In [ ]:

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import torch
import torch.nn as nn
import torch.optim as optim
import torch.utils.data as data
from torch.utils.data import TensorDataset, DataLoader


# Load the data and create a DataFrame
D = np.load('../data/Lorenz_data.npy')
df = pd.DataFrame(D)
df.columns = ["time", "x", "y","z"]
df
```

Out[ ]:

	time	x	y	z
0	0.000000	-11.783618	3.527359	12.356182
1	0.008008	-10.662556	2.077418	11.846309
2	0.016016	-9.735181	0.733408	11.482696
3	0.024024	-8.978741	-0.513167	11.232045
4	0.032032	-8.373017	-1.673767	11.069919
...	...	...	...	...
995	7.967968	11.734805	10.459749	32.546310
996	7.975976	11.616529	9.940428	32.802521
997	7.983984	11.466892	9.409619	32.994403
998	7.991992	11.287723	8.875007	33.121536
999	8.000000	11.081295	8.343971	33.184732

1000 rows × 4 columns

In [ ]:

```
# Extract x, y, and z time series
timeseries_x = df[["x"]].values.astype('float32')
timeseries_y = df[["y"]].values.astype('float32')
timeseries_z = df[["z"]].values.astype('float32')

# train-test split for time series
train_size = int(len(timeseries_x) * 0.85)
test_size = len(timeseries_x) - train_size

# Extract time series for overall data
times = df[["time"]].values.astype('float32')
timeseries =  df[["x", "y","z"]]


# train-test split for x, y, and z time series
x_train, x_test = timeseries_x[:train_size], timeseries_x[train_size:]
y_train, y_test = timeseries_y[:train_size], timeseries_y[train_size:]
z_train, z_test = timeseries_z[:train_size], timeseries_z[train_size:]
t_train, t_test = times[:train_size], times[train_size:]
```

In [ ]:

```
# Function to create the dataset
def create_dataset(dataset_x, dataset_y, dataset_z, lookback):
    X, y = [], []
    for i in range(len(dataset_x)-lookback):
        # Create feature by stacking lookback points of x, y, and z
        feature = np.column_stack((dataset_x[i:i+lookback], dataset_y[i:i+lookback], dataset_z[i:i+lookback]))
        # Create target by stacking lookback+1 points of x, y, and z
        target = np.column_stack((dataset_x[i+1:i+lookback+1], dataset_y[i+1:i+lookback+1], dataset_z[i+1:i+lool
        X.append(feature)
        y.append(target)
    return torch.tensor(X), torch.tensor(y)

lookback = 5
X_train, y_train  = create_dataset(x_train, y_train, z_train, lookback=lookback)
```

In [ ]:

```
history = []

