) # Remove Segment column to understand how it was build
segmentation |> head(n = 5)

# A tibble: 5 x 6
age gender income kids ownHome subscribe
```

```
# A tibble: 5 x b

age gender income kids ownHome subscribe

<dbl> <chr> <dbl> <dbl> <chr> <dbl> <chr> <dbl> <dbl> <chr> <dbr > </d> 

    1 47.3 Male 49483. 2 ownNo subNo
    2 ownYes subNo

    3 43.2 Male 44169. 0 ownYes subNo

    4 37.3 Female 81042. 1 ownNo subNo

    5 41.0 Female 79353. 3 ownYes subNo
```

Inspect data

```
segmentation |> glimpse()
```

Transform data

```
segmentation <- segmentation |>
 mutate(gender = factor(gender, ordered = FALSE).
        kids = as.integer(kids),
        ownHome = factor(ownHome, ordered = FALSE),
        subscribe = factor(subscribe, ordered = FALSE))
segmentation |> head(n = 5)
# A tibble: 5 x 6
   age gender income kids ownHome subscribe
 <dbl> <fct> <dbl> <int> <fct>
                                 <fct>
1 47.3 Male 49483. 2 ownNo
                                 subNo
2 31.4 Male 35546. 1 ownYes subNo
3 43.2 Male 44169. 0 ownYes subNo
 37.3 Female 81042. 1 ownNo
                                 subNo
```

5 41.0 Female 79353. 3 ownYes subNo

Summarize data

 Ups the table is really big!!! Try it in your console to see the complete table

segmentation |> skim()

Table 1: Data summary

Name	segmentation
Number of rows	300
Number of columns	6
Column type frequency:	_
factor	3
numeric	3
Group variables	None

Variable type: factor

skim_variable	n_missing	complete_rate	ordered	n_unique	top_counts	
gender	0	1	FALSE	2	Fem: 157, Mal: 143	
ownHome	0	1	FALSE	2	own: 159, own: 141	
subscribe	0	1	FALSE	2	sub: 260. sub: 40	
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Segmentation

- Classification (We will not cover this topic)
 - Supervised learning
 - Dependent variable is known and the goal is to predict the dependent variable from the independent variables
 - Naive bayes, Random Forest
- Clustering (This topic will be covered)
 - Unsupervised learning
 - Dependent variable is unknown and the goal is to discover it from the independent variables
 - Model-based clustering, Latent Class Analysis (We will not cover these methods)
 - Hierarchical clustering, k-means (These methods will be covered)

Clustering

- Grouping a set of observations in such a way that observations in the same group (cluster) are more similar to each other than to those in other groups (clusters).
- A notion of how "close" 2 observations is necessary to group objects where this is formalized using the concept of distance (known as metric¹ in mathematics)
 - There are many notions of distance (Deza and Deza 2016) where in this chapter the Euclidean and the Gower distance will be used

Euclidean distance: it can only be used for numerical data

•
$$x = (x_1, x_2, \dots, x_n)$$

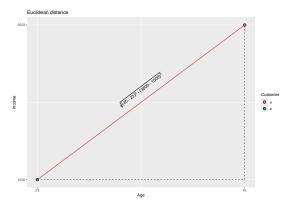
•
$$y = (y_1, y_2, \dots, y_n)$$

$$\begin{split} d(x,y) &= \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + \ldots + (x_n-y_n)^2} \\ &= \sqrt{\sum_{k=1}^n (x_k-y_k)^2} \end{split}$$

- An example:
 - 2 customers characteristic by age and income
 - a = (45, 3500)
 - b = (23, 1500)

Manual calculation

•
$$d(a,b) = \sqrt{(45-23)^2 + (3500-1500)^2} = 2000.121$$



Using R

