Segmentation: Clustering

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Please Read Me

- This presentation is based on (Chapman and Feit 2019, chap. 11)

Purpose

- Find groups of customers that differ in different dimensions to engage in more effective promotion

Consumer segmentation survey

- **age**: age of the consumer in years
- **gender**: if the consumer is male of female
- **income**: yearly disposable income of the consumer
- **kids**: number of children of the consumer
- **ownHome**: if the consumer owns a home
- **subscribe**: if the consumer is subscribed or not

Consumer segmentation survey

- **Import data**

```
segmentation <- read_csv(file = "http://goo.gl/qw303p") |>
  select(-Segment) # Remove Segment column to understand how it was build
segmentation |> head(n = 5)
```

```
# A tibble: 5 x 6
    age gender income  kids ownHome subscribe
  <dbl> <chr>   <dbl> <dbl> <chr>   <chr>
1  47.3 Male   49483.     2 ownNo   subNo
2  31.4 Male   35546.     1 ownYes  subNo
3  43.2 Male   44169.     0 ownYes  subNo
4  37.3 Female 81042.     1 ownNo   subNo
5  41.0 Female 79353.     3 ownYes  subNo
```

Consumer segmentation survey

- **Inspect data**

```
segmentation |> glimpse()
```

```
Rows: 300
Columns: 6
$ age       <dbl> 47.31613, 31.38684, 43.20034, 37.31700, 40.95439, 43.03387, ~
$ gender    <chr> "Male", "Male", "Male", "Female", "Female", "Male", "Male", ~
$ income    <dbl> 49482.81, 35546.29, 44169.19, 81041.99, 79353.01, 58143.36, ~
$ kids      <dbl> 2, 1, 0, 1, 3, 4, 3, 0, 1, 0, 0, 0, 2, 3, 1, 3, 0, 0, 1, 2, ~
$ ownHome   <chr> "ownNo", "ownYes", "ownYes", "ownNo", "ownYes", "ownYes", "o~
$ subscribe <chr> "subNo", "subNo", "subNo", "subNo", "subNo", "subNo", "subNo~
```

Consumer segmentation survey

- Transform data

```r
segmentation <- segmentation |>
  mutate(gender = factor(gender, ordered = FALSE),
         kids = as.integer(kids),
         ownHome = factor(ownHome, ordered = FALSE),
         subscribe = factor(subscribe, ordered = FALSE))

segmentation |> head(n = 5)
```

```
# A tibble: 5 x 6
    age gender income  kids ownHome subscribe
  <dbl> <fct>   <dbl> <int> <fct>   <fct>
1  47.3 Male   49483.     2 ownNo   subNo
2  31.4 Male   35546.     1 ownYes  subNo
3  43.2 Male   44169.     0 ownYes  subNo
4  37.3 Female 81042.     1 ownNo   subNo
5  41.0 Female 79353.     3 ownYes  subNo
```

Consumer segmentation survey

- **Summarize data**
  - Ups the table is really big!!! Try it in your console to see the complete table

```
segmentation |> skim()
```

Table 1: Data summary

Name	segmentation
Number of rows	300
Number of columns	6
_______________________	
Column type frequency:	
factor	3
numeric	3
________________________	
Group variables	None

Variable type: factor

skim_variable	n_missing	complete_rate	ordered	n_unique	top_counts
gender	0	1	FALSE	2	Fem: 157, Mal: 143
ownHome	0	1	FALSE	2	own: 159, own: 141
subscribe	0	1	FALSE	2	sub: 260, sub: 40

Consumer segmentation survey

Segmentation

- Classification (**We will not cover this topic**)
  - Supervised learning
    - Dependent variable is known and the goal is to predict the dependent variable from the independent variables
    - Naive bayes, Random Forest
- Classification (**This topic will be covered**)
  - Unsupervised learning
    - Dependent variable is unknown and the goal is to discover it from the independent variables
    - Model-based clustering, (**We will not cover these methods**)
    - Hierarchical clustering, k-means (**These methods will be covered**)

Consumer segmentation survey

- Clustering
  - Grouping a set of observations in such a way that observations in the same group (cluster) are more similar to each other than to those in other groups (clusters).
  - A notation of how **"close"** 2 observations is necessary to group objects where this is formalized using the concept of **distance** (know as metric[1] in mathematics)
    - There are many notations of distance (Deza and Deza 2016) where in this chapter the **Euclidean** and the **Gower** distance will be used