class RNNModel(nn.Module):
    def __init__(self):
        super().__init__()
        self.lstm = nn.LSTM(input_size=3, hidden_size=100, num_layers=1, batch_first=True)
        self.linear = nn.Linear(100, 3)
```

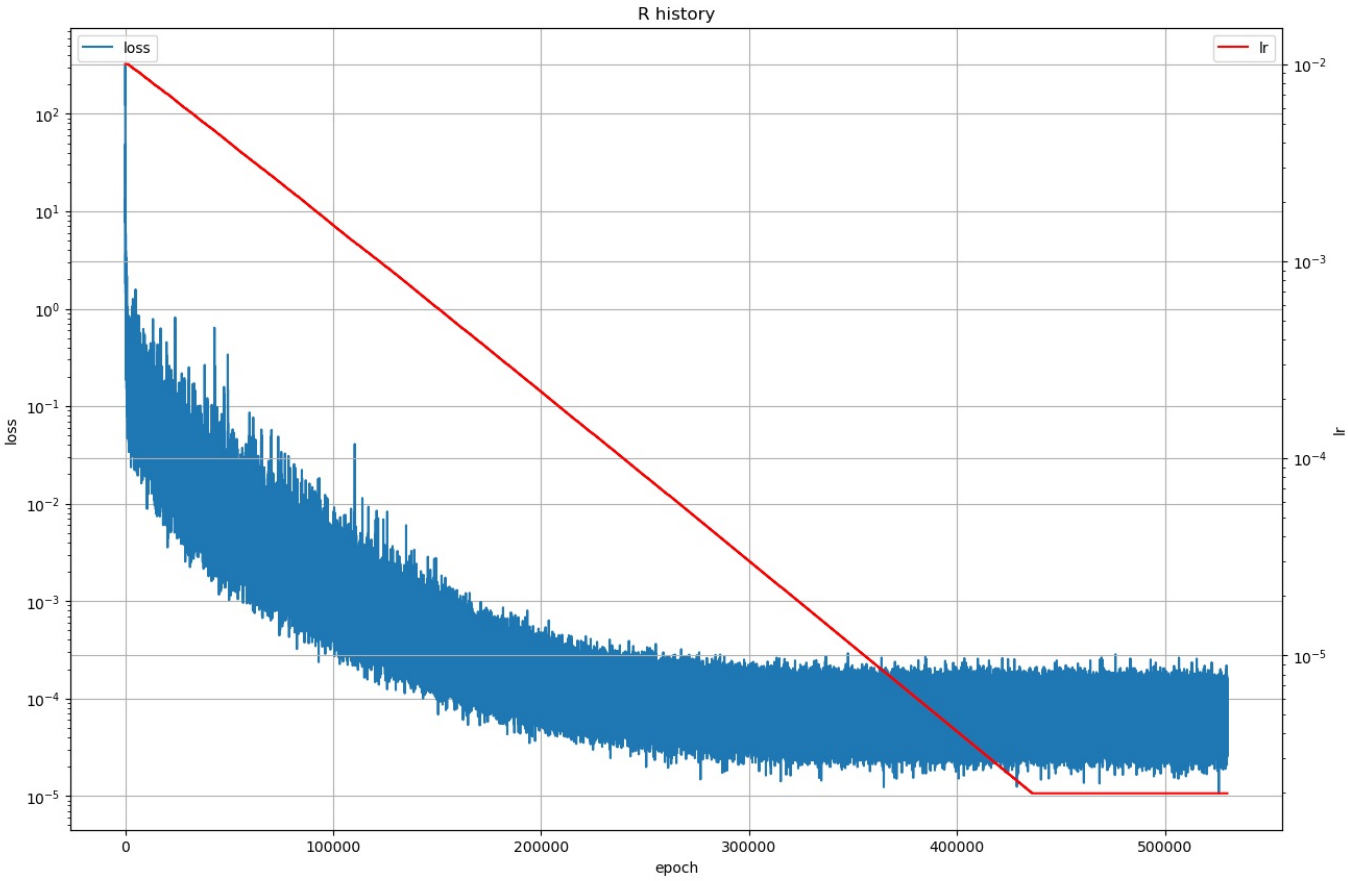
```python
    def forward(self, x):
        x, _ = self.lstm(x)
        x = self.linear(x)
        return x
```

In [ ]:
```python
# Learning rate and scheduler
lr = 0.01
factor = 0.995
patience = 250

history_RNN = []
model = RNNModel()
optimizer = optim.Adam(model.parameters(),lr=lr)
loss_fn = nn.MSELoss()
scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(optimizer,mode=  'min', factor=factor, patience=patience

loader = DataLoader(TensorDataset(X_train, y_train), shuffle=True, batch_size=8)

n_epochs = 5000
for epoch in range(n_epochs):
    model.train()
    for X_batch, y_batch in loader:
        y_pred = model(X_batch)
        loss = loss_fn(y_pred, y_batch)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
        scheduler.step(loss)
        history_RNN.append([loss.item(), optimizer.param_groups[0]['lr']])
    if epoch % 100 != 0:
        continue
    model.eval()
    with torch.no_grad():
        y_pred = model(X_train)
        train_rmse = loss_fn(y_pred, y_train)

    print("Epoch %d: train MSE %.4e, lr %.4e" % (epoch, train_rmse,optimizer.param_groups[0]['lr']))