```
customers <- tibble(Customer = c("a", "b"),
                   Age = c(45, 23),
                  Income = c(3500, 1500)
customers
# A tibble: 2 x 3
 Customer Age Income
 <chr> <dbl> <dbl>
1 a
          45 3500
2 b
    23 1500
library(cluster)
customers |>
 select(-Customer) |>
 daisy(metric = "euclidean")
Dissimilarities :
2 2000 121
```

Metric : euclidean Number of objects: 2

- Gower distance: it can be used for categorical, numerical data and missing values
 - $x = (x_1, x_2, \dots, x_n)$
 - $\bullet \ y=(y_1,y_2,\ldots,y_n)$

$$\begin{split} d(x,y) &= \left[\frac{w_1 \delta_{x_1 y_1}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_1 y_1}^1 + \left[\frac{w_2 \delta_{x_2 y_2}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_2 y_2}^2 + \ldots + \left[\frac{w_n \delta_{x_n y_n}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k}\right] d_{x_n y_n}^n \\ &= \frac{\sum_{k=1}^n w_k \delta_{x_i y_i}^k d_{x_i y_i}^k}{\sum_{k=1}^n w_k \delta_{x_i y_i}^k} \end{split}$$

Where:

$$w_k \in \mathbb{R}$$
 for $k = 1, 2, \dots, n$

$$\sum_{k=1}^n w_k \delta_{x_i y_i}^k = w_1 \delta_{x_1 y_1}^1 + w_2 \delta_{x_2 y_2}^2 + \ldots + w_n \delta_{x_n y_n}^n$$

- Gower distance: it can be used for categorical, numerical data and missing values
 - $x = (x_1, x_2, \dots, x_n)$
 - $y = (y_1, y_2, \dots, y_n)$

$$d(x,y) = \frac{\sum_{k=1}^n w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^n w_k \delta_{x_k y_k}^k}$$

Where²:

$$\delta_{x_ky_k}^k = \begin{cases} 0 & \text{if } x_k \text{ or } y_k \text{ is a missing value} \\ 0 & \text{if } x_k, y_k \text{ represent an asymmetric binary variable and } x_k = y_k = 0 \\ 1 & \text{otherwise} \end{cases}$$

²See (Kaufman and Rousseeuw 1990, 25–27) for a definition of asymmetric binary variable

 Gower distance: it can be used for categorical, numerical data and missing values

$$\bullet \ x=(x_1,x_2,\dots,x_n)$$

$$\bullet \ y=(y_1,y_2,\dots,y_n)$$

$$d(x,y) = \frac{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^k}$$

Where:

$$d_{x_{k}y_{k}}^{k} = \begin{cases} 0 \\ 1 \\ \frac{|x_{k} - y_{k}|}{\max(x_{k}, y_{k}) - \min(x_{k}, y_{k})} \end{cases}$$

 $d^k_{x_ky_k} = \begin{cases} 0 & \text{if } x_k, y_k \text{ represent a nominal or binary variable and } x_k = y_k \\ 1 & \text{if } x_k, y_k \text{ represent a nominal or binary variable and } x_k \neq y_k \end{cases}$ otherwise

If x_k, y_k represent an ordinal variable they are replaced by their integer codes. For example if $x_k \lesssim y_k$ then 1 is assigned to x_k and 2 is assigned to y_k

• An example:

- 2 customers characteristic by sex (nominal), income (numerical), satisfaction (ordinal with levels $Low \preceq Medium \preceq High$) and age (with a missing value (NA))
 - $\bullet \ \ a = (Female, 3500, Medium, 45)$
 - $\bullet \ b = (Male, 1500, High, NA)$

Manual calculation:

- \bullet In R $w_k=1$ for every k as a default value where in this example k=1,2,3,4
- $\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k = 1 * 1 + 1 * 1 + 1 * 1 + 1 * 0 = 1 + 1 + 1 + 0 = 3$
- $\bullet \ \sum_{k=1}^4 w_k \delta^k_{x_k y_k} d^k_{x_k y_k} = 1*1+1*\frac{|3500-1500|}{3500-1500} + 1*\frac{|2-3|}{3-2} + 0 = 3$
- $d(x,y) = \frac{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k d_{x_k y_k}^k}{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^k} = \frac{3}{3} = 1$

• Gower distance range:

- $d(x,y) \in [0,1]$ • If $d(x,y) \longrightarrow 0$ is more similar • If $d(x,y) \longrightarrow 1$ is more dissimilar
- Using R

Using R

```
customers2 |>
  select(-Customer) |>
  daisy(metric = "gower")