[1] https://en.wikipedia.org/wiki/Metric_space

Consumer segmentation survey

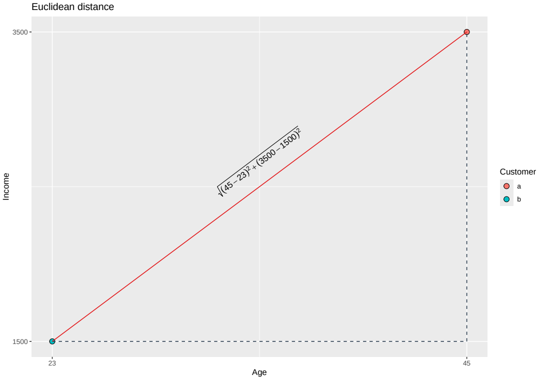
- **Euclidean distance**: it can only be used for numerical data
    - $x = (x_1, x_2, \dots, x_n)$
    - $y = (y_1, y_2, \dots, y_n)$

$$
\begin{aligned}
d(x, y) &= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2} \\
&= \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}
\end{aligned}
$$

- An example:
    - 2 customers characteristic by age and income
        - $a = (45, 3500)$
        - $b = (23, 1500)$

Consumer segmentation survey

- Manual calculation
  - $d(a,b) = \sqrt{(45-23)^2 + (3500-1500)^2} = 2000.121$



Consumer segmentation survey

- Using R

```
customers <- tibble(Customer = c("a", "b"),
                    Age = c(45, 23),
                    Income = c(3500, 1500))
customers
```

```
# A tibble: 2 x 3
  Customer   Age Income
  <chr>    <dbl>  <dbl>
1 a           45   3500
2 b           23   1500
```

```
library(cluster)
customers |>
  select(-Customer) |>
  daisy(metric = "euclidean")
```

```
Dissimilarities :
         1
2 2000.121

Metric :  euclidean
Number of objects : 2
```

Consumer segmentation survey

- **Gower distance**: it can be used for categorical, numerical data and missing values
  - $x = (x_1, x_2, \dots, x_n)$
  - $y = (y_1, y_2, \dots, y_n)$

$$
\begin{aligned}
d(x, y) &= \left[\frac{w_1 \delta^k_{x_1 y_1}}{\sum_{k=1}^n w_k \delta^k_{x_i y_i}}\right] d^1_{x_1 y_1} + \left[\frac{w_2 \delta^k_{x_2 y_2}}{\sum_{k=1}^n w_k \delta^k_{x_i y_i}}\right] d^2_{x_2 y_2} + \dots + \left[\frac{w_n \delta^k_{x_n y_n}}{\sum_{k=1}^n w_k \delta^k_{x_i y_i}}\right] d^n_{x_n y_n} \\
&= \frac{\sum_{k=1}^n w_k \delta^k_{x_i y_i} d^k_{x_i y_i}}{\sum_{k=1}^n w_k \delta^k_{x_i y_i}}
\end{aligned}
$$

Where:

$$w_k \in \mathbb{R} \text{ for } k = 1, 2, \dots, n$$

$$\sum_{k=1}^n w_k \delta^k_{x_i y_i} = w_1 \delta^1_{x_1 y_1} + w_2 \delta^2_{x_2 y_2} + \dots + w_n \delta^n_{x_n y_n}$$

Consumer segmentation survey

- **Gower distance**: it can be used for categorical, numerical data and missing values
  - $x = (x_1, x_2, \ldots, x_n)$
  - $y = (y_1, y_2, \ldots, y_n)$

$$d(x,y) = \frac{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^{k} d_{x_k y_k}^{k}}{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^{k}}$$

Where[2]:

$$\delta_{x_k y_k}^{k} = \begin{cases} 0 & \text{if } x_k \text{ or } y_k \text{ is a missing value} \\ 0 & \text{if } x_k, y_k \text{ represent an asymmetric binary variable and } x_k = y_k = 0 \\ 1 & \text{otherwise} \end{cases}$$

[2] See (Kaufman and Rousseeuw 1990, 25–27) for a definition of **asymmetric binary variable**

Consumer segmentation survey

- **Gower distance**: it can be used for categorical, numerical data and missing values
  - $x = (x_1, x_2, \dots, x_n)$
  - $y = (y_1, y_2, \dots, y_n)$

$$d(x,y) = \frac{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^{k} d_{x_k y_k}^{k}}{\sum_{k=1}^{n} w_k \delta_{x_k y_k}^{k}}$$

Where:

$$d_{x_k y_k}^{k} = \begin{cases} 0 & \text{if } x_k, y_k \text{ represent a nominal or binary variable and } x_k = y_k \\ 1 & \text{if } x_k, y_k \text{ represent a nominal or binary variable and } x_k \neq y_k \\ \frac{|x_k - y_k|}{max(x_k, y_k) - min(x_k, y_k)} & \text{otherwise} \end{cases}$$

If x_k, y_k represent an ordinal variable they are replaced by their integer codes. For example if $x_k \precsim y_k$ then 1 is assigned to x_k and 2 is assigned to y_k

Consumer segmentation survey

- An example:
  - 2 customers characteristic by sex (nominal), income (numerical), satisfaction (ordinal with levels $Low \precsim Medium \precsim High$) and age (with a missing value (NA))
    - $a = (Female, 3500, Medium, 45)$
    - $b = (Male, 1500, High, NA)$
- Manual calculation:
  - In R $w_k = 1$ for every k as a default value where in this example $k = 1, 2, 3, 4$
  - $\sum_{k=1}^{4} w_k \delta_{x_k y_k}^{k} = 1 * 1 + 1 * 1 + 1 * 1 + 1 * 0 = 1 + 1 + 1 + 0 = 3$
  - $\sum_{k=1}^{4} w_k \delta_{x_k y_k}^{k} d_{x_k y_k}^{k} = 1 * 1 + 1 * \frac{|3500-1500|}{3500-1500} + 1 * \frac{|2-3|}{3-2} + 0 = 3$
  - $d(x, y) = \frac{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^{k} d_{x_k y_k}^{k}}{\sum_{k=1}^{4} w_k \delta_{x_k y_k}^{k}} = \frac{3}{3} = 1$

Consumer segmentation survey

- **Gower distance** range:
    - $d(x, y) \in [0, 1]$
    - If $d(x, y) \longrightarrow 0$ is more similar
    - If $d(x, y) \longrightarrow 1$ is more dissimilar
- Using R

```
customers2 <- tibble(Customer = c("a", "b"),
                     Sex = c("Female", "Male"),
                     Income = c(3500, 1500),
                     Satisfaction = c("Medium", "High"),
                     Age = c(45, NA)) |>
  mutate(Sex = factor(x = Sex,
                      ordered = FALSE),
         Satisfaction = factor(x = Satisfaction,
                               levels = c("Low", "Medium", "High"),
                               ordered = TRUE))
customers2
```

```
# A tibble: 2 x 5
  Customer Sex    Income Satisfaction   Age
  <chr>    <fct>   <dbl> <ord>        <dbl>
1 a        Female   3500 Medium          45
2 b        Male     1500 High            NA
```

Consumer segmentation survey

- Using R

```r
customers2 |>
  select(-Customer) |>
  daisy(metric = "gower")
```

```
Dissimilarities :
  1
2 1

Metric :  mixed ;  Types = N, I, O, I
Number of objects : 2
```

- In this case:
  - `Metric: mixed` because it includes categorical and numerical data
  - For `Types = N, I, O, I` check out `?cluster::dissimilarity.object`[3]
    - `N`: Nominal (factor)
    - `I`: Interval scaled (numeric)
    - `O`: Ordinal (ordered factor)

[3] See (Stevens 1946) and Level of measurement

Consumer segmentation survey

- Using R

```
customers2 |>
  select(-Customer) |>
  daisy(metric = "gower")
```

```
Dissimilarities :
  1
2 1

Metric :  mixed ;  Types = N, I, O, I
Number of objects : 2
```

- In this case:
  - `Number of objects : 2`
    - There are 2 observations that correspond to customers **a** and **b**: $a = (Female, 3500, Medium, 45)$ and $b = (Male, 1500, High, NA)$

Consumer segmentation survey

- The original dissimilarity matrix is of dimension 300×300
  - Showing only the relation between the first 5 observations
  - The position (i, j) means the dissimilarity between the observations i and j
    - For example $(4, 3)$, which is equal to 0.425, is the dissimilarity between the observations 4 and 3

```r
segmentation_dist <- segmentation |>
  daisy(metric = "gower")

segmentation_dist |>
  as.matrix() |>
  as_tibble() |>
  select(`1`:`5`) |>
  slice(1:5)
```

```
# A tibble: 5 x 5
    `1`    `2`    `3`   `4`   `5`
  <dbl>  <dbl>  <dbl> <dbl> <dbl>
1 0      0.253  0.233  0.262 0.416
2 0.253 0       0.0680 0.413 0.301
3 0.233 0.0680 0       0.425 0.293
4 0.262 0.413  0.425  0     0.227
5 0.416 0.301  0.293  0.227 0
```

Consumer segmentation survey

```r
customers3 <- tibble(Customer = c("a", "b", "c", "d", "e"),
                     Sex = c("Female", "Male", "Female", "Female", "Male"),
                     Income = c(3500, 1500, 200, 450, 5000),
                     Satisfaction = c("Medium", "High", "Low", "Low", "Medium"),
                     Age = c(45, NA, 34, 23, 55)) |>
  mutate(Sex = factor(x = Sex,
                      ordered = FALSE),
         Satisfaction = factor(x = Satisfaction,
                               levels = c("Low", "Medium", "High"),
                               ordered = TRUE))

customers3
```

```
# A tibble: 5 x 5
  Customer Sex    Income Satisfaction   Age
  <chr>    <fct>   <dbl> <ord>        <dbl>
1 a        Female   3500 Medium          45
2 b        Male     1500 High            NA
3 c        Female    200 Low             34
4 d        Female    450 Low             23
5 e        Male     5000 Medium          55
```

Consumer segmentation survey

- Hierarchical clustering
  - **Method**: Complete Linkage Clustering

```
customers3_dist <- daisy(x = select(customers3, -Customer),
                        metric = "gower")

customers3_dist
```

```
Dissimilarities :
           1          2          3          4
2 0.63888889
3 0.38281250 0.75694444
4 0.45572917 0.73958333 0.09895833
5 0.40625000 0.40972222 0.78906250 0.86197917

Metric :  mixed ;  Types = N, I, O, I
Number of objects : 5
```

```
customers3_hc <- hclust(d = customers3_dist,
                        method = "complete")

customers3_hc
```

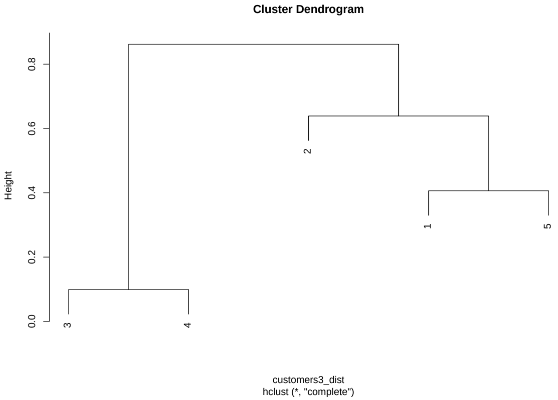
```
Call:
hclust(d = customers3_dist, method = "complete")

Cluster method   : complete
Number of objects: 5
```

Consumer segmentation survey

- Hierarchical clustering
  - **Method**: Complete Linkage Clustering

```
plot(customers3_hc)
```



Consumer segmentation survey

- Compare each observation and find the pair that is more similar

	1	2	3	4	5
1	0.0000000	0.6388889	0.3828125	0.4557292	0.4062500
2	0.6388889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.4557292	0.7395833	0.09895833	0.0000000	0.8619792
5	0.4062500	0.4097222	0.7890625	0.8619792	0.0000000

Consumer segmentation survey

- Now we have the first cluster that includes the observations 3 and 4: $C(3,4)$
- Then we need to create clusters with observations 1, 2 and 5 and the cluster $C(3,4)$
  - How we compare a cluster with an observation
    - **Complete Linkage Clustering**: Use the maximum distance between an observation and an observation that belongs to the cluster

Consumer segmentation survey

- Compare each observation, including the clusters build, and find the pair that is more similar
  - In our case 1, 2, 5 and $C(3,4)$
    - The distance between 1 and $C(3,4)$ is 0.45572917
    - The distance between 2 and $C(3,4)$ is 0.7569444
    - The distance between 5 and $C(3,4)$ is 0.8619792

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000

Consumer segmentation survey

- Now we have the second cluster that includes the observations 1 and 5: $C(1,5)$
- Then we need to create clusters with observation 2 and clusters $C(3,4)$ and $C(1,5)$
    - How we compare a cluster with another cluster
        - **Complete Linkage Clustering**: Use the maximum distance between an observation that belongs to the first cluster and an observation that belongs to the second cluster

Consumer segmentation survey

- Compare each observation, including the clusters build, and find the pair that is more similar
  - In our case 2, $C(3,4)$ and $C(1,5)$
    - The distance between 2 and $C(3,4)$ is 0.7569444
    - The distance between 2 and $C(1,5)$ is 0.6388889

	1	2	3	4	5
1	0	0.6388889	0.3828125	0.4557292	0.4062500
2	0.63888889	0.0000000	0.75694444	0.7395833	0.4097222
3	0.3828125	0.7569444	0	0.0989583	0.7890625
4	0.45572917	0.7395833	0.09895833	0.0000000	0.8619792
5	0.40625	0.4097222	0.7890625	0.8619792	0.0000000

Consumer segmentation survey

- Now we have the third cluster that includes the observation 2 and the cluster $C(1,5)$: $C(2,C(1,5))$
- Then we need to create clusters with cluster $C(2,C(1,5))$ and cluster $C(3,4)$
  - This is the cluster that includes all the observations

Consumer segmentation survey

- Compare each observation, including the clusters build, and find the pair that is more similar
  - In our case $C(3,4)$ and $C(2,C(1,5))$
    - The distance between $C(3,4)$ and $C(2,C(1,5))$ is 0.86197917

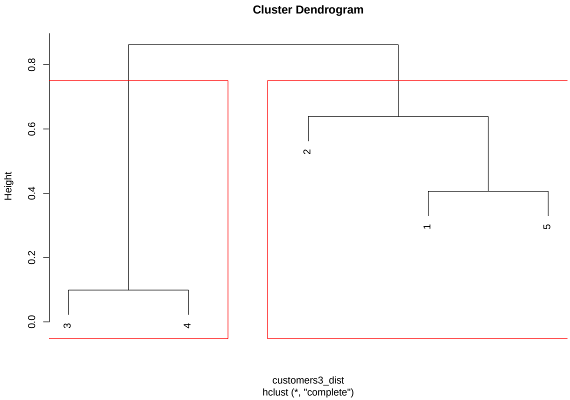
	1	2	3	4	5
1	0	0.6388889	0.3828125	0.45572917	0.4062500
2	0.63888889	0.0000000	0.75694444	0.73958333	0.4097222
3	0.3828125	0.7569444	0	0.09895833	0.7890625
4	0.45572917	0.7395833	0.09895833	0	0.8619792
5	0.40625	0.4097222	0.7890625	0.86197917	0.0000000

- The heights of the **Cluster Dendrogram** are: 0.09895833, 0.40625, 0.63888889 and 0.86197917

Consumer segmentation survey

- Select a number of clusters, for example: 2 clusters

```
plot(customers3_hc)
rect.hclust(customers3_hc, k = 2, border = "red")
```



Consumer segmentation survey

- Extract clusters and assign them to observations

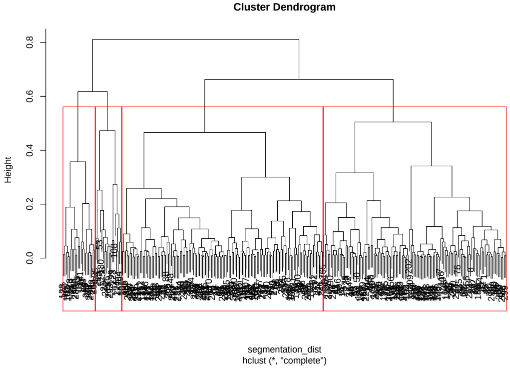
```
customers3_hc_clusters <- cutree(customers3_hc, k = 2)
customers3 |>
  mutate(cluster = customers3_hc_clusters)
```

```
# A tibble: 5 x 6
  Customer Sex    Income Satisfaction   Age cluster
  <chr>    <fct>   <dbl> <ord>        <dbl>   <int>
1 a        Female   3500 Medium          45       1
2 b        Male     1500 High            NA       1
3 c        Female    200 Low             34       2
4 d        Female    450 Low             23       2
5 e        Male     5000 Medium          55       1
```

Consumer segmentation survey

- Select a number of clusters, using `segmentation`, for example: 4 clusters

```
segmentation_hc <- hclust(d = segmentation_dist,
                          method = "complete")
plot(segmentation_hc)
rect.hclust(segmentation_hc, k = 4, border = "red")
```



Consumer segmentation survey

- Extract clusters and assign them to observations, using `segmentation`

```
segmentation_hc_clusters <- cutree(segmentation_hc, k = 4)
segmentation |>
  mutate(cluster = segmentation_hc_clusters)
```

```
# A tibble: 300 x 7
     age gender income  kids ownHome subscribe cluster
   <dbl> <fct>   <dbl> <int> <fct>   <fct>       <int>
 1  47.3 Male   49483.     2 ownNo   subNo           1
 2  31.4 Male   35546.     1 ownYes  subNo           1
 3  43.2 Male   44169.     0 ownYes  subNo           1
 4  37.3 Female 81042.     1 ownNo   subNo           2
 5  41.0 Female 79353.     3 ownYes  subNo           2
 6  43.0 Male   58143.     4 ownYes  subNo           1
 7  37.6 Male   19282.     3 ownNo   subNo           1
 8  28.5 Male   47245.     0 ownNo   subNo           1
 9  44.2 Female 48333.     1 ownNo   subNo           2
10  35.2 Female 52568.     0 ownYes  subNo           2
# i 290 more rows
```

Consumer segmentation survey

References

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