torch.save(model.state_dict(), '../models/Lorenz_LSTM.pt')
```

```
Epoch 0: train MSE 7.3412e+00, lr 1.0000e-02
Epoch 100: train MSE 4.1760e-02, lr 8.4331e-03
Epoch 200: train MSE 4.5006e-02, lr 6.9356e-03
Epoch 300: train MSE 1.4037e-02, lr 5.6472e-03
Epoch 400: train MSE 1.2970e-02, lr 4.6212e-03
Epoch 500: train MSE 3.9816e-03, lr 3.7627e-03
Epoch 600: train MSE 7.9136e-03, lr 3.0791e-03
Epoch 700: train MSE 2.3862e-03, lr 2.5197e-03
Epoch 800: train MSE 2.2578e-03, lr 2.0516e-03
Epoch 900: train MSE 1.0562e-03, lr 1.6621e-03
Epoch 1000: train MSE 7.7279e-04, lr 1.3533e-03
Epoch 1100: train MSE 4.6614e-04, lr 1.1075e-03
Epoch 1200: train MSE 4.7215e-04, lr 9.0627e-04
Epoch 1300: train MSE 3.9817e-04, lr 7.3791e-04
Epoch 1400: train MSE 6.7137e-04, lr 5.9483e-04
Epoch 1500: train MSE 3.6473e-04, lr 4.8433e-04
Epoch 1600: train MSE 1.9187e-04, lr 3.9435e-04
Epoch 1700: train MSE 2.2374e-04, lr 3.1949e-04
Epoch 1800: train MSE 1.4926e-04, lr 2.6014e-04
Epoch 1900: train MSE 1.6410e-04, lr 2.1075e-04
Epoch 2000: train MSE 1.4311e-04, lr 1.7074e-04
Epoch 2100: train MSE 1.1544e-04, lr 1.3833e-04
Epoch 2200: train MSE 9.4828e-05, lr 1.1263e-04
Epoch 2300: train MSE 8.7091e-05, lr 9.1249e-05
Epoch 2400: train MSE 7.7721e-05, lr 7.4297e-05
Epoch 2500: train MSE 7.6802e-05, lr 6.0192e-05
Epoch 2600: train MSE 7.2363e-05, lr 4.8765e-05
Epoch 2700: train MSE 7.0414e-05, lr 3.9508e-05
Epoch 2800: train MSE 6.7315e-05, lr 3.2007e-05
Epoch 2900: train MSE 6.6064e-05, lr 2.5931e-05
Epoch 3000: train MSE 6.5160e-05, lr 2.1008e-05
Epoch 3100: train MSE 6.3837e-05, lr 1.7020e-05
Epoch 3200: train MSE 6.3418e-05, lr 1.3789e-05
Epoch 3300: train MSE 6.2846e-05, lr 1.1115e-05
Epoch 3400: train MSE 6.1873e-05, lr 9.0051e-06
Epoch 3500: train MSE 6.1594e-05, lr 7.2956e-06
Epoch 3600: train MSE 6.0971e-05, lr 5.9106e-06
Epoch 3700: train MSE 6.0879e-05, lr 4.7885e-06
Epoch 3800: train MSE 6.0703e-05, lr 3.8794e-06
Epoch 3900: train MSE 6.0467e-05, lr 3.1272e-06
Epoch 4000: train MSE 6.0357e-05, lr 2.5335e-06
Epoch 4100: train MSE 6.0349e-05, lr 2.0526e-06
Epoch 4200: train MSE 6.0254e-05, lr 1.9918e-06
Epoch 4300: train MSE 6.0225e-05, lr 1.9918e-06
Epoch 4400: train MSE 6.0108e-05, lr 1.9918e-06
Epoch 4500: train MSE 6.0141e-05, lr 1.9918e-06
Epoch 4600: train MSE 5.9961e-05, lr 1.9918e-06
Epoch 4700: train MSE 5.9950e-05, lr 1.9918e-06
Epoch 4800: train MSE 5.9914e-05, lr 1.9918e-06
Epoch 4900: train MSE 5.9811e-05, lr 1.9918e-06
```

In [ ]:

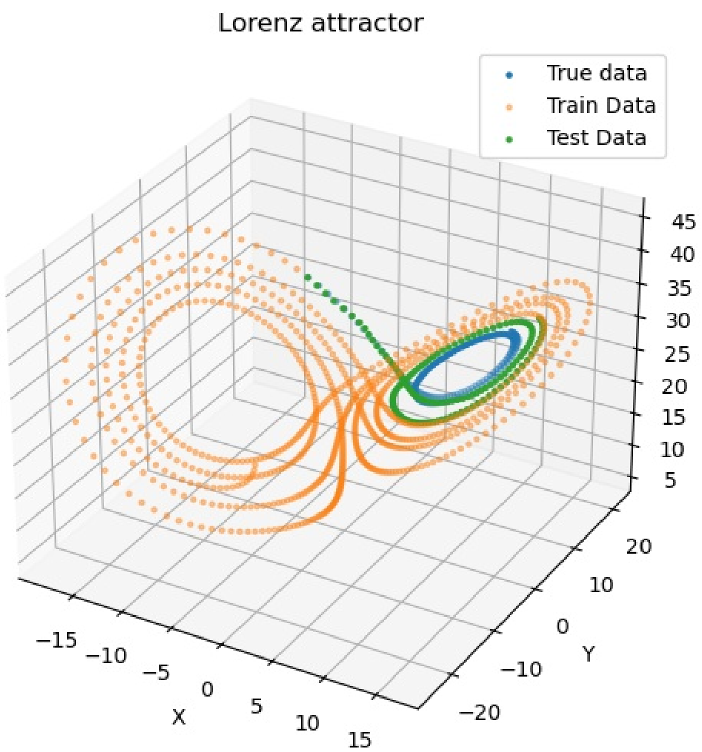
```python
# plot history_FFNN loss and lr in two subplots
history = np.array(history_RNN)

fig, ax = plt.subplots(figsize=(15, 10))
# plot the loss
ax.plot(history[:, 0], label='loss')
ax.legend(loc='upper left')
ax.set_yscale('log')
ax.set_xlabel('epoch')
ax.set_ylabel('loss')
plt.grid()