Dissimilarities:
  1
2 1
```

Metric: mixed; Types = N, I, O, I Number of objects: 2

- In this case:
 - Metric: mixed because it includes categorical and numerical data
 - For Types = N, I, O, I check out
 ?cluster::dissimilarity.object3
 - N: Nominal (factor)
 - I: Interval scaled (numeric)
 - 0: Ordinal (ordered factor)

³See (Stevens 1946) and Level of measurement

Using R

```
customers |>
    select(-Customer) |>
    daisy(metric = "gower")

Dissimilarities:
    1
2.1
```

In this case:

Number of objects: 2

Metric: mixed; Types = N, I, O, I

- Number of objects : 2
 - ullet There are 2 observations that correspond to customers ullet and ullet: a=(Female,3500,Medium,45) and b=(Male,1500,High,NA)

- ullet The original dissimilarity matrix is of dimension 300 imes 300
 - ullet Showing only the relation between the first 5 observations
 - \bullet The position (i,j) means the dissimilarity between the observations i and j
 - For example (4,3), which is equal to 0.425, is the dissimilarity between the observations 4 and 3

```
segmentation_dist <- segmentation |>
daisy(metric = "gower")

segmentation_dist |>
as.matrix() |>
as_tibble() |>
select('1':'5') |>
slice(1:5)
```

```
customers3 <- tibble(Customer = c("a", "b", "c", "d", "e"),</pre>
                     Sex = c("Female", "Male", "Female", "Female", "Male").
                     Income = c(3500, 1500, 200, 450, 5000).
                     Satisfaction = c("Medium", "High", "Low", "Low", "Medium"),
                     Age = c(45, NA, 34, 23, 55)) |>
 mutate(Sex = factor(x = Sex.))
                      ordered = FALSE),
         Satisfaction = factor(x = Satisfaction,
                               levels = c("Low", "Medium", "High"),
                                ordered = TRUE))
customers3
```

```
# A tibble: 5 x 5
 Customer Sex
              Income Satisfaction
 <chr> <fct> <dhl> <ord>
                               <dh1>
1 a
       Female 3500 Medium
                                  45
2 b
       Male 1500 High
                                 NA
     Female 200 Low
3 с
                                  34
     Female 450 Low
4 d
                                 23
5 e
      Male 5000 Medium
                                  55
```

Hierarchical clustering

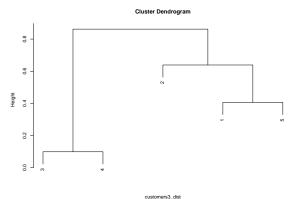
• Method: Complete Linkage Clustering

```
customers3_dist <- daisy(x = select(customers3, -Customer),</pre>
                        metric = "gower")
customers3_dist
Dissimilarities :
2 0.63888889
3 0 38281250 0 75694444
4 0 45572917 0 73958333 0 09895833
5 0.40625000 0.40972222 0.78906250 0.86197917
Metric: mixed; Types = N, I, O, I
Number of objects: 5
customers3 hc <- hclust(d = customers3 dist.
                        method = "complete")
customers3 hc
```

```
Call:
hclust(d = customers3_dist, method = "complete")
Cluster method : complete
```

- Hierarchical clustering
 - Method: Complete Linkage Clustering

plot(customers3_hc)



hclust (*, "complete")

• Compare each observation and find the pair that is more similar

	1	2	3	4	5
1	0.0000000	0.6388889	0.3828125	0.4557292	0.4062500
2	0.6388889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.4557292	0.7395833	0.09895833	0.0000000	0.8619792
5	0.4062500	0.4097222	0.7890625	0.8619792	0.0000000

- \bullet Now we have the first cluster that includes the observations 3 and 4 : C(3,4)
- \bullet Then we need to create clusters with observations $1,\,2$ and 5 and the cluster C(3,4)
 - How we compare a cluster with an observation
 - Complete Linkage Clustering: Use the maximum distance between an observation and an observation that belongs to the cluster

- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case 1, 2, 5 and C(3,4)
 - ullet The distance between 1 and C(3,4) is 0.45572917
 - ullet The distance between 2 and C(3,4) is 0.7569444
 - ullet The distance between 5 and C(3,4) is 0.8619792

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000

- \bullet Now we have the second cluster that includes the observations 1 and $5\colon\thinspace C(1,5)$
- Then we need to create clusters with observation 2 and clusters C(3,4) and C(1,5)
 - How we compare a cluster with another cluster
 - Complete Linkage Clustering: Use the maximum distance between an observation that belongs to the first cluster and an observation that belongs to the second cluster

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- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case 2, C(3,4) and C(1,5)
 - The distance between 2 and C(3,4) is 0.7569444
 - ullet The distance between 2 and C(1,5) is 0.6388889

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000

- Now we have the third cluster that includes the observation 2 and the cluster $C(1,5)\colon C(2,C(1,5))$
- \bullet Then we need to create clusters with cluster C(2,C(1,5)) and cluster C(3,4)
 - This is the cluster that includes all the observations

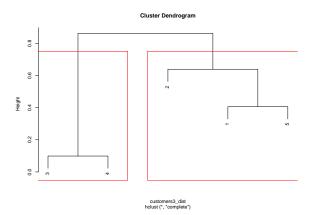
- Compare each observation, including the clusters build, and find the pair that is more similar
 - In our case C(3,4) and C(2,C(1,5))
 - ullet The distance between C(3,4) and C(2,C(1,5)) is 0.86197917

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.45572917	0.4062500
2	0.63888889	0.0000000	0.75694444	0.73958333	0.4097222
3	0.3828125	0.7569444	0	0.09895833	0.7890625
4	0.45572917	0.7395833	0.09895833	0	0.8619792
5	0.40625	0.4097222	0.7890625	0.86197917	0.0000000

 \bullet The heights of the **Cluster Dendrogram** are: 0.09895833, 0.40625, 0.63888889 and 0.86197917

• Select a number of clusters, for example: 2 clusters

```
plot(customers3_hc)
rect.hclust(customers3_hc, k = 2, border = "red")
```

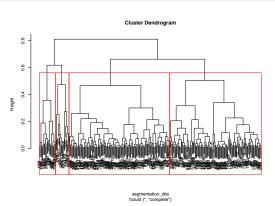


Extract clusters and assign them to observations

```
customers3_hc_clusters <- cutree(customers3_hc, k = 2)
customers3 |>
mutate(cluster = customers3_hc_clusters)
```

```
# A tibble: 5 x 6
 Customer Sex
              Income Satisfaction
                                Age cluster
 <chr>>
        <fct> <dbl> <ord>
                               <dbl>
                                     <int>
       Female 3500 Medium
    Male 1500 High
3 c
    Female 200 Low
                                34
    Female 450 Low
                                23
       Male
                5000 Medium
                                 55
```

 Select a number of clusters, using segmentation, for example: 4 clusters



 Extract clusters and assign them to observations, using segmentation

```
segmentation_hc_clusters <- cutree(segmentation_hc, k = 4)
segmentation |>
 mutate(cluster = segmentation_hc_clusters)
# A tibble: 300 x 7
     age gender income kids ownHome subscribe cluster
   <dbl> <fct> <dbl> <int> <fct>
                                     <fct>
                                                  <int>
 1 47.3 Male 49483.
                           2 ownNo
                                     subNo
  31.4 Male 35546.
                           1 own Yes subNo
3 43.2 Male 44169. 0 ownYes
4 37.3 Female 81042. 1 ownNo
5 41.0 Female 79353. 3 ownYes
                           O ownYes subNo
                                     subNo
                           3 own Yes subNo
6 43.0 Male 58143. 4 ownYes subNo
7 37.6 Male 19282.
                                    subNo
                           3 ownNo
8 28 5 Male 47245 0 own No
                                    subNo
  44.2 Female 48333.
                       1 ownNo
                                     subNo
  35.2 Female 52568.
                           O ownYes subNo
```

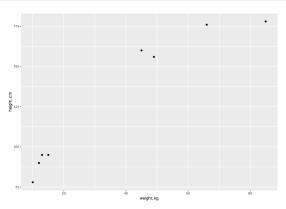
i 290 more rows

• K-means clustering example (Kaufman and Rousseeuw 1990, 5)

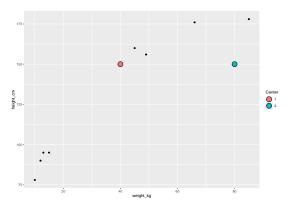
```
# A tibble: 8 x 3
              weight_kg height_cm
  name
  <chr>>
                  <dh1>
                             <dh1>
1 Ilan
                      15
                                95
2 Jacqueline
                               156
3 Kim
                     13
                                95
4 Lieve
                     45
                               160
                     85
                               178
5 Leon
6 Peter
                     66
                               176
7 Talia
                     12
                                90
8 Tina
                                78
                      10
```

• K-means clustering example (Kaufman and Rousseeuw 1990, 5)

```
kaufman_example |>
ggplot() +
geom_point(aes(x = weight_kg, y = height_cm))
```

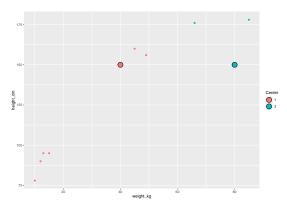


- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - \bullet Choose k centers or the computer will choose k centers at random, in our case we choose k=2



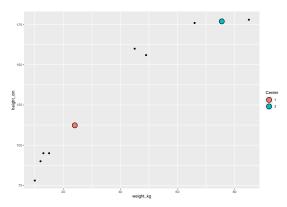
- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - ullet Calculate the squared euclidean distance for each point to the k centers and assign each point to the nearest center
 - \bullet For example for the point Ilan=(15,95) the distance to $Center_1=(40,150) \text{ is } (15-40)^2+(95-150)^2=3650 \text{ and the distance to } Center_2=(80,150) \text{ is } (15-80)^2+(95-150)^2=7250$
 - \bullet Therefore Ilan=(15,95) is assigned to $Center_1$

- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm

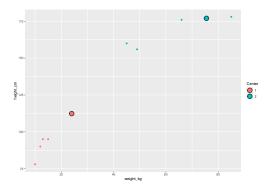


- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - Now calculate new centers using the assigned points by using the mean
 - \bullet For example for the new $Center_1$ the new position will be $x=\frac{15+49+13+45+12+10}{6}=24$ and $y=\frac{95+156+95+160+90+78}{6}\approx 112.33$
 - Therefore we update as $Center_1 \approx (24, 112.33)$

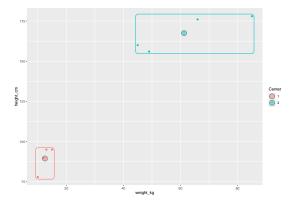
- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm



- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - \bullet Repeat the process by calculating the squared euclidean distance for each point to the new k centers and assign each point to the nearest center



- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Lloyd's algorithm
 - \bullet Repeat the process until the k centers don't change and assign each point to the nearest final center



- K-means clustering example (Kaufman and Rousseeuw 1990, 5)
 - Applying the Hartigan-Wong algorithm

```
kaufman_example_kmeans <- kaufman_example |>
 select(weight kg, height cm) |>
 kmeans(centers = 2.
         algorithm = "Hartigan-Wong") # R uses this algorithm by default
kaufman example kmeans
K-means clustering with 2 clusters of sizes 4, 4
Cluster means:
 weight kg height cm
      61.25
              167.5
     12.50
                89.5
Clustering vector:
[1] 2 1 2 1 1 1 2 2
Within cluster sum of squares by cluster:
[1] 1371.75 206.00
 (between SS / total SS = 91.5 %)
Available components:
[1] "cluster"
                   "centers"
                                                                 "tot withinss"
                                  "totss"
                                                  "withinss"
[6] "betweenss"
                                                  "ifault"
                  "size"
                                  "iter"
```

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Extract clusters and assign them to observations

```
kaufman_example_kmeans_clusters <- kaufman_example |>
    mutate(cluster = kaufman_example_kmeans$cluster)
kaufman_example_kmeans_clusters
```

A tibble: 8 x 4

***	A CIDDIC.	O A I		
	name	weight_kg	${\tt height_cm}$	cluster
	<chr></chr>	<dbl></dbl>	<dbl></dbl>	<int></int>
1	Ilan	15	95	2
2	Jacqueline	49	156	1
3	Kim	13	95	2
4	Lieve	45	160	1
5	Leon	85	178	1
6	Peter	66	176	1
7	Talia	12	90	2
8	Tina	10	78	2

- Select a number of clusters, using segmentation, for example: 4 clusters
 - · k-means only work with numerical data
 - A possible solution is to transform categorical data into numerical data
 - If a variable is nominal only works if you have 2 categories

Segmentation: Clustering

- If a variable is ordinal you assume that the notion of distance between them is constant or you need to specify integers to determine what distance is appropriate
- Also you need to scale the variables taking into account that you are mixing categorical and numerical variables

- Convert binary nominal data to numerical data
 - Only make sense when you have 2 categories

```
segmentation_numeric <- segmentation |>
 mutate(gender = as.integer(gender),
        ownHome = as.integer(ownHome),
        subscribe = as.integer(subscribe))
segmentation_numeric
# A tibble: 300 x 6
    age gender income kids ownHome subscribe
  <dbl> <int> <dbl> <int>
                             <int>
                                      <int>
 1 47 3
            2 49483
  31.4
            2 35546.
 3 43.2
         2 44169.
        1 81042.
4 37.3
        1 79353.
5 41.0
6 43.0
          2 58143.
7 37.6
          2 19282.
          2 47245.
   28.5
```

1 483333.

1 52568

44.2

i 290 more rows

10 35.2

- Scale data to map each variable to a common scale
 - We are going to scale each variable to [0,1]
 - Use across and rescale

```
segmentation numeric scale <- segmentation numeric |>
 mutate(across(.cols = age:subscribe,
                # scales is a package that is
                # installed with the tidvverse
                # but it is not loaded automatically
                # You can use a particular function of a package using the notation
                ## <package>::<function>
                .fns = scales::rescale))
segmentation_numeric_scale |> head()
# A tibble: 6 x 6
    age gender income kids ownHome subscribe
 <db1> <db1> <db1> <db1>
                             <db1>
                                        <db1>
1 0.458
        1 0.458 0.286
```

```
2 0.198 1 0.341 0.143
3 0.391 1 0.413 0
4 0.295 0 0.722 0.143
      0 0.708 0.429
5 0 354
      1 0.530 0.571
6 0.388
```

2024-03-16

- Apply k-means with k=4 and Hartigan-Wong algorithm
 - k-means start with k=4 random centers so you need to fix this initial decision using set.seed if the clusters tend to change

Segmentation: Clustering

```
set.seed(seed = 1234)
segmentation_numeric_scale_kmeans <- segmentation_numeric_scale |>
 kmeans(centers = 4.
        algorithm = "Hartigan-Wong")
segmentation numeric scale kmeans |> str()
List of 9
$ cluster : int [1:300] 2 3 3 4 1 3 2 2 4 1 ...
$ centers : num [1:4, 1:6] 0.431 0.278 0.446 0.298 0 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : chr [1:4] "1" "2" "3" "4"
  .. ..$ : chr [1:6] "age" "gender" "income" "kids" ...
 $ totss
              · num 218
$ withinss : num [1:4] 18.6 17.5 14.4 15.4
$ tot.withinss: num 65.9
$ betweenss : num 152
$ size : int [1:4] 76 78 65 81
            : int 3
$ iter
$ ifault
            : int 0
```

- attr(*, "class")= chr "kmeans"

Extract clusters and assign them to observations

```
1 47 3 Male 49483
                       2 ownNo
                                subNo
   31.4 Male 35546.
                       1 ownYes subNo
  43.2 Male 44169
                       O ownYes subNo
  37.3 Female 81042 1 ownNo
                                subNo
  41.0 Female 79353.
                       3 ownYes subNo
  43.0 Male 58143.
                       4 own Yes subNo
   37.6 Male 19282.
                       3 ownNo
                                subNo
   28.5 Male 47245.
                       0 ownNo
                               subNo
   44.2 Female 48333.
                    1 ownNo
                                subNo
   35.2 Female 52568.
                       0 ownYes
                               subNo
# i 290 more rows
```

References

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