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Herramientas computacionales avanzadas
12 de marzo de 2023

1.3 Calcular los coeficientes de la aproximación $D_3f(x) = Af_{i+1} + Bf_i + Cf_{i-1} + Df_{i-2}$

$$f'_i = A\left(f_i + hf'_i + \frac{h^2}{2}f''_i + \frac{h^3}{6}f'''_i\right) + Bf_i + C\left(f_i - hf'_i + \frac{h^2}{2}f''_i - \frac{h^3}{6}f'''_i\right) + D\left(f_i - 2hf'_i + \frac{4h^2}{2}f''_i - \frac{8h^3}{6}f'''_i\right)$$

$$f'_i = (A+B+C+D)f_i + (A-C-2D)hf'_i + \left({}^{A}/_{2} + {}^{C}/_{2} + 2D\right)h^2f''_i + \left({}^{A}/_{6} - {}^{C}/_{6} - {}^{4D}/_{3}\right)h^3f'''_i$$

- *Para que ambos lados de la ecuación anterior sean iguales se debe cumplir lo siguiente:*

$$\begin{aligned} A + B + C + D &= 0 \\ A - C - 2D &= {}^{1}/_{h} \\ {}^{A}/_{2} + {}^{C}/_{2} + 2D &= 0 \\ {}^{A}/_{6} - {}^{C}/_{6} - {}^{4D}/_{3} &= 0 \end{aligned}$$

- *Construcción de la matriz para resolver el sistema de ecuaciones*

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ \frac{1}{2} & 0 & \frac{1}{2} & 2 \\ \frac{1}{6} & 0 & \frac{-1}{6} & \frac{-4}{3} \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ {}^{1}/_{h} \\ 0 \\ 0 \end{pmatrix}$$

F2−1*F1 → F2

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & {}^{1}/_{h} \\ 1/2 & 0 & 1/2 & 2 & 0 \\ 1/6 & 0 & -1/6 & -4/3 & 0 \end{array}\right)$$

F3−(1/2)*F1 → F3

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & {}^{1}/_{h} \\ 0 & -1/2 & 0 & 3/2 & 0 \\ 1/6 & 0 & -1/6 & -4/3 & 0 \end{array}\right)$$

F4−(1/6)*F1 → F4

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1/h \\ 0 & -1/2 & 0 & 3/2 & 0 \\ 0 & -1/6 & -1/3 & -3/2 & 0 \end{array}\right)$$

F3−(1/2)*F2 → F3

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1/h \\ 0 & 0 & 1 & 3 & -1/2h \\ 0 & -1/6 & -1/3 & -3/2 & 0 \end{array}\right)$$

F4−(1/6)*F2 → F4

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1/h \\ 0 & 0 & 1 & 3 & -1/2h \\ 0 & 0 & 0 & -1 & -1/6h \end{array}\right)$$

Construyendo el sistema de ecuaciones:

$$\begin{cases} A+B+C+D = 0 \\ -B-2C-3D = 1/h \\ C+3D = -1/2h \\ -D = -1/6h \end{cases}$$

Entonces

$$D = 1/6h$$

$$C = -1/2h - 3\left(1/6h\right) = --1/h$$

$$-B = 1/h + 2\left(-1/h\right) + 3\left(1/6h\right) = \left(-1/2h\right) \rightarrow \text{B=} 1/2h$$

$$A = -1/2h - \left(-1/h\right) - \left(1/6h\right) = \left(1/3h\right)$$

$$A = 1/3h$$

$$B = 1/2h$$

$$C = -1/h$$

$$D = 1/6h$$