# plot the learning rate
ax2 = ax.twinx()
ax2.plot(history[:, 1], label='lr', color='r')
ax2.set_yscale('log')
ax2.set_ylabel('lr')
# legend to the right
ax2.legend(loc='upper right')
plt.grid()
plt.title('R history')

# save the figure
plt.savefig('../plot/Lorenz_RNN_history.png')
```



In [ ]:
```
model = RNNModel()
model.load_state_dict(torch.load('../models/Lorenz_LSTM.pt'))
model.eval()

# Initialize an empty plot for position, velocity, and time
train_plot_x = np.ones_like(timeseries_x) * np.nan
train_plot_y = np.ones_like(timeseries_y) * np.nan
train_plot_z = np.ones_like(timeseries_z) * np.nan
test_plot_x = np.ones_like(timeseries_x) * np.nan
test_plot_y = np.ones_like(timeseries_y) * np.nan
test_plot_z = np.ones_like(timeseries_z) * np.nan

with torch.no_grad():
    # Generate the model predictions for training and testing data
    train_last_x = model(X_train)[:, -1, 0].numpy()
    train_last_y = model(X_train)[:, -1, 1].numpy()
    train_last_z = model(X_train)[:, -1, 2].numpy()
    train_plot_x[lookback:lookback + len(train_last_x)] = train_last_x.reshape(-1, 1)
    train_plot_y[lookback:lookback + len(train_last_y)] = train_last_y.reshape(-1, 1)
    train_plot_z[lookback:lookback + len(train_last_z)] = train_last_z.reshape(-1, 1)

    input_seq_x = torch.from_numpy(train_last_x[-lookback:])
    input_seq_y = torch.from_numpy(train_last_y[-lookback:])
    input_seq_z = torch.from_numpy(train_last_z[-lookback:])
    input_seq = torch.stack([input_seq_x, input_seq_y, input_seq_z], dim=1)
    input_seq = input_seq.view(1, lookback, 3)
    x_test, y_test, z_test = [], [], []
    for i in range(len(timeseries_x) - (train_size + lookback)):
        predicted = model(input_seq)
        x_test.append(predicted[:, -1, 0].item())
        y_test.append(predicted[:, -1, 1].item())
        z_test.append(predicted[:, -1, 2].item())
        new_line = predicted[:, -1, :].unsqueeze(0)
        input_seq = torch.cat([input_seq, new_line], dim=1)
        input_seq = input_seq[:, 1:, :]

    test_last_x = np.array(x_test)
    test_last_y = np.array(y_test)
    test_last_z = np.array(z_test)
    test_plot_x[train_size:len(timeseries_x)-lookback] = test_last_x.reshape(-1, 1)
    test_plot_y[train_size:len(timeseries_y)-lookback] = test_last_y.reshape(-1, 1)
    test_plot_z[train_size:len(timeseries_z)-lookback] = test_last_z.reshape(-1, 1)
```

In [ ]:
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

test_true_x = timeseries_x[train_size:len(timeseries_x)-lookback]
```

```python
test_true_y = timeseries_y[train_size:len(timeseries_x)-lookback]
test_true_z = timeseries_z[train_size:len(timeseries_x)-lookback]


# Create a 3D plot
fig = plt.figure(figsize=(10, 6))
ax = fig.add_subplot(111, projection='3d')

# Plot the training data
ax.scatter(test_true_x,test_true_y,test_true_z,marker='.',label = 'True data')
ax.scatter(train_plot_x, train_plot_y, train_plot_z,marker = '.',  label='Train Data',alpha = 0.4)

# Plot the testing data
ax.scatter(test_plot_x, test_plot_y, test_plot_z,marker = '.',  label='Test Data',alpha=0.9)

# Set labels and title
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Lorenz attractor')

# Add a legend
ax.legend()

plt.savefig('../plot/Lorenz_RNN.pdf')
# Show the plot
plt.show()
```



In [ ]:
```python
from sklearn.metrics import mean_squared_error

# Remove nan values from the arrays
clean_test_true_x = test_true_x[~np.isnan(test_true_x)]
clean_test_true_y = test_true_y[~np.isnan(test_true_y)]
clean_test_true_z = test_true_z[~np.isnan(test_true_z)]
clean_test_pred_x = test_plot_x[~np.isnan(test_plot_x)]
clean_test_pred_y = test_plot_y[~np.isnan(test_plot_y)]
clean_test_pred_z = test_plot_z[~np.isnan(test_plot_z)]

# Calculate RMSE for x, y, z coordinates
mse_x = mean_squared_error(clean_test_true_x, clean_test_pred_x)
mse_y = mean_squared_error(clean_test_true_y, clean_test_pred_y)
mse_z = mean_squared_error(clean_test_true_z, clean_test_pred_z)

loss_RNN = (mse_x + mse_y + mse_z) / 3

print("mse: ",loss_RNN)
```

mse: 2.1510194142659507

In [ ]: